

**Determining the Relation between the
Mergers and Acquisitions of Firms with the
Announced Total Value: A Bi-variate
Generalized Poisson Regression Approach**

A Thesis

Submitted to the Graduate School

in Partial Fulfillment of the Requirements

for the Degree

Master of Science

by

Mina Mahbub Hossain

Dr. Munni Begum - Advisor

Ball State University

Muncie, Indiana

May 2017

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1 Abstract

Mergers and Acquisitions ($M\&A$) are major strategic initiatives undertaken by many organisations. The choice of payment type in ($M\&A$) – Cash, Stock Debt, or a combination of these – can have a substantial impact on the successful completion of the transaction. In this thesis, we provide an empirical investigation into the determinants of the Number of Mergers by introducing the Announced Total Value (ATV) according to the payment type investment characteristics of the companies involved as additional variables which have not been considered in previous studies. We considered the payment types and TVA of a particular payment type of the companies involved in ($M\&A$) to explore their effects on the number of Mergers completed in given year.

It is generally regarded to be preferable by shareholders of the target company to receive a Cash payment rather than shares of the bidding company as we can see 67.33% of the total Mergers were completed by Cash. We have focused on how the number of Mergers depends on ATV in between the bidding companies. From the univariate analysis of total number of Mergers, number of Mergers by Cash and number of Mergers by Stock on the Mean, Median and the Standard Deviation of the ATV, we have found that Negative binomial regression model performs better than the Poisson regression model and all of these three response variables have positive weak but significant association with the standard deviation of the ATV. So we can recommend that the Standard Deviation of the ATV can be treated as significant predictor to model the number of Mergers in a given year.

The Bi-variate Poisson regression model shows the positive effect of the mean ATV for Mergers by Stock on the joint distribution of the number of Mergers by Cash and Stock, which is statistically significant. After the Adjustment of Overdispersion Parameters for the Mergers by Cash ($\phi_1 = 0.08267357$) and the Number of Mergers by Stock ($\phi_2 = 0.08267357$) respectively, the negative impact of the standard deviation of ATV by Stock on their joint distribution of the number of Mergers by Cash and Stock becomes statistically significant. Here we have found that, the standard deviation of ATV by Stock influence the joint distribution of the number of Mergers by Cash and Stock in the presence of overdispersion. The research can be further extended in particular at how mixed offers of the payment types is determined by considering investment characteristics such as the target growth rate, relative size of the company, debt ratio, investment portfolio, tax effects and budget constraints etc.

2 Introduction

The terminology ‘Mergers and Acquisitions ($M\&A$)’ is used jointly to indicate a change in company ownership without alluding to the conditions of the transaction, that is it could be either a merger or an acquisition that represents a common way of restructuring assets. Between, 1965 and 1989, there were more than 75000 mergers in the United States of America alone. Jim Rogers, Duke Energy CEO, in a press release explained the goal of the announced merger between Duke Energy Corporation and Progress Energy Inc. in 2011 as: “By merging our companies, we can do that more economically for our customers, improve shareholder value and continue to grow.” Progress Energy CEO, Bill Johnson, also said: “It makes clear, strategic sense and creates exceptional value for our shareholders” [39]. Firms use mergers as a strategy to increase market share, improve shareholder value, exploit efficiencies, diversify product portfolios, obtaining market power, entering into new markets, and spreading risk. During the last decade, ($M\&A$) were recognised as the preferred vehicle for expansion into the global construction market. Major European and international construction organisations use mergers or acquisitions to increase their geographical coverage and business portfolio. There is a long tradition of academic research on ($M\&A$) within finance, business and economics. Throughout the recent wave of mergers, there has been no shortage of explanations for the increase in activity in the market for corporate control. Some explanations emphasize the positive role that mergers and takeovers play in the allocation of resources in society. For example, corporate acquisitions may lead to the replacement of a poor management team; they may facilitate the contraction of an industry in which no firm would voluntarily adopt a reduction in size; they may generate synergies through the combination of complementary resources. The theories of mergers can be classified into two groups. The first group claims that mergers create values. According to the group, the value of the combined firms exceeds the sum of the values of the firms operating separately when mergers are completed. Values are created through increased market power or synergies resulting from mergers. The second group, however, views mergers as non-value-creating transactions. One popular theory is Managerial theory, Mueller (1969). It contends that managers’ own personal interests in increasing their own wealth or prestige by managing a larger post-merger entity motivate mergers. Other is the economic disturbance theory, Gort (1969). It suggests that mergers may be caused by valuation differentials among market participants triggered by economic

shocks. The modern finance theory (Manne, 1965), states that shareholder wealth maximization can be reached by firms that invest when the sum of the present values of the present cash flows exceeds the initial investment. Translating this to (*M&A*), a firm has to undertake a merger if there is added value from the bidders perspective. In other words, the added value exceeds the transaction costs and the acquisition premium. The added value consists of operating synergies for both the bidder and the target. He also suggested that there are several different ways that companies may reduce taxes through a merger or acquisition, and tax benefits can accrue at both the corporate and the shareholder levels. However, in some cases the tax benefits from a corporate combination are also available by other means, and such benefits should not be attributed to the merger process alone. Managerial motives can be important motives to engage in MA activity, as they try to maximize their own utility and perhaps practice ‘empire building’ in some cases instead of maximizing shareholder wealth. Interestingly, merger activity seems to increase in years in which the stock market does well, which is counter to what we would expect if the primary motive for acquisitions were undervaluation. There also seems to be a tendency for mergers to be concentrated in a few sectors; in the early 1980s, many of the mergers involved oil companies, whereas the focus shifted to food and tobacco companies in the latter half of the decade and shifted again to media and financial service firms in the early 1990s. When analyzing investment decisions of firms by looking at the different forms taken by mergers and acquisitions, we tried to model the existing relationship between the number of Mergers and ATV of the firms in our present research.

The study of mergers and acquisitions represents a broad interdisciplinary field of research. Mergers and acquisitions are ever present in the corporate world, and they have become an increasingly important part of corporate strategies. The data lend themselves to the execution of the four pronged inquiries of the present study:

- To explore the basic features of our response and predictor variables.
- What is the effect of the mean, median and standard deviation of total announced value on the total number of mergers?
- Do the the effect of the mean, median and standard deviation of total announced value on the total number of mergers varies with the types of mergers?

- Is it possible to fit a bivariate regression model considering the number of mergers by cash and the number of mergers by Stock as response variables while taking the mean, median and standard deviation of total announced value separately for each of these two responses?

3 Literature Review

Mergers Acquisitions is a topic that attracts lots of attention, in both economic research and in the public news. A wide range of additional factors have been investigated and found to be relevant for the payment form in mergers and acquisitions, however, refute the importance of the number of mergers and acquisitions of the companies involved in a given year. Golbe and White(1988) attempted to discuss the available time-series data on mergers and acquisitions and their suitability and limitations for time-series analysis; also they presented a historical perspective on the current merger wave.

The Impact of Taxation on Mergers and Acquisitions was first addressed by Auerbach A.J. and David Reishus, D.(1988). To evaluate the magnitude of the tax benefits from mergers and acquisitions, they compiled a sample of large mergers and acquisitions that occurred between 1968 and 1983. Their results suggest that, for mergers and acquisitions in the 1970s and early 1980s between large public corporations in the United States, the potential transfer of unused tax credits and tax losses was the most significant potential tax-related factor. This was particularly so in cases where the benefits were used by an acquiring company to shelter the income of acquired taxable companies. The benefits from stepping up asset bases are less discernible. Likewise, purported gains from increasing leverage appear to be refuted by the stability of debt-equity ratios measured before and after mergers. Even where potential tax benefits have been identified, they have failed to find any evidence that they have played an important role in the structure and frequency of mergers and acquisitions.

Davutyan N. (1989) has modeled the number of bank failures per year as a Poisson process. The author proposed that in explaining bank failures the Poisson method provides the correct approach because it takes due account of their discreet and non-negative nature. A number of improvements are possible such as, one would be to try to devise a variable to measure the mismatch between loans and obligations.

Jaggia S. and Thosar S.(1993) employed count data methodology in which the dependent

variable represents the number of bids after the initial bid (count) received by the target firm, which was relatively a new methodology in the area of corporate finance. For parametric estimation, they assumed that the dependent variable follows a Poisson distribution.

Cameron and Trivedi(1996) discussed models for count data, such as Poisson and negative binomial are presented, with emphasis placed on the underlying count process and links to dual data on durations. They have also offered a self-contained discussion of regression techniques for the standard models is given, in the context of financial applications.

Buehler S. et. al.(2005)examines the determinants of mergers and bankruptcies, using firm level data from the Swiss Business Census and the Dun Bradstreet exit database for Switzerland (1995-2000). They found considerable differences in the determinants of mergers and bankruptcies, in particular with respect to firm size, location and the impact of macroeconomic conditions. Their results also supports the notion that mergers are often undertaken to seize growth opportunities.

Focarelli D. et al. (2008) studied how MAs in the financial industry affect systematic volatility by analyzing more than 1,400 large transactions in over 75 countries from 1988 to 2007. They found that a company's systematic risk, and therefore its cost of capital, increases on average after a merger or an acquisition.

Ismail A. and Krause A.(2010) empirically investigated the determinants of the payment form in mergers and acquisitions and introduced new variables on the target and acquirer investment characteristics to evaluate whether the concerns of target and acquirer shareholders are taken into account. The authors have established the relevance of a previously unreported variable for the determination of the payment form, the correlation of returns between target and acquirer, besides the more established determinants hostile takeovers, and defence mechanisms; weak evidence was found for the significance of budget constraints and no evidence for asymmetric information or tax considerations being a relevant factor.

McCarthy(2011)discussed the effect of firm size on mergers and acquisitions (*M&A*) in European context. In this paper they focused to include an important but long ignored sector of the economy, by explicitly considering the activity of the small and medium sized enterprises (SME) and their mergers and acquisitions. They have showed direct and indirect evidence which suggests not only that the behaviour and financial success of mergers by SMEs may significantly differ from larger public firms, but also that the underlying merger theories which motivate these ventures might need to be revisited to account for this discrepancy.

Erdogan (2012) examines the financial variables that predict the merger and acquisition targets in Turkey. The author used Cox regression with segmented time-dependent covariates used to determine the factors that predict target companies for mergers and acquisitions. The author has found that a lower pretax profit margin is associated with an increased chance of being a merger or an acquisition target. In addition, the lower the debt ratio, the more likely the firm will be a target for a merger or an acquisition.

Vyas V. et.al. (2012) attempted to find out the determinants of MA in Indian pharmaceutical industry with the help of cross-tabulations and Logit analysis. The results of the Logit analysis suggests that large and multinational affiliated firms are investing more in MA activities. Similarly, firms reporting excess capacity and high RD investments are relying heavily on MA to restructure and consolidate their position in the industry.

Mergers and acquisitions are driven by the same motive: synergy creation and realization of larger value by combining companies. However, numerous merger and acquisition efforts show a lack of critical success elements: attention directed towards integration of employees and work processes. Srbinoska(2016) focus on the analysis of the meaning of post-transactional integration of mergers and acquisitions as a determinant to the business deal success, with particular emphasis on the Republic of Macedonia. The success of the merger i.e. acquisition act depends on how the deal is conducted, i.e. on the success of the integration process, which the author has demonstrated through a field research across several Macedonian enterprises.

Bhagwat V. et. al.(2016) proposed a new link between market conditions and merger activity. Specifically, they predicted that higher uncertainty will decrease deal activity. There are many reasons why uncertainty might affect merger activity, but they focused on its deal-specific effects during the delay between deal announcement and completion. Firm- and industry-level uncertainty measures reveal similar findings, ruling out an unobserved macro variable. They concluded that interim uncertainty contributes to understanding the timing and intensity of public firms' merger activity.

While most of the determinants of the payment types considered thus far are based on misvaluations, moral hazard, asymmetric information, tax effects and budget constraints, we have consider the mean, median and standard deviation of the total announced value by the types of mergers as explanatory variables into our model.

4 Methodology

In some financial studies the dependent variable is a count, taking non negative integer values. Examples include the number of takeover bids received by a target firm, the number of unpaid credit installments (useful in credit scoring), the number of accidents or accident claims (useful in determining insurance premium) and the number of mortgage loans prepaid (useful in pricing mortgage-backed securities). When investigating the payment types of mergers and acquisitions, we have considered the number mergers taken place in a given year as the response variable. The nature of our response variables suggest the use of Poisson regression model.

4.1 Univariate Poisson Model

Let Y be the number of Total Mergers in a given time interval which has a Poisson distribution with parameter, μ , the probability distribution function is written as

$$g(y) = \frac{e^{-\mu} \mu^y}{y!}, \quad y \geq 0 \quad (1)$$

for some $\mu \geq 0$. Suppose that we have a sample of n observations $y_1, y_2, y_3, \dots, y_n$ which can be treated as realizations of independent Poisson random variables, with $Y_i \sim P(\mu_i)$ with variance function $Var(Y_i) = \mu_i$. Let us consider that the mean μ_i depend on a vector of explanatory variables, X_i . Then the canonical link function can be expressed as (McCullagh and Nelder, 1989) below:

$$g(\mu_i) = \ln(\mu_i) = \eta_i = X' \beta \quad (2)$$

Then the univariate Poisson regression model can be expressed as:

$$\mu = \exp(X' \beta) \quad (3)$$

where, $X' = (1, X_1, X_2, X_3, \dots, X_p)$ and $\beta' = (1, \beta_0, \beta_1, \beta_2, \dots, \beta_p)$ The log-likelihood function for the model can be written as

$$\ln L(\beta) = \sum_{i=1}^n [-\exp(\beta' X_i) + y_i(\beta' X_i) - \ln(y_i!)] \quad (4)$$

The Estimating equations are

$$\frac{\partial \ln L}{\partial \beta_j} = \sum_{i=1}^n X_i (y_i - e^{X_i' \beta}) = 0, j = 0, 1, 2, \dots, p \quad (5)$$

The Goodness of fit for the Poisson model is measured by the Deviance(D) statistic, which is given by

$$D = 2 \sum_{i=1}^n [y_i \ln\left(\frac{y_i}{\hat{\mu}_i}\right) - (y_i - \hat{\mu}_i)] \quad (6)$$

An alternative measure of Goodness of fit named Pesrson's chi-squared statistic is also used, which is defined as

$$\chi_p^2 = \sum_{i=1}^n \frac{(y_i - \hat{\mu}_i)^2}{\hat{\mu}_i} \quad (7)$$

4.2 Univariate Negative Binomial Model

In the Poisson regression model we assume that the conditional mean function equals the condition variance function. But count data very often diverts from this assumption and this problem is commonly known as overdispersion or underdispersion. The Poisson model does not allow for overdispersion or underdispersion. A richer model is obtained by using the negative binomial distribution instead of the Poisson distribution (Hilbe, 2011). Negative binomial regression is a type of generalized linear model in which the dependent variable Y is a count of the number of times an event occurs. A convenient parametrization of the negative binomial distribution is expressed as

$$P(Y = y) = \frac{\Gamma(y + \frac{1}{\alpha})}{\Gamma(y + 1) \Gamma(\frac{1}{\alpha})} \left[\frac{1}{1 + \alpha \mu} \right]^{\frac{1}{\alpha}} \left[\frac{\alpha \mu}{1 + \alpha \mu} \right]^y \quad (8)$$

where $\mu > 0$ is the mean of Y and $\alpha > 0$ is the heterogeneity parameter and $Var(Y) = \mu(1 + \alpha\mu)$. The traditional negative binomial regression model is written as

$$\ln(\mu) = X' \beta \quad (9)$$

where, $X' = (1, X_1, X_2, X_3, \dots, X_p)$ and $\beta' = (1, \beta_0, \beta_1, \beta_2, \dots, \beta_p)$

The log-likelihood function for the model can be written as

$$\ln L(\alpha, \beta) = \sum_{i=1}^n [y_i \ln \alpha + y_i (X_i \beta) - [y_i + \frac{1}{\alpha}] \ln(1 + \alpha e^{X_i \beta}) + \ln \Gamma(y_i + \frac{1}{\alpha}) - \ln \Gamma(y_i + 1) - \ln \Gamma(\frac{1}{\alpha})] \quad (10)$$

The Estimating equations are

$$\frac{\partial \ln L(\beta, \alpha)}{\partial \beta_j} = \sum_{i=1}^n \frac{(y_i - \mu_i) X_{ij}}{(1 + \alpha \mu_i)} = 0, j = 0, 1, 2, \dots, p \quad (11)$$

$$\frac{\partial \ln L(\beta, \alpha)}{\partial \alpha} = \sum_{i=1}^n \left[\sum_{r=1}^{y_i-1} \left(\frac{r}{1 + \alpha r} \right) \right] + \alpha^{-2} \ln(1 + \alpha \mu_i) - \frac{(y_i + \alpha^{-1}) \mu_i}{(1 + \alpha \mu_i)} = 0, j = 0, 1, 2, \dots, p \quad (12)$$

The Goodness of fit for the Poisson model is measured by the Deviance(D) statistic, which is given by

$$D = 2 \sum_{i=1}^n \left(\ln L(y_i^*, y_i) - \ln L(\mu_i^*, y_i) \right) \quad (13)$$

where $\ln L(\mu_i^*, y_i)$ is our log-likelihood function (10), and $\ln L(y_i^*, y_i)$ is the log-likelihood function with y_i replacing μ_i . An alternative measure of Goodness of fit named Pearson's chi-squared statistic is also used, which is defined as

$$\chi_p^2 = \sum_{i=1}^n \frac{(y_i - \hat{\mu}_i)^2}{\hat{\mu}_i + \hat{\alpha} \hat{\mu}_i^2} \quad (14)$$

4.3 Bi-variate Poisson Regression Model

Y_1 be the Number of mergers by Cash in a given time interval which has a Poisson distribution with parameter, λ_1

$$g_1(y_1) = \frac{e^{-\lambda_1} \lambda_1^{y_1}}{y_1!}, \quad y_1 \geq 0 \quad (15)$$

for some $\lambda_1 > 0$

Let Y_2 be the Number of mergers by Stock in a given time interval which has a Poisson distribution with parameter, λ_2

$$g_2(y_2) = \frac{e^{-\lambda_2} \lambda_2^{y_2}}{y_2!}, \quad y_2 \geq 0 \quad (16)$$

for some $\lambda_2 > 0$

The conditional distribution of Y_2 given Y_1 has a Poisson distribution with parameter $\lambda_2 y_1$

$$g_3(y_2|y_1) = \frac{e^{-\lambda_2 y_1} (\lambda_2 y_1)^{y_2}}{y_2!} \quad y_2 \geq 0 \quad (17)$$

Then the joint distribution of Cash-mergers and Stock-mergers can be written as

$$g_4(y_1, y_2) = \frac{e^{-\lambda_1} \lambda_1^{y_1} e^{-\lambda_2 y_1} (\lambda_2 y_1)^{y_2}}{y_1! y_2!} \quad y_1 \geq 0 \quad y_2 \geq 0 \quad (18)$$

Univariate Generalized Linear Model(GLM) for Poisson

The exponential form of univariate Poisson for Y_1 can be written as

$$g_1(y_1) = e^{-\lambda_1 + y_1 \ln \lambda_1 - \ln y_1!} \quad (19)$$

with the link function

$$\ln \lambda_1 = X' \beta_1 \quad (20)$$

where, $X' = (1, X_1, X_2, X_3, \dots, X_p)$ and $\beta'_1 = (1, \beta_{10}, \beta_{11}, \beta_{12}, \dots, \beta_{1p})$ Similarly, the exponential form for the marginal distribution of y_2 can be written as

$$g_2(y_2) = e^{-\lambda_2 + y_2 \ln \lambda_2 - \ln y_2!} \quad (21)$$

with the link function

$$\ln \lambda_2 = X' \beta_2 \quad (22)$$

where, $X' = (1, X_1, X_2, X_3, \dots, X_p)$ and $\beta'_2 = (1, \beta_{20}, \beta_{21}, \beta_{22}, \dots, \beta_{2p})$

From the conditional model the exponential form can be written as

$$g_3(y_2|y_1) = e^{-\lambda_2 y_1 + y_2 \ln(\lambda_2 y_1) - \ln y_2!} \quad (23)$$

By denoting

$$\lambda_2 y_1 = \lambda_3 \quad (24)$$

the link function can be expressed as

$$\ln \lambda_3 = X' \beta_3 \quad (25)$$

where, $X' = (1, X_1, X_2, X_3, \dots, X_p)$ and $\beta'_3 = (1, \beta_{30}, \beta_{31}, \beta_{32}, \dots, \beta_{3p})$

Bivariate Generalized Linear Model for Poisson-Poisson

The joint distribution of Y_1 and Y_2 can be written as

$$g_4(y_1, y_2) = e^{y_1 \ln \lambda_1 + y_2 \ln \lambda_2 - \lambda_1 - \lambda_2 y_1 + y_2 \ln y_1 - \ln y_1! - \ln y_2!} \quad (26)$$

The link functions are

$$\ln \lambda_1 = X' \beta_1 \quad (27)$$

where, $X' = (1, X_1, X_2, X_3, \dots, X_p)$ and $\beta'_1 = (1, \beta_{10}, \beta_{11}, \beta_{12}, \dots, \beta_{1p})$

$$\ln \lambda_1 = X' \beta_2 \quad (28)$$

where, $X' = (1, X_1, X_2, X_3, \dots, X_p)$ and $\beta'_2 = (1, \beta_{20}, \beta_{21}, \beta_{22}, \dots, \beta_{2p})$

The mean of Y_1 and Y_2 can be written as

$$E(Y_1) = \lambda_1 = e^{X' \beta_1} \quad (29)$$

$$E(Y_2) = \lambda_2 = e^{X' \beta_2} \quad (30)$$

The log likelihood function for the bivariate distribution is

$$L = \prod_{i=1}^n g_4(y_{1i}, y_{2i}) = \prod_{i=1}^n e^{y_{1i} \ln \lambda_1 + y_{2i} \ln \lambda_2 - \lambda_1 - \lambda_2 y_{1i} + y_{2i} \ln y_{1i} - \ln y_{1i}! - \ln y_{2i}!} \quad (31)$$

The log likelihood function is

$$\ln L = \sum_{i=1}^n (y_{1i}(X'_i \beta_1) + y_{2i}(X'_i \beta_2) - e^{X'_i \beta_1} - (e^{X'_i \beta_2}) y_{1i} + y_{2i} \ln y_{1i} - \ln y_{1i}! - \ln y_{2i}!) \quad (32)$$

The Estimating equations are

$$\frac{\partial \ln L}{\partial \beta_{1j}} = \sum_{i=1}^n x_{ij}(y_{1i} - e^{X_i' \beta_1}) = 0, j = 0, 1, 2, \dots, p \quad (33)$$

$$\frac{\partial \ln L}{\partial \beta_{2j}} = \sum_{i=1}^n x_{ij}(y_{2i} - y_{1i} e^{X_i' \beta_1}) = 0, j = 0, 1, 2, \dots, p \quad (34)$$

The second derivatives are

$$\frac{\partial^2 \ln L}{\partial \beta_{1j} \partial \beta_{1j'}} = - \sum_{i=1}^n x_{ij} x_{ij'} e^{X_i' \beta_1} = 0; j, j' = 0, 1, 2, \dots, p \quad (35)$$

$$\frac{\partial^2 \ln L}{\partial \beta_{2j} \partial \beta_{2j'}} = - \sum_{i=1}^n y_{1i} x_{ij} x_{ij'} e^{X_i' \beta_1} = 0; j, j' = 0, 1, 2, \dots, p \quad (36)$$

4.4 Overdispersion in Bi-variate Poisson Regression Model

The overdispersion in the data set can be measured by

$$\hat{\phi}_k = \frac{1}{n-p} \sum_{i=1}^n \frac{(y_i - \hat{\mu}_{ki})^2}{V(\hat{\mu}_{ki})} = \frac{\chi_{k,p}^2}{(n-p)}, k = 1, 2; \quad (37)$$

where, $V(\hat{\mu}_{ki}) = \hat{\mu}_{ki}$

$$V \hat{\beta}_k = \hat{\phi}_k (X' W X)^{-1} \quad (38)$$

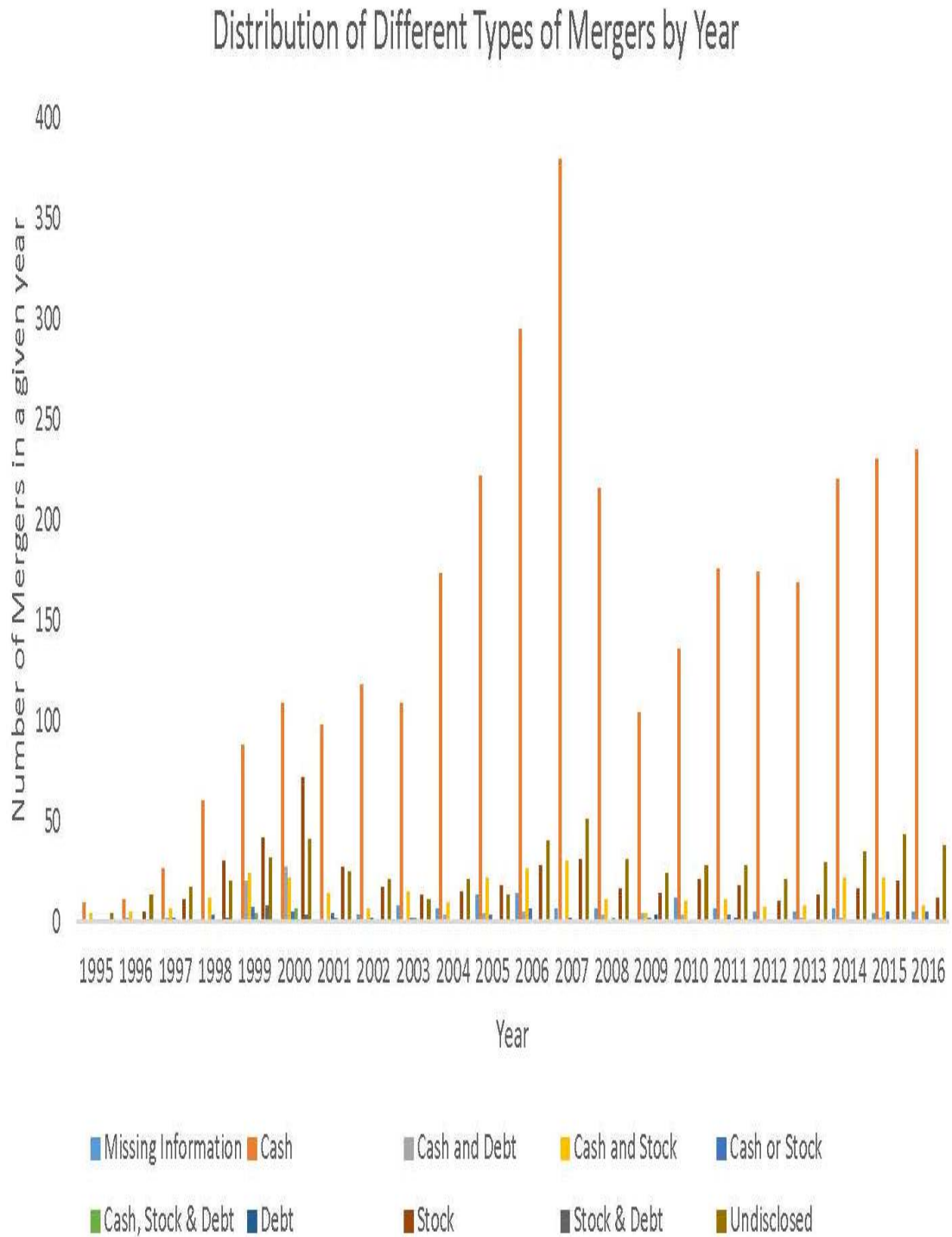
where, $W_k = \text{diag}(\mu_{k1}, \mu_{k2}, \mu_{k3}, \mu_{k4}, \dots, \mu_{kn})$; $k = 1, 2$

5 Results

5.1 Exploratory Data Analysis

In our present research we have used Mergers & Acquisition Data from 1995 to 2016, Bloomberg [40]. Our data set has 4987 Merger and Acquisitions (*M&A*), that took place from the year 1995 to 2016 by different types of payment, such as; Not reported (Missing Types of Payment), Cash, Cash and Debt, Cash & Stock, Cash or Stock, Cash & Stock & Debt, Debt, Stock, Stock & Debt, and Undisclosed (types of payment was not disclosed).

Figure 1: Distribution of Different Types of Mergers by Year



We considered Total number of mergers, Number of number of mergers by Cash and number of mergers by Stock individually as our response variables and the mean, median and the standard deviation of the total Announced value as our explanatory variables for univariate analysis. For bivariate analysis, the number of mergers by Cash and number of mergers by Stock were considered as responses and the mean, median and the standard deviation of the Announced total value in a given year were calculated separately for each of these two responses.

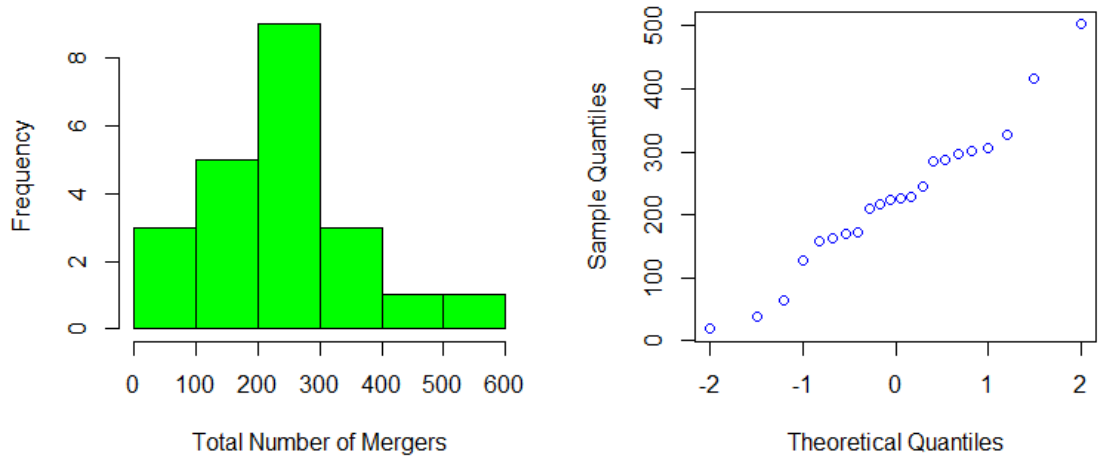
From figure-1, we can see the maximum number of mergers were completed in the year 2007. Among all the different types of mergers, the highest number of mergers were completed by the payment type, Cash(3358) and the lowest number of mergers were completed by the payment type, Debt(9) in our study period. Undisclosed (586) was the second and Stock(450) was the third respectively according to the number of mergers. Among all the types of payment, we have focused on Cash and Stock as the payment type to take place merger and acquisitions. In next few graphs, we have explored some features about the Total number of mergers, the number of mergers by Cash, and the number of mergers by Stock in a given year.

Figure-2 portrays the distributional properties of the total number of mergers in a given year. The Histogram and Normal Q-Q plot shows that the total number of mergers in a given year has a skew distribution and it does not satisfy the normality assumption. Time plot shows a trend over the study period and the values are dependent to previous lagged values. The cumulative distribution of total number of mergers in a given year is in steady state position when the number of mergers are just above 350. The Box-whisker plot show a moderate level of dispersion of the total number of mergers.

From the table-1 and figure-3, we can see that the total numbers of mergers in a given year has positive relationship with the mean and Standard Deviation of announced total value in a given year, whereas negative relationship with the median of Announced Total Value in a given year. A Poisson regression model for the total mergers on the mean, standard deviation, and median of the announced total value.

Figure 2: Distribution of Total Number of Mergers

Histogram of Total Number of Mergers Normal Q-Q plot of Total number of Mergers



Time plot of Total number of Mergers Time plot of Total number of Mergers at lag=1



Empirical Cumulative Distribution of Total Number Box-Whisker plot of Total Number of Mergers

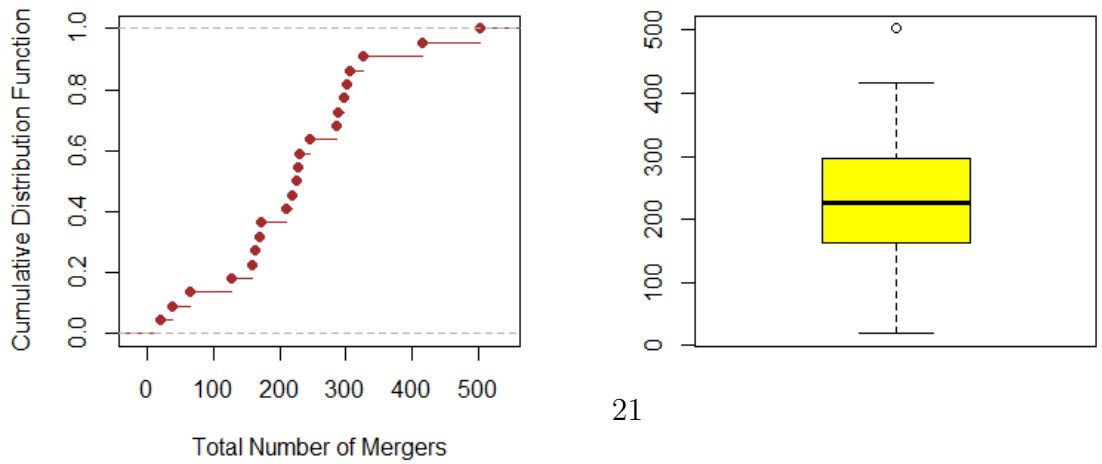


Table 1: Correlation of the Total Number of Mergers with the Mean ,Median and Standard Deviation of the Announced Total Value given in a Year

	Mean	Meadin	Standard Deviation
Number of Total Mergers	0.136524	-0.40043	0.3635178

Figure 3: Simple Scatter plot Matrix of Total Number of Mergers with the Mean, Median and Standard Deviation of the Announced Total Value

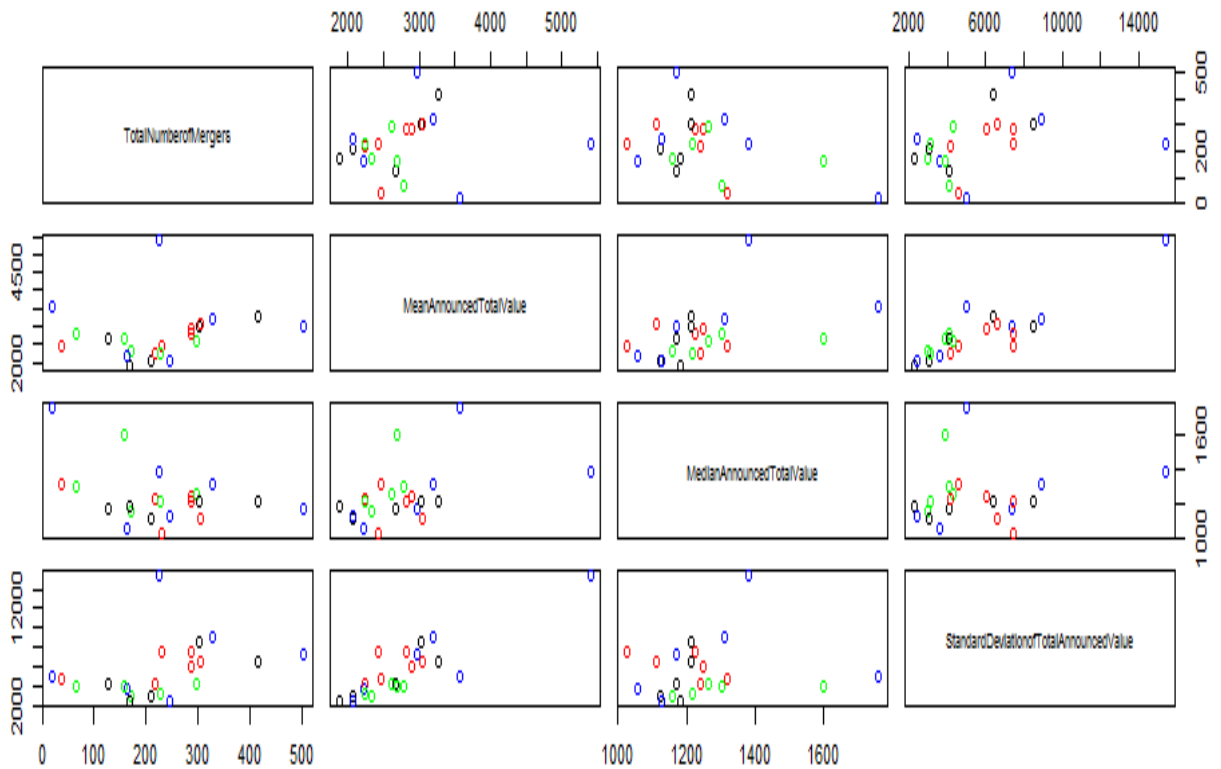
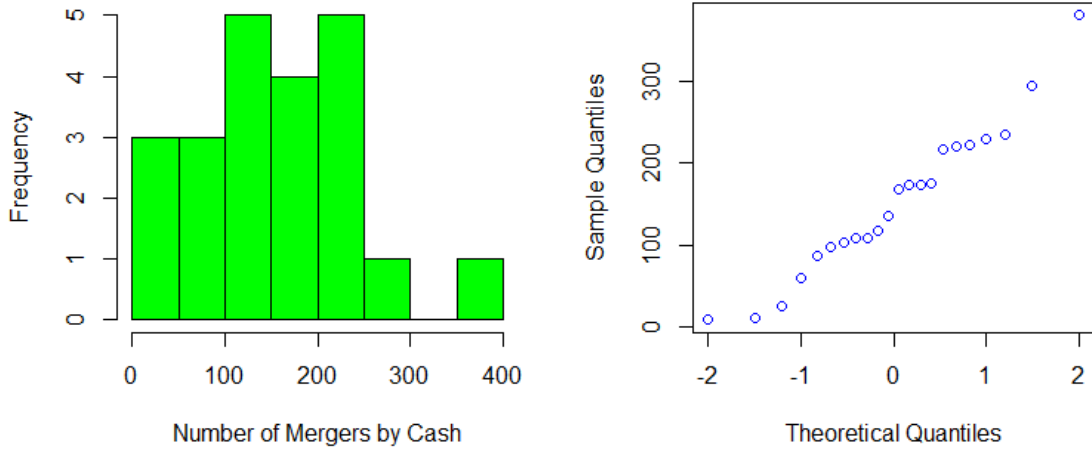


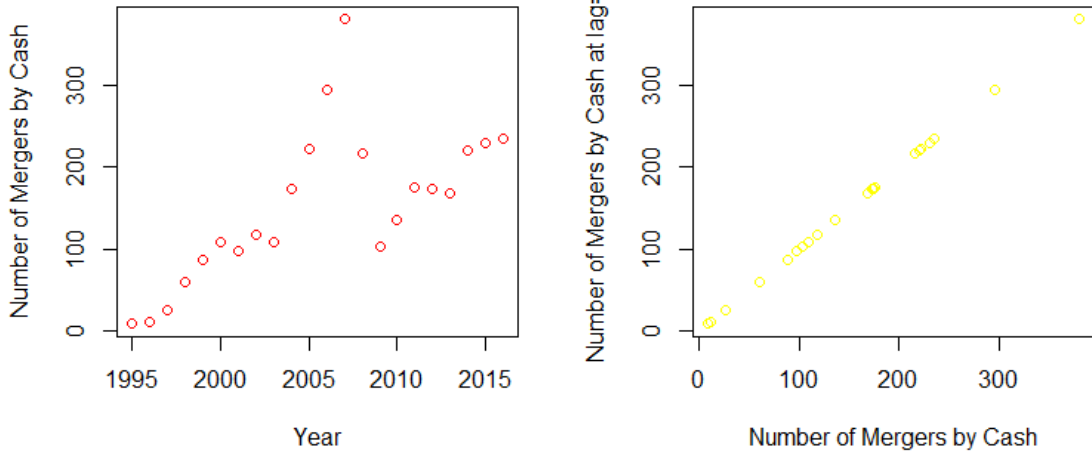
Figure-4 provides some basic characteristics of the number of mergers by Cash. It has a left-skewed distribution as such does not satisfy the normality assumptions. It has a trend over time. Though the number of mergers by Cash has moderate dispersion its distribution its steady around 250.

Figure 4: Distribution of Number of Mergers by Cash

Histogram of Number of Mergers by Cash | Normal Q-Q plot of Number of Mergers by Cash



Time plot of Number of Mergers by Cash | Time plot of Number of Mergers by Cash at lag=1



Cumulative Distribution of Number of Mergers by Cash | Box-Whisker plot of Number of Mergers by Cash

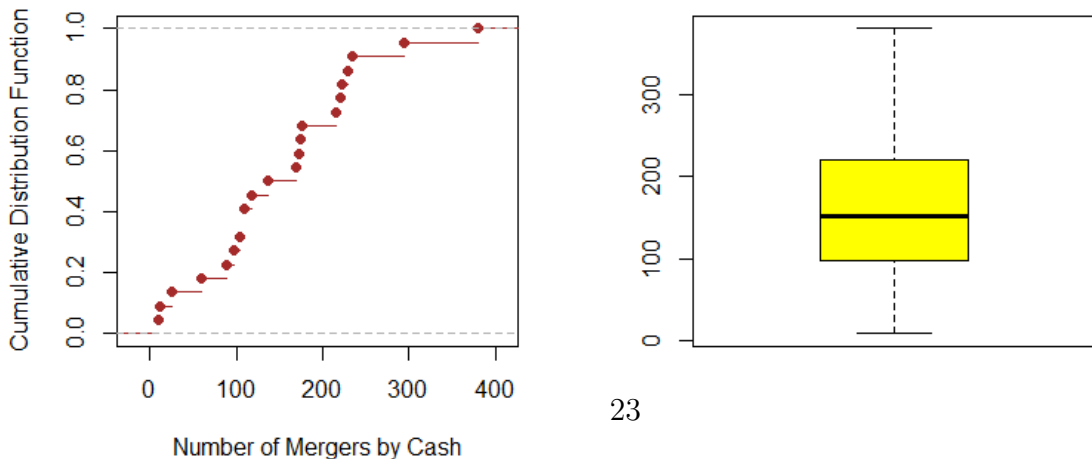


Table 2: Correlation of the Number of Mergers by Cash with the Mean ,Median and Standard Deviation of the Announced Total Value in a given Year

	Mean	Median	Standard Deviation
Number of Mergers by Cash	0.182994	-0.3511	0.5004258

Figure 5: Simple Scatter plot Matrix of Number of Mergers by Cash with the Mean, Median and Standard Deviation of the Announced Total Value in a given year

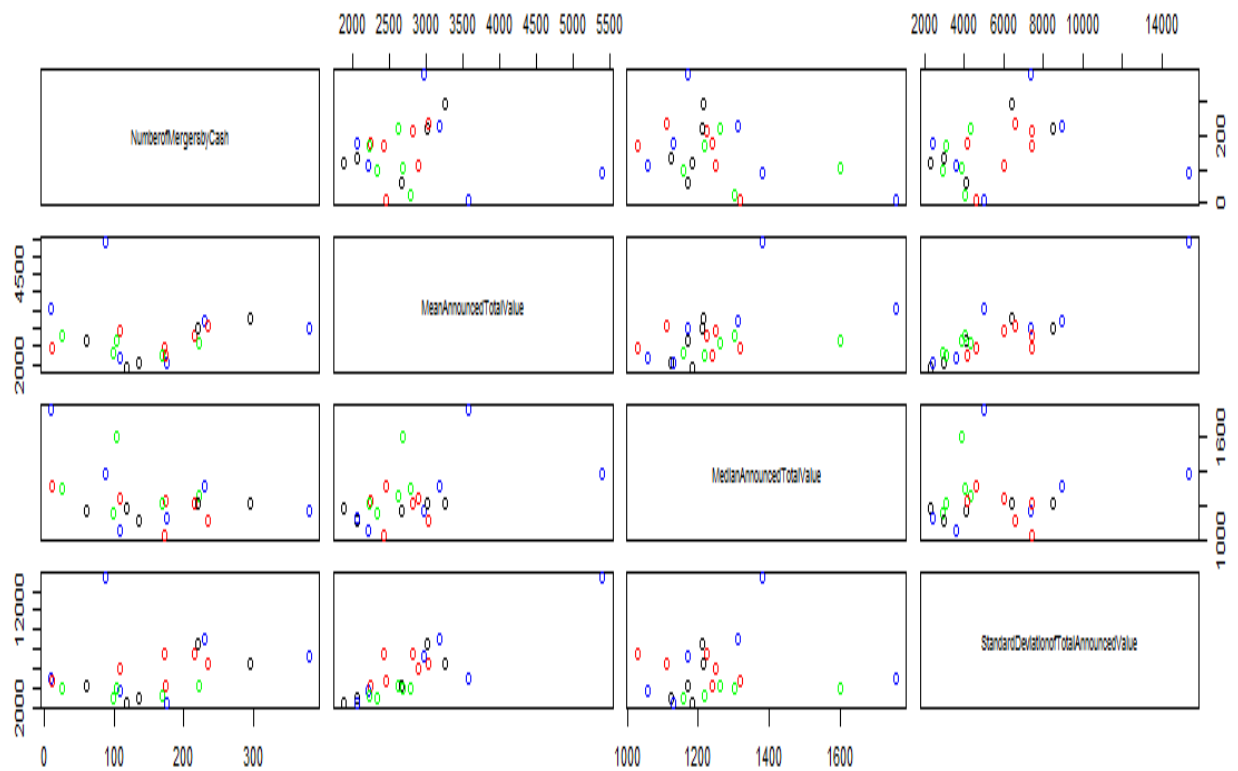


Table-2 and Figure-5 give the correlation of the number of mergers by Cash with the mean , median, and standard deviation of the announced total value in a given year. The number of mergers by Cash is positively related with the standard deviation of announced total value in a given year and the mean announced total value in a given year but negatively related with median announced total value in a given year.

Figure 6: Simple Scatter plot Matrix of Number of Mergers by Stock with the Mean, Median and Standard Deviation of the Announced Total Value in a given year

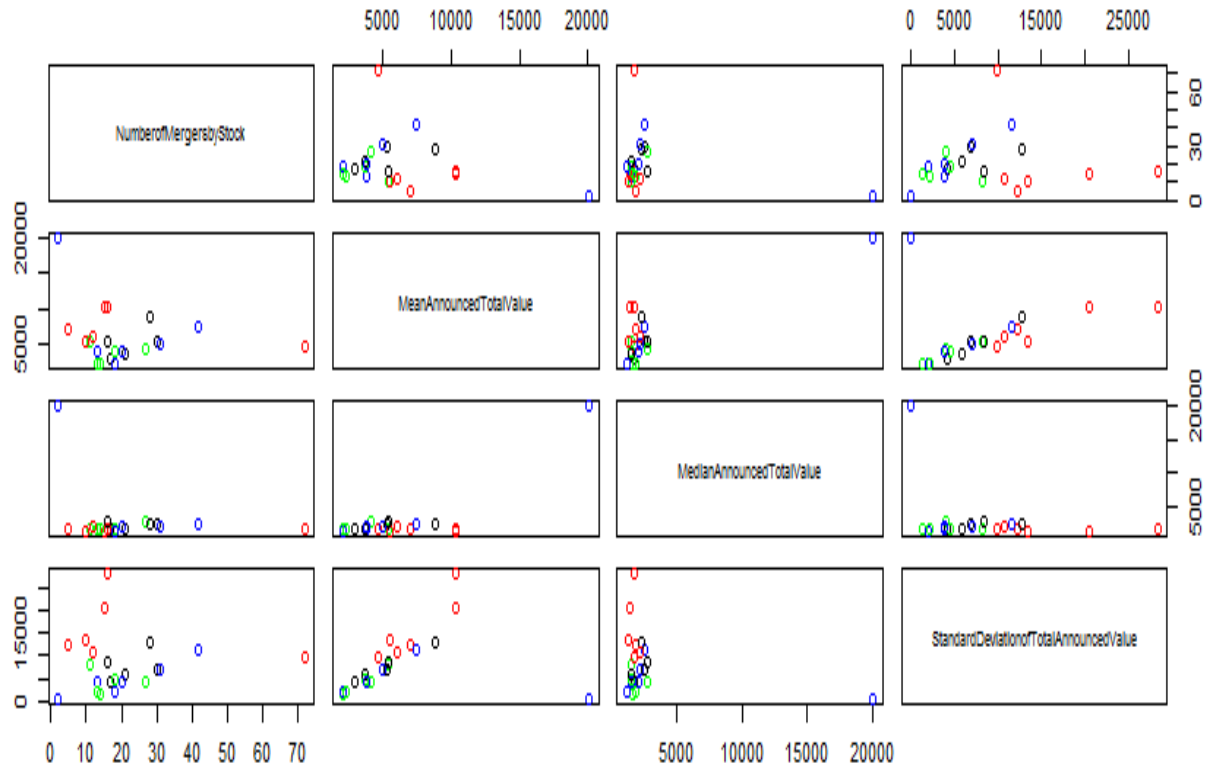


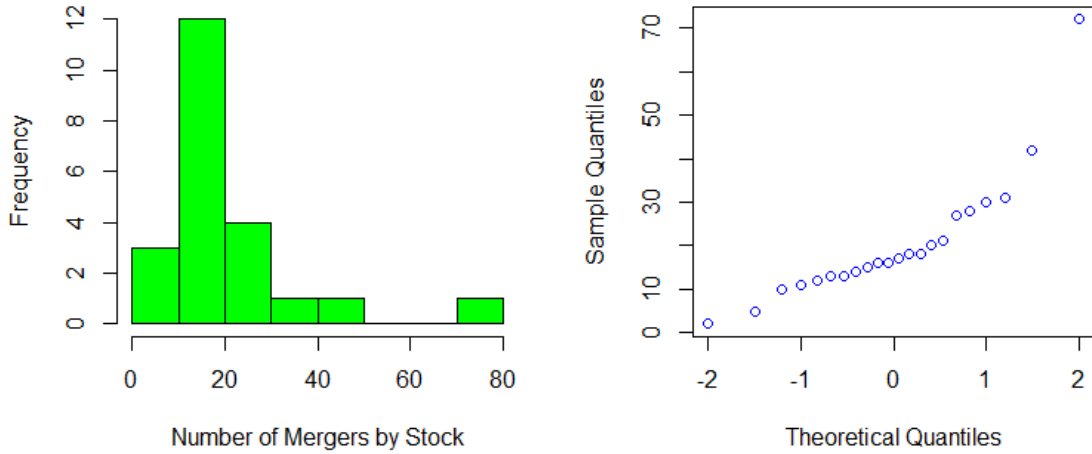
Table 3: Correlation of the Number of Mergers by Stock with the Mean ,Median and Standard Deviation of the Announced Total Value in a given Year

	Mean	Median	Standard Deviation
Number of Mergers by Stock	-0.20866	-0.2452	0.07790895

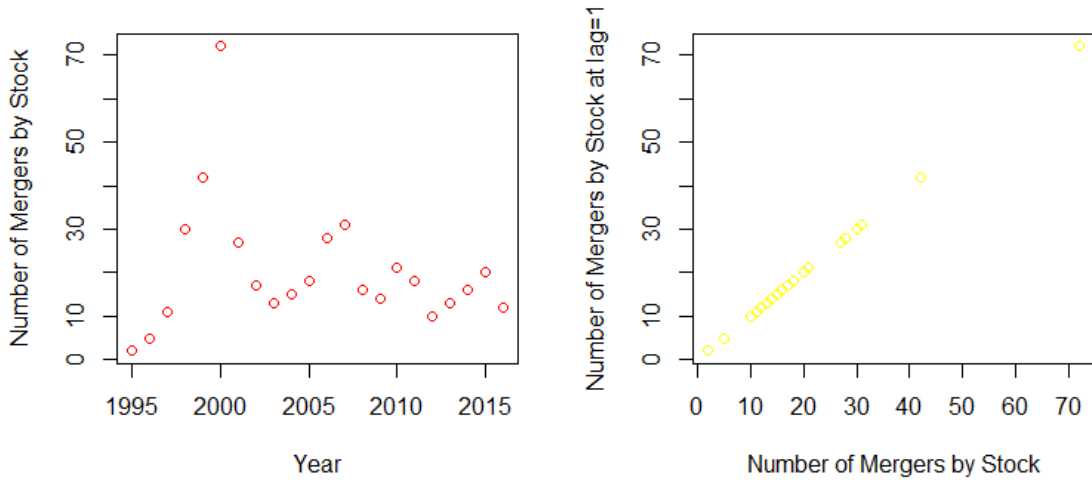
Figure-6 and Table-3 give the nature of association of the number of mergers by Stock with the mean, median and standard deviation of the announced total value in a given year. The number of mergers by Stock has a positive relation with the standard deviation of announced total value in a given year only whereas negative relationship with the mean and median of the announced total value in a given year.

Figure 7: Distribution of Number of Mergers by Stock

Histogram of Number of Mergers by Stock **Normal Q-Q plot of Number of Mergers by Stock**



Time plot of Number of Mergers by Stock **Time plot of Number of Mergers by Stock at lag=1**



Cumulative Distribution of Number of Mergers by Stock **Box-Whisker plot of Number of Mergers by Stock**

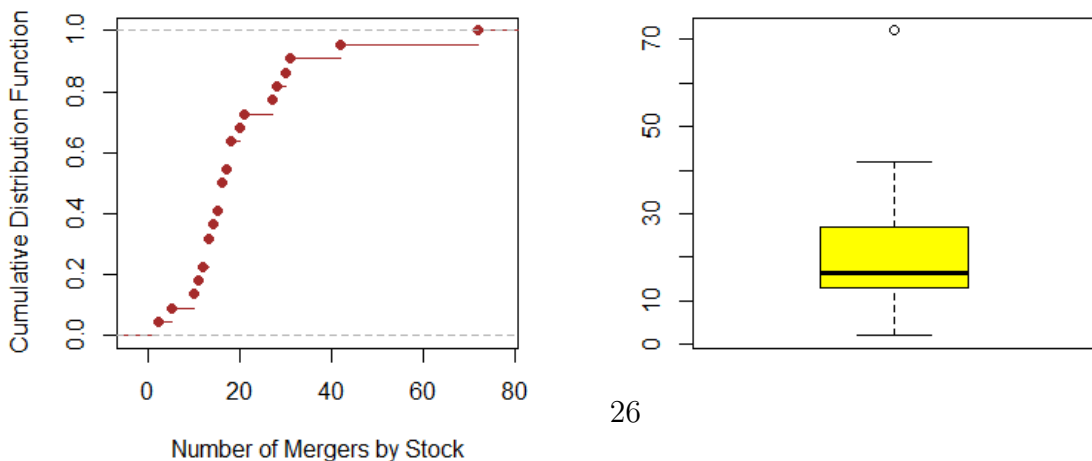


Figure-7 shows a skewed distribution for the number of mergers by Stock. The Normal Q-Q plot also confirms violation of Normality assumptions. The number of mergers by Stock has trend overtime and also lagged with its previous values. The distribution is steady state after 35 and Box-Whisker plot shows a moderate level of dispersion in the data set.

Figure 8: Distribution of the Number of Mergers by Cash, Number of Mergers by Stock, Number of Mergers by both Cash and Stock, and the Total Number of Mergers from 1995 to 2015

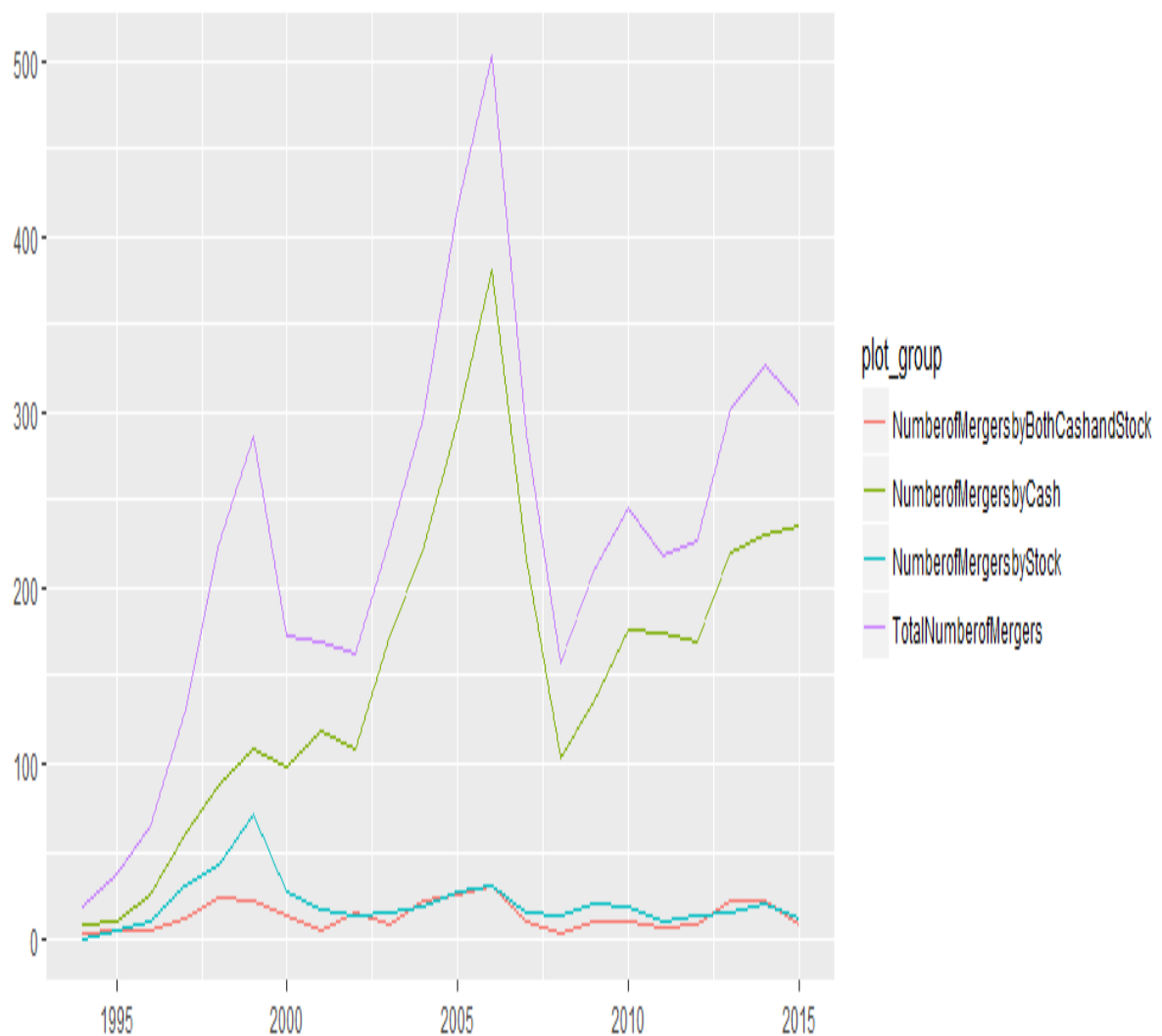


Figure-8 gives us the overall pattern of the total number of mergers, the number of mergers by Cash, the number of mergers by Stock, and the number of mergers by both Cash and

Stock in a given year simultaneously. The total number of mergers and the number of mergers by Cash has similar trend over Time. The number of mergers by Stock had got its maximum value(72) in year 2000. On the other hand, the total number of mergers and the number of mergers by Cash had maximum value of 502 and 380 respectively in the year 2007. Surprisingly, the number of mergers by Stock, the number of mergers by both Cash and Stock has very close pattern of movement over time.

Figure-9 and table-4 shows positive relationship between the total number of mergers, the number of mergers by Cash, and the number of mergers by Stock in a given year. The correlation between between the total number of mergers and the number of mergers by Cash is higher than the correlation between between the total number of mergers and the number of mergers by Stock whereas the correlation between the number of mergers by Cash and the number of mergers by Stock is the lowest among them.

Figure 9: Simple Scatter plot Matrix of Total Number of Mergers, with the Number of Mergers by Cash and the Number of Mergers by Stock

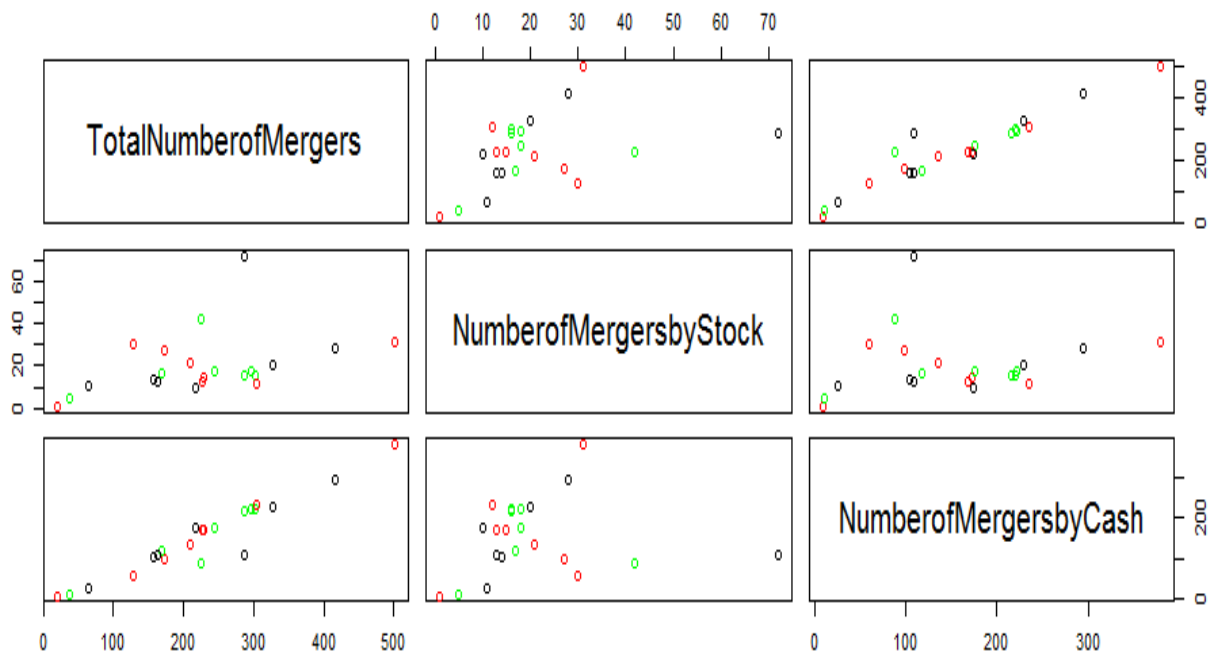


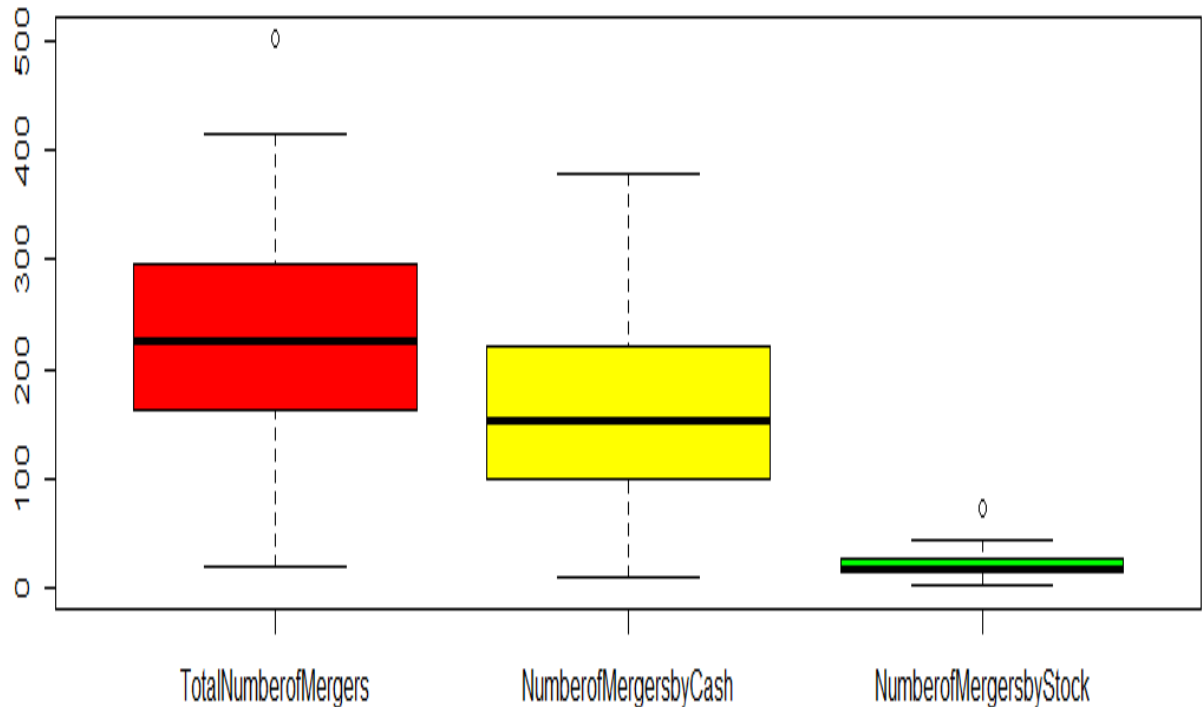
Table 4: Correlation between the Total Number of Mergers, Number of Mergers by Cash, and Number of Mergers by Stock

	Total Mergers	# Mergers by Cash	#Mergers by Stock
Total Mergers	1	0.955358	0.400337
# Mergers by Cash	0.955358	1	0.122953
# Mergers by Stock	0.400337	0.122953	1

Table 5: Mean and Variance of the Number of Total Mergers, Number of Mergers by Cash and Number of Mergers by Stock

Response Variables	Mean	Variance
Total number of mergers	226.6818	13136.8
Number of mergers by Cash	152.6364	8566.814
Number of mergers by Stock	20.45455	217.0216

Figure 10: Box-Whisker plot of the Number of Total Mergers, with the Number of Mergers by Cash and the Number of Mergers by Stock



From figure-10 and table-5, We can get an idea about the dispersion in the number of Total mergers, the number of mergers by Cash and the number of mergers by Stock in a given year. The amount of dispersion is the highest for the number of Total mergers and lowest for the number of mergers by Stock whereas the number of mergers by Cash has medium level of dispersion in comparison to total mergers and the number of mergers by Stock. Table-5 measure the mean and standard deviation individually for the number of Total mergers, the number of mergers by Cash and the number of mergers by Stock in a given year. Each of these response variables has lower mean than variance; that is, variance and mean are unequal to each other. Since all of these responses are count in nature, a Poisson regression model is a reasonable choice for these responses. Table-5 show the presence of overdispersion which calls for Negative Binomial Regression model.

5.2 Univariate Poisson and Binomial Regression Analysis for the Total Number of Mergers

Table-06 summarizes the output of Poisson regression considering the Total number of Mergers as response variable on the Mean, Median and Standard deviation of the Announced Total Value in a given year. All the regression coefficients are significant at 5 percent level of significance. The regression coefficient of Mean Announced Total Value in a given year is negative (-.000104). That is, The marginal effect of the mean Announced Total Value on the expected number of mergers can be interpreted as follows: For a 1-unit increase in mean Announced Total Value, the estimated total number of mergers decreases by a factor of $\exp(-0.0001408) = 0.9998$. Thus, the total number of mergers is decreases by 0.01 percent per year due to an increase in the mean Announced Total Value . Similarly, we have the marginal effect of its median Announced Total Value on the expected number of mergers, For a 1-unit increase in the median Announced Total Value, the estimated total number of mergers decreases by a factor of $\exp(-.0016) = 0.9983$. That is, the total number of mergers is decreases by 0.16 percent per year due to an increase in median Announced Total Value. On the other hand, The regression coefficient of the Standard deviation of Announced Total Value is positive (0.000101). For a 1-unit increase in the standard deviation of Announced Total Value the estimated number of mergers increases by a factor of $\exp(0.000101) = 1.0001$.

So, the number of total mergers is not changing with respect to the standard deviation of announced total value, that growing at the rate of 0% per year. However, these results may not be reliable as the large residual deviance indicates poor fitting of Poisson model to the data.

Table 6: Univariate Poisson Regression Model for the Total Number of Mergers

Coefficients	Estimate	Std. Error	z value	P-value
Intercept	7.21E+00	1.37E-01	52.494	$\leq 2e - 16$
Mean Total Announced Value	-1.41E-04	5.51E-05	-2.556	0.0106
Median Total Announced Value	-1.61E-03	1.48E-04	-10.931	$\leq 2e - 16$
Standard Deviation of Total Announced Value	1.01E-04	1.23E-05	8.251	$\leq 2e - 16$

Table-07 shows the summary results of the negative binomial regression of the total number of mergers on the mean, median and standard deviation of the Announced Total Value in a given year. The regression coefficient of the Mean Announced Total Value is negative(-0.000364) and statistically insignificant. This means that for each one-unit increase in the Mean of Announced Total Value, the difference in the logs of expected counts would be expected to decrease by 0.000364 units, while holding the other variables in the model constant. Similarly, the Median Announced Total Value has got a negative regression coefficient(-0.00186). That is, For each one-unit increase in the Median of Announced Total Value, the difference in the logs of expected counts would be expected to decrease by 0.00186 units, while holding the other variables in the model constant. The standard deviation of the Announced Total Value in a given year is positively associated with the total number of mergers. Here, for each one-unit increase in the Median of Announced Total Value, the difference in the logs of expected counts would be expected to increase by 0.000158 units, while holding the other variables in the model constant. The model's Null deviance value is 35 which has degrees of freedom 21 and the model's Residual deviance value is 23.069 which has degrees of freedom 18. The residual deviance has a p-value of 0.18. That is, The null hypothesis(our model is correctly specified) is significant and we have strong evidence to reject that hypothesis. So we have strong evidence that our model fits well for this data set. That is, Smaller residual deviance provides that Negative binomial model fits well the data set.

Both the regression coefficient of the Median Announced Total Value in significant and the regression coefficient for Standard deviation of Announced Total Value is insignificant at 5

Table 7: Univariate Negative Binomial Regression for the Number of Total Mergers

Coefficients	Estimate	Std. Error	z value	P-Value
Intercept	7.81E+00	8.35E-01	9.355	$\leq 2e - 16$
Mean Total Announced Value	-3.64E-04	4.30E-04	-0.848	0.3965
Median Total Announced Value	-1.86E-03	9.25E-04	-2.011	0.0444
Standard Deviation of Total Announced Value	1.58E-04	9.56E-05	1.653	0.0982

percent level of significance. If we compare between these two models, the negative binomial regression model best fits our data set as the responses present overdispersion problem (mean of total number of mergers is not equal to the variance of total number of mergers). In a nut shell, the standard deviation of Announced Total Value has weak but positive effect on the total number of mergers in a given year.

5.3 Univariate Poisson and Binomial Regression Analysis for the Number of Mergers by Cash

Table-08 summarizes the output of Poisson regression considering the number of Mergers by Cash as response variable on the Mean, Median and Standard deviation of the Announced Total Value in a given year. All the regression coefficients are significant at 5 percent level of significance except for mean Announced Total Value. Though insignificant, the regression coefficient of Mean Announced Total Value in a given year is positive (.000127). That is, The marginal effect of the mean Announced Total Value on the expected number of mergers by cash is for a 1-unit increase in mean Announced Total Value, the estimated total number of mergers increases by a factor of $\exp(0.000127) = 1.0001$. so, the total number of mergers is not changing that is increasing 0 percent per year due to an increase in the mean Announced Total Value . Again, the marginal effect of its median Announced Total Value on the expected number of mergers is for a 1-unit increase in the median Announced Total Value, the estimated number of mergers by cash decreases by a factor of $\exp(-.0016) = 0.9980$. That is, the number of mergers by cash is decreasing by 0.19 percent per year due to an increase in median Announced Total Value. On the other hand, The regression coefficient of the Standard deviation of Announced Total Value is positive (0.000137). The marginal effect of standard deviation of Announced Total Value is for a 1-unit increase in the standard deviation of Announced Total Value the estimated number of mergers by cash increases by a factor of $\exp(0.000137) = 1.0001$. So, the number of total mergers is not significantly

changing, that is, growing at the rate of 0 percent per year. The model's Null deviance value is 1308.63 which has degrees of freedom 21 and the model's Residual deviance value is 793.78 which has degrees of freedom 18. Finally, the large residual deviance indicates a p-value 0, that is our model poorly fitted for this data set.

Table 8: Univariate Poisson Regression Model for the Number of Mergers by Cash

Coefficients	Estimate	Std. Error	z value	P-value
Intercept	6.54E+00	1.35E-01	48.428	$\leq 2e - 16$
Mean Total Announced Value	1.27E-04	9.46E-05	1.34	0.18
Median Total Announced Value	-1.93E-03	1.70E-04	-11.322	$\leq 2e - 16$
Standard Deviation of Total Announced Value	1.37E-04	1.81E-05	7.568	3.79e-14

Table 9: Univariate Negative Binomial Regression Model for the Number of Mergers by Cash

Coefficients	Estimate	Std. Error	z value	P-value
Intercept	7.0745667	0.8626674	8.201	$\leq 2.39e - 16$
Mean Total Announced Value	-0.0003904	0.0007453	-0.524	0.6004
Median Total Announced Value	-0.0018491	0.0011834	-1.563	0.1182
Standard Deviation of Total Announced Value	0.00029	0.0001563	1.855	0.04635

Table-09 shows the summary results of the negative binomial regression of the number of mergers by Cash on the mean, median and standard deviation of the Announced Total Value in a given year. The regression coefficient of the Mean Announced Total Value is negative (-0.0003904) and statistically insignificant. This means that for each one-unit increase in the Mean of Announced Total Value, the difference in the logs of expected counts would be expected to decrease by 0.0003904 units, while holding the other variables in the model constant. Similarly, the Median Announced Total Value has got a negative regression coefficient (-0.0018491) as well as insignificant. That is, for each one-unit increase in the Median of Announced Total Value, the difference in the logs of expected counts would be expected to decrease by 0.0018491 units, while holding the other variables in the model constant. The standard deviation of the Announced Total Value in a given year is positively associated with the total number of mergers by cash. Here, for each one-unit increase in the Median of Announced Total Value, the difference in the logs of expected counts would be expected to increase by 0.00029 units, while holding the other variables in the model constant. The model's Null deviance value is 37.736 which has degrees of freedom 21 and the model's Residual deviance value is 23.587 which has degrees of freedom 18. The residual deviance has a

p-value of 0.18. That is, The null hypothesis(our model is correctly specified) is significant and we have strong evidence to reject that hypothesis. So we have strong evidence that our model fits well for this data set.

Though the regression coefficient of the Mean Announced Total Value is significant and the regression coefficient of the Median Announced Total Value are insignificant but the regression coefficient for Standard deviation of Announced Total Value is significant at 5 percent level of significance. If we compare in between these two models, the negative binomial regression model best fits our data set as it has overdispersion problem(mean of total number of mergers is not equal to the variance of total number of mergers). In a nut shell, the Standard deviation of Announced Total Value has weak but positive effect on the total number of mergers in a given year.

5.4 Univariate Poisson and Binomial Regression Analysis for the Number of Mergers by Stock

Table-10 summarizes the output of Poisson regression considering the number of Mergers by Stock as response variable on the Mean, Median and Standard deviation of the Announced Total Value in a given year. All the regression coefficients are insignificant at 5 percent level of significance. The regression coefficient of Mean Announced Total Value in a given year is negative (-.0000775). That is, The marginal effect of the mean Announced Total Value on the expected number of mergers by stock is for a 1-unit increase in mean Announced Total Value, the estimated number of mergers by stock decreases by a factor of $\exp(-0.0000775) = 0.9999$. so, the number of mergers by stock is unaffected(decreasing by approximately 0 percent per year) due to an increase in the mean Announced Total Value . Similarly, the marginal effect of the median Announced Total Value on the expected number of mergers is for a 1-unit increase in the median Announced Total Value, the estimated total number of mergers decreases by a factor of $\exp(-.00000604) = 0.9999$. That is, the total number of mergers is also unchanged (decreasing by almost 0 percent per year) due to an increase in median Announced Total Value. On the other hand, The regression coefficient of the Standard deviation of Announced Total Value is positive (0.0000275). The marginal effect of standard deviation of Announced Total Value is for a 1-unit increase in the standard deviation of Announced Total Value the estimated number of mergers by stock increases by

a factor of $\exp(0.0000275)=1.0000277$. Again the number of total mergers is not changing with respect to the standard deviation of Announced Total Value, that is growing at the rate of 0 percent per year. The standard deviation of the Announced Total Value has a significant positive but weak relation with the number of mergers by Stock. Zero p-value of the he residual deviance indicates the poor fitting of our model. The model's Null deviance value is 182.34 which has degrees of freedom 21 and the model's Residual deviance value is 161.88 which has degrees of freedom 18. Finally, the residual deviance has a p-value 0, that is our model poorly fitted for this data set.

Table 10: Univariate Poisson Regression Model for the Number of Mergers by Stock

Coefficients	Estimate	Std. Error	z value	P-value
Intercept	3.23E+00	1.01E-01	32.11	$\leq 2e - 16$
Mean Total Announced Value	-7.75E-05	6.38E-05	-1.214	0.225
Median Total Announced Value	-6.04E-06	6.76E-05	-0.089	0.929
Standard Deviation of Total Announced Value	2.77E-05	2.28E-05	1.212	0.0225

Table 11: Univariate Negative Binomial Regression Model for the Number of Mergers by Stock

Coefficients	Estimate	Std. Error	z value	P-value
Intercept	3.28E+00	2.52E-01	13.013	$\leq 2e - 16$
Mean Total Announced Value	-6.49E-05	1.66E-04	-0.391	0.696
Median Total Announced Value	-4.80E-05	1.69E-04	-0.284	0.777
Standard Deviation of Total Announced Value	2.47E-05	6.07E-05	0.407	0.684

Table-11 shows the summary results of the negative binomial regression for the number of mergers by stock on the mean, median and standard deviation of the Announced Total Value in a given year. The regression coefficient of the Mean Announced Total Value is negative(-0.0000649) and statistically insignificant. This means that for each one-unit increase in the Mean of Announced Total Value, the difference in the logs of expected counts would be expected to decrease by 0.0000649 units, while holding the other variables in the model constant. Similarly, the Median Announced Total Value has got a negative regression coefficient(-0.000048). That is, for each one-unit increase in the Median of Announced Total Value, the difference in the logs of expected counts would be expected to decrease by 0.000048 units, while holding the other variables in the model constant. The standard deviation of the Announced Total Value in a given year is positively associated with the number of mergers

by stock. Here, for each one-unit increase in the Median of Announced Total Value, the difference in the logs of expected counts would be expected to increase by 0.0000274 units, while holding the other variables in the model constant. The residual deviance has a p-value of 0.26. That is, The null hypothesis (our model is correctly specified) is significant and we have strong evidence to reject that hypothesis. So we have strong evidence that our model fits well for this data set. Both the regression coefficient of the Median Announced Total Value and the regression coefficient of the Standard deviation of Announced Total Value are insignificant but the regression coefficient for Standard deviation of Announced Total Value is significant at 5 percent level of significance. If we compare in between these two models, the negative binomial regression model best fits our data set as it has overdispersion problem (mean of total number of mergers is not equal to the variance of total number of mergers). In a nut shell, the Standard deviation of Announced Total Value has positive weak effect on the total number of mergers in a given year. The model's Null deviance value is 27.582 which has degrees of freedom 21 and the model's Residual deviance value is 21.939 which has degrees of freedom 18. The residual deviance has a p-value of 0.18. That is, The null hypothesis (our model is correctly specified) is significant and we have strong evidence to reject that hypothesis. So we have strong evidence that our model fits well for this data set.

5.5 Bi-variate Poisson Regression Analysis for the Number of Mergers by Cash and the Number of Mergers by Stock

From the univariate analysis of total number of mergers, number of mergers by Cash and number of mergers by Stock on the mean, median and the standard deviation of the total Announced value, we have found that Negative binomial regression performs better than the Poisson regression approach and finally all of these three response variables has positive weak but significant association with the standard deviation of the total announced value. We consider a bi-variate Poisson regression model for the number of mergers by Cash and the number of mergers by Stock, mean, median and the standard deviation of the total Announced value for each type of mergers addressing correlation among the number of merger by Cash and Stock. The methodology described in section 4.3 by following Leiter and Hamdan(1973) and Islam and Chowdhury(2015).

Table 12: Bi-variate Poisson Regression Model for the Number of Mergers by Cash and the Number of Mergers by Stock

Coefficients	Estimate	Std. Error	z value	P-value
Intercept	0.100675	7.820918	0.012872	0.989730
Mean Announced Total Value for Mergers by Cash	-0.124349	5.099422	-0.024385	0.980546
Median Announced Total Value for Mergers by Cash	-0.201194	3.884816	-0.051790	0.958696
Standard Deviation of Announced Total Value for Mergers by Cash	0.478903	1.748120	0.273953	0.784120
Intercept	0.391994	4.185772	0.093649	0.925388
Mean Announced Total Value for Mergers by Stock	0.908196	0.377324	2.406940	0.016087
Median Announced Total Value for Mergers by Stock	-0.458748	NA	NA	NA
Standard Deviation of Announced Total Value for Mergers by Stock	-0.741141	1.396703	-0.530636	0.595671

Table-12 shows the summary results of the bivariate Poisson regression for the number of mergers by Cash and the number of mergers by Stock on the mean, median and standard deviation of the Announced Total Value in a given year for these two response variables respectively. The regression coefficients of the Mean Announced Total Value and the Median Announced Total Value for the number of merger by cash are negative but regression coefficients for the standard deviation of Announced Total Value for the number of merger by cash is positive. These three regression coefficients are statistically insignificant. This means that in a bivariate scenario, the mean and median of the Announced Total Value in a given year for the number of mergers by cash interacts negatively for a given number of mergers by Stock. But the standard deviation of the Announced Total Value has positive association with the number of mergers by Cash for a given value of number of mergers by Stock. The results follow the similar pattern as before like the univariate case. Now if we look at the regression coefficients of median and standard deviation of the Announced Total Value in a given year for the number of mergers by Stock , we see that the median and the standard deviation of the Announced Total Value in a given year for the number of mergers by Stock interacts negatively for a given number of mergers by Cash and vice versa. Surprisingly , the mean Announced Total Value has a significant and positive association with the number of mergers by Stock for a given value of number of mergers by Cash. The

standard error for the the regression coefficients of median Announced Total Value in a given year for the number of mergers by Stock was not found due to computational inefficiency by the statistical packages.

5.6 Adjustment for Overdispersion

The bivariate model parameters and their corresponding standard error were further updated by estimating overdispersion parameters for these two responses separately as illustrated in Table-13. After the adjustments of overdispersion, the regression coefficients of the mean, median and standard deviation of the Announced Total Value in a given year for the number mergers by cash for a given number of mergers by Stock behaves in similar fashion before the adjustment. But the the regression coefficients of the mean, median and standard deviation

Table 13: Bi-variate Poisson Regression Model for the Number of Mergers by Cash and the Number of Mergers by Stock after the Adjustment of Overdispersion Parameters for the Mergers by Cash($\phi_1 = 0.08267357$) and the Number of Mergers by Stock ($\phi_2 = 0.08267357$) respectively

Coefficients	Estimate	Std. Error	z value	P-value
Intercept	0.100675	2.248753	0.044769	0.964291
Mean Announced Total Value for Mergers by Cash	-0.124349	1.466242	-0.084808	0.932414
Median Announced Total Value for Mergers by Cash	-0.201194	1.117004	-0.180120	0.857059
Standard Deviation of Announced Total Value for Mergers by Cash	0.478903	0.502639	0.952779	0.340702
Intercept	0.391994	2.291523	0.171063	0.864174
Mean Announced Total Value for Mergers by Stock	0.908196	0.700358	1.296760	0.194714
Median Announced Total Value for Mergers by Stock	-0.458748	0.925712	-0.495563	0.620203
Standard Deviation of Announced Total Value for Mergers by Stock	-0.741141	0.332552	-2.228644	0.025838

of the Announced Total Value in a given year for the number mergers by Stock for a given number of mergers by Cash behaves in slightly different way than before the adjustment. We can see that the standard deviation of the Announced Total Value in a given year for the number of mergers by Stock interacts negatively for a given number of mergers by Cash and became statistically significant. Whereas the effect of mean Announced Total Value in a

given year for the number of mergers by Stock given a number of mergers by cash became insignificant. We were unable to get the standard error for the the regression coefficients of median Announced Total Value by Stock before the adjustment for overdispersion. Among all the predictors, only the effect of the standard deviation of the announced total value by Stock is found negative but statistically significant for our bi-variate model.

6 Conclusions and Limitations of the Study

From the individual univariate analysis of total number of mergers, number of mergers by Cash and number of mergers by Stock on the mean, median and the standard deviation of the total Announced value, we have found that Negative binomial regression performs better than the Poisson regression approach. All three response variables have positive weak but significant association with the standard deviation of the total Announced value. So we can recommend that the standard deviation of the total Announced value can be treated as significant predictor to model the number of mergers in a given year.

The positive effect of the mean ATV for Mergers by Stock significantly influence the joint distribution of the number of Mergers by Cash and Stock in the Bi-variate Poisson model. But, the Adjustment of Overdispersion Parameters for the Mergers by Cash ($\phi_1 = 0.08267357$) and the Number of Mergers by Stock ($\phi_2 = 0.08267357$) respectively in our model results in a statistically significant negative impact of the standard deviation of ATV by Stock on their joint distribution of the number of Mergers by Cash and Stock. Finally our bivariate analysis concludes that the standard deviation of ATV by Stock more strongly controls the joint distribution of the number of Mergers by Cash and Stock than the mean ATV for Mergers by Stock.

There are important limitations that must be acknowledged regarding our study. WE were unable to calculate the standard error for the the regression coefficients of median Announced Total Value in a given year for the number of mergers by Stock to computational inefficiency by the statistical packages. In addition, due to unavailability of data, we were not able to include other predictor variables such as firm size, debt ratio, etc in our analysis.

Additional research with a larger data set of enlisted firms is required to verify the results. Future studies can be extended by incorporating qualitative variables that represent non-financial characteristics of the merger and acquisition activities of a firm.

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8 Appendix-A

R-codes Used to Analyze the Data Set

```
##### Data Preparation\\
install.packages("lubridate")
library("lubridate")
year<-format(as.Date(date1, format="%m/%d/%Y"), "%Y")
pmt<-mydata$Payment.Type
data2<-xtabs(~ year+pmt, mydata)
mydata$pmtnew[pmt=='Cash'] <- 'Cash'
mydata$pmtnew[pmt=='Stock'] <- 'Stock'
mydata$pmtnew[pmt=='Cash and Debt' | pmt=='Cash and Stock'
| pmt=='Cash or Stock' | pmt == 'Cash, Stock & Debt' | pmt=='Debt' |
pmt == 'Stock & Debt'
| pmt=='Undisclosed' | pmt==''] <- 'Other'

data3<-xtabs(~ year+pmtnew, mydata)
library(Hmisc)
g <- function(mydata) c(Mean=mean(mydata, na.rm=TRUE), Median=median
(mydata, na.rm=TRUE), sd=sd(mydata, na.rm=TRUE))
summarize(mydata$Announced.Total.Value, llist(year, mydata$pmtnew), g)
mApply(mydata$Announced.Total.Value, llist(year, mydata$pmtnew), g)\\
##### Exploratory Analysis\\
# creating 4-plots for a univariate distribution
par(mfrow=c(3,2))
hist(d1$Total, col = "green")
qqnorm(d1$Total, col = "blue")
plot(d1$year, d1$Total, col="red")
plot(lag(d1$Total, k=1), d1$Total, col = "yellow")
Total.ecdf = ecdf(d1$Total)
plot(Total.ecdf, xlab = '', ylab = '',
```

```

main = 'Empirical Cumulative Distribution ', col="brown")
boxplot(d1$Total, col = "yellow")
pdf("Total.pdf")
pairs(~ d1$Total+d1$Mean+d1$Median+d1$sd, data=d1,
main="Simple Scatterplot Matrix", col= c("blue", "red", "green", "black"))
##### Univariate Poisson and Binomial Regression Model\\
library(MASS)
library(foreign)
library(ggplot2)
library(sandwich)
library(msm)
m1 <- glm(d1$Total ~ d1$Mean + d1$Median + d1$sd,
family="poisson", data=d1)\\
summary(m1)
cov.m1 <- vcovHC(m1, type="HC0")
std.err <- sqrt(diag(cov.m1))
r.est <- cbind(Estimate= coef(m1), "Robust SE" = std.err,
"Pr(>|z|)" = 2 * pnorm(abs(coef(m1)/std.err),
lower.tail=FALSE),
LL = coef(m1) - 1.96 * std.err,
UL = coef(m1) + 1.96 * std.err)

r.est
with(m1, cbind(res.deviance = deviance, df = df.residual,
p = pchisq(deviance, df.residual, lower.tail=FALSE)))

m2 <- glm.nb(d1$Total ~ d1$Mean + d1$Median + d1$sd, data = d1)
summary(m2)
est <- cbind(Estimate = coef(m2), confint(m2))

#####Final Bivariate Poisson model for cash and stock\\

```

```

l = function(th) {

  for (i in 1:22){
    sum1 = sum1 +mydata$Cash[i]*t(c(1, mydata$MeanC[i], mydata$MedianC[i],
    mydata$SDC[i]))
    %*% c(beta_01, beta_11, beta_21, beta_31)
    sum2 = sum2 +mydata$Stock[i]*t(c(1, mydata$MeanS[i], mydata$MedianS[i],
    mydata$SDS[i]))
    %*% c( beta_02, beta_12, beta_22, beta_32)
    sum3 = sum3 -exp(t(c(1, mydata$MeanC[i], mydata$MedianC[i],
    mydata$SDC[i]))
    %*% c(beta_01, beta_11, beta_21, beta_31))
    sum4 = sum4 -exp(t(c(1, mydata$MeanS[i], mydata$MedianS[i],
    mydata$SDS[i]))
    %*% c( beta_02, beta_12, beta_22, beta_32))*mydata$Cash[i]
    sum5 = sum5 + mydata$Stock[i]*log(mydata$Cash[i])
    sum6 = sum6 - log(factorial(mydata$Cash[i]))
    sum7 = sum7 - log(factorial(mydata$Stock[i]))
  }

  sum1+sum2+sum3+sum4+sum5+sum6+sum7
}

```

```

l.prime = function(th){

```

```

  for (i in 1:22){

    sum8 = sum8 + 1*(mydata$Cash[i]-exp(t(c(1, mydata$MeanC[i],
    mydata$MedianC[i], mydata$SDC[i])) %*% c(beta_01, beta_11, beta_21, b
    sum9 = sum9 + mydata$MeanC[i]*(mydata$Cash[i]-exp(t(c(1,
    mydata$MeanC[i], mydata$MedianC[i], mydata$SDC[i])) %*%

```

```

c(beta_01 , beta_11 , beta_21 , beta_31 )))
sum10 = sum10 + mydata$MedianC [ i ]*(mydata$Cash [ i ]-exp
(t(c(1 , mydata$MeanC [ i ] , mydata$MedianC [ i ] , mydata$SDC [ i ])) %*%
c(beta_01 , beta_11 , beta_21 , beta_31 )))
sum11 = sum11 + mydata$SDC [ i ]*(mydata$Cash [ i ]-exp(t(c(1 , mydata$MeanC
c(beta_01 , beta_11 , beta_21 , beta_31 )))

```

```

sum12 = sum12 + 1*(mydata$Stock [ i ]-(mydata$Cash [ i ]*exp(t(c(1 , mydata$M
c( beta_02 , beta_12 , beta_22 , beta_32 ))))
sum13 = sum13 + mydata$MeanS [ i ]* (mydata$Stock [ i ]-(mydata$Cash [ i ]*
exp(t(c(1 , mydata$MeanS [ i ] , mydata$MedianS [ i ] , mydata$SDS [ i ])) %*% c(
sum14 = sum13 + mydata$MedianS [ i ]*(mydata$Stock [ i ]-(mydata$Cash [ i ]*
exp(t(c(1 , mydata$MeanS [ i ] , mydata$MedianS [ i ] , mydata$SDS [ i ])) %*% c(
sum15 = sum13 + mydata$SDS [ i ]*(mydata$Stock [ i ]-(mydata$Cash [ i ]*
exp(t(c(1 , mydata$MeanS [ i ] , mydata$MedianS [ i ] , mydata$SDS [ i ])) %*% c(

```

```

}

```

```

c(dldbета_01 , dldbета_11 , dldbета_21 , dldbета_31 ,
dldbета_02 , dldbета_12 , dldbета_22 , dldbета_32)

```

```

}

```

```

B=matrix(c(dld2beta_01 , dldbета_01beta_11 , dldbета_01beta_02 , dldbета_01be
dld2beta_11 , dldbета_11beta_21 , dldbета_11beta_31 ,
dldbета_21beta_01 , dldbета_21beta_11 ,
dld2beta_21 , dldbета_21beta_31 ,
dldbета_31beta_01 ,
dldbета_31beta_11 , dldbета_31beta_21 ,
dld2beta_31) , nrow=4 , ncol=4 , byrow=T)

```

```

C=matrix(c(dld2beta_02 , dldbета_02beta_12 , dldbета_02beta_22
, dldbета_02beta_32 ,
dldbета_12beta_02 , dld2beta_12 , dldbета_12beta_22 ,
dldbета_12beta_32 ,
dldbета_22beta_02 , dldbета_22beta_12 , dld2beta_22 ,
dldbета_22beta_32 , dldbета_32beta_02 ,
dldbета_32beta_12 , dldbета_32beta_22 , dld2beta_32 ) , nrow=4, ncol=4, byrow=T)

```

```

D=matrix(c(rep(0,16)) , nrow=4, ncol=4, byrow=T)

```

```

E=cbind(B,D)

```

```

G=cbind(D,C)

```

```

U = rbind(E,G)

```

```

U

```

```

}

```

```

theta = c(beta_010 , beta_110 , beta_210 , beta_310 ,
beta_020 , beta_120 , beta_220 , beta_320)

```

```

l(theta)

```

```

l.prime(theta)

```

```

l2.prime(theta)

```

```

itr = 100

```

```

library(MASS)

```

```

for(i in 1:itr){theta = theta - l.prime(theta)%*% ginv(l2.prime(theta))}

```

```

optim.fit <- optim(c(beta_010 , beta_110 , beta_210 ,

```

```

beta_310 , beta_020 , beta_120 , beta_220 , beta_320) , l ,

```

```

l.prime , hessian = TRUE, method = "BFGS" , control=list(fnscale=-1))

```

```

optim.fit
optim.fit$par
z<-(-optim.fit$hessian)
z
z1<-ginv(z)
z1[7,7]<-0.4111
z1

summary.optim1 <- function ( optim.fit ){

  Estimate = optim.fit$par
  cov.mat = z1
  Std.Error = sqrt(diag(cov.mat))
  z.value = Estimate/Std.Error
  p.value = 2*( 1-pnorm(abs(z.value)))

  ds.out1 = cbind(Estimate, Std.Error, z.value, p.value)
  colnames(ds.out1) <- c('Estimate', 'Std Error', 'z value', 'p value')

  ds.out1
}
ds.out1

##### Measuring for overdispersion
## 0.1006746 -0.1243491 -0.2011944 0.4789034 0.3919941
0.9081960 -0.4587482 -0.7411409
mu=rep(0,22)
sum1=0
for (i in 1:22){
  mu[i] =exp(t(c(1, mydata$MeanC[i], mydata$MedianC[i],
mydata$SDC[i]))%*%c( 0.1006746, -0.1243491, -0.2011944, 0.4789034 ))
  sum1=sum1+((mydata$Cash[i]-mu[i])^2/mu[i])
}

```



```

}
print(sum1)
W = diag(mu)
phi1=sum1/(22-3)
phi1
y=t(rep(1,22))
x1=mydata$MeanC
x2=mydata$MedianC
x3=mydata$SDC
x =t(rbind(y, x1, x2, x3))
z=t(x)%*%W%*%x
solve(z)
beta1=phi1*solve(z)
beta1
z2=t(x2)%*%W1%*%x2
z2
solve(z2)
phi2=sum2/(22-3)
phi2
beta2=phi2*solve(z2)
beta2

##### New Covariance matrix after adjustment for overdispersion

D=matrix(c(rep(0,16)),nrow=4, ncol=4, byrow=T)
E=cbind(beta1,D)
G=cbind(D,beta2)
A = rbind(E,G)
A
##### Final estimate and p-values of parameters
optim.fit
summary.optim1 <- function ( optim.fit ){

```

```
Estimate = optim.fit$par
cov.mat = A
Std.Error = sqrt(diag(cov.mat))
z.value = Estimate/Std.Error
p.value = 2*( 1-pnorm(abs(z.value)))

ds.out2 = cbind(Estimate, Std.Error, z.value, p.value)
colnames(ds.out2) <- c('Estimate', 'Std Error', 'z value', 'p value')

ds.out2
}
```