RECREATION OF THE BULLET CLUSTER (1E 0657-56) MERGING EVENT VIA N-BODY COMPUTER SIMULATION

A THESIS SUBMITTED TO THE GRADUATE SCHOOL IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE MASTER OF SCIENCE

BY

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Table of Contents

Table of Contents ................................................................. ii
Acknowledgements ............................................................... iv
List of Figures ........................................................................ v
List of Tables ........................................................................... v
Abstract ..................................................................................... viii
Chapter 1 Introduction ............................................................. 1
  1.1. What Are Clusters of Galaxies? ............................................. 1
  1.2. The Virial Theorem and Dark Matter ................................. 2
  1.3. Cluster Potentials and Methods of Mass Determinations .... 5
  1.4. Structure of Clusters: Intra-Cluster Medium and Mass Distributions ............................... 8
  1.5. Merging Galaxy Clusters: Most Energetic Events Since Big Bang ................. 10
  1.6. The Bullet Cluster (1E-0657-56) ........................................... 13
  1.7. Thesis Preview .................................................................. 18
Chapter 2 Methodology ........................................................... 19
  2.1. N-Body Techniques and Gravitational Physics .................. 19
  2.2. Collisionless Systems ...................................................... 23
Chapter 3 Cluster Setup .......................................................... 25
  3.1. Schechter Luminosity Function .......................................... 25
  3.2. Stability Check and Mass Distributions ......................... 27
Chapter 4 Re-creating the Merger Event ............................... 33
  4.1. Run #1: Head-On Collision From a Parabolic Orbit ............ 33
  4.2. Run #2: Head-On Collision From a Sub-Parabolic Orbit .... 37
  4.3. Data Reduction ............................................................. 42
Chapter 5 Results ..................................................................... 43
  5.1. Run #1: Head-On Collision From a Parabolic Orbit ............ 44
  5.1.1. Velocity Dispersion .................................................... 50
  5.1.2. Cluster Radial Velocities of Galaxies ......................... 55
  5.2. Run #2: Head-On Collision From a Sub-Parabolic Orbit .... 67
  5.2.1. Velocity Dispersion .................................................... 73
  5.2.2. Cluster Radial Velocities of Galaxies ........................... 77
  5.3. Error Analysis ............................................................... 86
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I am also very happy for the support, love, and care that my family and my significant other has provided me over the last couple of years!

'We began as wanderers, and we are wanderers still. We have lingered long enough on the shores of the cosmic ocean. We are ready at last to set sail for the stars.'

-- Carl Sagan
List of Figures

1.1: Multi-wavelength observations of the central region of galaxy cluster Abell 1689 .......... 2
1.2: Best fit circular orbit velocity curves to the observed values for the spiral galaxy
    NGC 3198 ............................................................................................................. 4
1.3: Motion of a celestial object, as viewed from an observer ........................................ 7
1.4: Image of MACS J0025.4, showing a composite of the optical, X-ray (pink),
    and dark matter distribution (blue). ....................................................................... 10
1.5: Mass reconstruction of A520 .................................................................................. 12
1.6: Multi-wavelength observations of the Bullet Cluster .............................................. 13
1.7: Mass distribution contours (red) as determined by Clowe et al. 2006 ...................... 14
1.8: Snapshots of the simulations performed by MB and SF ........................................ 16
2.1: Number of force calculations scales directly with number of particles.................. 20
2.2: A 2D example of the tree method of Barnes & Hut for N-body Simulations .......... 21
3.1: The Schechter luminosity function, showing the transition from power law
    \( (\alpha = -1.0) \) to exponential behavior .............................................................. 26
3.2: Two-Body effects shown, with the Lagrangian radii, for a galaxy evolved in isolation..... 27
3.3: Constant radii, no Two-Body effects, due to a best-fit softening parameter, with a
    value of \( \epsilon = 0.00095 \) .................................................................................... 28
3.4: The Main and the Bullet clusters of the re-created Bullet Cluster system ............... 32
4.1: Relative and center-of-mass (CM) setup .............................................................. 34
4.2: Initial conditions for Run #1 ................................................................................ 36
4.3: Decoupling from the Hubble Flow and the resulting Bullet Cluster merger event ...... 37
4.4: Initial conditions for Run #2 ................................................................................ 41
5.1: Cluster positions at \( t = 0 \) for Run #1 ................................................................. 45
5.2: Same as Figure 5.1, except \( t = 0.75 \) .................................................................. 46
5.3: Same as Figure 5.1, except \( t = 1.00 \) ................................................................. 47
5.4: Same as Figure 5.1, except \( t = 1.175 \) ............................................................... 48
5.5: Same as Figure 5.1, except \( t = 1.5 \) (end of simulation) and with a zoomed in view
    of the central simulation region ............................................................................. 49
5.6: Temporal evolution of the velocity dispersion along the line of sight (along z-axis) of
    galaxies in each cluster as a function of simulation time ......................................... 50
5.7: LEFT PANEL: Center-of-mass (CM) separations between the two clusters, calculated from dark matter particles and from galaxies. RIGHT PANEL: Difference between the Dark CM and Galaxy CM calculations. $t_{\text{collision}} \sim 1.1$, when the two cluster centers overlap. The small differences provides evidence that the galaxies trace the dark matter ...

5.8: The number of galaxies present during the simulation ................................................................. 54

5.9: Calculation of the cluster radial velocity of a galaxy from cluster CM........................................ 55

5.10: Average cluster radial velocity of 30 most massive (blue) and 60 least massive galaxies (red) in the sub-cluster .................................................................................................................. 56

5.11: Average cluster radial velocity of 50 most massive (blue) and 110 least massive galaxies (red) in the main cluster .................................................................................................................. 58

5.12: Variety of sampling of galaxies for the sub-cluster (left column) and the main cluster (right column) .......................................................................................................................... 60

5.13: Region dimensions for each cluster, with values given in code length units................................. 61

5.14: Average cluster radial velocity of galaxies in each three region, with respect to their respective cluster center............................................................................................................................................. 63

5.15: Velocity dispersion of galaxies along the line-of-sight (LOS) in each of the three regions.................................................................................................................................................................... 64

5.16: Average cluster radial velocities of the galaxies that were present in each region at $t_{\text{collision}} = 1.1$ ....................................................................................................................................................................................................... 66

5.17: Cluster positions at time $t = 0$ for Run #2 ..................................................................................... 68

5.18: Same as Figure 5.17, except $t = 1.00$ ............................................................................................ 69

5.19: Same as Figure 5.17, except $t = 1.35$ (top panel) and $t = 1.5$ (bottom panel)......................... 70

5.20: Same as Figure 5.17, except $t = 2.0$ ............................................................................................. 71

5.21: Same as Figure 5.17, except $t = 2.5$ (end of simulation)............................................................ 72

5.22: Temporal evolution of the velocity dispersion along the LOS (along z-axis) of galaxies in each cluster as a function of simulation time ................................................................. 73

5.23: LEFT PANEL: Center-of-mass (CM) separations between the two clusters, calculated from dark matter particles and from galaxies. RIGHT PANEL: Difference between the Dark CM and Galaxy CM separations ................................................................. 75

5.24: The number of galaxies present during the simulation................................................................. 76

5.25: Average cluster radial velocity of 30 most massive (blue) and 60 least massive galaxies (red) in the sub-cluster .................................................................................................................................................................. 77

5.26: Cluster radial velocity of 50 most massive (blue) and 110 least massive galaxies
5.27: Variety of sampling of galaxies for the sub-cluster (left column) and the main cluster (right column).

5.28: Average cluster radial velocity of galaxies in inner, middle, and outer regions, as defined in the text.

5.29: Velocity dispersion of galaxies along the line-of-sight in each of the three regions.

5.30: Average cluster radial velocities of galaxies originally present in each region at $t_{\text{collision}} = 1.3$.
List of Tables

Table 1.1: Measured mass of the Bullet Cluster, compiled from literature.......................... 15
Table 1.2: Simulation parameters of Mastropietro and Burkert (2008) for the model that
most accurately recreated the observed properties of the Bullet Cluster .................. 16
Table 1.3: Simulation parameters of Springel and Farrar (2008) for the model that most
accurately recreated the observed properties of the Bullet Cluster.......................... 17
Table 3.1: Properties of the two clusters .............................................................................. 31
Table 4.1: Parameters for Run #1. All values are given in code units ................................. 35
Table 4.2: Parameters for Run #2. All values are given in code units .................................. 40
Table 5.1: Comparison between simulation velocity dispersion and velocity dispersion
found in literature ............................................................................................................. 52
Table 5.2: Comparison between simulation velocity dispersion and velocity dispersion
found in literature for Run #1 and Run #2 ..................................................................... 75
Abstract

Merging galaxy clusters are excellent laboratories where astronomical theories, like the presence of dark matter, can be tested. Merging events are the most energetic phenomena to occur since the Big Bang, with a single merger event releasing $10^{63-64}$ ergs of energy. Observations of isolated clusters strongly support the hypothesis that the Intra Cluster Medium (ICM) is in hydrostatic equilibrium with the gravitational potential well, where the gravitational potential is traced to the dominating matter.

I present N-body simulations that recreate a two-cluster merging system known as the Bullet Cluster (1E 0657-56). An N-body simulation is a simulation of N particles, whose motion is governed by Newtonian gravity. The computer model is self-consistent, meaning that all gravitational forces are determined by the distribution of the particles. Collisional parameters are taken from existing literature and are used to calculate initial conditions. Initial positions and velocities of the two clusters are determined by solving a two-body problem under the assumption that each cluster is a point mass.

The main goal of this thesis is to re-create the Bullet Cluster merger event and to characterize the accompanied signatures - such as changes in radial velocities of galaxies - to independently confirm a post-merging event in the Bullet Cluster.
Chapter 1: Introduction

1.1. What Are Clusters of Galaxies?

Clusters of galaxies are the largest gravitationally bound systems and are visually comprised of numerous member galaxies with numbers ranging from a few hundred to a few thousand (Fig 1.1). Early attempts concentrated on cataloging galaxy clusters, in order to gain a greater understanding of these systems. A commonly cited catalogue is the Abell (1958) Catalogue, whereby Abell searched the sky northward of declination $\delta = 20^\circ$ S for enhancements in number densities of galaxies. In this catalogue, 2712 galaxy clusters were compiled and characterized by the following criteria (Abell 1958):

- *richness* – number of galaxies
- *distance* – as estimated by the 10th brightest galaxy in the cluster.

Other catalogues were developed as well, for example the Zwicky Catalogue (Zwicky et al. 1961-1968). The development of such catalogues was influenced by the need to gain a greater understanding of these systems, as demonstrated by the pioneering work of Zwicky (1937), who showed that the mass of the Coma cluster of galaxies could not be accounted for by the luminous matter alone.
1.2. The Virial Theorem and Dark Matter

Zwicky’s (1937) study of the Coma cluster made the assumption that the Coma cluster was a gravitationally bound system in dynamic equilibrium. Under this assumption, a cluster of galaxies is well approximated by the scalar Virial Theorem. The scalar virial theorem states that

$$2K + U = 0,$$  \hspace{1cm} (1.1)

where $K$ is the total kinetic energy and $U$ is the total potential energy of the cluster system. These quantities are mathematically defined as

$$K = \frac{1}{2} \sum_{i=1}^{N_{gal}} M_i v_i^2,$$  \hspace{1cm} (1.2)

and

$$U = - \sum_{i=1}^{N_{gal}} \left( \sum_{j=1, j\neq i}^{N_{gal}} \frac{G M_i M_j}{|\vec{r}_{ij}|} \right),$$  \hspace{1cm} (1.3)
where $N_{gal}$ is the total number of galaxies in the cluster, $|\vec{r}_{ij}| = |\vec{r}_i - \vec{r}_j|$ and is the separation between the $i^{th}$ and $j^{th}$ galaxy, and $M_i$ is the mass of the $i^{th}$ galaxy.

The virial theorem establishes a connection between the cluster’s total kinetic energy and total potential energy. Assuming a spherical cluster system, one can rewrite equation 1.1 in terms of total mass $M$ and the motion of galaxies within the cluster as

$$< v^2 > = \alpha \frac{GM}{R},$$

where $\alpha$ is $3/5$ for a spherical system and $R$ is the gravitational radius or the size of the cluster as defined by the mean harmonic separation (see section 3.2). The quantity $< v^2 >$ is represented by the velocity dispersion, which is determined by measuring the Doppler-shift of galaxy spectra (Binney & Tremaine 2008). Assuming an isotropic velocity distribution, it is easy to show that equation 1.4 becomes

$$3 < v_r^2 > = \alpha \frac{GM}{R},$$

where $< v_r^2 >$ is the line-of-sight (LOS) velocity dispersion. Then the total mass of the cluster becomes

$$M_{cluster} = \frac{3R<v_r^2>}{aG}.$$  

The virial theorem provides an accurate estimate for the mass of a cluster. Zwicky employed the virial theorem and showed that the luminous matter only accounted for about 10% of the matter required to maintain virial equilibrium, while the unaccounted for ~90% would become known as dark matter (Zwicky 1937).
Further investigation in the 1970s and 1980s of rotational velocities of spiral galaxies showed a similar discrepancy to the previous study by Zwicky. Measurements of

![Figure 1.2: Best fit circular orbit velocity curves to the observed values for the spiral galaxy NGC 3198. Top curve shows the total circular orbit velocity curve resulting from the summation of the luminous disk component and the dark matter halo component. Individual curves, showing the contribution of each component independently, are also shown and are labeled. Measured circular orbit velocities are shown by dark circles, along with 1σ error bars (adopted from Van Albada et al. 1985).](image-url)
orbital velocities of stars in spiral galaxies indicated that the luminous matter could not account for the observed orbital velocities and required a significant distribution of mass at large radii from the galaxy center (Rubin & Ford 1980). The observed rotation curves (Fig 1.2) became an empirical proof that observations did not match expected velocity profiles as determined by luminous mass distribution.

1.3. Cluster Potentials and Methods of Mass Determinations

Clusters of galaxies have enormous amounts of gravitational potential energy due to their size and mass. For example, the Coma cluster is about $5h^{-1}\,\text{Mpc}$ in diameter and has a mass of roughly $1.8 \times 10^{15} \, h^{-1}M_\odot$, where $h$ is the dimensionless Hubble constant* and the assumed value of the Hubble constant is 70 km/s/Mpc (Briel et al. 1992). The gravitational potential, as seen in section 1.1, takes the form of

$$W \propto \frac{GM^2}{R}$$  \hspace{1cm} (1.7)

(Binney & Tremaine 2008). A cluster with mass on the order of $10^{15}h^{-1}M_\odot$ has a potential binding energy on the order of $10^{64}h^{-1}\text{ergs}$. For comparison, the average energy released during a supernova event is on the order of $10^{51}\text{ergs}$. This demonstrates that galaxy clusters are the largest gravitational bound systems known and are the most energetic systems, with order of magnitude estimates $\sim 10^{64}h^{-1}\text{ergs}$, since the Big Bang itself.

\* $h = \frac{H_0}{100\text{km/s/Mpc}}$, where $H_0$ is the Hubble constant.
Numerous techniques have been developed to approximate masses of clusters, including but not limited to (Binney & Tremaine 2008):

- **Weak Gravitational Lensing.**
- **Strong Gravitational Lensing.**
- **Sunyaev-Zeldovich effect,** and
- **Line-of-sight velocity data.**

I will briefly discuss each below.

**Weak Gravitational Lensing** refers to the bending of light as it passes near a massive object, like a cluster of galaxies. The galaxy cluster acts like a lens and distorts the view of the background galaxies. The amount of distortion, or **shear,** is measured and used to determine the enclosed mass. **Strong Gravitational Lensing** differs from weak lensing in that the source that causes the distortion is great enough to cause multiple images of the object to appear (for more details, see Kaiser et al. 1993 and Kochanek 2006). A closer inspection of Figure 1.1 reveals several strong gravitationally lensed background galaxies in the central region of Abell 1689, while weak lensing is observed in the outer regions (*not visible in Fig 1.1*).

The **Sunyaev-Zeldovich** effect is a change in the measured temperature of the cosmic microwave background (CMB) radiation in the direction of the cluster. Because the intracluster medium (ICM) is hot and energetic, the photons, through a mechanism called inverse Compton scattering, will be up-scattered to higher energies. This will result in a decrement of the photons in the CMB radiation, resulting in a change in the measured temperature (Sunyaev & Zel’dovich 1980). This technique can be used to
measure various properties of the cluster environment and thereby allow an estimate of the cluster mass.

Velocities projected along the line of sight are known as *LOS* (or radial) velocities. Henceforth, we will refer to velocities projected along the line of sight as LOS velocities. Velocities that are projected from the cluster center-of-mass along the radial vector will be referred to as *cluster radial velocities*. Angular velocities that are observed in the plane of the sky are known as proper motion (Fig 1.3). Given the vast distances associated with cluster of galaxies, proper motions are not observable. The galaxy’s peculiar velocity is the true velocity of the galaxy in the reference frame of the cluster. As discussed in section 1.2, LOS velocities are used *via* the virial theorem to estimate cluster masses (see also Smith 1936 or Arnaboldi et al. 2004).

**Figure 1.3:** Motion of a celestial object, as viewed from an observer. The Hubble flow is the recessional velocity resulting from the expansion of the universe.
1.4. Structure of Clusters: Intra-Cluster Medium and Mass Distributions

Clusters contain a hot, X-ray emitting gas that permeates the space between galaxies. This gas is known as the Intra-Cluster Medium (ICM), and Cavaliere et al. (1971) first suggested its presence. This ICM extensively has been observed in numerous clusters and has been shown to have typical gas temperatures $kT_{\text{gas}} \sim 5 - 10 \text{ keV}$ and luminosities $L \sim 10^{44-45} \text{ ergs/s}$ (Forman & Jones 1982). Because most isolated clusters are considered virialized (or relaxed) systems, the traditional assumption is the ICM is in hydrostatic equilibrium. Hydrostatic equilibrium is the condition where the outward ICM gas pressure is equal to the inward gravitational force. Numerous studies have supported this conclusion (e.g., Evrard et al. 1996 and Briel et al. 1992). Several heating mechanisms of this gas have been proposed and range from shock fronts formed by the motion of the galaxies through the ICM to shocks formed resulting from the natural formation process of the cluster itself (Sarazin 1986). To this day, the source of this heating mechanism remains controversial.

According to this picture, once the ICM has achieved hydrostatic equilibrium with the cluster, the gravitational potential well determines the state of the gas (i.e. temperature and density). Consequently, the gas is a tracer of the cluster-wide potential well and the associated mass. Therefore, we can use the gas to determine numerous interesting properties of the cluster, for example its total mass and distribution of matter.
Dark matter halos in clusters and galaxies appear to follow a spherically averaged power-law profile for its mass distribution, similar to luminous distributions (Navarro, Frenk, & White 1997 and Binney & Tremaine 2008):

$$\rho(r) = \frac{\rho_0}{\left(\frac{r}{a}\right)^\alpha \left(1 + \frac{r}{a}\right)^{\beta - \alpha}}$$

(1.8)

where $r$ is the distance from center of the system; furthermore

- $\alpha$ and $\beta$ are the inner and outer power-law slopes, respectively,
- $a$ is a characteristic radius that marks the transition between the inner and outer power-law slopes,
- $\rho_0$ is a constant used to scale the density.

This density profile allows one to characterize numerous models through the parameters $\alpha$ and $\beta$. For example, $(\alpha, \beta) = (1, 3)$ is known as the Navarro, Frenk, & White (NFW) universal density model which was shown to accurately describe dark matter halos in cosmological numerical simulations (Navarro, Frenk, & White 1997). Other models exist, for example the analytic King (1962) model, which can be described by $(\alpha, \beta) = (0, 3)$ and will be discussed later.

Because dark matter does not radiate in any observable electromagnetic band, its presence is inferred through the mass as determined through the cluster gravitational potential well (Clowe et al. 2006). As shown above, the distribution of galaxies in the cluster and the ICM are both tracers of the cluster-wide gravitational potential well. Unlike the galaxies, the ICM follows the laws of hydrodynamics and will experience forces like ram pressure and friction, and other related processes. Galaxies are well
approximated by collisionless systems and thus do not experience these forces. Despite this difference, it is expected that both the ICM and the galaxies in the cluster will show a similar distribution.

### 1.5. Merging Galaxy Clusters: Most Energetic Events since Big Bang

A widely accepted model of structure formation is the *hierarchical merging scenario*, whereby clusters of galaxies grow in mass by accreting smaller, less massive clusters of galaxies. In the accretion process, clusters of galaxies will redistribute the orbital energy through various mechanisms into the internal dynamics of the cluster itself, that are on the order of the gravitational potential binding energy discussed in section 1.2 (Sarazin 2004).

![Figure 1.4: Image of MACS J0025.4, showing a composite of the optical, X-ray (yellow contours), and dark matter distribution (red contours). The total I-band light is shown with white contours. The ICM distribution is determined from the X-ray emission. The dark matter distribution is determined from weak lensing results. Note the differing ICM and dark matter distributions (figure adopted from Bradac et al. 2008).](image_url)
An accretion event is called a cluster merger event and has been shown to be an excellent laboratory that allows the testing of theories concerning dark matter (Nusser 2008).

Several cluster merging events have been observed to date and are studied extensively, like MACS J0025.4 – 1222 (Fig 1.4). MACS J0025.4 – 1222 is a massive merging cluster, located at a redshift of $z = 0.586$, colliding almost in the plane of the sky (Bradac et al. 2008).

Figure 1.4 shows an optical image of the MACS J0025.4, overlaid with reconstruction of the dominant mass distribution (red contours) using weak gravitational lensing. A recent study by Bradac et al. (2008) showed MACS J0025.4 – 1222 to be a post-merger event. It is clear from Figure 1.4 that the X-ray emitting ICM does not spatially coincide with the dark matter distribution as determined from weak lensing. This clearly demonstrates that the ICM of the respective clusters exhibits collisional properties as a result of direct interaction between ICMs, whereby the galaxies and dark matter do not (Bradac et al. 2008).

During cluster merger events, shocks are also driven into the ICM, stirring up the hot gas and driving the cluster dynamics away from equilibrium (Roettiger et al. 1996). This makes the use of the virial theorem uncertain, because the virial theorem should be applied to dynamically stable systems.

A recent lensing study by Jee et al. (2012) provides a counter-example to the dark matter behavior seen in other merger clusters: the dark matter components in the A520 cluster have not passed cleanly through one another, exhibiting properties that indicate the dark matter to be collisional. As Figure 1.5 shows, the detected dark matter (labeled 3
in the figure) from weak lensing is concentrated in between the two dominant luminous mass clumps (labeled 2 and 4 in the figure). The dark core is detected at $> 10\sigma$, with an upper limit of cross-sectional interaction of $\sim 3.8 \pm 1.1 \, cm^2/gm$ (Jee et al. 2012). The derived cross-sectional interaction for the Bullet Cluster is $\sim 1 \, cm^2/gm$ (Markevitch et al. 2004).

![Figure 1.5: Mass reconstruction of A520. Chandra (X-ray) emission is depicted in red. Mass contours are depicted in white. Detected mass peaks are labelled 1 – 6. Number 3 is the dark matter clump, coinciding with the X-ray luminosity (adopted from Jee et al. 2012).](image)

The dynamics of A520 are very difficult to explain, especially in the presence of the available analysis of clusters whose dark matter is collisional, like the Bullet Cluster (section 1.6). To date, a successful theory explaining the collisional versus collisionless properties observed in clusters of galaxies remains unknown.
1.6. The Bullet Cluster (1E-0657-56)

Another example of a cluster merger event is the Bullet Cluster (Fig 1.6). Observations in X-ray show two distinct emission peaks, decoupled from the dominant mass distribution as determined from weak lensing, providing further evidence that dark matter exhibits collisionless properties and segregates from the ICM (Angus et al. 2007, Angus & McGaugh 2008, Clowe et al. 2006, Spring & Farrar 2008, Milosavljevic et al. 2007, and others).

Figure 1.6: Multi-wavelength observations of the Bullet Cluster. LEFT PANEL: X-ray emission peaks in yellow/red, with green contours showing dark matter distribution, also shown in the right panel. RIGHT PANEL: Optical image, showing mass distribution reconstruction via weak lensing. The bullet is on the right and the main cluster is on the left (Clowe et al. 2006).

The Bullet Cluster is located at a redshift of $z = 0.296$ with the merger event happening nearly in the plane of the sky, and an observed mass-to-light ratio of $\sim 217M_\odot/L_\odot$ (Barrena et al. 2002). A recent study by Clowe et al. (2006) indicates the Bullet Cluster is a post-merger event involving a massive main cluster and a less massive sub-cluster that passed through the main cluster. The time of collision, defined as the time
of center-of-mass passage, is estimated to be 100 million years prior (Mastropietro & Burkert 2008). Given its observed redshift, the two clusters collided ~9 billion years after the Big Bang (Angus & McGaugh 2008).

Estimates of the merging velocity are inferred from the velocity propagation of the bow shock through the ICM, resulting from the merger event (Clowe et al. 2006). Simulations show that this is consistent with the sub-cluster’s current velocity of \( v \sim 4740 \text{ km/s} \) (Springel & Farrar 2007).

![Mass distribution contours](image)

**Figure 1.7:** Mass distribution contours (red) as determined by Clowe et al. 2006. The solid black curves are the mass distribution contours determined by Angus 2007. The dark matter center-of-mass (CM1, CM2) and X-ray peaks (XR1, XR2) are also shown. Green shaded regions are the highly dense area of the clusters (for further details, see Angus 2007).
The observed separation between the two clusters’ centers-of-mass (Fig 1.7) as calculated from dark matter is $\sim 0.7 \, \text{Mpc}$ (Angus 2007). One can find in the literature the mass reconstruction of the Bullet Cluster from weak gravitational lensing and radial velocity measurements. As mentioned at the end of section 1.4, given the unstable and dynamic environment due to merging, obtaining masses from virial theorem can be misleading (Markevitch et al. 2004). The masses (within certain radii) of the two clusters are provided in Table 1.1, compiled from the available literature.

<table>
<thead>
<tr>
<th>Reference</th>
<th>$\text{Mass}<em>{\text{main}} , M</em>{\odot}$</th>
<th>$\text{Mass}<em>{\text{bullet}} , M</em>{\odot}$</th>
<th>$\frac{\text{Mass}<em>{\text{main}}}{\text{Mass}</em>{\text{bullet}}}$</th>
<th>Calculation Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bradac et al. 2006</td>
<td>$0.28 \times 10^{15}$ $&lt; 250 , \text{kpc}$</td>
<td>$0.23 \times 10^{15}$ $&lt; 250 , \text{kpc}$</td>
<td>$\sim 1$</td>
<td>Observation (lensing)</td>
</tr>
<tr>
<td>Clowe et al. 2004</td>
<td>–</td>
<td>$\sim 7.3 \times 10^{13}$ $&lt; 150 , \text{kpc}$</td>
<td>–</td>
<td>Observation (lensing)</td>
</tr>
<tr>
<td>Barrena et al. 2002</td>
<td>$1.24 \times 10^{15}$ $&lt; r_{200}$*</td>
<td>$(0.07 - 0.34) \times 10^{14}$ $&lt; r_{200}$</td>
<td>$\sim 62$</td>
<td>Velocity Dispersion/Virial</td>
</tr>
<tr>
<td>Girardi &amp; Mezzetti 2001</td>
<td>$8.39 \times 10^{14}$ $&lt; 1.5 , \text{Mpc}$</td>
<td>–</td>
<td>–</td>
<td>Velocity Dispersion/Virial</td>
</tr>
</tbody>
</table>

* $r_{200}$ is defined as the distance from the center where the matter density is 200 times the cosmological critical density (Binney & Tremaine 2008). $r_{200} \sim 2 \, \text{Mpc}$ (Barrena et al. 2002).

**Table 1.1:** Measured mass of the Bullet Cluster, compiled from literature.

The Bullet Cluster and MACS J0025.4 are among the merging systems that provide strong evidence for the presence of dark matter. Current observations clearly establish that these systems are not in an equilibrium state (not virialized). It is clear that normal virial estimates cannot be used. Simulations play a critical role in understanding these systems. Numerous studies exist that have simulated the Bullet Cluster, with a
successful reproduction of the observed properties, but opposing viewpoints exist about the initial cluster properties necessary to best reproduce the observed current state of this merging system.

**Mastropietro and Burkert (2008)**, hereafter MB, simulated the Bullet Cluster (Fig 1.8) with a non-zero impact parameter $b$ (not head-on collision). Their hydrodynamical simulation matches well with the currently observed state of the system. Table 1.2 lists the parameters and initial conditions used by MB.

<table>
<thead>
<tr>
<th>$M_{\text{mass,main}}$ ($10^{14}M_\odot$)</th>
<th>$M_{\text{mass,sub}}$ ($10^{14}M_\odot$)</th>
<th>$b$ (kpc)</th>
<th>$d_i$ (kpc)</th>
<th>$v_i$ (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.3</td>
<td>~1.38</td>
<td>150</td>
<td>5000</td>
<td>3000</td>
</tr>
</tbody>
</table>

**Table 1.2:** Simulation parameters of Mastropietro and Burkert (2008) for the model that most accurately recreated the observed properties of the Bullet Cluster.

**Figure 1.8:** Snapshots of the simulations performed by MB and SF. Left panel shows a time snapshot of the MB simulation, while the right panel shows a time snapshot of the SF simulation. Contours represent the dominant (dark matter) mass distribution (adopted from Mastropietro & Burkert 2008 and Springel & Farrar 2007).
Their dark matter distribution follows a NFW profile, with a concentration number* of $c = 6$ for the main cluster. *The authors mention that a choice of zero impact parameter does not recreate the Bullet System*, because a head-on collision does not produce the observed displacement between the gas and dark matter in the main system.

**Springel and Farrar (2007),** hereafter SF, assumed a direct, head-on collision (Fig 1.8). Table 1.3 lists the parameters and initial conditions used by SF for their best fit hydrodynamical model.

<table>
<thead>
<tr>
<th>Mass$<em>{\text{main}}$ (10$^{14}$M$</em>{\odot}$)</th>
<th>Mass$<em>{\text{sub}}$ (10$^{14}$M$</em>{\odot}$)</th>
<th>$b$ (kpc)</th>
<th>$d_0$ (kpc) initial separation</th>
<th>$v_1$ (km/s) relative to main cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1.5</td>
<td>0</td>
<td>$\sim$3400 (when the two clusters first touch)</td>
<td>1870</td>
</tr>
</tbody>
</table>

**Table 1.3: Simulation parameters of Springel and Farrar (2007) for the model that most accurately recreated the observed properties of the Bullet Cluster.**

SF also adopted a NFW profile for their dark matter mass distribution, with concentration numbers of $c = 2.0$ and $c = 7.2$. However, the authors suggest that *a zero impact parameter is essential for a successful recreation of the Bullet Cluster system.*

A reviewing the existing literature shows that the outcome of the simulation does not weigh heavily on the impact parameter. These simulations feature a wide range of mass ratios between the main and sub cluster. Nusser (2008) calculates the maximum possible mass ratio, in the framework of cosmology, to be 1.4. This is consistent with lensing observation from Bradac et al. (2006).

---

* The concentration number is defined to be the distance from halo center where mean density is 200 times the cosmological density ($r_{200}$) normalized by the NFW scale radius $a$: $c = r_{200}/a$
1.7. Thesis Preview

This thesis work focuses on two N-body simulations of two clusters of galaxies, with head-on collisions, one described by a parabolic orbit and the second described by a sub-parabolic orbit. The goal of this work is to re-create the Bullet Cluster merger event and to independently confirm a post-merger scenario. Chapter 2 introduces the methodology, explaining the N-body code and the basic physics framework described by Newtonian gravitational physics. Chapter 3 discusses the cluster setups. Galaxy and cluster assembly from luminosity function and dark matter distribution model are discussed. Chapter 4 presents the initial conditions and dynamics of Run #1, which is a collision from a parabolic orbit, and Run #2, which is a collision from sub-parabolic orbit. Chapter 5 presents the results of these two simulations. Velocity dispersions and cluster radial velocities are presented that reflect a post-collision scenario in a cluster merger system. Chapter 6 contains the summary of results and talk about possible future work on this thesis topic.
Chapter 2: Methodology

2.1. N-Body Techniques and Gravitational Physics

Many astrophysical scenarios currently observed are modeled with computer simulation, primarily to validate the underlying theories. A variety of computer simulations are used, a common one being Smoothed Particle Hydrodynamics (SPH) and N-Body. The following section describes the general N-Body simulation technique, although the reader is also referred to Monaghan (1992) for an extended discussion on SPH.

N-Body simulations are computer codes that calculate the motion of large number of particles due to the net gravitational force of the whole system. The Newtonian gravitational force between a pair of particles is given by

\[ \vec{F}_{grav} = -\frac{GM_1 M_2}{r^2} \hat{r}, \]

(2.1)

where the masses of the two particles are \( M_1 \) and \( M_2 \), separated by distance \( r \), and \( \hat{r} \) is the radial unit vector from the first particle to the second. For large systems, calculating the net force on a single particle can become prohibitively large because the number of force calculations required scales as \( \sim N^2 \), where \( N \) is the total number of particles in the system. For example, consider the particles in Figure 2.1. For three particles, three force
calculations are required; for four particles, six force calculations are required; for five particles, ten force calculations are required. The total number of force calculations required is \((N - 1)/2\).

To achieve an efficient means of computing the total number of force terms (the direct summation of all forces acting on one particle, for all particles) *Barnes and Hut* (1986) introduced the *tree* method. In this method, the number of force calculations scales as \(\sim N \log N\), a much more manageable and less computer intensive number. The

**Figure 2.1: Number of force calculations scales directly with number of particles.**
basic idea behind the tree method is to spatially sort the particles into a hierarchical system of groups. The tree method (Fig 2.2) is as follows.

First, a rectangular volume is placed around all the particles in the system; this is the root of the tree. The root is sub-divided equally in each dimension, resulting in eight cubes (oct tree), or four squares (quad tree) in 2-dimensions. Each cube becomes a node. This process continues until each particle is alone in a node.

Figure 2.2: A 2D example of the tree method of Barnes & Hut for N-body simulations.
The force on each particle is calculated by ‘walking’ the tree, according to the criterion

$$\frac{s}{d} < \theta,$$  

(2.2)

where $d$ is the separation between a target particle and the center-of-mass of the node that acts on the target particle, $s$ is the size of the node defined to be the length of the side of the node under consideration (Fig 2.2), and $\theta$ is the user supplied opening angle criterion for the node (Hernquist 1987). If for a given target particle the above criterion is met, then the node is treated as a particle, with a combined mass of each particle within the node, whose position coincides with the CM of the node. If the criterion is not satisfied, then the node is opened and the force contribution from each node is calculated. This recursive process is repeated at each time step. The opening angle criterion for the simulations presented in this thesis is $\theta = 0.7$.

The gravitational force of two particles, equation 2.1, can have an infinite value if the separation distance between two particles approaches zero. This problem can be resolved with the introduction of a softening parameter, $\epsilon$ (Hernquist 1987). The softened potential between two particles now becomes

$$\phi = -\frac{GM_1M_2}{(r^2 + \epsilon^2)^{3/2}},$$  

(2.3)

and from this equation the force can be easily calculated, since $\vec{F} = -\vec{\nabla}\phi$. Even if $r = 0$, the softening parameter keeps the force term finite. The softening parameter also functions as the minimum spatial resolution in the simulation.
2.2. Collisionless Systems

Simulating a large collection of particles (i.e. a galaxy with \(N \sim 10^{11}\) stars, clusters) requires identifying the system being modeled as a collisional system or a collisionless system. Consider a subject star in a galaxy. The force on this subject star does not vary rapidly; instead, its acceleration is smoothed due to the gravitational density distribution of the entire galaxy (Binney & Tremaine 2008). However, passing encounters of other stars causes this subject star’s velocity to alter slightly. The amount of time required for a star to have its velocity altered by an amount that is comparable to itself is known as the relaxation time (Binney & Tremaine 2008). The number of galaxy crossings that is required for star’s velocity to change by of order itself is

\[
n_{\text{relax}} \approx \frac{N}{8 \ln \left( \frac{b_{\text{max}}}{b_{\text{min}}} \right)},
\]

(2.4)

where \(b_{\text{max}}\) and \(b_{\text{min}}\) are the farthest and closest approaches of other stars (impact parameters), respectively. The relaxation time is given by (Binney & Tremaine 2008)

\[
t_{\text{relax}} \sim 0.1 \frac{N}{\ln N} t_{\text{cross}},
\]

(2.5)

where \(t_{\text{cross}} = R/v\), defined to be the crossing time – with a galaxy radius of \(R\) and star velocity \(v\). After many encounters, the star’s velocity will be altered, and as these interactions occur, it will forget its initial conditions (Binney & Tremaine 2008).

For a galaxy of \(N \sim 10^{11}\) stars and crossing times on the order of millions of years,

\[
t_{\text{relax}} \sim 10^{17} \text{ years}.
\]
The age of the Universe is approximately \((13 - 14) \times 10^9 \text{ years}\). These relaxation times are therefore unimportant and stellar encounters can be ignored. However, for a globular cluster \(N \sim 10^5\) and stars having crossing times of \(\sim 1 \text{ Myr}\), the relaxation time becomes \(t_{relax} \sim 10^8 \text{ years}\).

The lifetime of a typical globular cluster is \(10 \times 10^9 \text{ years}\), so relaxation effects become important (Binney & Tremaine 2008).

For clusters of galaxies, the crossing time of a galaxy through the cluster is (Bahcall 1977)

\[
t_{cross} = (6 \times 10^8 \text{ yr}) \left( \frac{R}{\text{Mpc}} \right) \left( \frac{v_r}{10^3 \text{ km sec}^{-1}} \right),
\]

(2.6)

where \(v_r\) is the measured LOS velocity and \(R\) is the radius of the cluster. As long as the timescale of the system dynamics is approximately less than or equal to the relaxation time of the system itself, then the system is said to be collisionless (Binney & Tremaine 2008). The simulations presented in this thesis involve clusters of galaxies.

Because galaxies have long relaxation times and thus are considered collisionless systems, simulating galaxies with some number of particles* requires that the N-body system be also collisionless. This is accomplished via the softening parameter discussed in the previous section. The discussion for establishing a collisionless N-body system is discussed further in section 3.2.

*In this work, a single particle in the N-body simulation might represent a parcel of matter that is comparable to a large number of stars.
Chapter 3: Cluster Setup

3.1. Schechter Luminosity Function

Assuming a constant mass-to-light ratio, one can obtain the mass spectrum for galaxies contained in a cluster, provided that the observed luminosity distribution can be matched with an empirically fitted function. A common luminosity function is the Schechter (1976) luminosity function,

\[ \phi(L)dL = \phi_\star \left( \frac{L}{L_\star} \right)^\alpha e^{-L/L_\star} \frac{dL}{L_\star}, \]  

(3.1)

which fits the observed galaxy luminosity data through the parameters \((\alpha, \phi_\star, L_\star)\) and gives \(\phi(L)dL\) galaxies in the interval \(L \rightarrow L + dL\), as shown in Figure 3.1. The parameters that describe this luminosity function are:

- \(\phi_\star\) = normalization constant
- \(L_\star\) = characteristic luminosity that marks the transition from faint end power law to exponential cutoff
- \(\alpha\) = faint-end power-law slope \((\alpha = -1.25)\)
Assuming a constant mass-to-light ratio and integrating downward, one can obtain the mass spectrum of galaxies from the Schechter luminosity function. For N-body setups, integrating the Schechter luminosity function with number of particles provides the mass spectrum of each galaxy in terms of the number of particles in a galaxy. The integration is done by several key parameters, one being $N_\star = 3600$ (particles), the galaxy of both clusters that corresponds to the exponential cutoff. Because of this exponential cutoff and our given value for the faint-end power-law slope $\alpha = -1.25$, there are more low-luminosity galaxies (fewer particles) than high-luminosity galaxies (more particles). As noted in figure 3.1, which shows a Schecter luminosity whose faint-end power-law is

![Figure 3.1: The Schechter luminosity function, showing the transition from power law ($\alpha = -1.0$) to exponential behavior. Vertical axis is the number of galaxies (log space) and the horizontal axis is the normalized luminosity (log space).](image)
described by $a = -1.0$, there are more low-luminous galaxies than galaxies whose galaxies are $> L_\ast$.

### 3.2. Stability Check and Mass Distributions

It was shown in section 2.2 that galaxies are collisionless and should not exhibit any two-body effects. To ensure that the constructed galaxies from the N-body particles also do not exhibit collisional behavior, the least massive, most massive, and the galaxy corresponding to the cutoff (called the $L_\ast$ galaxy) on the Schechter function are evolved in time. Two-body relaxation time is the time in which collisions can produce a large change in the original velocity distribution (Bahcall 1977 and references therein).

![Image of radii evolution over time with logarithmic scale on the y-axis and time on the x-axis.](chart)

**Figure 3.2:** Two-Body effects shown, with the Lagrangian radii, for a galaxy evolved in isolation. Notice the increasing 90% + 95% radii with time.
For N-body simulations, two-body effects can be recognized when the inner region of the galaxy shrinks while the outer region expands (Fig 3.2). The softening parameter $\epsilon$ is tweaked until no two-body effects are seen, as shown in Figure 3.3.

Figure 3.3: Constant radii, no Two-Body effects, due to a best-fit softening parameter, with a value of $\epsilon = 0.00095$. Time scale is in code units. Vertical axis is $ln(radii)$ in code units. The galaxies are evolved in time for a much longer period than the simulations presented in this thesis ($t \sim 2.5$).

The particles that make up the galaxies in these simulations are only luminous particles. The clusters are assembled with 90% of the cluster mass found within the cluster-wide
dark matter distribution and the remaining 10% of the mass is assigned to the galaxies (luminous particles). The mass distribution of galaxies and the clusters themselves follows a King (1966) model, as well as the dark matter particles, which are inserted into the core and halo regions of the clusters.

The King density model is given by (King 1966 and Binney & Tremaine 2008)

\[
f_{\text{King}}(\varepsilon) = \begin{cases} 
\rho_1 (2\pi \sigma^2)^{-3/2} \left( \frac{\varepsilon}{\sigma^2} - 1 \right), & \varepsilon > 0 \\
0, & \varepsilon \leq 0 
\end{cases},
\]

(3.2)

where \(f_{\text{King}}(\varepsilon)\) is a probability function that depends on the relative energy, defined as

\[
\varepsilon = \psi - \frac{1}{2} v^2,
\]

(3.3)

and where \(\psi\) is the relative potential

\[
\psi = -\phi + \phi_0.
\]

(3.4)

The quantity \(\phi\) is the potential and \(\phi_0\) is a constant chosen so that \(f_{\text{King}}\) has a positive value for \(\varepsilon > 0\) and is zero for \(\varepsilon \leq 0\). The King model represents a lowered isothermal sphere, lowered meaning that the Maxwellian distribution function is truncated at high energies by subtraction of the constant in equation 3.2. This is necessary because galaxies in clusters (or stars in globular clusters) have energies that are less than the escape energy of the system, thereby producing a gravitationally bound system. Plugging equation 3.3 back into 3.2 and integrating over all velocities, the density function becomes

\[
\rho_K = \int f d^3 v,
\]

\[
\rho_K(\psi) = \rho_1 \left[ \frac{\psi}{e^{\sigma^2}} \text{erf} \left( \frac{\sqrt{\psi}}{\sigma} \right) - \sqrt{\frac{4\psi}{\pi \sigma^2}} \left(1 + \frac{2\psi}{3\sigma^2}\right) \right].
\]

(3.5)
The term $\sqrt{\psi/\sigma^2}$ is known as the King parameter, often designated as $W_0$. In these models, I adopt a King parameter of $W_0 = 6$ for the dark matter halo of the clusters and the galaxy distribution, and $W_0 = 8.25$ for the particles distributions in the galaxies. These values are selected to match the observational properties of stars within galaxies and galaxies within a cluster. By combining equations 3.2 - 3.5 and assuming spherical symmetry, one can re-write Poisson’s equation as

$$\nabla^2 \phi = 4\pi G \rho$$

$$\frac{d}{dr} \left( r^2 \frac{d\psi}{dr} \right) = -4\pi G \rho_1 r^2 \left[ e^{\psi/\sigma^2} \operatorname{erf} \left( \frac{\sqrt{\psi}}{\sigma} \right) - \frac{4\psi}{\pi \sigma^2} \left( 1 + \frac{2\psi}{3 \sigma^2} \right) \right]. \quad (3.6)$$

Integrating equation 3.6 provides the necessary position and velocity data that particles in the simulation system will need to have in order to follow the King profile. Table 3.1 summarizes the properties of the two clusters.

The Mean Harmonic Separation (MHS) is used in this thesis to quantify the size of the clusters. Because clusters of galaxy do not have a well-defined boundary where one can say the cluster ends, the MHS is a weighted average of the separation of particles that make up the cluster. In the simulations, it is given by

$$\text{MHS} = 0.5 \, N \left( \frac{N-1}{s} \right), \quad (3.7)$$

where $N$ is the number of particles in a cluster and $s$ is given by

$$s = \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \frac{1}{r_{ij}} \quad (3.8)$$

Figure 3.4 shows the constructed main cluster and the smaller sub-cluster.
Table 3.1: Properties of the two clusters.

<table>
<thead>
<tr>
<th>Properties of the <strong>main cluster:</strong></th>
<th>Properties of the <strong>smaller cluster:</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>➢ Radius: (Mean Harmonic Separation –MHS-) ~ 0.41</td>
<td>➢ Radius: (Mean Harmonic Separation – MHS-) ~0.21</td>
</tr>
<tr>
<td>➢ Mass: 1.0</td>
<td>➢ Mass: 0.5</td>
</tr>
<tr>
<td>➢ # of galaxies: 250</td>
<td>➢ # of galaxies: 125</td>
</tr>
<tr>
<td>➢ # of total particles ($N_{250}$):</td>
<td>➢ # of total particles ($N_{125}$):</td>
</tr>
<tr>
<td>333049</td>
<td>164720</td>
</tr>
<tr>
<td>➢ Mass per particle: $1/N_{250}$</td>
<td>➢ Mass per particle: $1/(2N_{125})$</td>
</tr>
<tr>
<td>➢ Non-luminous: 90%</td>
<td>➢ Non-luminous: 90%</td>
</tr>
<tr>
<td>➢ Luminous: 10%</td>
<td>➢ Luminous: 10%</td>
</tr>
<tr>
<td>Total # of particles in the simulation:</td>
<td>497769</td>
</tr>
<tr>
<td>Softening parameter:</td>
<td>0.00095</td>
</tr>
</tbody>
</table>
Figure 3.4: The Main and the Bullet clusters of the re-created Bullet Cluster system.
Chapter 4: Re-creating the Merger Event

4.1. Run #1: Head-On Collision From a Parabolic Orbit

A review of literature shows that most simulations of the Bullet Cluster merger event are modeled with initial conditions adopted by treating the two clusters as point masses, infalling from infinity, on a parabolic orbit ($E_{\text{orbit}} = 0$) (SF 2007; Milosavljevic et al. 2007; MB 2008). The cluster merger then breaks down into a two-body problem.

Consider the two clusters having point masses of $M_{125} = 0.5$ and $M_{250} = 1.0^*$, separated a distance $r_i$. If we turn on gravity and allow the two point masses to gravitate towards one another, then from energy conservation principles (Fig 4.1):

$$\frac{1}{2} \mu \dot{r}_i^2 - \frac{G M_{250} M_{125}}{r_i} = E_{\text{orbit}},$$  \hspace{1cm} (4.1)

where $\mu = \frac{M_{250} M_{125}}{M_{250} + M_{125}}$, which is the reduced mass of the system. But for a head-on, parabolic orbit, $E_{\text{orbit}} = 0$, thus

$$\frac{1}{2} \mu \dot{r}_i^2 - \frac{G M_{250} M_{125}}{r_i} = 0,$$  \hspace{1cm} (4.2)

Solving for $\dot{r}_i$,

* 125 is the sub-cluster (or Bullet) and the 250 is the main cluster.
\[
\dot{\mathbf{r}}_i = \frac{d\mathbf{r}_i}{dt} = \sqrt{\frac{2GM_{250}M_{125}}{\mu r_i}},
\]

(4.3)

where \( G = 1 \) in the simulation, \( M_{250} = 1.0, M_{125} = 0.5, \) and \( \mu = 1/3 \)

\[
\dot{r}_i = v_i = \frac{3}{\sqrt{r_i}} \quad \text{(in code units, relative to the main cluster)}
\]

(4.4)

Figure 4.1: Relative and center-of-mass (CM) setup. The red dot is the \( CM_{\text{system}} \)

Further integrating equation 4.2, the separation distance can be found in terms of the collision time:

\[
r_i \approx 1.8899 \left( t_{\text{collide}} \right)^{2/3} \quad \text{(in code units)}
\]

(4.5)
Equations 4.2 and 4.3 are relative to one cluster, but it is trivial to move into the center-of-mass frame:

\[
\frac{x \, x_{\text{CM}}} = \frac{x_{125}M_{125} + x_{250}M_{250}}{M_{125} + M_{250}}
\]

\[
\frac{v \, v_{\text{CM}}} = \frac{v_{125}M_{125} + v_{250}M_{250}}{M_{125} + M_{250}}.
\]  

(4.6)

Moving the entire system into the center-of-mass frame is a necessary step to ensure that the system will not ‘wander’ off during the simulation. Velocities and positions are always with respect to the center-of-mass of the two merging clusters (Fig 4.2).

The motion of the two clusters is along the x-direction, with the observer looking down along the z-direction, onto the xy plane. The xy-plane in these simulations can be thought of as being the plane of the sky for an observer. For Run #1, the initial conditions are listed in Table 4.1, where equations 4.4 and 4.5 (in CM frame) are used to compute initial speed and time to collision. Figure 4.2 shows the cluster merger setup at \( t = 0 \).

Unless otherwise stated, code units are used.

<table>
<thead>
<tr>
<th>Initial Separation</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial In-Fall Speed:</td>
<td>Center-of-Mass Frame</td>
</tr>
<tr>
<td>Sub-cluster</td>
<td>+0.8165</td>
</tr>
<tr>
<td>Main cluster</td>
<td>-0.4082</td>
</tr>
<tr>
<td>Time to collision</td>
<td>1.09</td>
</tr>
<tr>
<td># of time steps</td>
<td>3000</td>
</tr>
<tr>
<td>Time Step (( \Delta t ))</td>
<td>0.0005</td>
</tr>
<tr>
<td>Simulation Time</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**Table 4.1:** Parameters for Run #1. All values are given in code units.
Figure 4.2: Initial conditions for Run #1. Cluster centers-of-mass (from dark matter) separation is 2.00. Luminous particles are given in black and dark matter particles are given in orange. Red arrows represent the motion of the clusters, towards the center-of-mass (red dot).
4.2. Run #2: Head-On Collision from a Sub-Parabolic Orbit

The outward recessional flow of all the matter in the Universe is often referred to as the *Hubble flow*, which is considered to be the expansion of the Universe (Binney & Tremaine 2008). Because merger events between galaxies and clusters of galaxies are

![Diagram of Hubble Flow and Sub-Parabolic Orbit]

**Figure 4.3:** Decoupling from the Hubble Flow and the resulting Bullet Cluster merger event. The two clusters begin falling towards each other after $d_{\text{max}}$, the maximum separation between them, on a sub-parabolic orbit.
observed, all the matter that created these galaxies and clusters decoupled from the Hubble flow and collided from a Keplerian orbit (Sarazin 2004). Figure 4.3 shows the decoupling for the Bullet Cluster merger, with a merge time of $\sim 9$ billion years (Angus & McGaugh 2008).

Using Kepler’s Third Law, the maximum separation between any two clusters is given by

$$T_{merge}^2 \approx \frac{4\pi^2 (d_{\text{max}})^3}{G(M_1 + M_2)}, \quad (4.7)$$

where $M_1$ and $M_2$ are the masses for two clusters, $d_{\text{max}}$ is the maximum separation between them, and $T_{merge}$ is the time required for the two clusters to collide since the Big Bang. Solving for $d_{\text{max}},$

$$d_{\text{max}} = [2G(M_1 + M_2)]^{1/3} \left( \frac{T_{merge}}{\pi} \right)^{2/3}. \quad (4.8)$$

Conserving energy and orbital angular momentum, Sarazin (2004) finds the relative velocity between the two clusters, at a separation distance $d$, to be

$$v^2 \approx 2G(M_1 + M_2) \left( \frac{1}{d} - \frac{1}{d_{\text{max}}} \right) \left[ 1 - \left( \frac{b}{d_{\text{max}}} \right)^2 \right]^{-1}, \quad (4.9)$$

where $b$ is the impact parameter.

Applying equation 4.8 to the Bullet Cluster system with the following values,

$$M_{\text{bullet}} \approx 1.5 \times 10^{14} \ M_\odot,$$

$$M_{\text{main}} \approx 1.3 \times 10^{15} \ M_\odot,$$
\[ t_{\text{merge}} \approx 9 \times 10^9 \text{ years}, \text{ and} \]

\[ d_{\text{max}} \text{ becomes} \]

\[ d_{\text{max}} \approx 5 \text{ Mpc} = 5000 \text{ kpc}. \]

The mass of the bullet and the main cluster used in equation 4.8 is an average value obtained from the available literature, while the time to merge was adopted from Angus & McGaugh (2008). To convert this astrophysical distance quantity into code length units, I consider the virial radius of SF 2007 and MB 2008, as well as the mass reconstruction (Fig 1.7) of Angus & McGaugh (2008), of the main cluster. I adopt a value of \( \approx 800 \text{ kpc} \) for the size of the main cluster. Since the Mean Harmonic Separation of my constructed main cluster is \( \approx 0.41 \), the conversion becomes

\[
\frac{800 \text{ kpc}}{2(0.41) \text{ code length units}} \approx 976 \text{ kpc / code length units}
\]

For converting velocities from code units to astrophysical velocity units, I adopt the following method: the constructed main cluster in the simulations has 250 galaxies, which is comparable to the Shapley 8 (SC1325-311) cluster. The measured galaxy velocity dispersion of the Shapley 8 cluster is 983 km/s (Metcalfe et al. 1987), while the calculated galaxy velocity dispersion of the constructed main cluster is 0.61 ± 0.03 velocity code units. The conversion then becomes

\[
\frac{983 \text{ km/s}}{0.61 \text{ velocity code units}} \approx 1612 \frac{\text{km/s}}{\text{velocity code units}}.
\]
Converting the maximum separation into code units, the value of $d_{max}$ now becomes

$$d_{max} = 5000 \text{ kpc} \times \left(\frac{\text{code length units}}{976 \text{ kpc}}\right) \approx 4.8 \text{ code units}.$$ 

Equation 4.9 can now be used in code units to find the merger velocity at a separation $d$, letting the impact parameter $b$ be zero. In Run #2 the initial separation is also set to 2.00, but the relative velocity at this separation, given by equation 4.9, now becomes:

$$v \approx 0.9376.$$ 

In Run #1, the initial relative velocity is 1.2247 at the same separation distance. Note that the relative speed of Run #2 is smaller by a factor of $\sqrt{2} \approx 1.4$. Table 4.2 contains the initial conditions used for Run #2, in code units. Velocities are moved into the CM frame.

Figure 4.4 shows the cluster merger setup at $t = 0$ for Run #2.

<table>
<thead>
<tr>
<th>Initial Separation</th>
<th>2.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial In-Fall Speed:</td>
<td>Center-of-Mass Frame</td>
</tr>
<tr>
<td>Sub-cluster</td>
<td>+0.6251</td>
</tr>
<tr>
<td>Main cluster</td>
<td>-0.3125</td>
</tr>
<tr>
<td>Time to collision</td>
<td>~1.3</td>
</tr>
<tr>
<td># of time steps</td>
<td>5000</td>
</tr>
<tr>
<td>Time Step ($\Delta t$)</td>
<td>0.0005</td>
</tr>
<tr>
<td>Simulation Time</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Table 4.2: Parameters for Run #2. All values are given in code units.**
Figure 4.4: Initial conditions for Run #2. Cluster centers-of-mass (from dark matter) separation is 2.00. Luminous particles are given in black and dark matter particles are given in blue. Red arrows represent the motion of the clusters, towards the center-of-mass (red dot).
4.3. Data Reduction

During the course of the simulation, a data file is created that stores mass, position, and velocity information about each particle, at each time step, in the entire simulation. Upon completion of the simulation, this data file is processed through a program that finds galaxies (groups of particles) via an algorithm that rolls particles towards the maximum calculated density gradient. Each galaxy is then tracked throughout the simulation. The tracking information is written out to a new file which contains the mass, position, and velocity of each galaxy, for each time step.

Any two or more galaxies can merge or spatially coincide. This causes the tracking program to momentarily lose the position of the galaxies. Manual identification and correction – via a cubic spline interpolation scheme – was necessary to re-assign position, velocity, and mass information for the lost galaxies in the file.
Chapter 5: Results

In this section I present the results of the two simulations, discussing the velocity dispersion and the dynamics of the galaxies in the pre- and post-collision time frame. I adopt the following analysis and definitions used throughout this section:

- For figures in which data are presented, $1\sigma$ error bars, as computed from bootstrapping, are used throughout. Error analysis is explained in section 5.3.

- Collision time is the time when the separation between the two clusters’ center-of-mass is zero, or the time when the centers-of-mass of both clusters spatially coincide.

- The center-of-mass of each cluster is computed using both the dark matter and galaxy (luminous) distributions, although it will be shown that the dark matter distribution coincides well with the luminous distribution. Data analysis is done with cluster centers-of-mass computed from dark matter distributions.

- Interaction time is the time when the boundaries of the two clusters, as given by the MHS, first touch. MHS was defined in section 3.2.

- Cluster radial velocity vector of a galaxy is the radial velocity vector that is anchored to the center-of-mass of the cluster to which the galaxy belongs.
5.1. Run #1: Head-On Collision from a Parabolic Orbit

As mentioned in section 4.1, the cluster-cluster collision occurs along the x direction. Initial cluster separation is 2.00 code units. The sub-cluster and the main cluster have an initial velocity of +0.8165 and -0.4082, respectively, in the center-of-mass frame. Run #1 lasts 1.5 time code units, during which time the sub-cluster sees only one passage through the main cluster. Figures 5.1 – 5.5 show various time snapshots of the simulation, from the beginning \((t = 0)\) until the end \((t = 1.5)\), where time is given in time code units.
Figure 5.1: Cluster positions at $t = 0$ for Run #1. Dark matter particles are orange and the luminous particles are black.
Figure 5.2: Same as Figure 5.1, except $t = 0.75$. 
Figure 5.3: Same as Figure 5.1, except $t = 1.00$. Note that the cluster boundaries, as defined by the MHS, first touch.
Figure 5.4: Same as Figure 5.1, except $t = 1.175$. The CM of the two clusters overlap, which corresponds to a $t_{\text{collision}} \approx 1.175$. The two-body calculated collision time is $t_{\text{collision}} \approx 1.09$, which is in excellent agreement with the simulation.
Figure 5.5: Same as Figure 5.1, except $t = 1.5$ (end of simulation) and with a zoomed in view of the central simulation region. Note that the smaller cluster (the bullet) passed through the main cluster. This morphology is consistent with the currently observed state of the Bullet Cluster (Fig 1.6).
5.1.1. **Velocity Dispersion**

During a cluster merger event, the ICM’s X-ray temperature increases as a result of shocks driven into it (Barrena et al. 2002, Pinkney et al. 1996). The orbital energies of the galaxies themselves will also be altered due to the time-changing gravitational potential well of the merging cluster system. These changes in dynamics will manifest in the velocity dispersion of the galaxies in the cluster (e.g., Roettiger et al. 1993, Owen et al. 1997, and Crone & Geller 1995). Figure 5.6 shows the temporal evolution of the velocity dispersion along the line of sight of the galaxies within each cluster.

![Figure 5.6: Temporal evolution of the velocity dispersion along the line of sight (along z-axis) of galaxies in each cluster as a function of simulation time. The yellow region signifies the post-collision time frame ($t_{\text{collision}} \sim 1.1$) and the grey region is the interaction time period. The two cluster boundaries (from MHS) first touch at $t \sim 1.0$. The vertical dashed line shows the corresponding time for greatest LOS velocity dispersion values.](image)
The average velocity dispersion, both clusters, before the collision is $\sim 0.67$, reaching a peak value of $1.09 \pm 0.07$ at time $t \sim 1.23$, which is a 61% increase. The currently observed state of the Bullet Cluster system involves only the first passage through the Main cluster (Clowe et al. 2006). The current dark-matter centers-of-mass separation from each cluster is $\sim 700 \text{ kpc}$ (Barrena et al. 2002). This translates to $\sim 0.7 \text{ code units}$. However, the maximum separation between the two dark matter CM after initial passage in the simulation occurs at $t = 1.5$, with 0.61 code units, which is approximately 595 kpc. At $t = 1.5$, the two clusters are still moving apart, but the simulation ends at this time.

The separation distance (calculated from both dark matter and luminous distributions) between the two clusters’ CM is shown in Figure 5.7. Note that the small difference in the dark matter CM and luminous CM provides evidence that these two mass distributions spatially coincide. Data analysis for Run #1 is independent of CM determination (CM or luminous) and does not affect the results presented henceforth.

Because the separation distance between the two clusters is closest to the actual observed separation of the Bullet cluster at $t = 1.5$ (595 kpc versus 700 kpc) I choose the $t = 1.5$ frame to represent the currently observed state of the system (also comparing Fig 5.5 and 1.6). At $t = 1.5$, the velocity dispersion of the Bullet is $0.71 \pm 0.04$ and of the Main cluster is $0.84 \pm 0.04$. In astrophysical units these values are $1355 \pm 65 \text{ km/s}$ for the Main cluster and $1145 \pm 65 \text{ km/s}$ for the Bullet. Table 5.1 compares my calculated velocity dispersion values to the given values in literature.
Figure 5.7: LEFT PANEL: Center-of-mass (CM) separations between the two clusters, calculated from dark matter particles and from galaxies. RIGHT PANEL: Difference between the Dark CM and Galaxy CM calculations. $t_{\text{collision}} \sim 1.1$, when the two cluster centers overlap. The small differences provide evidence that the galaxies trace the dark matter.

<table>
<thead>
<tr>
<th></th>
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</tr>
<tr>
<td><strong>Simulation @ $t = 1.5$</strong></td>
<td><strong>$1355 \pm 65$</strong></td>
<td><strong>$1145 \pm 65$</strong></td>
</tr>
</tbody>
</table>

Table 5.1: Comparison between simulation velocity dispersion and velocity dispersion found in literature.

Barrena et al. (2002), using spectroscopic and photometric surveys applied to a total of 78 galaxies from the Bullet Cluster (71 galaxies for the main cluster and 7 galaxies for the Bullet), measures a velocity dispersion of $\sim 1200$ km/s for the main
cluster and $\sim 200 \, km/s$ for the Bullet. Although Bradac et al. (2006) quotes 1400 $km/s$ for the Main and 1200 $km/s$ for the Bullet. My simulation values are in good agreement with the quoted literature velocity dispersions, with the exception of the Bullet for Barrena et al. (2002).

The large amount of energy that is released in a merger event perturbs the entire cluster environment (section 1), with the overlapping cluster potentials altering the dynamics of the galaxies (Sarazin 2004; Binney & Tremaine 2008). This sets up scenarios where galaxies can interact with one another.

In a cluster-cluster collision event, galaxies can merge into a larger galaxy via tidal stripping and dynamical friction (Binney & Tremaine 2008). The number of galaxies present in the simulation is shown in Figure 5.8, where one merger is observed during the course of this simulation. The total of 375 galaxies in the two clusters are present until $t = 1.0$, which signifies the start of the interaction time, after which the total number of galaxies drops down to 374.
Figure 5.8: The number of galaxies present during the simulation. Notice how galaxies merge close to $t_{\text{collision}} \sim 1.1$. 
5.1.2. Cluster Radial Velocities of Galaxies

As Figure 5.9 shows, the radial velocity (with respect to the cluster CM) of a galaxy is defined in this thesis as the cluster radial velocity vector*. It can be calculated from simple vector addition, taking into account the position vectors as anchored to the system center-of-mas, which is at the origin of the coordinate system.

![Diagram of cluster radial velocity](image)

**Figure 5.9:** Calculation of the cluster radial velocity of a galaxy from cluster CM. The cluster radial velocities of galaxies depend on the angle $\theta$ associated with the velocity vector, as shown in the figure.

The cluster radial velocity of a galaxy belonging to its respective cluster can be calculated by

$$v_{rad} = \vec{v}_{galaxy} \cdot \left( \frac{\vec{r}_{rad}}{|\vec{r}_{rad}|} \right),$$

where $\vec{r}_{rad}$ is the unit radial vector as defined from the cluster center-of-mass.

---

*The cluster radial velocity can be positive (galaxy moving outward from the cluster) or negative (galaxy moving inward, towards the cluster CM).
Figure 5.10 shows a plot of the average cluster radial velocities of both low and high mass galaxies in the sub-cluster. The 60 least massive (red) and 30 most massive (blue) galaxies were sampled from the sub-cluster.

Figure 5.10: Average cluster radial velocity of 30 most massive (blue) and 60 least massive galaxies (red) in the sub-cluster. The radial velocities are computed with respect to the cluster center-of-mass, which was obtained from dark matter particles. Yellow region signifies the post-collision period and the grey region is the time period when the two cluster boundaries (from MHS) first touch.
Before collision ($t = 0 \rightarrow t \sim 1.0$), the cluster dynamics represents an isolated cluster. If the collision time would have occurred later, the galaxies would have enough time to settle into their cluster potential before collision. However, since the initial separation is 2.00 code units, the galaxies do not have enough time to achieve equilibrium with the cluster potential well. When the two cluster boundaries (as given by the MHS) first touch ($t \sim 1.0$), the massive galaxies from the sub-cluster receive an impulse that makes them move outward from the cluster, followed by the least massive galaxies, which move inward. After the collision ($t \sim 1.3$) the least massive galaxies also have a strong positive radial velocity vector. This occurs because as these low-mass galaxies move inward, they eventually pass through the cluster CM. After passing through the CM, their radial velocity vectors are now flipped. These low-mass galaxies are now moving outward. In the post-collision period, both the high-mass and low-mass galaxies of the sub-cluster experience a positive cluster radial velocity, which suggest an outward expansion of the Bullet. Note that these observations are not statistically significant due to only $(1 - 1.5)\sigma$ deviations from the zero cluster radial velocity line. It is not possible to determine which galaxies receive a stronger impulse due to the collision.

Figure 5.11 shows a plot of the average cluster radial velocities of both low and high mass galaxies in the main cluster. The 110 least massive (red) and 50 most massive (blue) galaxies were sampled from the main cluster. The least massive galaxies experience a reversal of their radial velocity vector, becoming positive at $t \sim 1.0$. This is also observed in Figure 5.10 for the sub-cluster but at a later time. However, these observations are once again not statistically significant due to only $(1 - 1.5)\sigma$ deviations.
from the zero cluster radial velocity line. No statistically significant results can be deduced between the least massive and most massive galaxies in the main cluster.

Figure 5.11: Average cluster radial velocity of 50 most massive (blue) and 110 least massive galaxies (red) in the main cluster. Yellow region signifies the post-collision period and the grey region is the time period when the two cluster boundaries (from MHS) first touch. Due to \((1 - 1.5)\sigma\) deviations, no statistically significant difference can be established between the least and most massive galaxies.

The sampling number (how many galaxies are picked for the least and most massive groups) can affect the results statistically. To ensure that the observed trends are not a consequence of the sampling numbers, additional samplings of low-mass and high-mass galaxies are done. Since the galaxy mass spectrum conforms to the Schechter luminosity function, the cluster population is dominated by the low-mass galaxies, due to
the $\alpha = -1.25$ power-law slope for the faint end of the Schechter luminosity function. As a result, the sampling is adopted to reflect the large number of low-mass galaxies relative to the high-mass galaxies. The following least massive/most massive galaxy samples were also analyzed for the sub-cluster: 100 / 20, 80 / 40, and 40 / 15 and for the main cluster: 150 / 80, 90 / 60, and 50 / 50.

Figure 5.12 shows the average cluster radial velocities of the above sampling, where blue represents the most massive galaxies and red represents the least massive galaxies. The above samplings for the sub-cluster shows (at $2\sigma - 3\sigma$ significance) that both the least massive and the most massive galaxies have their radial velocities altered in the post-collision period, where the change corresponds to an outward expansion from the cluster center due to the impulse delivered from the cluster-cluster collision. The main cluster shows very poor statistical significance ($\leq 1\sigma$) between the pre- and post-collision period and no conclusion can be made about the behavior of the least massive and most massive galaxies due to the cluster-cluster collision.
Figure 5.12: Variety of sampling of galaxies for the sub-cluster (left column) and the main cluster (right column). Vertical scale represents the average radial velocity in code units and the horizontal scale represents the simulation time in code units.
The sampling error was calculated using *random sampling with replacement*, also known as bootstrapping. This is discussed in further detail in section 5.3.

Next, the post-collision galaxy dynamics are analyzed from a standpoint of location within the cluster: inner region, middle region, and outer region. The average cluster radial velocities from the center-of-mass of the cluster of the galaxies occupying these region at each time is calculated. The size of the regions is not allowed to change throughout the simulation, although galaxies will migrate from one region to another. The three regions were chosen such that the number of galaxies in each region is sufficient to avoid large sampling errors and to have an even distribution of galaxies. Figure 5.13 below shows the regions selected for the sub-cluster and the main cluster for Run #1.

**Figure 5.13**: Region dimensions for each cluster, with values given in code length units. Blue is the inner region, red is the middle region, green is the outer region.
Figure 5.14 shows the average cluster radial velocity of the galaxies in each of the three regions, where the inner region galaxies are in blue, middle region galaxies are in red, and outer region galaxies are in green. As the cluster-cluster interaction proceeds and culminates with collision, galaxies steadily migrate from one region to another. The galaxies present in each region are updated at each time step and the average cluster radial velocity is recomputed. The signature of a cluster-cluster collision event is apparent in the graphs as changes in the cluster radial velocities of the galaxies. Pre-collision radial velocities average to zero in both clusters. After $t_{\text{collision}} \sim 1.1$, the inner and middle galaxies have a strong outward expansion from the cluster centers, while the outer galaxies show a statistically significant ($\geq 4\sigma$) strong negative radial movement, as compared between pre- and post-collision period. At $t \sim 1.3$ and $t \sim 1.5$, for the sub-cluster and main cluster, respectively, the outer galaxies pass through the cluster CM and their cluster radial velocity vector flips direction. The inner and middle galaxies continue expanding outward from the cluster CM, albeit at a slower rate due to gravitational deceleration.

A second investigation involved looking at the LOS velocity dispersion of the galaxies present in each region at each time, as shown in Figure 5.15. All three regions show an increase in the velocity dispersion along the line-of-sight in both clusters at approximately $t = 1.2$, with a steady decrease towards pre-collisional values towards the end of the simulation. The velocity dispersions of the three regions of the sub-cluster in the post-collision period have very poor statistical significance amongst each region, which prevents any statistically significant comparison amongst the regions. However, the main cluster’s velocity dispersion does show a statistically significant (at $> 4\sigma$
significance) difference between the inner and outer regions at $t = 1.2$, where the inner galaxies exhibit approximately twice the velocity dispersion of the outer galaxies.

Figure 5.14: Average cluster radial velocity of galaxies in each three region, with respect to their respective cluster center. Red represents the galaxies in the inner region, blue represents the galaxies in the middle regions, and green represents galaxies in the outer region, as determined at each time step. Note the large negative radial velocity of the outer galaxies in each cluster after collision.
Figure 5.15: Velocity dispersion of galaxies along the line-of-sight (LOS) in each of the three regions. Galaxies present in each region are updated every time step. Note that the inner galaxies show the highest velocity dispersion. Yellow region signifies post-collision period.
The third investigation was performed on the galaxies that occupy each region at $t_{\text{collision}} \sim 1.1$. Galaxies that originally occupied the inner, middle, and outer regions at $t_{\text{collision}} \sim 1.1$ are then tracked throughout the simulation time and their average cluster radial velocity recomputed at each time step. The results are shown in Figure 5.16. All three regions show evidence of a cluster merger when comparing pre-collision and post-collision time periods ($\geq 3\sigma$ significance), where pre-collision cluster radial velocities average out to zero. It is evident in both clusters that galaxies present in the inner region receive a strong impulse that sends these galaxies moving outward from the cluster center. The middle and outer galaxies move inward, towards the center of their clusters, although after $t \sim 1.2$ the middle and outer galaxies eventually attain a positive radial velocity due to passage through the center-of-mass. The strong differences between the inner, middle, and outer galaxies are statistically significant ($> 4\sigma$) in the post-collision period and even at the end of the simulation, where the inner and middle galaxies have radial velocities that are twice of the outer galaxies.

Because of the length of this simulation (1.5 code time units), the curves in Figure 5.16 do not provide enough information to surmise the state of the galaxies after collision. This is because only the initial passage of the bullet through the main cluster is observed in Run #1.
Figure 5.16: Average cluster radial velocities of the galaxies that were present in each region at $t_{\text{collision}} = 1.1$. These galaxies were then tracked for the entire simulation. Yellow region signifies post-collision period.
5.2. Run #2: Head-On Collision From a Sub-Parabolic Orbit

As discussed in Section 4, Run #2 differs from Run #1 because it follows the two clusters from a sub-parabolic orbit until the moment of impact. This results in a smaller initial relative velocity between the two clusters. Initial cluster separation is 2.00 code units. The sub-cluster and the main cluster have an initial velocity of +0.6251 and -0.3125, respectively, in the center-of-mass frame. Run #2 lasts 2.5 time code units, during which the sub cluster sees two passages through the main cluster, with the last passage signifying the start of the two clusters merging together. Figures 5.17 – 5.21 show various time snapshots of the simulation, from the beginning ($t = 0$) until the end ($t = 2.5$), where time is given in time code units.
Figure 5.17: Cluster positions at time $t = 0$ for Run #2. Luminous particles are black and the dark matter particles are blue.
Figure 5.18: Same as Figure 5.17, except $t = 1.00$. The outer boundaries, as defined by the MHS, of the two clusters start to touch.
Figure 5.19: Same as Figure 5.17, except $t = 1.35$ (top panel) and $t = 1.50$ (bottom panel). Note that the cluster centers-of-mass overlap at $t_{\text{collision}} = 1.35$ and pass through each other at $t \sim 1.5$. 
Figure 5.20: Same as Figure 5.17, except $t = 2.0$. Note that $t = 2$ and subsequent times shown are beyond the currently observed status of the Bullet Cluster. See section 5.2 for further details.
Figure 5.21: Same as Figure 5.17, except $t = 2.5$ (end of simulation). Note how a large massive galaxy is forming at the combined cluster potential well.
5.2.1. Velocity Dispersion

Because Run #2 was allowed to run for a longer time period, the evolution of the two clusters is seen further in time, which allows for a more thorough investigation of the velocity dispersion. Figure 5.22 shows the temporal evolution of the velocity dispersion of Run #2.

Figure 5.22: Temporal evolution of the velocity dispersion along the LOS (along z-axis) of galaxies in each cluster as a function of simulation time. The yellow region signifies the post-collision period and the grey region is the time period when the two cluster boundaries (MHS) first touch ($t \sim 1.0$). The vertical dashed line shows the corresponding time where velocity dispersion is highest.
The maximum value of the velocity dispersion occurs at \( t \sim 1.4 \), with a value of \( 1.07 \pm 0.06 \) for the sub-cluster (bullet) and \( 0.99 \pm 0.04 \) for the Main cluster. Because we know that the currently observed state of the Bullet Cluster shows only the first passage (Clowe et al. 2006), the apparent dark-matter CM separation between the main and sub-cluster is used to find the best simulation time snapshots that represent the Bullet Cluster, similarly as in Run #1 analysis.

The currently observed centers-of-mass separation as calculated from dark matter is \( \sim 700 \text{ kpc} \) (Barrena et al. 2002). This translates to \( \sim 0.7 \text{ code units} \). However, as Figure 5.23 shows, there is a difference between the CM separations calculated form dark matter and the CM separations calculated from luminous matter. The maximum separation of the CM of the two clusters after initial passage in the simulation occurs at \( t = 1.9 \), with \( 0.63 \text{ code units} \) (614 kpc) from dark matter and \( 0.58 \text{ code units} \) (566 kpc) from luminous matter calculations. The maximum difference between the CM separations from the luminous calculation and dark matter calculation is 19\%, occurring at \( t \sim 2.3 \) (right panel, Fig 5.23). A 19\% difference still provides evidence that the dark matter distribution traces the luminous distribution in the simulation for Run #2.

Choosing the \( t = 1.9 \) time snapshot as the currently observed state of the Bullet Cluster system, the velocity dispersion is \( 0.75 \pm 0.03 \) for the Main cluster and \( 0.63 \pm 0.04 \) for the Bullet. A comparison with the literature and Run #1 is given in Table 5.2. Run #2 shows smaller velocity dispersion for both clusters, which reflects the less energetic sub-parabolic collision. In addition, simulation values are still consistent with the ones from literature, showing that the Bullet has smaller velocity dispersion in each
run. An investigation of the number of galaxies present in the simulation (Fig 5.24) shows six merger events.

Figure 5.23: LEFT PANEL: Center-of-mass (CM) separations between the two clusters, calculated from dark matter particles and from galaxies. RIGHT PANEL: Difference between the dark matter CM and luminous matter CM separations ($t_{collision} \approx 1.3$, when the two cluster centers-of-mass overlap).

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</tr>
<tr>
<td>Simulation Run #1 @ $t = 1.5$</td>
<td>$1355 \pm 65$</td>
<td>$1145 \pm 65$</td>
</tr>
<tr>
<td>Simulation Run #2 @ $t = 1.9$</td>
<td>$1209 \pm 48$</td>
<td>$1016 \pm 65$</td>
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</tbody>
</table>

Table 5.2: Comparison between simulation velocity dispersion and velocity dispersion found in literature for Run #1 and Run #2.
Figure 5.24: The number of galaxies present during the simulation. Notice how galaxies merge starting when the two clusters fully collide, from $t \sim 1.3$ onwards.
5.2.2. Cluster Radial Velocities of Galaxies

Similarly to Run #1, the dynamics of galaxies, as observed via their masses or their location in the cluster potential, is analyzed. Figure 5.25 shows a plot of the average cluster radial velocity of both low and high mass galaxies in the sub-cluster. The 60 least massive (red) and 30 most massive (blue) galaxies were sampled from the sub-cluster, similarly to Run #1. The vertical dashed line corresponds to the time that matches best with the currently observed state of the Bullet Cluster system. Due to poor statistical significance, it is not possible to say whether the low-mass or high-mass galaxies of the

![Average cluster radial velocity of 30 most massive (blue) and 60 least massive galaxies (red) in the sub-cluster. Note the outward movement of galaxies in the post-collision period, signified by the yellow region. The vertical dashed line corresponds to the time that matches best with the currently observed state of the Bullet Cluster system (t~1.9).](image)

Figure 5.25: Average cluster radial velocity of 30 most massive (blue) and 60 least massive galaxies (red) in the sub-cluster. Note the outward movement of galaxies in the post-collision period, signified by the yellow region. The vertical dashed line corresponds to the time that matches best with the currently observed state of the Bullet Cluster system (t~1.9).
sub-cluster have a stronger outward or inward radial velocity throughout the post-collision time frame. However, post-collision period does clearly show that both the high-mass and low-mass galaxies are moving outward from the cluster center (at $\sim 2\sigma$ significance) due to the cluster-cluster collision, with a maximum radial velocity value of $\sim 0.7$ occurring at $t \sim 2.1$, after which galaxies are slowly decelerating and attain pre-collisional value at the end of the simulation. This maximum at $t \sim 2.1$ is remarkably close to the simulation time corresponding to the Bullet Cluster.

Figure 5.26: Cluster radial velocity of 50 most massive (blue) and 110 least massive galaxies (red) in the main cluster. Yellow region signifies the post-collision period. Note the strong outward movement of all galaxies after collision. The vertical dashed line corresponds to the time that matches best with the currently observed state of the Bullet Cluster system ($t \sim 1.9$).
Figure 5.26 shows a plot of the average cluster radial velocities of both low and high mass galaxies in the main cluster. The 110 least massive (red) and 50 most massive (blue) galaxies were sampled from the main cluster. Due to poor statistical significance, it is very difficult to extract any information pertaining to the galaxies that receive the greatest impulse during the collision time. However, throughout the remaining simulation, there is a signature that shows both the high-mass and low-mass galaxies expanding outward from the cluster center, beginning at $t$~1.7.

To ensure that any observable trend is not due to statistical noise, the same various batches of samples as in Run #1 are also analyzed. Figure 5.27 shows the radial velocities of these additional samplings (blue = most massive, red = least massive). The early post-collision time frame, $t = 1.3 - 1.6$, clearly shows that the most massive galaxies have a negative cluster radial velocity in the main cluster, compared to pre-collision. However, this observation shows weak statistical significance. In the sub-cluster, post-collision period differs from pre-collision period because the galaxies see a strong increase (at $2\sigma - 3\sigma$ significance) in their cluster radial velocities: galaxies are moving outward from the cluster. In both clusters, the changes in the cluster radial velocities following initial collision suggests that most galaxies are expanding outward from the cluster center, peaking at $t = 2.3$. After $t = 2.3$, the expansion rate slows due to deceleration (from gravity of the final merged system) and reduces back down to pre-collisional values.
Figure 5.27: Variety of samplings of galaxies for the sub-cluster (left column) and the main cluster (right column). Vertical and horizontal scales are the same as Figure 5.12.
Figure 5.28 shows the average cluster radial velocity of the galaxies in each of the three regions, where the inner region galaxies are in blue, middle region galaxies are in red, and outer region galaxies are in green. Galaxy population in each region is updated and average cluster radial velocity recomputed at each time step. For the sub-cluster, I bin the galaxies radially into the following bins: the inner region is $r \leq 0.10$, the middle region is $0.10 < r \leq 0.30$, and outer region is $r > 0.30$. For the main cluster, the inner region is $r \leq 0.20$, the middle region is $0.20 < r \leq 0.45$, and the outer region is $r > 0.45$. There is a noticeable change in the cluster radial velocities ($> 2\sigma$), compared between pre-collision and post-collision. Pre-collision cluster radial velocities average out to zero. For the sub-cluster, the inner and middle galaxies have a slight positive cluster radial velocity after collision, whereas the outer galaxies have a significant negative cluster radial velocity. These outer galaxies have their cluster radial velocity vector suddenly flipped (due to passage through the CM) at $t = 1.5$, peaking at $t = 1.7$.

Towards the end of the simulation, the curves show that all the galaxies decelerate, due to the gravitational influence of the newly merged cluster system, which is also clearly seen in Figure 5.21. Note that the data for inner galaxies lacks error bar due to a lack of galaxies in the inner region at the end of the simulation. For the main cluster, the inner galaxies gain an outward expansion, while the middle and outer galaxies move inward. At $t = 2.3$ the inner and middle galaxies steadily decelerate, along with the outer galaxies in the sub-cluster, while the outer galaxies in the main cluster continue expanding outward at a constant rate.
Figure 5.28: Average cluster radial velocity of galaxies in inner, middle, and outer regions, as defined in the text. Galaxies are updated in each region at every time step. For the sub-cluster, missing error bars on the inner galaxies toward the end of the simulation is due to few or no galaxies discovered in the inner region. Regions are labeled in the bottom panel for both panels. The lack of error bars on the inner region is explained in the text.
Figure 5.29: Velocity dispersion of galaxies along the line-of-sight in each of the three regions. Note that the inner galaxies show the highest velocity dispersion in the main cluster. Yellow region signifies post-collision period. For the sub-cluster, missing error bars and zero velocity values in the inner galaxies toward the end of the simulation are due to few or no galaxies discovered in the inner region.
An investigation of the velocity dispersion of the galaxies that occupy each region is shown in Figure 5.29. Similarly to Run #1, Run #2 also shows high velocity dispersion (Figure 5.29) for the inner and middle galaxies immediately after collision \( t = 1.3 - 1.6 \), while the outer galaxies have the smallest velocity dispersion, five times smaller, being statistically significant \( (> 3\sigma) \). At around \( t = 1.6 \), the outer galaxies in the sub-cluster attain a strong velocity dispersion, although this is most likely not statistically significant. Inner galaxies exhibit the highest velocity dispersion values in the main cluster. It is interesting to note that at the end of the simulation, the velocity dispersion is comparable the pre-collision velocity dispersion values. Note that the inner galaxies in the sub-cluster show zero velocity with no error bars at the end of the simulation. This is due to few \(< 10\) or no galaxies discovered in the inner region.

Galaxies that originally occupied the inner, middle, and outer regions at \( t_{\text{collision}} = 1.3 \) were also tracked throughout the simulation time and their average cluster radial velocity recomputed at each time step. The results are shown in Figure 5.30. Galaxies present in the inner region show a strong outward radial velocity \( (> 3\sigma) \) in both the sub-cluster and the main cluster immediately after collision. The middle galaxies in the sub-cluster move outward from the cluster center at a constant rate, decelerating after \( t = 2.3 \). The outer galaxies in the sub-cluster have twice the radial velocity as compared to the galaxies of the main cluster \( (1.2 \text{ versus } 0.6, \text{ respectively}) \). The outer galaxies in the sub-cluster decelerate, while the galaxies in the main cluster exhibit a constant outward movement with a radial velocity value of \(~0.6\). In the main cluster, the middle galaxies receive the most impulse, having a very strong positive radial velocity at
Figure 5.30: Average cluster radial velocities of galaxies originally present in each region at $t_{\text{collision}} = 1.3$. These galaxies were then tracked for the entire simulation. Upper panel corresponds to the sub-cluster and the lower panel corresponds to the main cluster. $t = 1.9$ corresponds to the Bullet Cluster system (vertical dashed line).
\[ t = 1.8 (> 4\sigma), \text{ while at the same time in the sub-cluster the outer galaxies receive the most impulse. In both clusters, the inner and middle galaxies move outward immediately after collision, while the outer galaxies move inward. After } t = 1.5, \text{ all galaxies expand outward, with steady deceleration occurring as time advances.} \]

5.3. Error Analysis

The calculated values given for the velocity dispersions and cluster radial velocities in the previous sections are given with error bars. These error bars were calculated using random sampling with replacement, also known as bootstrapping. The bootstrapping method is a robust technique to estimate sample errors when the underlying distribution function is unknown.

The bootstrapping method is described as follows: from a given original set of galaxies, a new phantom set is made by drawing randomly from the original set, such that every time a galaxy is drawn, it is placed back into the original set. This makes it possible for a galaxy to be picked numerous times. With the advancement in computer power, the number of phantom sets created should be as high as possible. For each phantom set, the average value of the quantity of interest (velocity dispersion or average cluster radial velocities) is computed. Then from all these phantom sets the standard deviation error is calculated using

\[
\sigma_{\text{final}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - x_{\text{avg}})^2}
\]
where $x_i$ is the quantity of interest of each $i^{th}$ phantom set, $x_{avg}$ is the final average values of all phantom sets, and $N=10,000$ phantom samples for this thesis work.
Chapter 6: Summary and Future Work

6.1. Summary

I have presented two N-body simulations, each with different initial conditions. The motivation for this study was to re-create the Bullet Cluster merging system. I present the following findings.

- The velocity dispersion along the LOS (z-axis) of galaxies of both Run #1 and Run #2 is altered significantly due to the cluster merger event. The velocity dispersion is increased by a factor of ~2 for both the sub-cluster and the main cluster after the collision event. The increase in velocity dispersion reflects a change in the orbital dynamics of the galaxies and is consistent with literature on cluster merger events. My computed values are consistent with the values from literature, and show that the main cluster features a higher velocity dispersion than the sub-cluster.

- A robust correlation between galaxy masses and dynamics was not established because of a lack of statistical significance between the least massive and most massive galaxy data. However, for Run #1, there are signs in the sub-cluster that
indicate the least massive galaxies have a stronger negative cluster radial velocities at the collision time compared to the more massive galaxies. For Run #2, in the main cluster, the least massive galaxies exhibit a positive cluster radial velocity, while the most massive galaxies exhibit a negative cluster radial velocity, immediately after collision \((t = 1.2 - 1.6)\).

- Average cluster radial velocities of galaxies located in the inner region, middle region, and outer regions of the clusters differ significantly in both Run #1 and Run #2. In both simulations and both clusters, at the time of collision the inner galaxies receive an impulse that directs them outward from the cluster centers, while the middle and outer galaxies have a negative cluster radial velocity, showing an inward migration towards the cluster centers. These middle and outer galaxies pass through their cluster CM, flipping their cluster radial velocity vector and moving outward from the cluster along with the inner galaxies.

- Post-collision period indicates that the Bullet Cluster system will tend toward a relaxed, merged system. The end of Run #2 shows that the rate of outward expansion slows due to deceleration of the combined gravitational pull of the system, with cluster radial velocities values matching or becoming more negative than the pre-collision values. The end of Run #2 also shows the formation of the merged system of the two clusters.

- The velocity dispersion along the LOS of the galaxies located in inner, middle, and outer regions of the clusters also reflects a change in the galaxy dynamics due to the cluster-cluster collision. Galaxies present in the inner and middle regions at the collision time have their velocity dispersion increased by at least a factor of \(~2\)
for both Run #1 and Run #2. The galaxies present in the outer regions of the cluster exhibit the least amount of change to their velocity dispersion, although this change is more pronounced in the sub-cluster.

- The average cluster radial velocities of the galaxies that were originally present at the collision time in each region were also analyzed throughout the simulations. Run #1 and Run #2 show a very strong positive cluster radial velocity of the inner galaxies at the time of collision, while the middle and outer galaxies exhibit a very strong negative cluster radial velocity. As the simulation time progresses, the middle and outer galaxies pass through their cluster CM and exhibit a similar positive cluster radial velocity.

- Galaxies merge with one another, thereby reducing the total number of galaxies during the merger event. One merger is seen in Run #1 and six mergers are seen in Run #2. The merger rate is highest during the transient period when clusters collide and the galaxies pass through their respective cluster CM. The passage through the CM results in an increase in the number density of galaxies, thereby also increasing the interactions between galaxies and resulting in galaxy mergers.

- Both Run #1 and Run #2 successfully re-create the currently observed morphological condition of the Bullet Cluster merging system: the Bullet cleanly passed through the Main cluster. Run #1 and Run #2 re-create the currently observed separation distance between the main cluster and sub-cluster with a percent difference of approximately 15% (average of ~600kpc from both runs versus ~700kpc from Barrena et al. 2002).
6.2. Future Work

The work done in this thesis can be expanded with the addition of particles that resemble gas in the clusters. This would allow a study of the ICM environment during a cluster-cluster collision. For the ICM, various properties of interest could be calculated from the simulation and then compared to observation of the ICM in galaxy clusters. A different density profile for the N-body cluster setups could also be adopted: a NFW profile might be used instead of the King profile. Comparison between different density models and the simulation outcomes is beneficial, because it can help to further diminish the difference between the inferred dark matter distribution from observations and those from simulations and better our understanding of dark matter.

The Bullet Cluster merging system provides an astrophysical scenario where dark matter is collisionless and dominates the cluster potential, properties that were adopted by this thesis work. As mentioned in Chapter 1, evidence also exists where dark matter is consistent with a collisional model, as in A520 cluster (Jee et al. 2012). It might be interesting to alter the simulations such that dark matter exhibits collisional properties and then attempt to re-create observed properties of A520. No current explanation exists that successfully explains this observed discrepancy between the collisional properties of dark matter.
Chapter 7: References


