AN INVESTIGATION OF THE ENVIRONMENTAL EFFECTS ON THE MERGER RATES IN GALAXY CLUSTERS

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ERIN E. LUECK

ADVISOR: DR. ROBERT BERRINGTON

BALL STATE UNIVERSITY
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ABSTRACT

I present a study of the merger rate of galaxies in various cluster environments. Mergers are thought to be an important process in galaxy formation and evolution. The frequency of mergers in clusters is disputed, but they are thought to create the large elliptical galaxies found at the centers of clusters, brightest cluster galaxies (BCGs). I study how the cluster environment influences the formation of BCGs through the use of N-body simulations containing different numbers of galaxies. Merger rates will depend on the cluster environment: a higher cluster mass reflects a deeper potential well and a greater velocity dispersion. This decreases the chance of a collision resulting in a merger. This study investigates the merger rate of galaxies along with other properties such as mass evolution, radial distribution and velocity dispersion of the galaxies within the cluster, and the formation of BCGs. I find that mergers occur less frequently in larger clusters.
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1.1 Overview of Clusters

Galaxies are found in many environments, ranging from small groups of a few galaxies to rich clusters of hundreds of galaxies. A typical cluster has a radius of $3h^{-1}$ Mpc, a velocity dispersion of 800 km s$^{-1}$, a mass of $10^{15}h^{-1}$, and a mass-to-light ratio of $400h$ $M_{\text{sun}}/L_{\text{sun}}$ (Binney & Tremaine 1987, Carroll & Ostlie 2007). Because of the high concentration of galaxies, clusters are a prime environment to observe galaxy interactions and mergers, and to study the dense regions that exist at the centers of some clusters.

Galaxy clusters are the largest gravitationally bound structures in the universe. Clusters were at first assumed to be simply a higher density of galaxies in the field, but it was discovered that many of the dense regions were in fact spatially connected. Abell (1958) catalogued many rich clusters, and further studies have attempted to classify clusters in several ways: by richness (the number of galaxies in the cluster), by spatial distribution of galaxies, or by properties of the brightest member. Rood & Sastry (1971) described a tuning fork diagram that demonstrates how clusters evolve. According to this diagram, clusters start as an irregular distribution of galaxies and a large central galaxy forms over time. Similarly, Bautz & Morgan (1970) classify clusters according to the
brightest members, with types ranging from a cluster with no dominant galaxies (Type III) to a cluster with a large cD galaxy (Type I). Many clusters have this type of large elliptical or cD galaxy at the center and are classified as cD clusters. cD galaxies are distinguished from normal ellipticals by an extended outer envelope. Normal ellipticals follow the de Vaucouleurs profile where the surface brightness falls off as $R^{1/4}$ (de Vaucouleurs 1948) but in cD galaxies the outer halo flattens in comparison to an $R^{1/4}$ surface brightness profile and is therefore extended.

As cD galaxies are always found in the presence of other galaxies and not in isolation, this suggests that they are a product of the cluster environment. However, the question of how they formed is still being investigated. The following sections in this chapter review the results of observations and simulations in the literature, discussing the formation of central galaxies and the merger rate of galaxies. My simulations are described in Chapter 2, with results and discussion in Chapter 3 and conclusions in Chapter 4.

1.2 Brightest Cluster Galaxies

Brightest Cluster Galaxies (BCGs) are bright elliptical galaxies often found at the centers of clusters. These massive galaxies are frequently located at the center of the dark matter potential well (Zabludoff & Mulchaey 1998). BCGs are of interest due to several properties. There is a very low dispersion in average brightness of BCGs, in the range 0.25 - 0.3 mag (Sandage 1972, Postman & Lauer 1995, Aragón-Salamanca, Baugh, & Kauffmann 1998), which led to their use as standard candles. The location at the kinematic center of the cluster suggests that BCGs are inherently related to the cluster
environment. They seem to be drawn from a different luminosity function than normal ellipticals (Oemler 1976) and they also have a steeper Faber-Jackson relation (Lauer et al. 2007). The Faber-Jackson relation defines the correlation between velocity dispersion ($\sigma$) and luminosity ($L$) as $L \propto \sigma^4$ (Faber & Jackson 1976).

Because of their differences from the rest of the galaxy population, this suggests a unique formation history for BCGs as compared to normal elliptical galaxies, and the commonality of many characteristics suggests a similar formation history for all BCGs.

There are several proposed mechanisms for the formation of BCGs: cooling flows, collisional stripping, formation during cluster collapse, and galactic cannibalism, or mergers between galaxies. Each of these scenarios will be discussed in turn.

All clusters of galaxies have a hot, tenuous intra-cluster medium (ICM) that emits in the X-ray region of the electromagnetic spectrum via thermal free-free (Bremsstrahlung) radiation (Felten et al. 1966). As the central region emits X-ray radiation and cools, the pressure of the surrounding gas causes an inward cooling flow down the potential well (e.g. Fabian 1994). The cooling of this gas should be evident in the spectrum as emission lines, but this is not seen. Other observations show that not enough material is deposited from cooling flows to account for the massive cD galaxies (McNamara & O’Connell 1992).

Collisional or tidal stripping occurs when material is stripped off of a galaxy by another nearby galaxy. Repeated stripping can lead to the disruption of the galaxy, called galaxy harassment (Moore et al. 1996). This process may contribute to the extended envelope associated with cD galaxies, but is unlikely to form the galaxy itself.
A hierarchical formation scenario suggests that BCGs formed from the highest peaks of initial density fluctuations (e.g. Springel et al. 2005), establishing BCGs as the dominant galaxies in the cluster at early times. Indeed, several studies indicate that BCGs formed very early during cluster formation, or from mergers of clusters or subclusters (Merritt 1984, 1985). Observations indicate the presence of substructure in many clusters, which is the result of mergers between groups or clusters (Bird 1994).

Mergers between galaxies are suggested as a formation mechanism for BCGs after the cluster itself has been formed. Ostriker & Tremaine (1975) introduced the idea of galactic cannibalism, where BCGs grow by accreting small galaxies via dynamical friction. However, some have argued that the dynamical friction timescales needed to build BCGs are too long (Merritt 1984, Dubinski 1998). In addition, merging should be suppressed in rich clusters (Ostriker 1980) because a higher velocity dispersion resulting from a deeper potential well means that fewer galaxies will have the low velocities needed for mergers to occur.

Despite the controversy, mergers remain a favored mechanism for the formation of BCGs. In the literature, two main types of mergers are defined, based on the mass ratio of the merging galaxies. A minor merger occurs when one galaxy is significantly more massive than the other, commonly defined as having a mass ratio of 1:3 or less. Mergers with mass ratios above 1:3 are called major mergers. It is unclear whether the more important type is minor mergers (Dubinski 1998, De Lucia & Blaizot 2007, Bernardi 2009), or major mergers (Rines, Finn, & Vikhlinin 2007, Liu et al. 2009).

The mass evolution of BCGs is also unclear. Many agree that the stars that make up BCGs formed early, by a redshift of $z \sim 2 – 3$ (De Lucia & Blaizot 2007, Stott et al.
but results for assembly histories differ widely. Many observational studies (Collins & Mann 1998, DePropris et al. 1999, Burke, Collins, & Mann 2000, Conselice 2006, Whiley et al. 2008, Collins et al. 2009, Stott et al. 2010, Wen & Han 2011) find that BCGs do not experience significant mass growth since \( z = 1 \), doubling in mass instead of the factor of 4 mass growth predicted by current models. There is also little observed luminosity evolution over this time period (Aragón-Salamanca, Baugh, & Kauffmann 1998, Whiley et al. 2008), confirming that the stellar population must grow through accretion or mergers rather than star formation. There is a decrease in star formation after \( z \sim 1 \) (Madau et al. 1996) but this does not necessarily mean a decrease in mergers (D’Onghia, Mapelli, & Moore 2008). Wet mergers (between galaxies with a gaseous component) can trigger star formation, form quasars, and change disks to ellipticals. Dry or dissipationless mergers (without gas) simply increase the mass of the galaxy (Lin et al. 2008).

Models of BCG formation (Bower et al. 2006, De Lucia & Blaizot 2007) predict later assembly times and larger recent mass growth, with BCGs accumulating old stellar populations. This shows that dissipationless mergers are important to BCG formation (Tran et al. 2008). There is evidence for recent merging activity (Gao et al. 2004, Rines, Finn, & Vikhlinin 2007, Tran et al. 2008); however, Stott et al. (2010) say that recent dry mergers have had little effect on the mass of BCGs. They suggest this discrepancy can be reconciled if there is a large growth in intracluster light (Conroy et al. 2007). Whiley et al. (2008) suggest that models have trouble producing realistic BCGs because of the discrepancy in predicted stellar mass growth rates between observations and simulations. They suggest that the underlying physical processes are not yet fully understood.
1.3 Merger Rate

The current model of galaxy formation describes a hierarchical process—galaxies grow from the build-up of smaller objects that formed from initial density fluctuations in the early universe. Dark matter is important to this scenario, as gas trapped in dark matter potential wells cools and forms into stars and galaxies. Mergers and interactions of galaxies are also an essential part of this process.

Mergers are recognized as a very important factor in galaxy formation and evolution. They can explain a variety of galactic phenomena, such as quasars, starbursts, and supermassive black holes (e.g. Hopkins et al. 2006). However, the importance of mergers has been historically difficult to quantify, and we are still unsure of the extent to which mergers affect galaxy formation and evolution.

Many studies have tried to quantify the merger rate as a function of time. This is essential to understanding how groups, clusters, and individual galaxies formed. The remainder of the chapter will discuss results for the merger rate for both observations and simulations.

1.3.1 Observations

Mergers are most commonly observed in one of two ways, either by looking at close pairs of galaxies and estimating when they will merge, or by studying remnants of mergers.

Close pair studies define a separation distance, inside of which it is assumed the galaxies will merge. The pair fraction is determined, which must be converted into a merger rate. This is done by estimating the timescale in which the galaxies will merge,
using dynamical friction arguments (e.g. Binney & Tremaine 2008). Timescale estimates vary widely, around 0.5 Gyr with uncertainties of a factor of 2 (Le Fevre et al. 2000, Bell et al. 2006). Recently, simulations have been done to calibrate the timescales in close pair studies. These show that timescales used in the literature are about a factor of 2 larger than simulated timescales, leading to an overestimate of the merger rate (Boylan-Kolchin, Ma, & Quataert 2008, Kitzbichler & White 2008). There are other problems associated with close pair studies. For example, some galaxies appear close because of random projection. This can be minimized if actual redshifts can be obtained. Another method for reducing projection effects is given by Patton et al. (2002), who studied dynamically confirmed pairs by using only those with small rest-frame line-of-sight velocity differences.

Another method of finding mergers is to look at remnants. Galaxies that show asymmetry are assumed to have undergone recent mergers. For this method, it must be known how long the effects of mergers are visible. Bell et al. (2006a) put this timescale at ~ 150 million years by looking at merger remnants in N-body simulations (Naab, Khochfar, & Burkert 2006). These two methods are preferable in different scenarios: asymmetry is better at high redshifts because of high-resolution imaging, while pair counts are better at low redshifts (de Propris et al. 2007). There is a connection between close pairs and asymmetry; however, the overlap is small. Both methods give similar results, so de Propris et al. (2007) suggest that they are tracing different stages of the merger process.

Evolution in the merger rate is often given as a function of redshift (z):

\[ (1 + z)^m \]  

(1.1)
with values of $m$ between 0 and 5. The wide range of $m$ values can be explained by varying definitions of close pairs, different methods of linking pair fraction to merger rate, survey completeness, cosmic variance, and other factors (Patton et al. 2000, Lin et al. 2004).

Low values of $m$ correspond to little or no variation of the merger rate with redshift, and several studies have found $m < 1$ (e.g. Carlberg et al. 2000, Lin et al. 2004, Lotz 2007, Lin et al. 2008). Many values are in the range $2 < m < 3$ (e.g. Patton et al. 1997, Le Fevre et al. 2000, Patton et al. 2002, Conselice 2006, De Ravel et al. 2008). Conselice et al. (2003) find a steeper slope of $4 < m < 6$ for more massive galaxies only. As an example, the value of $m = 2.2$ in Patton et al. (2002) means that 15% of galaxies have undergone a major merger since $z \sim 1$.

Merger rates are also given in other forms, some with a more complicated dependence on redshift (e.g. Conselice 2006). Results per unit time per co-moving volume obtained from pair calculations are on the order of $10^{-4} \, h^3 \, \text{Mpc}^{-3} \, \text{Gyr}^{-1}$ (Lin et al. 2004, De Propris et al. 2007).

1.3.2 Simulations

Simulations are a way to check whether observations agree with standard models of galaxy formation and evolution. A common type is N-body simulations, which are discussed further in Chapter 2. One of the first N-body simulations was performed in 1972 by Toomre & Toomre. These types of simulations generally have good agreement with semi-analytical models (e.g. Lacey & Cole 1994, Mihos & Hernquist 1996, Kang et al. 2005).
The Millennium Simulation (MS) (Springel et al. 2005) is one of the largest simulations to date, following over $10^{10}$ particles from a redshift of $z = 127$ to $z = 0$ in an attempt to study the formation of large-scale structure. This was a cosmological simulation, in which the equations of motion are solved in a co-moving system to account for the expansion of the universe. The MS provided useful merger rate data in the form of dark halo merger trees. The resolution was not high enough to give merger rates of luminous galaxies.

Simulations like the MS often use the halo merger rate rather than the luminous galaxy merger rate. However, the halo and galaxy merger rates cannot be directly compared, as the number of galaxies that occupy each halo is unknown. Because of this, simulations often find higher values of $m$ (as defined in equation 1.1) than observations (Lotz 2007, Lin et al. 2008). There are also two ways to define the merger rate in simulations: per progenitor halo and per descendant halo, which correspond to close pairs and asymmetry, respectively (Genel et al. 2009). Some studies use subhalos, which may be more comparable to the galaxy merger rate. Berrier et al. (2006) finds that the number of close companions per galaxy can constrain the occupation distribution of galaxies in halos.

One of the few studies of environmental dependence of the merger rate is part of a series of papers. Fakhouri & Ma (2008) studied the merger rate of dark matter halos in the Millennium Simulation and found a very weak dependence on halo mass, a power law dependence in progenitor mass ratio ($\xi^2$), and a redshift dependence of $(1+z)^n$ with $n = 2-2.3$. They compared their results to the Extended Press-Schechter model (Lacey & Cole 1993), which is an extension of the semi-analytical model of Press & Schechter.
Semi-analytical models are used to approximate complex physical processes and have generally shown good agreement with N-body simulations (e.g. Lacey & Cole 1993). Fakhouri & Ma (2008) found that these models overestimate the major merger rate and underestimate the minor merger rate. Fakhouri & Ma (2009) studied the environmental dependence of the merger rate and found that halos in the densest regions merge around 2.5 times more often than halos in voids. Their results for merger rate dependence on various parameters change little when extended to the Millennium II Simulation (Fakhouri, Ma, & Boylan-Kolchin 2010), which extends the lower range of halo masses.

I have run a series of N-body simulations to study the merger rate in various cluster environments. I look specifically at the luminous galaxy merger rate, as the entire cluster is contained within one dark matter halo.
2. Methodology

I present twenty N-body simulations, ten simulating a 100-galaxy cluster environment, and ten simulating a 200-galaxy cluster environment. I also re-analyze the data from nine simulations of a 50-galaxy cluster (Humphrey 2010). In this chapter, I will discuss the simulation techniques used in the models, cluster model set-ups, the initial conditions, and data reduction procedures.

2.1 N-Body Techniques

N-body simulations are designed to solve the equation of motion for N particles:

\[ m_j \frac{d^2 \vec{r}_j}{dt^2} = \sum_{j \neq i}^N \vec{F}_{gij} \]  

(2.1)

where a particle \( j \) has position \( \vec{r}_j \) and mass \( m_j \), and \( \vec{F}_{gij} \) is the gravitational force exerted on particle \( i \) by particle \( j \). The gravitational force on every particle is calculated, and the particles moved accordingly. With large numbers of particles this calculation scales as \( \frac{N(N-1)}{2} \), which is computationally intensive. For example, \( N = 4 \) needs 6 force calculations, \( N = 10 \) needs 45, and \( N = 1,000 \) needs 499,500. For 500,000 particles,
1.25 x 10^{11} calculations are needed. This quickly becomes prohibitively long, given the duration of the simulation.

My simulations use a hierarchical tree method, first developed by Barnes & Hut (1986). The tree code was developed to reduce the computational time; it scales as N log N. The tree method starts by drawing a cube around the entire simulation. This cube is divided into 8 smaller cubes, and this process continues until each cube contains only one particle (Fig. 2.1). Monopole and quadrupole contributions for the gravitational force are calculated for each particle. If a cell is within a certain distance of the particle, all of its subcells are also evaluated. The distance criterion is determined by a parameter

\[ \theta > \frac{l}{d} \]

where \( l \) is the size of a cell and \( d \) is the distance to the center of mass of the particles. As close particles will have a much greater effect, the contribution from distant particles can be approximated by using the center of mass of that cell. In this case, the particles are treated like a single particle with the total mass of the particles in that cell, located at the center of mass of those particles. The calculations are repeated to find the total force on each particle, and then all particles are moved. In my simulations, quadrupole contributions are not calculated, but the cell opening criteria is increased to account for this.

### 2.2 Basic Properties

The simulations in my study are of 100-galaxy and 200-galaxy clusters. I also use data from a 50-galaxy simulation done by Humphrey (2010). The 50-galaxy cluster has \( n = 600,000 \) particles, the 100-galaxy cluster has \( n = 250,080 \) particles, and the 200-galaxy cluster has \( n = 500,163 \) particles. In each simulation, 90% of the particles
Fig. 2.1 – 2-dimensional example showing the tree code method of dividing a group of particles into cells. Each cell contains one particle and cells are not divided further once they contain only one particle.
represent the dark matter intra-cluster background. The remaining 10% of the particles represent the luminous matter. There is no dark matter included in the galaxies themselves, so the 10% of particles which are luminous are all included in the galaxies. The total mass of each cluster is normalized to 1, and the particles all have the equal mass of 1/n.

2.3 Individual Galaxies

The galaxy masses are chosen to conform to the Schechter luminosity function (SLF) (Schechter 1976). The SLF provides a good fit to data, providing a convenient way to parameterize the luminosity function for comparison amongst clusters. With the assumption of a constant mass-to-luminosity ratio, the luminosity function can be uniquely correlated with a mass distribution function. The SLF can be written as

\[ \phi(L) dL = \phi_\star \left( \frac{L}{L_\star} \right)^\alpha \exp \left( -\frac{L}{L_\star} \right) dL/L_\star \]  

(2.2)

where \( \phi(L) dL \) is the number of galaxies in the interval \( L \) to \( L + dL \) (Schechter 1976). Low \( \log(L/L_\star) \) is well described by a power law of slope \( \alpha = -1.25 \), and \( L_\star \) marks the transition from the faint end power law to the high \( L \) exponential cutoff. This function provides a good fit to the observed luminosity function, though other studies have found more complicated functions (e.g. De Filippis et al. 2011). The use of this function ensures a realistic distribution of both high and low mass galaxies. The SLF is shown in Fig. 2.2.

Under the assumption of a constant mass-to-luminosity ratio, the mass distribution is obtained by integrating the SLF. To perform the integration, several numbers are used: \( N_{\text{top}} \) is the number of particles in the largest galaxy, \( N_{\text{bot}} \) is the number
Fig. 2.2 – The Schechter luminosity function. The parameter $L_*$ is indicated at the “knee” of the function. Low $\log(L/L_*)$ is well described by a power law of slope $= -1.25$, and $L_*$ marks the transition from the faint end power law to the high $L$ exponential cutoff.
of particles in the smallest galaxy, and \( N_* \) is the number of particles in the \( N_* \) galaxy. \( N_* \) is a characteristic galaxy with mass \( M_* \) and luminosity \( L_* \), and it is found on the SLF at the transition between the power law slope and the exponential cutoff, as seen in Fig. 2.2. \( N_{\text{bot}}/N_* \) and \( N_{\text{top}}/N_* \) are the limits of integration. To accurately represent the high mass end of the SLF, I integrate downward until the required number of galaxies is reached. This allows the flexibility whereby, for example, the masses of the 100 largest galaxies from the 200-galaxy simulation are the same as the masses of the galaxies used in the 100-galaxy simulation.

Next, the individual galaxies, whose numbers of particles were determined previously, are created. In these simulations, the individual galaxies as well as the dark matter halo are set up as King models. These models represent a lowered isothermal sphere. This provides a more realistic physical description than an isothermal sphere, because an isothermal sphere requires infinite binding potential and therefore infinite mass. By lowering the binding potential, there is a finite mass and escape velocity.

Starting with a galaxy of mass

\[
M(R) = \int_0^R 4\pi r^2 \rho(r) \, dr
\]  

(2.3)

and potential

\[
\phi(R) = -\frac{GM(R)}{R},
\]  

(2.4)

a relative potential \( \Psi \) and relative energy \( \mathcal{E} \) are defined,

\[
\psi = -\phi + \phi_0
\]  

(2.5)

\[
\mathcal{E} = \Psi - \frac{1}{2}v^2
\]  

(2.6)
where \( \phi_o \) is chosen so that when \( \varepsilon \leq 0 \), \( f_K(\varepsilon) = 0 \), so that the particles are unbound. In a similar fashion to Eq. 2.1, the gravitational potential of a particle is calculated as

\[
\phi_j(\vec{r}_j) = \sum_{i=1, i \neq j}^{N} \frac{G M_i}{|\vec{r}_{ij}|} \tag{2.7}
\]

where \( \vec{r}_{ij} \) is the radial separation

\[
\vec{r}_{ij} = \vec{r}_i - \vec{r}_j. \tag{2.8}
\]

The equation for the distribution function of a King model is given in Binney and Tremaine (2008), hereafter BT 2008:

\[
f_K(\varepsilon) = \begin{cases} 
\rho(2\pi \sigma^2)^{-\frac{3}{2}} \left( e^{\frac{\varepsilon}{\sigma^2}} - 1 \right), & \varepsilon > 0 \\
0 & \varepsilon \leq 0
\end{cases} \tag{2.9}
\]

where \( \sigma \) is the velocity dispersion.

Each particle’s position and velocity is assigned according to the King model. This includes the particles in the individual galaxies and the dark matter particles, as the cluster potential well is also set up as a King model.

The simulations are run for a total of 16,000 time steps, and cluster snapshots are written out every 50 steps. There are a total of 321 models over the duration of the simulation. The time steps are divided into 8 code units, each unit corresponding to 2.7 billion years for \( h_o = 50 \). This is set to be the crossing time for a galaxy, defined as the time it takes for a typical galaxy to cross the cluster.

The dark and luminous matter each have a softening parameter, \( \varepsilon \), which can be thought of as a minimum resolution for the simulation. The dark softening parameter has been kept constant for the three simulations at 0.008 and I have adjusted the luminous softening parameter for each simulation. The gravitational force on a particle is
where $\varepsilon$ is the softening parameter. This keeps the particles from experiencing strong accelerations resulting from two-body collisions. I am using a collisionless simulation because of negligible two-body effects observed in elliptical galaxies. The softening parameter for luminous matter is adjusted as follows.

After the galaxies are created, the largest, smallest, and $N_*$ galaxies are tested for stability. Each galaxy is evolved in isolation, away from the influence of other galaxies and the cluster potential as a whole. The behavior of different mass radii (from 5% to 95%) vs. time is graphed, as seen in Figs. 2.3 and 2.4. Changing mass radii reflect a change in a galaxy’s structure and thereby an unstable model. The smallest allowable softening parameter was chosen to maximize spatial resolution and reflect no structural changes in the galaxy models for the duration of the simulation. If the softening parameter is too small, two-body relaxation effects are seen. Two-body relaxation effects occur when stars interact with each other and their orbits are disrupted through strong gravitational interactions, i.e. they lose the "memory" of their initial conditions. The time it takes for this to happen is called the relaxation time,

$$t_{relax} \approx \frac{0.1N}{\ln N} t_{cross}$$

where $N$ is the number of particles and $t_{cross}$ is the crossing time.

Small systems such as globular clusters have $N \approx 10^5$ and crossing time $t_{cross} \approx 1$ Myr, giving a relaxation time of around 1 Gyr, which is much shorter than the lifetime of the object (10 Gyr), meaning that two-body relaxation effects are important (BT 2008). However, galaxies have $N \approx 10^{11}$ and $t_{cross} \approx 100$ Myr, giving a relaxation
time of $10^{16}$ years, which is very long compared to a Hubble time. Thus, the effects of these stellar encounters can be ignored. Two-body relaxation effects are not seen in galaxies and therefore collisionless equations are used. For a rich galaxy cluster, $N \approx 10^3$ and $t_{\text{cross}} \approx 1 \text{ Gyr}$, giving a relaxation time of 10 Gyr. Depending on the age of the cluster, significant relaxation effects may be observed.

The effects of two-body relaxation are characterized by the production of a small dense core and a halo of low density (BT 2008). These effects are reflected in Fig. 2.3 as the outer radii expand and the inner radii shrink. The softening parameter $\epsilon$ is chosen to minimize two-body interactions. When the galaxies are stable, it is clear that any trends observed in the data are not due to the effects of two-body interactions. Fig. 2.4 shows the plot for a luminous softening parameter of 0.0009, which is used in the simulations of 200 galaxies. The luminous softening parameter for the 100-galaxy simulation was 0.003, reflecting a smaller value of $N$.

### 2.4 Cluster Dark Matter Particles

The cluster dark matter particle positions and velocities are chosen to conform to a King model in the same way as the individual galaxies (see Section 2.3). In these simulations, the dark matter represents 90% of the total amount of matter. The 50-galaxy cluster contains 600,000 particles, of which 540,000 represent the dark matter. The 100-galaxy cluster contains 250,080 particles, of which 225,072 represent the dark matter. The 200-galaxy cluster contains 500,163 particles, of which 450,142 represent the dark
Fig. 2.3 – Various mass radii vs. time for an $M_*$ galaxy with a softening parameter of $\epsilon = 0.0006$.

The mass radii are shown on the right-hand side of the graph. Solid lines are best-fit lines. This galaxy shows instability over the course of the simulation due to two-body relaxation effects.
Fig. 2.4 – Various mass radii vs. time for an $M_*$ galaxy with a softening parameter of $\varepsilon = 0.0009$. The mass radii are shown on the right-hand side of the graph. Solid lines are best-fit lines. This galaxy shows minimal two-body relaxation effects, indicating stability over the course of the simulation.
matter. The 50-galaxy cluster has a radius of 1.5 Mpc, a total mass of $10^{14} \, M_{\odot}$, and a total simulation time of 21.6 Gyr. These properties are scaled for the 100-galaxy and 200-galaxy clusters by a factor determined by integrating the SLF. The scaled properties are shown in Table 2.1. The initial distribution of dark matter particles is seen in Figs. 2.5, 2.7, and 2.9.

### 2.5 Galaxy Positions and Velocities

The placement of galaxies into the cluster is determined by randomly choosing a number of dark matter particles equal to the number of galaxies and replacing these particles with the individual galaxies. The position and velocity of the removed dark matter particles become the center of mass position and velocity of the inserted galaxies. The resulting initial positions and velocities of all the particles are used as the starting point for the tree code. Figs. 2.5 and 2.6 show an example of the initial positions for the 50-galaxy simulation, Figs. 2.7 and 2.8 show an example for the 100-galaxy simulation, and Figs. 2.9 and 2.10 show an example for the 200-galaxy simulation. In each of the runs, the initial positions and velocities are different.

Figs. 2.5, 2.7, and 2.9 show the initial positions of all the particles in the 50-galaxy, 100-galaxy, and 200-galaxy clusters, respectively. The blue points represent the cluster-wide dark matter and the black points represent the luminous matter. These figures show a radius of 1 code unit of length. Figs. 2.6, 2.8, and 2.10 show the central region of the cluster for the 50-galaxy, 100-galaxy, and 200-galaxy clusters. Only the galaxies are shown in these figures.
Table 2.1 – Scaling parameters for the different cluster sizes.

<table>
<thead>
<tr>
<th></th>
<th>50 galaxies</th>
<th>100 galaxies</th>
<th>200 galaxies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (Gyr)</td>
<td>21.6</td>
<td>16.6</td>
<td>12.8</td>
</tr>
<tr>
<td>Mass ($M_{\odot}$)</td>
<td>$1 \times 10^{14}$</td>
<td>$1.3 \times 10^{14}$</td>
<td>$1.7 \times 10^{14}$</td>
</tr>
<tr>
<td>Radius (Mpc)</td>
<td>1.50</td>
<td>1.64</td>
<td>1.79</td>
</tr>
</tbody>
</table>
Fig. 2.5 – Initial x and y positions of all the particles of a representative example of a 50-galaxy cluster. Blue points are dark matter and black points are luminous matter. This simulation contains 600,000 particles, with 90% of those representing the dark matter and 10% representing luminous matter. Galaxies are composed entirely of luminous particles.
Fig. 2.6 – A close-up of the initial positions of galaxies in the central region of the 50-galaxy cluster shown in Fig. 2.5. Only the luminous particles are shown.
Fig. 2.7 - Initial x and y positions of all the particles of a representative example of a 100-galaxy cluster. Blue points are dark matter and black points are luminous matter. This simulation contains 250,080 particles, with 90% of those representing the dark matter and 10% representing luminous matter. Galaxies are composed entirely of luminous particles.
Fig. 2.8 – A close-up of the initial positions of the galaxies in the central region of the 100-galaxy cluster shown in Fig. 2.7. Only the luminous particles are shown.
Fig. 2.9 – Initial x and y positions of all the particles of a representative example of a 200-galaxy cluster. Blue points are dark matter and black points are luminous matter. This simulation contains 500,163 particles, with 90% of those representing the dark matter and 10% representing luminous matter. Galaxies are composed entirely of luminous particles.
Fig. 2.10 – A close-up of the initial positions of the galaxies in the central region of the 200-galaxy cluster shown in Fig. 2.9. Only the luminous particles are shown.
2.6 Data Reduction

The data reduction process for these simulations is as follows: density and density gradients for each particle are calculated and particles are moved along the gradient until a local maximum is found. Particles are found by a friends-of-friends (FOF) algorithm (Davis et al. 1985) which links particles within a defined linking length (L) into the same group. This technique isolates groups within an isodensity surface (L^3). I have adopted an adjustable linking length that is adjusted in length at every step so that 90% of the particles are associated with a group. Any group that does not exceed the minimum size of a group (which is defined as 17 particles for the 100-galaxy simulation and 34 particles for the 200-galaxy simulation) is discarded. The galaxies are identified by the FOF algorithm and followed for the duration of the simulation. Data recorded at each time step for each galaxy’s center of mass includes x, y, and z positions, x, y, and z velocities, x, y, and z velocity dispersions, 3-dimensional velocity dispersion, and mass-to-luminosity ratio which for these simulations is always 1.

Data files were inspected for missed galaxies and were corrected by visual inspection of raw particle position data. Missed galaxies occasionally occur because of the difficulty in distinguishing between two galaxies located at similar spatial locations (colliding). In subsequent time steps these galaxies are located by the FOF algorithm and I interpolate the missing time steps by using a standard cubic spline interpolation scheme.
3. Results and Discussion

I present the results of ten 100-galaxy simulations and ten 200-galaxy simulations, along with a re-analysis of nine 50-galaxy simulations (Humphrey 2010). I discuss the galaxy merger rates, mass evolution, formation of a large central galaxy, evolution of the velocity dispersion distribution function (VDDF), radial distribution within the cluster, and mass segregation.

3.1 Mergers

In Figs. 3.1 – 3.5 I present the number of galaxies remaining vs. time for the 50-galaxy, 100-galaxy, and 200-galaxy models. Galaxy merger events are the result of a decrease in galaxy numbers. Fig. 3.1 shows the merger history for run 1 of the 100-galaxy simulation. Five mergers occurred in this particular run. Figs. 3.2 – 3.4 show the composite merger histories of the 50-galaxy, 100-galaxy, and 200-galaxy simulations, respectively, where all runs from a given simulation are shown on the same graph. The number of mergers in each run and the averages including standard deviation are given in Table 3.1.
Fig. 3.1 – Number of remaining galaxies versus time for run 1 of a 100-galaxy simulation.

Punctuated galaxy losses are seen at various times over the duration of the simulation. A galaxy loss is assumed to be the result of two galaxies merging into a single galaxy – a merger event. A total of five merger events are seen in this simulation.
Fig. 3.2 – The number of galaxies remaining as a function of time. Time is given in code units.

All nine simulations of the 50-galaxy cluster are shown. This is a re-analysis of the data by Humphrey (2010). The number of mergers in a single simulation ranges from 6 to 14. Before time step $t = 4$, all simulations have experienced at least one merger event.
Fig. 3.3 – The number of galaxies remaining as a function of time. Time is given in code units.

All ten simulations of the 100-galaxy cluster are shown in this figure. The number of mergers in a single simulation ranges from 2 to 14. By time step $t = 5.5$ all simulations have experienced at least one merger event.
Fig. 3.4 - The number of galaxies remaining as a function of time. Time is given in code units. All ten simulations of the 200-galaxy cluster are shown. The number of mergers in a single simulation ranges from 0 to 7. Some of the simulations show no merger events for the duration of the simulation.
Fig. 3.5 – The normalized merger rate, given as the fraction of galaxies remaining, as a function of time. Time is given in code units. The results from all of the simulations are shown together.

The 50-galaxy simulations are shown by the black curve, the 100-galaxy simulations are shown by the blue curve, and the 200-galaxy models are shown by the red curve. It is determined that the fractional merger rate is highest for the 50-galaxy cluster and lowest for the 200-galaxy cluster.
<table>
<thead>
<tr>
<th>Simulation Number</th>
<th>50 Galaxies</th>
<th>100 Galaxies</th>
<th>200 Galaxies</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>N/A</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>11</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>10</td>
<td>4</td>
</tr>
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<td>14</td>
<td>9</td>
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<td>7</td>
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</tr>
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<td>9</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>Average Number</td>
<td>9.77 ± 2.54</td>
<td>8.90 ± 3.70</td>
<td>3.50 ± 2.22</td>
</tr>
<tr>
<td>Average Percent</td>
<td>19.54 ± 5.08%</td>
<td>8.90 ± 3.70%</td>
<td>1.75 ± 1.11%</td>
</tr>
</tbody>
</table>

Table 3.1 – The number of mergers for each simulation. Both average number and percentage of mergers for each cluster size is reported along with 1σ error bars. All error bars assume a Gaussian distribution. Not Applicable (N/A) was reported for Run 2 of the 50-galaxy cluster simulation because it was shown to be a duplicate of Run 4.
For the 50-galaxy simulation the number of mergers ranged from a minimum of 6 to a maximum of 14, with an average of 9.8. This represents a percentage range of 12% to 28% of galaxies lost due to merger events. For the 100-galaxy simulation, the number of mergers ranged from a minimum of 2 to a maximum of 14, with an average of 8.9, representing a percentage range of 2% to 14% of galaxies lost due to merger events. For the 200-galaxy simulation, the number of mergers ranged from a minimum of 0 to a maximum of 7, with an average of 3.5. This represents a percentage range of 0% to 3.5% of galaxies lost due to merger events.

Fig. 3.5 shows the fractional number of luminous galaxies remaining for each simulation, to enable comparison between different clusters. Merger histories are assumed from the number of galaxies lost vs. time in the simulations; I assume the loss of a galaxy is the result of a merger event. For the 50-galaxy and 100-galaxy clusters, the numbers of mergers experienced in each simulation, as well as the averages, are comparable. This represents twice the percentage of mergers in the 50-galaxy simulation compared to the 100-galaxy simulation.

As shown in Fig. 3.5, there is a small region of overlap between the 50-galaxy and 100-galaxy clusters. Visual inspection shows that during this time the central density of the cluster is enhanced, leading to a higher probability of mergers. This overlap region reflects the time frame over which the 50-galaxy and 100-galaxy share this density enhancement mechanism as a dominant mechanism for influencing cluster dynamics.

In a similar fashion, there is also a region of overlap between the 100-galaxy and 200-galaxy clusters. This overlap extends to a later time than the overlap between the 50-galaxy and 100-galaxy clusters.
In Fig. 3.5, it is clear that the 50-galaxy cluster has the largest fractional merger rate and the 200-galaxy cluster has the lowest. The trend of fewer mergers with increasing cluster size is expected, due to the increase in velocity dispersion with cluster size. The data confirms the expectation that mergers are less common in larger clusters.

3.2 Mass Evolution

Previous observational studies of galaxies and clusters have concentrated on the luminosity evolution of the galaxies in the cluster. Under the presupposition that one can deduce a mass from a given luminosity (i.e. a given mass-to-luminosity ratio), one can determine the mass evolution of galaxies within the cluster. Unfortunately, numerous observational studies show that galaxies exhibit significant luminosity evolution (e.g. Butcher & Oemler 1984). What is not known is whether the evolution observed in the luminosity function is a result of this luminosity evolution or of mass evolution. Galaxy masses are affected not only by growth due to merger events, but also by the loss or accretion of particles resulting from tidal stripping from interactions with other galaxies. If most of the mass evolution were due to mergers, I would expect higher mass evolution in smaller clusters where more mergers occur. Figs. 3.6 – 3.8 show the number of galaxies per mass bin at four time slices, t = 0, 2, 4, and 8, for each of the 50-galaxy, 100-galaxy, and 200-galaxy clusters. Each curve in Figs. 3.6 – 3.8 shows the summed log(frequency) as a function of mass bin, where galaxy frequency is the number of galaxies in a given bin. Unless otherwise stated, the histograms in this chapter use the combined number of galaxies from all simulations of a given size. The results are
Fig. 3.6 – The mass evolution at 4 time steps (t = 0, 2, 4, and 8 code units) for the 50-galaxy cluster. Log(frequency) is shown as a function of normalized mass ($M/M_*$). Error bars are 1σ error bars given by the square root of the frequency. An increase in the number of low-mass galaxies is seen, likely because of mass loss due to tidal stripping. A few massive galaxies are starting to form at $t = 4$ and several very high-mass galaxies are seen at $t = 8$. 
Fig. 3.7 – The mass evolution at 4 time steps (t = 0, 2, 4, and 8 code units) for the 100-galaxy cluster. Log(frequency) is shown as a function of normalized mass (M/M_\ast). Error bars are 1\sigma error bars given by the square root of the frequency. An increase in the number of low-mass galaxies is seen, likely because of mass loss due to tidal stripping. More mass evolution happens in the second half of the simulation. By t = 8, a few very massive galaxies have formed.
Fig. 3.8 – The mass evolution at 4 time steps (t = 0, 2, 4, and 8 code units) for the 200-galaxy cluster. Log(frequency) is shown as a function of normalized mass (M/M_\star). Error bars are 1\sigma error bars given by the square root of the frequency. An increase in the number of low-mass galaxies is seen, likely because of mass loss due to tidal stripping. Little mass evolution is seen in the rest of the simulation.
presented in mass bins, and all error bars shown are 1σ error bars assuming Poissonian statistics, equivalent to the square root of the frequency. All error bars in this chapter are calculated in this way, unless otherwise noted. The masses are normalized by dividing by the appropriate $M_\ast$ corresponding to the cluster of a given size. This enables a comparison amongst the different cluster sizes.

In the 50-galaxy cluster (Fig. 3.6), the $t = 0$ curve shows the initial mass function of the cluster. I see little mass evolution between time steps $t = 0$ and $t = 2$. This reflects the low occurrence of mergers in the first two time steps (Fig. 3.2) where galaxies are moving toward the center of the cluster but rarely merging. In contrast, I start to see some mass evolution from time steps $t = 2$ to $t = 4$. A few massive galaxies with $M/M_\ast > 2$ have formed at $t = 4$. A closer inspection of Fig 3.6 shows the number of mid-mass galaxies dropping and the appearance of a few massive galaxies. The most mass evolution is seen between time steps $t = 4$ and $t = 8$. This is reflected in Fig. 3.2 when comparing the number of mergers that occur from $t = 0$ to $t = 4$ to the number that occur from $t = 4$ to $t = 8$. The number of galaxies found in the lowest mass bin shows a steady increase with simulation time. This increase in low mass galaxies is due to the mass loss of the mid-mass to low-mass galaxies which is a result of tidal stripping from interactions with other galaxies in the cluster. This is easily seen in Fig 3.6 as the second and third mass bins show a decrease in mass at each time step. At $t = 8$, I see a large number of high mass galaxies. These galaxies clearly exhibit strong mass evolution over the duration of the simulation. Given the number of galaxies at high mass in Fig. 3.6, each cluster should have on average one massive galaxy. This will be confirmed later upon visual inspection (see Section 3.4). It should be noted that at $t = 8$, these high-mass
galaxies appear discontinuous from the mass function of the cluster. The high-mass galaxies seem to grow at the expense of the mid-mass galaxies. The t = 8 curve shows the greatest decrement in the number of mid-mass galaxies.

In the 100-galaxy cluster (Fig. 3.7), the appearance of large galaxies with $M/M_\ast > 2$ is seen as early as $t = 2$. This reflects the occurrence of several mergers during this time (Fig. 3.3), unlike the 50-galaxy cluster which experiences fewer mergers from $t = 0$ to $t = 2$. From time steps $t = 0$ to $t = 4$, not much mass evolution is seen. There is a drop in the number of mid-mass galaxies ($0.2 < M/M_\ast < 2$) from $t = 4$ to $t = 8$, likely due to the high number of mergers during this time (Fig. 3.3) and the buildup of massive galaxies. By time step $t = 8$, a few high-mass galaxies have formed, with $M/M_\ast > 2.5$. These galaxies exhibit strong mass evolution over the duration of the simulation. As in the 50-galaxy cluster, the high-mass galaxies grow at the expense of the mid-mass galaxies. The number of galaxies in the smallest mass bin grows at each time step. This growth is greatest between time steps $t = 4$ and $t = 8$. This is the result of tidal stripping from interactions with other galaxies in the cluster.

In the 200-galaxy cluster (Fig. 3.8), little mass evolution is seen throughout the simulation. As in the 50-galaxy and 100-galaxy simulations (Figs. 3.6 – 3.7), the number of low-mass galaxies increases due to mass loss from tidal stripping. The number of mid-mass galaxies with $0.6 < M/M_\ast < 1.2$ steadily decreases throughout the simulation. In contrast to the 50-galaxy and 100-galaxy clusters, the decrease in number of mid-mass galaxies does not contribute to the growth of high-mass galaxies, as no buildup of massive galaxies is seen. In fact, the number of galaxies in the largest mass bin decreases at each time step. This suggests that in the 200-galaxy cluster, mass loss is occurring.
without a mechanism for building up the mass. The galaxies lose mass due to tidal stripping from interactions with other galaxies, but there are few occurrences of mergers that would form massive galaxies. The small amount of mass evolution is likely correlated with the low number of mergers in this simulation (Fig. 3.4).

Comparing the results from all three cluster sizes, it is clear that the 50-galaxy cluster shows the most significant mass increase in the largest galaxies. In the 100-galaxy cluster simulations, I also see the buildup of massive galaxies, but with fewer than in the 50-galaxy cluster. In the 50-galaxy and 100-galaxy clusters, a few very massive galaxies form, with $M/M_\ast > 2.5$. In contrast, the 200-galaxy evolution shows little mass increase in the highest mass galaxies and exhibits mass loss in the most massive galaxies. All simulations show an increase in the number of galaxies in the lowest mass bin, demonstrating the importance of tidal stripping in the cluster environment. Most of the galaxies lose mass to the cluster environment as time evolves. All of this is supported by the merger histories shown in Fig. 3.5, with the 50-galaxy cluster showing the highest merger rates and the 200-galaxy cluster showing the lowest merger rates.

### 3.3 VDDF Evolution

The VDDF is defined as the number of galaxies whose stellar velocity dispersion ($\sigma_{\text{gal}}$) lies in the velocity dispersion interval $\sigma$ to $\sigma + d\sigma$. The VDDF is a useful tool because it measures the depth of the gravitational potential well of a stellar system and thereby its mass with minimal dependence on luminosity evolution. The VDDF also has a strength in that measured observational quantities are directly comparable to quantities calculated in numerical simulations. In addition to its sensitivity with a galaxy’s mass, the
VDDF is not only influenced but also degenerate with both the cluster environmental effects and galaxy-galaxy interactions.

Figs. 3.9 – 3.11 show the VDDF evolution as a function of time. Four time steps are shown, at \( t = 0, 2, 4, \) and \( 8 \). Each curve shows \( \log(\text{frequency}) \) as a function of velocity dispersion bins. Velocity dispersions are normalized by \( \sigma_* \), where \( \sigma_* \) is the velocity dispersion of the \( M_* \) galaxy.

In the VDDF for the 50-galaxy cluster (Fig. 3.9), there is a significant change from the initial VDDF shown by the \( t = 0 \) curve to the \( t = 2 \) curve. The galaxy models used in my simulations are shown to be in equilibrium when in isolation (see Section 2.3). These galaxy models are no longer isolated, but in the galaxy cluster environment, and after \( t = 0 \) they quickly establish an equilibrium with the cluster environment. This is reflected in the large drop in the VDDF from \( t = 0 \) to \( t = 2 \). As seen in Fig. 3.9, the VDDF curves at later times (\( t = 4 \) and \( t = 8 \)) reflect the evolution observed in the mass function seen in Fig. 3.6. The increase in the number of galaxies with high \( \sigma_{\text{gal}} \) at later times reflects an increase in mass due to mergers. An increase in the number of galaxies with low \( \sigma_{\text{gal}} \) is also seen. This trend is likely a result of environmental effects and galaxy-galaxy interactions that do not result in mergers.

Similarly, the VDDF of the 100-galaxy cluster (Fig. 3.10) also reflects the galaxies achieving an equilibrium with their environment from \( t = 0 \) to \( t = 2 \). Little evolution is seen between time steps \( t = 2 \) and \( t = 4 \). For the later time steps, \( t = 4 \) and \( t = 8 \), the velocity dispersions reflect a slow mass buildup at the high-\( \sigma_{\text{gal}} \) end. The mid-\( \sigma_{\text{gal}} \) galaxies are consistently dropping as some will end up in the low-\( \sigma_{\text{gal}} \) and some in the high-\( \sigma_{\text{gal}} \) bins. At \( t = 8 \), a small number of galaxies have a large \( \sigma_{\text{gal}} \) with
Fig. 3.9 – The VDDF evolution at 4 time steps (t = 0, 2, 4, and 8 code units) for the 50-galaxy cluster. Log(frequency) is shown as a function of normalized velocity dispersion ($\sigma_{\text{gal}}/\sigma_*$). Error bars are $1\sigma$ error bars given by the square root of the frequency. The number of galaxies with low $\sigma_{\text{gal}}$ increases due to interactions between galaxies that do not result in mergers. At $t = 8$ there is a build-up of galaxies with high $\sigma_{\text{gal}}$, reflecting an increase in mass.
Fig. 3.10 – The VDDF evolution at 4 time steps (t = 0, 2, 4, and 8 code units) for the 100-galaxy cluster. Log(frequency) is shown as a function of normalized velocity dispersion (\(\sigma_{\text{gal}}/\sigma_*\)). Error bars are 1σ error bars given by the square root of the frequency. The number of galaxies with low \(\sigma_{\text{gal}}\) increases due to interactions between galaxies that do not result in mergers. At t = 8 there are several galaxies with high \(\sigma_{\text{gal}}\), reflecting an increase in mass.
Fig. 3.11 – The VDDF evolution at 4 time steps (t = 0, 2, 4, and 8 code units) for the 200-galaxy cluster. Log(frequency) is shown as a function of normalized velocity dispersion (σ_{gal}/σ_*). Error bars are 1σ error bars given by the square root of the frequency. The number of galaxies with low σ_{gal} increases due to interactions between galaxies that do not result in mergers. There is otherwise little VDDF evolution in these simulations.
$\sigma_{\text{gal}}/\sigma_* \geq 1.5$. An increase in the number of galaxies with low $\sigma_{\text{gal}}$ reflects galaxy-galaxy interactions as well as environmental effects due to tidal interactions with the cluster environment. The increase in the high-$\sigma_{\text{gal}}$ end reflects the mass evolution of the massive galaxies in the cluster (Fig. 3.7).

The 200-galaxy cluster (Fig. 3.11) shows a strikingly different evolution, as the increase in the high-$\sigma_{\text{gal}}$ galaxies does not occur throughout the simulation. Minimal evolution is observed for the mid-$\sigma_{\text{gal}}$ galaxies, as the VDDF drops slightly throughout the simulation. An increase in the number of galaxies with low $\sigma_{\text{gal}}$ is observed, as in the 50-galaxy and 100-galaxy clusters (Figs. 3.9 – 3.10). This minimal evolution can be understood by the lack of merger events in the 200 galaxy cluster (Fig. 3.4). This supports mass evolution dominated by tidal stripping resulting from interactions with the cluster environment and other galaxies, rather than a mass evolution characterized by galaxy mergers.

In comparing the VDDF evolution of the 50-galaxy and 100-galaxy cluster to that of the 200-galaxy cluster, it is clear that the 200-galaxy cluster shows much less evolution during the simulation. There is very little change except for the growth of low-$\sigma_{\text{gal}}$ galaxies, which is due to interactions that did not result in mergers. In the 50-galaxy and 100-galaxy clusters, the VDDF shows an increase in galaxies with high $\sigma_{\text{gal}}$ at the last time step, similar to the increase in galaxies with high mass. This reflects a correlation between the mass function and the VDDF. This is expected because the VDDF is a direct measure of the depth of the gravitational potential well of the galaxy, which is determined by the distribution of matter and thereby confirms that the VDDF traces the galaxy mass function.
3.4 Galaxy Dynamics

A visual inspection of the temporal evolution of the galaxy distribution for each simulation was performed. In Figs. 3.12 – 3.14 I show representative samples of positions at four time steps. These are the same simulations shown in Figs. 2.5 – 2.10 of Section 2.5, where the initial positions are shown. Figs. 3.12 – 3.14 show a close-up of the central region with a radius of 0.25 code units. To investigate whether or not a large central galaxy formed, I visually inspected the positions of the galaxies at $t = 8$ for each simulation. I examined the $x$-$y$, $y$-$z$, and $z$-$x$ planes for each simulation to check for projection effects.

All of the 50-galaxy cluster simulations formed a large galaxy at the center. Surrounding this galaxy is a haze of particles which have been stripped off of other galaxies. Fig. 3.12 shows the positions of the galaxies in the $x$-$y$ plane for time steps $t = 0, 2, 4, \text{ and } 8$. The region shown has a radius of 0.25 code units. Over time, the galaxies move toward the center and merge to create a large central galaxy.

For the 100-galaxy cluster, large galaxies appear to form until the central region is viewed more closely. There are many galaxies in the center but they generally have not actually merged. There is a haze of particles but this is less extensive than in the 50-galaxy cluster. Fig. 3.13 shows the positions of the galaxies in the $x$-$y$ plane for time steps $t = 0, 2, 4, \text{ and } 8$. The region shown has a radius of 0.25 code units. As in the 50-galaxy cluster, the galaxies move toward the center throughout the simulation, but fewer mergers occur so a large central galaxy is rarely formed.

The 200-galaxy simulations show much less merging and therefore hardly any build-up at the center. A minimal haze of particles is seen, and does not appear to be tied
Fig. 3.12 – Positions of the galaxies in the x-y plane for time steps \( t = 0, 2, 4, \) and 8 for a 50-galaxy cluster. Starting in the upper left-hand panel with \( t = 0, \) times progress across each row. This is the same example simulation seen in Figs. 2.5 – 2.6. These clusters have 600,000 particles, of which 10% are luminous. Only luminous particles are shown. The radius shown is 0.25 code units. Galaxies move toward the center over the length of the simulation. At \( t = 8, \) a large central galaxy is seen, with haze of particles that have been stripped from other galaxies.
Fig. 3.13 – Positions of the galaxies in the x-y plane for time steps $t = 0, 2, 4, \text{ and } 8$ for a 100-galaxy cluster. Starting in the upper left-hand panel with $t = 0$, times progress across each row. This is the same example simulation seen in Figs. 2.7 – 2.8. These clusters have 250,080 particles, of which 10% are luminous. Only luminous particles are shown here. The radius shown is 0.25 code units. Galaxies move toward the center over the length of the simulation. At $t = 8$, several galaxies are seen in the center, with a faint haze of particles.
Fig. 3.14 – Positions of the galaxies in the x-y plane for time steps $t = 0, 2, 4, \text{ and } 8$ for a 200-galaxy cluster. Starting in the upper left-hand panel with $t = 0$, times progress across each row.

This is the same example simulation seen in Figs. 2.9 – 2.10. These clusters have 500,163 particles, of which 10% are luminous. Only luminous particles are shown here. The radius shown is 0.25 code units. Galaxies move toward the center over the length of the simulation. At $t = 8$, galaxies are seen in the central region but have not merged.
to any single galaxy, unlike the 50-galaxy cluster. Fig. 3.14 shows the positions of the galaxies in the x-y plane for time steps $t = 0, 2, 4, \text{ and } 8$. The region shown has a radius of 0.25 code units. Galaxies show some movement toward the center of the cluster, but a large galaxy is never formed.

The differences in the distribution of galaxies in the central region of the cluster between the 50-galaxy, 100-galaxy, and 200-galaxy simulations reflect the number of mergers that each cluster experiences (Fig. 3.5). This seems to show that a large central galaxy can be formed through mergers, but this only appears to happen in smaller clusters.

3.5 Radial Distribution

The initial position of each galaxy is determined by the King model (see Chapter 2), and over time I expect the galaxies to become more centrally located, due to dynamical friction and the shape of the gravitational potential well. Unless otherwise specified, radius will refer to a true three-dimensional radius. For each simulation, I have defined the cluster center as the centroid based on the positions of all the galaxies in the cluster. Figs. 3.15-3.17 show the number of galaxies per radial bin for the 50-galaxy, 100-galaxy, and 200-galaxy runs, respectively. The points are plotted at the centers of the bin ranges, except the first bin which is plotted at 0 and does not represent the center of this first bin.

Fig. 3.15 shows the 50-galaxy cluster. The shape of the curves at $t = 0$ corresponds to the initial distribution of the galaxies. Galaxies with radii $R > 0.2$ show a
Fig. 3.15 – The distance of the galaxies from the cluster center at 4 time steps (t = 0, 2, 4, and 8 code units) for the 50-galaxy cluster. Log(frequency) is on the y-axis and radial distance (in code units) is on the x-axis. Error bars are $1\sigma$ error bars given by the square root of the frequency.

From $t = 0$ to $t = 8$, the number of galaxies drops for each radial bin except the smallest one. This shows that the galaxies tend to move toward the center of the cluster.
Fig. 3.16 – The distance of the galaxies from the cluster center at 4 time steps (t = 0, 2, 4, and 8 code units) for the 100-galaxy cluster. Log(frequency) is on the y-axis and radial distance (in code units) is on the x-axis. Error bars are 1σ error bars given by the square root of the frequency. From t = 0 to t = 8, the number of galaxies drops for almost every radial bin except the smallest one. This shows that the galaxies tend to move toward the center of the cluster.
Fig. 3.17 – The distance of the galaxies from the cluster center at 4 time steps (t = 0, 2, 4, and 8 code units) for the 200-galaxy cluster. Log(frequency) is on the y-axis and radial distance (in code units) is on the x-axis. Error bars are $1\sigma$ error bars given by the square root of the frequency. From $t = 0$ to $t = 8$, the number of galaxies drops for almost every radial bin except the smallest one. This shows that the galaxies tend to move toward the center of the cluster.
trend of decreasing number with each subsequent time step. This is consistent with the majority of the points, except at large radii (R > 1), where small number statistics are probably dominant, as demonstrated by the large error bars. The central bin, R = 0, shows a trend of increasing number with increasing time. The increase in number is much larger than the error bars. This increase represents galaxies settling into the potential well as a result of dynamical friction.

In the 100-galaxy simulation (Fig. 3.16), the decreases at radii (R > 0.2) are not as pronounced as in the 50-galaxy cluster. Similarly, the increase in number at the smallest radial bin is not as pronounced. The 50-galaxy cluster shows a fractional change of about 0.95 in the smallest radial bin, whereas the 100-galaxy cluster shows a fractional change of about 0.6. The height of the initial curve at t = 0 is higher for the 100-galaxy cluster, reflecting the higher total number of galaxies.

The results for the 200-galaxy cluster (Fig. 3.17) look very similar to those of the 100-galaxy cluster. The number of galaxies with 0.2 < R < 1 generally decreases at each time step, and this change is comparable to that seen in the 100-galaxy cluster. The number of galaxies with R > 1 shows a more pronounced decrease at each time step. As in the 50- and 100-galaxy clusters, the number of galaxies in the smallest radial bin increases at each time step. The fractional change of 0.5 is slightly smaller than that of the 100-galaxy cluster.

The curves for each of the clusters at radii 0.2 < R < 1 generally show a drop at each subsequent time step. The number of galaxies in the smallest radial bin increases over time for all three clusters. The fractional increase of this bin is highest in the 50-galaxy cluster and lowest in the 200-galaxy cluster.
3.6 Mass Segregation

As a galaxy orbits in the cluster potential well, it interacts with other galaxies and the dark matter. This interaction opposes the galaxy’s motion in the cluster and resembles a frictional force known as dynamical friction (Chandrasekhar 1943). Chandrasekhar (1943) showed that this frictional force is directly proportional to the mass of the galaxy. Therefore, more massive galaxies will experience larger frictional forces and settle to the bottom of the potential well faster than lower mass galaxies. This prediction is known as mass segregation. Recent studies support the presence of mass segregation in clusters (De Lucia et al. 2006).

To investigate this trend, I plotted the average normalized mass vs. radius. All masses are normalized by the Schechter function’s $M_*$ mass. Figs. 3.18 – 3.20 show the average normalized mass for the summed number of galaxies for the all of the runs for the 50-galaxy, 100-galaxy, and 200-galaxy clusters, respectively, at time steps $t = 0, 2, 4,$ and $8$. For each simulation, I have defined the center of the cluster as the centroid based on the positions of all the galaxies in the cluster. The radial bins are spaced evenly in $\sqrt{R}$ space so that each radial bin represents an equal amount of surface area. I show $1\sigma$ error bars for the average normalized mass per radial bin. The points are plotted at the centers of the bin ranges, except the first bin which is plotted at 0 and does not represent the center of this first bin.

Fig. 3.18 shows the temporal evolution of the average normalized mass per radial bin for the 50-galaxy cluster models. The curve at $t = 0$ demonstrates the initial distribution of the galaxies. At points with $R > 0.2$, the points are within error bars, but the $t = 8$ curve is consistently the lowest of these values. In contrast, the central radial bin
Fig. 3.18 – Average $M/M_*$ as a function of radial distance at 4 time steps ($t = 0, 2, 4,$ and 8 code units) for the 50-galaxy cluster. The radial bins are spaced evenly in $\sqrt{R}$ space. Error bars show $1\sigma$ errors according to Poissonian statistics. A build-up of mass can be seen at the center of the cluster.
Fig. 3.19 – Average $M/M_*$ as a function of radial distance at 4 time steps ($t = 0, 2, 4,$ and $8$ code units) for the 100-galaxy cluster. The radial bins are spaced evenly in $\sqrt{R}$ space. Error bars show 1σ errors according to Poissonian statistics. A build-up of mass can be seen at the center of the cluster.
Fig. 3.20 – Average $M/M_*$ as a function of radial distance at 4 time steps ($t = 0, 2, 4,$ and 8 code units) for the 200-galaxy cluster. The radial bins are spaced evenly in $\sqrt{R}$ space. Error bars show $1\sigma$ errors according to Poissonian statistics. A build-up of mass can be seen at the center of the cluster.
initially shows an average mass comparable to the other bins, but at later times the mass grows, showing a build-up of large galaxies in the central bin by $t = 8$. This supports the presence of mass segregation.

Similarly, the 100-galaxy cluster (Fig. 3.19) exhibits an increase in mass in the smallest radial bin at each subsequent time step. For larger bins, the curve at time step $t = 8$ is usually, but not always, lower than the other curves. As in the 50-galaxy cluster, these points are all within error bars at each radial bin.

The 200-galaxy cluster (Fig. 3.20) also shows an increase in the smallest radial bin at each time step. In larger bins, the $t = 8$ curve is not always lower than the other curves, but all points are within the error bars, as in the 50- and 100-galaxy clusters. The smaller error bars with increasing cluster size reflect the increased number of galaxies in the clusters. In all three cluster sizes, the average mass in the smallest radial bin increases over time, showing the presence of mass segregation.
4. Summary, Conclusions, and Future Work

4.1 Summary and Conclusions

I presented ten N-body simulations of a 100-galaxy cluster, ten simulations of a 200-galaxy cluster, and re-analyzed nine simulations of a 50-galaxy cluster by Humphrey (2010). The simulations were performed using the tree method, and galaxies were tracked throughout the simulation, giving positions and velocities at every time step. I investigated the occurrence of mergers, the mass and VDDF evolution, formation of central galaxies, radial distribution within the cluster, and mass segregation. I compared the results between clusters of different sizes.

I conclude the following points from this study:

1. The number of mergers that occurred in a simulation was related to the size of the cluster. Larger clusters had a lower normalized merger rate. In the 50-galaxy cluster, an average of around 20% of the galaxies merged. In the 100-galaxy cluster, an average of 9% of the galaxies merged, and in the 200-galaxy cluster only an average of 2% of the galaxies merged. This trend is likely due to the higher velocity dispersion in more massive clusters.
2. The 200-galaxy cluster showed significantly less mass evolution leading to the build-up of massive galaxies than the 100-galaxy and 50-galaxy clusters. Several very massive galaxies formed in the 50-galaxy and 100-galaxy clusters, but not in the 200-galaxy cluster. The connection between merger rate and mass evolution signifies that mergers are an important contributor to the formation of these massive galaxies.

3. The 200-galaxy cluster showed the least amount of evolution in the VDDF, reflecting the minimal mass evolution and low number of merger events seen in this cluster.

4. Large central galaxies formed reliably in the 50-galaxy simulation. In the 100-galaxy and 200-galaxy simulations, several galaxies were usually located near the center but they did not merge. This implies that mergers can create large central galaxies or BCGs, but this appears to only happen in smaller clusters.

5. In all simulations, the number of galaxies in the central region increased over the length of the simulation, due to dynamical friction and the shape of the gravitational potential well. This increase was pronounced at each subsequent time step.

6. In all simulations, there was evidence of mass segregation due to dynamical friction. In each cluster, the number of massive galaxies in the central region increased over time.
4.2 Future Work

There are many future opportunities of study related to my project. Several variables in the simulations can be changed, including number of galaxies, number of particles, and dark matter fraction. Mergers of groups or clusters can also be studied, as BCGs may form from these types of mergers. Larger clusters should be studied, as 200 galaxies is still a relatively small cluster. From studies such as these, a model of merger statistics can be created, which may help to answer the many questions remaining on the topic of mergers.
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