

**NEW CLASSES OF GRACEFUL SPIDERS
AND RELATED COMPUTATIONAL RESULTS**

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1 Introduction

In general, a graph labeling is a mapping from the vertices or edges of a graph to the integers that satisfies some property. One of the most commonly studied labelings is the “graceful” labeling, first introduced by Rosa [14] in 1966. A graph is said to be gracefully labeled if each of its vertices is labeled with a unique integer between 0 and the number of edges in the graph, inclusive, such that every edge in the graph is uniquely identified by the absolute difference between the labels of the two nodes connected by that edge.¹ Rosa initially developed the concept as a method for attacking a conjecture of Ringel [13] that the complete graph K_{2n+1} can be cyclically decomposed into $2n + 1$ subgraphs that are isomorphic to a given tree with n edges. If a graceful labeling of the given tree can be found, then a cyclic decomposition of K_{2n+1} follows. First, label the vertices from 0 to $2n$. The i^{th} tree in the cyclic decomposition can be found by replacing each label m in the original graceful tree by $(m + i - 1) \bmod 2n + 1$, and then connecting the corresponding nodes in the complete graph.²

An unpublished result of Erdős (referenced by Gallian [7]) states that almost all graphs are non-graceful. In spite of this result, interest in the study of graceful graphs, particularly graceful trees, has remained high, as evidenced by the large number of related publications. Since their introduction, graceful labelings have found several practical applications in such fields as cryptography, computer networking, physics, and electrical engineering [7].

The Ringel-Kotzig conjecture that all trees are graceful (also known as the Graceful Tree Conjecture) has been the subject of a great deal of research [7] and remains one of the most famous open problems in graph theory to date. Several classes of trees have been shown to be graceful including paths [14], caterpillars (a connected graph such that the removal of all leaves yields a path) [14], symmetric trees (rooted trees where every level contains vertices of equal degree) [16], trees with diameter five or less [8], and all trees with 35 or fewer vertices (verified by computation) [6], among several others [7].

In spite of the large amount of research dedicated to the problem, there remain many open classes of trees for which gracefulness has not been proven. Alfalayleh, et al. [2] give a succinct list of several of the more commonly explored open classes of graphs, reproduced here in the authors’ perceived order of increasing difficulty:

- Paths of arbitrary lengths connected by a vertex (often referred to as “spiders”)
- All trees with 5 leaves
- The full class of lobsters (graphs which yield caterpillars when their leaves are removed)
- All trees of diameter 6
- All trees with maximum degree 3 (i.e. the class of binary trees)

¹This does not, however, place any explicit conditions on *which* induced labels can be assigned to any particular edge.

²Graceful trees are not the only way to cyclically decompose a graph, but they are the method that has gained the most popularity.

The majority of the remainder of this thesis will focus on exploring and extending what is known about spiders in particular. Using theoretical approaches, we show that several known classes of graceful spiders can be extended to larger classes of graceful spiders. Using a computational approach, we generate all possible graceful labelings for some low-order spiders and use these results to arrive at some additional conjectures and minor results relating to graceful spiders.

2 Literature Review

Definition 2.0.1. A spider is a tree with exactly one vertex of degree greater than two.³

Several classes of spiders have already been proven graceful (some directly, and others as the result of more general classes of graphs). We attempt now to enumerate the majority of classes of spiders that are already known to be graceful.

2.1 Terminology

For convenience, we have listed here some definitions which will be used throughout this thesis.

Definition 2.1.1. A labeling of a graph G with q edges is an injective mapping f from the vertices of G to the integers. The label of a vertex v is $f(v)$, and the induced label of an edge uv is $|f(u) - f(v)|$. V_f denotes the set of vertex labels, and E_f denotes the set of induced edge labels under f .

Definition 2.1.2. A labeling f is a graceful labeling if $V_f \subseteq \{0, \dots, q\}$ and $E_f = \{1, \dots, q\}$. A graph G is graceful if it has a graceful labeling.

Definition 2.1.3. A labeling f is an α -labeling with index k if f is a graceful labeling and k is an integer such that for each edge uv , either $f(u) \leq k < f(v)$ or $f(v) \leq k < f(u)$. A graph G is α -graceful if it has an α -labeling.

Definition 2.1.4. A zero-vertex of a graph G is any vertex v for which there exists a labeling f of G such that $f(v) = 0$.⁴ In general, a zero-vertex may refer to a vertex that can be labeled 0 under any specified labeling scheme. Here we generally use the term zero-vertex to refer to a zero-vertex under graceful labeling, although we also make use of zero-vertices under α -labeling at times.

Definition 2.1.5. The branch vertex (or simply branch) of a spider is the vertex with degree greater than two.

Definition 2.1.6. A path (denoted as P_n) is a connected graph on n vertices which has maximum degree two.

Definition 2.1.7. Within a spider, each path with n edges extending from the branch vertex to a leaf vertex is an n -leg. We often use the leg lengths of a spider to identify its isomorphism. $S(m_1, m_2, \dots, m_k)$ denotes a spider with k legs, each of length $m_1 \geq m_2 \geq \dots \geq m_k$.⁵

³Some sources define a spider to include trees with *no* vertices of degree greater than two. Of course, these are all path graphs, which are known to be universally graceful. Since these additional spiders are not particularly interesting for our purposes, we use the definition of a spider that requires a minimum of three leaf nodes.

⁴All graceful graphs have a zero-vertex, as without one, there would be no way to induce the largest edge label.

⁵Note that each n -leg is a path P_{n+1} . We adopt this notation because it makes certain operations more intuitive. For example, adding an n -leg to an existing spider requires that n vertices (and edges) be added to the existing graph.

2.2 The Number of Graceful Labelings of Spiders

In 2003, Aldred, Širáň and Širáň proved the following [1]:

Theorem 2.2.1. *The number of distinct graceful labelings of P_n is at least $(5/3)^n$.*

Since a graceful labeling of P_n can be turned into a gracefully labeled spider by appending two new vertices to the vertex in P_n with label zero and assigning the labels n and $(n + 1)$ to the new vertices, this gives us a lower bound on the number of graceful labelings for spiders:

Corollary 2.2.2. *The number of distinct graceful labelings of the class of spiders of order n is at least $(5/3)^{n-2}$.*

Similarly, we can derive an upper bound on the number of distinct graceful labellings for the class of spiders:

Theorem 2.2.3. *There are precisely $(n - 1)!$ distinct graceful labelings on the class of graphs with n vertices and $n - 1$ edges.⁶*

Proof. Since there are n vertices, and $n - 1$ edges, every label in $\{0, \dots, (n - 1)\}$ must be used exactly once in any graceful labeling. Begin with a graph G with n vertices, labeled $\{0, \dots, (n - 1)\}$. For each $i \in \{1, \dots, (n - 1)\}$, connect two vertices in G such that the absolute difference in their labels is i . It should be obvious that this process results in a graceful graph.

Notice that for any i , there are $(n - i)$ possible pairs of vertices that can be joined by this process. Hence, the number of graceful graphs is $\prod_{i=1}^{(n-1)} (n - i) = (n - 1)!$. \square

Since the process above yields some cyclic and disconnected graphs whenever $n > 3$, the following must hold:

Corollary 2.2.4. *The number of distinct graceful labellings of spiders on n vertices is strictly less than $(n - 1)!$.*

2.3 Caterpillars

Definition 2.3.1. *A caterpillar is any connected graph for which the removal of all of its leaves yields a path.*

In his original paper on graceful labelings, Rosa proved the following: [14]

⁶This does not guarantee that a graph on n vertices and $n - 1$ edges is graceful. In fact, many disconnected graphs that match this description are not graceful.

Theorem 2.3.2. *All caterpillars are α -graceful.*

This translates to spiders as the following:

Theorem 2.3.3. *Any spider with, at most, two legs longer than length 1 is α -graceful.*

2.4 Symmetrical Spiders

Definition 2.4.1. *A symmetric tree is a rooted tree such that the nodes within each level of the tree have equal degree.*

Stanton and Zarnke proved the following result in 1973 [16]:

Theorem 2.4.2. *All symmetrical trees are graceful. Furthermore, the root of a symmetrical tree is a zero-vertex.*

Symmetrical spiders (i.e. spiders with all legs of equal length) are a special case of symmetrical trees, so it naturally follows:

Corollary 2.4.3. *Let T be a symmetrical spider. T is graceful with its branch as a zero-vertex.*

2.5 Near-Symmetric Spiders

Spiders with legs of nearly-equal length have also been shown to be graceful. This result follows from the work of Poljak and Štúra in 1982 [12], as well as more recent work by Bahls, Lake, and Wertheim [4].

Theorem 2.5.1. *For an integer $m > 0$, let T be a spider with each leg of length m or $m + 1$. T is graceful. If T has an odd number of legs, its branch is a zero-vertex.⁷*

2.6 Spiders with Diameter 5 or Less

Spiders with diameter 2 are simply stars, and can easily be gracefully labeled by assigning the label 0 to the central vertex, and the remaining labels to the leaf nodes. Spiders with diameters 3 are caterpillars, which are known to be graceful. Spiders with diameter 4 are either $S(3, 1, 1, \dots)$ (which are also caterpillars), or spiders with exclusively 2- and 1-legs. To construct a graceful spider of the latter type, it's sufficient to construct a spider of order k with an equal number of 2-legs (but no 1-legs) with the branch node labeled zero (which can be done via Corollary 2.4.3). We can then attach an appropriate number of 1-legs to the branch node and assign these new nodes the labels $\{k, \dots, q\}$. Hrnčiar and Haviar proved the following in 2001 [8]:

⁷If T has an even number of legs, its root may still be a zero-vertex, but this is not guaranteed by the constructions used to prove this theorem.

Theorem 2.6.1. *All trees of diameter 5 are graceful, with the central vertices as zero-vertices.*

From this, and the fact that all diameter 5- spiders are either $S(4, 1, 1, \dots)$, for which zero-rooted labelings are simple to produce, or have a central vertex for a root vertex, it follows naturally that:

Theorem 2.6.2. *All spiders with diameter at most 5 are graceful with a zero-vertex branch.*

2.7 Spiders with Four Legs or Fewer

In 1982, Rosa, Kotzig, and Huang[9] showed the following:

Theorem 2.7.1. *All trees with 4 leaves or fewer are graceful.*

This immediately gives us:

Corollary 2.7.2. *All spiders with 4 legs or fewer are graceful.*

Rosa, Kotzig, and Huang show additional properties of these graphs, such as the fact that most of them are α -graceful. While these additional properties are interesting, they are not particularly useful for our purposes, so we will not explore them here in any further detail.

2.8 Olive Trees

An olive tree is a spider with n legs of lengths $\{1, \dots, n\}$ In 1978, Pastel and Raynaud proved the following by direct construction [11]:

Theorem 2.8.1. *All olive trees are graceful.*

2.9 Spiders with One “Long” Leg

In 1973, Kotzig showed the following [10]:

Theorem 2.9.1. *If any leaf of an arbitrary tree is joined to a leaf of a long-enough path, the resulting tree is α -graceful.*

While we provide no formal specification of what qualifies as “long-enough,” this result does give us the following:

Corollary 2.9.2. *Any spider with a single “long-enough” leg (relative to its order) is α -graceful.*

2.10 “Hairy” Spiders

In a similar vein as “long-legged” spiders, one can consider adding an arbitrary number of 1-legs (or “hairs”) to an arbitrary spider. In 2014, Superdock[17] showed the following:

Theorem 2.10.1. *Attaching sufficiently many leaves to any single vertex in an arbitrary tree yields a graceful tree.*

Specifically in the context of spiders, Superdock[17] also showed the following:

Theorem 2.10.2. *Let T be a spider with n vertices and branch v , and let the lengths of the legs of T be m_1, \dots, m_k , where $m_1 \geq \dots \geq m_k$. If, for each i , we have*

$$m_i \leq \max\left(1, \log_2\left(\frac{n}{2i-1}\right) + 1\right)$$

then T has a graceful labeling f with $f(v) = 0$.

While this is not expressly mentioned in relation to “hairy” spiders, it is clear upon examination that, at minimum, a quarter of the labels (all the odd labels greater than $q/2$) must be assigned to the leaves of 1-legs. This gives us a somewhat more concrete bound on the number of 1-legs that must be attached to an arbitrary spider to guarantee a graceful labeling.

3 Theory

This section is dedicated to developing some methods for constructing graceful graphs from smaller graceful graphs. We begin by working with graceful graphs in general, and then apply these general methods to known graceful classes of spiders to yield additional classes of necessarily graceful spiders.

3.1 Preliminaries

Before we begin with the main proofs, we provide a few additional preliminary results that will be used later.

Theorem 3.1.1. *If f is a graceful labeling of graph G with q edges, then labeling f' denoted by $f'(v) = q - f(v)$ is also a graceful labeling of G . Such a labeling f' is called the complementary labeling of f .*

Proof. Since f is a graceful labeling of G , f is injective onto $\{0, \dots, q\}$, $f'(v) = q - f(v)$ is also injective into $\{0, \dots, q\}$. Each induced edge label remains unchanged: $|f'(u) - f'(v)| = |(q - f(u)) - (q - f(v))| = |f(u) - f(v)|$. Therefore f' is a graceful labeling of G . \square

Theorem 3.1.2. *If f is an α -labeling with index k of graph G with q edges, then the labeling f' denoted by*

$$f'(v) = \begin{cases} k - f(v) & \text{if } f(v) \leq k \\ q + k + 1 - f(v) & \text{if } f(v) > k \end{cases}$$

is also an α -labeling with index k of G . Such a labeling f' is called the k -complement of α -labeling f .

Proof. For any edge uv , assume without loss of generality that $f(u) \leq k < f(v)$. Then $f'(u) \leq k < f'(v)$, and

$$\begin{aligned} f'(v) - f'(u) &= (q + k + 1 - f(v)) - (k - f(u)) \\ &= q + 1 - (f(v) - f(u)) \end{aligned}$$

Therefore, $E_{f'} = \{1, \dots, q\}$, so f' is an α -labeling with index k . \square

Theorem 3.1.3. *(Rosa [15]) Let v be a vertex of P_n . v is a zero-vertex of P_n under an α -labeling, unless v is the central vertex of P_5 .*

(We omit a proof for this theorem.)

3.2 Appending Alpha Graphs to Graceful Graphs

We concern ourselves here with the problem of joining two graceful graphs by the identification of two nodes, one from each graph. The theorems in this section were initially discovered by Huan, Kotzig, and Rosa [9], but were independently discovered here near the beginning of this work. We provide our own original proofs of these theorems here (although they are remarkably similar to the original proofs).

Theorem 3.2.1. *If G is a graceful graph and H is an α -graceful graph, then the graph W obtained by identifying any zero-vertex of G with any zero-vertex of H , is also graceful.*

Proof. Assume that G has q_G edges, f_G is a graceful labeling of G , and $f_G(v_G) = 0$. Assume also that H has q_H edges, and that v_H is a zero-vertex of H under α -labeling with index k . Since v_H is a zero-vertex of H under α -labeling, we can construct an α -labeling f_H such that $f_H(v_H) = k$ by k -complement (Theorem 3.1.2). We now construct W as follows:

Let W be the graph obtained by identifying the vertices v_G and v_H . Define f_W as:

$$f_W(v) = \begin{cases} f_G(v) + k & \text{if } v \in G \\ f_H(v) & \text{if } f(v) \leq k \\ f_H(v) + q_G & \text{if } f(v) > k \end{cases}$$

G is relabeled injectively with labels from $\{k, \dots, k + q_G\}$. H is relabeled injectively with labels from $\{0, \dots, k\} \cup \{k + q_G + 1, \dots, q_G + q_H\}$. Since the two vertices sharing label k have been identified into a single vertex and all other vertices have unique labels:

$$V_{f_W} \subseteq \{0, \dots, q_G + q_H\}$$

Since G has been relabeled with an additive constant, it maintains the edge labels $\{1, \dots, q_G\}$. Since H is α with index k , and each vertex v with $f_H(v) > k$ has been increased by q_G , the modified H has induced edge labels $\{q_G + 1, \dots, q_G + q_H\}$.

$$E_{f_W} = \{1, \dots, q_G + q_H\}$$

Since $q_G + q_H = q_W$, $V_{f_W} \subseteq \{0, \dots, q_W\}$ and $E_{f_W} = \{1, \dots, q_W\}$.

Therefore, by definition 2.1.2, f_W is a graceful labeling of W . □

Theorem 3.2.2. *If G and H are both α -graceful graphs with indices k_G and k_H , then the graph W obtained by identifying any zero-vertex of G with any zero-vertex of H , is also α -graceful with index $k_G + k_H$.*

Proof. Construct graceful labeling f_W as in the proof of Theorem 3.2.1. Note that since G and H are both α -graceful, $f_W(v) \leq k_G + k_H$ if and only if $f_G(v) \leq k_G$ or $f_H(v) \leq k_H$. Therefore, W is α -graceful with index $k_G + k_H$. \square

3.3 Appending Legs to a Spider

By applying the results from the previous section to spiders in particular, we present a method for extending certain classes of spiders which are known to be graceful. In particular, any spider which is known to be graceful with a zero root can immediately have two legs of *nearly* any length added, as well as an arbitrary number of 1-legs, while maintaining its gracefulness.

Theorem 3.3.1. *If a spider is graceful with its branch vertex as a zero-vertex, the spider obtained by appending any number of 1-legs, an i -leg, and a j -leg to the branch vertex is also graceful when $i, j \geq 0$ and at least one of $i, j \neq 2$.*

Proof. Let S be a labeled spider with q edges with the branch vertex labeled 0. Add n new vertices to S with labels $\{q + 1, \dots, q + n\}$ and connect them to the branch vertex, resulting in a new gracefully labeled spider with its branch vertex labeled 0. Construct a path P_{i+j+1} , with vertex v denoting a vertex distance i from a leaf. By Theorem 3.1.3, P_{i+j+1} has an α -labeling f such that $f(v) = 0$. By Theorem 3.2.1 the branch vertex of spider S (augmented with additional 1-legs) can be identified with v to form a new graceful spider S' as defined above. \square

Theorem 3.3.2. *For any spider with a leaf vertex v that is also a zero-vertex, the spider obtained by removing the leg containing v and appending any number of 1-legs, an i -leg and a j -leg to the branch vertex is also graceful when $i, j \geq 0$ and at least one of $i, j \neq 2$.*

Proof. Let S be a labeled spider as described above, with vertex v denoting the leaf with label 0. The vertex adjacent to v must have label q , as this is the only possible way to obtain the induced edge label q . Relabel S by its complement and then remove v to obtain a modified spider with either a zero-leaf or a zero-branch. Repeat this process until the branch vertex is labeled 0. Since i, j are not both equal to 2, we can construct S' by Theorem 3.3.1. \square

We can immediately use these results to extend some known classes of graceful spiders:

Corollary 3.3.3. *All spiders with $(2n + 1)$ legs all of lengths m or $m + 1$ ($m > 1$), an arbitrary number of 1-legs, and any two legs of lengths i, j with one of $i, j \neq 2$ are graceful.*

Proof. Since all near-symmetric spiders with an odd number of legs are graceful with a zero-branch, this result directly follows from Theorem 3.3.1. \square

Corollary 3.3.4. *If the removal of the two longest legs of a spider yields a graph which has diameter 5 or less, that spider is graceful.*

Proof. Since all spiders of diameter 5 or less are graceful with a zero-branch, we can safely add two legs of any length. If we add two legs both of length two, the new spider still has diameter 5 or less, and remains graceful. We can add two legs not both of length two by Theorem 3.3.1. \square

Corollary 3.3.5. *If the removal of some combination of two legs not both of length two from a spider S yields a “Superdock spider” (see Theorem 2.10.2), then S is graceful.*

Proof. This again, follows directly from the definition of Superdock’s class of spiders in conjunction with Theorem 3.3.1. \square

We can also use a slightly different approach to extend spiders which are also caterpillars:

Theorem 3.3.6. *If a spider is α -graceful with the branch vertex as a zero-vertex under α -labeling, the spider obtained by appending any number of 1-legs, an i -leg, and a j -leg to the branch vertex is also α -graceful when $i, j \geq 0$ and at least one of $i, j \neq 2$.*

Proof. The same proof as for Theorem 3.3.1, but with the application of Theorem 3.2.2 in the final step instead of Theorem 3.2.1. \square

Lemma 3.3.7. *Any spider $S(m, 1, 1, \dots)$ is α -graceful with a zero-branch.*

Proof. Let spider S be $S(m, 1, 1, \dots)$ with n 1-legs. Construct an α -labeled path of length $m + 1$ using the canonical labeling: $0, m, 1, (m - 1), 2, \dots$. Append any number of vertices to the vertex labeled 0, labeling these new vertices $(m + 1), \dots, (m + n)$. The result is an α -graceful labeling of S with a zero-branch. \square

Theorem 3.3.8. *Any spider with three or fewer legs of length longer than 1 is graceful. If at least one leg is longer than length 2, the spider is α -graceful.*

Proof. Assume S is a spider as defined above, with i, j, k as the lengths of the longest three legs. Assume without loss of generality that $i \geq j \geq k$.

If $k = 1$, S is a caterpillar, and thus α -graceful.

If $i > 2$, construct S by first creating the α -graceful spider $S(k, 1, 1, \dots)$ by Lemma 3.3.7, and appending two legs of length i and j by Theorem 3.3.6 to yield an α -labeling.

The only case that remains is when $i = j = k = 2$. In this case, no α -graceful labeling exists, but $S(2, 2, 2)$ is a symmetric spider, and has a zero-branch. Append to this (non- α -graceful) spider as many 1-legs as necessary by Theorem 3.3.1 to generate a graceful labeling of S . \square

3.4 Attaching Alpha-Graceful Graphs by an Edge

One major drawback of the above method for attaching an α -graceful graph to a graceful graph lies in the fact that the method necessarily moves the label 0 away from the identified vertices in nearly every case.⁸ This makes it difficult to iteratively apply the same method without first finding a method for moving the 0-label back to the identified vertex, which is a non-trivial problem. Here we present a method for connecting α -graceful graphs that avoids the problem of moving vertex labels.

Theorem 3.4.1. *Let G and H be α -labeled graphs, with indices k_G and k_H , respectively, and $k_G \leq k_H$. Suppose that there is a label i in G such that $0 \leq i \leq k_G$ and such that $(k_H - k_G + i)$ is a label in H . The graph W obtained by joining vertex i in G with vertex $(k_H - k_G + i)$ in H by an edge is also α -graceful. Furthermore, both of the connected vertices can maintain their original labels, although not necessarily simultaneously.*

Proof. Let G and H and i be as described above. Let q_G and q_H denote the number of edges in G and H , respectively. (We do not distinguish here between a vertex and its label.)

Define a labeling of W as follows:

$$\text{Relabel each vertex } v_G \text{ in } G \text{ by: } \begin{cases} v_G & \text{if } v_G \leq k_G \\ v_G + q_H + 1 & \text{if } v_G > k_G \end{cases}$$

$$\text{Relabel each vertex } v_H \text{ in } H \text{ by: } \begin{cases} v_H + k_G + q_H - k_H + 1 & \text{if } v_H \leq k_H \\ v_H + k_G - k_H & \text{if } v_H > k_H \end{cases}$$

The vertices of G are now mapped to unique labels from $\{0, \dots, k_G\} \cup \{k_G + q_H + 2, \dots, q_G + q_H + 1\}$, and the new induced edges of G are $\{q_H + 2, \dots, q_G + q_H + 1\}$. The vertices of H are now mapped to unique labels from $\{k_G + 1, \dots, k_G + q_H + 1\}$, and the induced edges of H remain $\{1, \dots, q_H\}$.

Connect vertex i in G to vertex $(k_H - k_G + i)$ in H by an edge. Note that vertex i in G maintains its original label, while vertex $k_H - k_G + i$ in H has a new label $q_H + 1 + i$, making the induced label of the new edge $q_H + 1$.

⁸A star graph can be attached by its central node without disturbing the placement of the label 0, but this is a trivial task.

Therefore, W has unique vertex labels from $\{0, \dots, q_G + q_H + 1\}$ and induced edge labels $\{1, \dots, q_G + q_H + 1\}$. Hence, W is graceful. It is simple to verify additionally that W is also α -graceful with index $k_W = k_G + q_H - k_H$.

As noted previously, vertex i in G maintained its label upon the construction of W . Vertex $k_H - k_G + i$ in H was relabeled $q_H + 1 + i$, which is equivalent to $((k_W + 1) + k_H - k_G + i$. By taking the α -inversion of the complement of W , we can retrieve the original label of $k_H - k_G + i$. \square

While this method of extending α -graceful graphs avoids the problem of moving vertex labels, we note that it does have some drawbacks of its own. For example, this method can only be used to join two α -graceful graphs together, while the previous method only requires that one of the graphs be α -graceful. In general, if we wish to successively append alpha graphs to a single vertex of a graph while maintaining that vertex's original label, each graph G that we append must have index k_G greater than or equal to the graph output by the previous iteration. If we desire to append a graph with a smaller index than this limit, in most cases, it once again becomes necessary to find methods for relabeling the nodes to be connected.

3.5 A New Class of Alpha-Graceful Spiders

By direct application of the above method for combining α -graceful graphs to the following lemma, we present a new class of α -graceful spiders.

Lemma 3.5.1. *(Cattell [5]) For a given path P_n and any $0 \leq i < n$, there exists an α labeling of P_n such that a leaf has label i , except when $n = 4s + 1$ and $i = s$ or $3s$.*

Corollary 3.5.2. *Let S be an α -graceful spider and k be one less than the number of vertices in the partition of S that contains the branch vertex.⁹ Let S' be a spider created by appending an m -leg to S at its branch, where $m \geq 2k - 1$. S' is α -graceful spider except when S has only two possible branch labels under α labeling, j and $q - j$. In this case, S' may still be α -graceful, but this is not guaranteed.*

When this construction is used, for any $i < k$, if i is a possible branch label of S , then i is also a possible branch label of S' . (Of particular note, if the branch of S is a zero-vertex, then the branch of S' is also a zero-vertex.)

As an example of an exception from above, spider $S(3, 3, 2, 2, 1, 1)$ can only be α -labeled with 2 and 10 at its root. Attempting to append a 9-leg to this spider by Theorem 3.4.1 would require that the label 2 be on a leaf of P_9 , which is impossible. Therefore, $S(9, 3, 3, 2, 2, 1, 1)$ cannot be guaranteed to be α -graceful.

⁹In other words, let k be the index of the α -labeled S when the branch node is assigned a label from the smaller partition.

Adding a 10-leg (or longer) to this spider is still perfectly valid, however, and will necessarily result in an α -graceful spider.

4 Enumerating Graceful Spiders

In an effort to gain insight into the graceful nature of spiders, we programmatically generated all possible graceful labelings for low-order spiders. Here we provide pseudocode that underlies our generation function along with a brief explanation of its correctness and a discussion of its performance. This code is inspired by, and loosely based on, a similar algorithm for generating gracefully labeled cycles, which is due to Bagga, Heinz, and Majumder [3].

4.1 Algorithm

```
1 Function enumGracefulSpiders(G, e)
   Data: G: a partially labeled graceful graph
   e: the next induced edge label to add
   Result: Enumerate graceful spiders recursively
2 foreach node (label) n1 and n2 such that  $n2 - n1 = e$  do
3   if potentiallyGraceful(G, n1, n2) then
4     add edge (n1, n2) to G
5     if e = 1 then
6       if isUniqueSpider(G) then
7         output(G)
8         output(complement(G))
9     else
10      enumGracefulSpiders(G, e - 1)
11      remove edge (n1, n2) from G
12 end
```

enumGracefulSpiders acts as the main entry point for our algorithm. To generate all graceful spiders of order n , we call **enumGracefulSpiders**($G_n, n - 1$), where G_n is a graph with n vertices labeled 0 to n . The algorithm then recursively constructs every possible graceful graph with n nodes and $n - 1$ edges. It should be obvious that this basic approach is a brute-force construction, and that all possible gracefully labeled spiders (and trees, for that matter) should lie somewhere along this computational branch.

Two major problems to consider under a brute-force approach are the large amounts of time spent on partial graceful graphs that can never become spiders and the idea that each graceful spider found has a complement that can be calculated directly rather than by brute-force. The function **potentiallyGraceful**, defined and discussed below, checks for dead-end conditions that would result from adding an edge to a partially constructed spider. If this check returns *false*, then adding the new edge to the partially labeled graph can never result in a graceful spider, so we prune that computational branch and continue on with the next possible edge. If **potentiallyGraceful** returns *true*, we cannot safely prune the branch, and we continue the brute-force assignment of edges.

Once a graceful graph has been completed (that is, after the edge labeled 1 has been added), **isUniqueSpider**, defined and discussed below, is called to verify that the graph *is*, in fact, a spider, and the spider is not an isomorphism of another spider generated by the process. If these conditions pass, then the graceful spider is output (in our case, saved to disk) along with its complement spider.

For use in pruning dead computational branches, we define **potentiallyGraceful** as follows:

```

1 Function potentiallyGraceful(G, n1, n2)
   Data: G: a partially labeled graceful graph
   n1: the smaller of two nodes to be connected
   n2: the larger of two nodes to be connected
   Result: If adding the edge (n1, n2) to G cannot possibly lead to a graceful spider, return false.
   Else, return true.
2   deg [ ] = degrees(G)
3   if componentCount(G) > n2 - n1 + 1 then
4     |   return false
5   if deg[n1] > 1 and deg[n2] > 1 then
6     |   return false
7   if max(deg) == 2 then
8     |   if deg[n1] == 2 and n1 > (order(G) - 1)/2 then
9       |   return false
10    |   if deg[n2] == 2 and n2 > (order(G) - 1)/2 then
11      |   return false
12    else if (deg[n1] == 2 or deg[n2] == 2) and max(deg) > 2 then
13      |   return false
14    return true

```

We follow with a brief explanation of each of the conditions of **potentiallyGraceful(*G*)**

If componentCount(*G*) > *n2* - *n1* + 1: It should be obvious that during the construction process, each added edge connects two components. We start with *n* components (the individual nodes), and *n* - 1 edges to add. Every time a valid edge is added, the remaining edge count and the component count both drop by one. The only way to add an edge without connecting two components is by creating a cycle, which cannot possibly lead to a graceful spider. Since we are adding edges in reverse order, when we prepare to add an edge (*n1*, *n2*) with label *m*, there are also (*n2* - *n1*) remaining edges in total. Hence, *componentCount(G)* > *n2* - *n1* + 1 implies a cycle in the graph, and we can safely prune the computational branch.

If deg(*n1*) > 1 and deg(*n2*) > 1: If the two nodes to be connected both have degree greater than one, then adding an edge between them would result in multiple nodes with degree greater than two. Since, by definition, a spider has exactly one node of degree greater than two, connecting these nodes cannot possibly result in a graceful spider, and we can safely prune the computational branch.

If max(deg) == 2: No branch node has been established yet, so if one of the nodes to be connected has

degree two, it will become the new branch node, which is normally acceptable, since we have already verified that the nodes are not both of degree two. If the new branch node has a label greater than $(\text{order}(G) - 1)/2$ (rounded down), however, we can prune the computational branch, because any spider with a “large” branch label (one from the upper half of the available labels) can be derived from a corresponding spider with a “small” branch label. Therefore, we can prune any computational branches where a spider would have a “large” branch label, approximately halving the total computation time.

Else if ($\text{deg}[n1] == 2$ or $\text{deg}[n2] == 2$) and $\max(\text{deg}) > 2$: A branch node has already been established, and adding an edge to a node of degree two would create a second branch point, which cannot possibly result in a spider. The computational branch can safely be pruned.

Finally, we define **isUniqueSpider** as follows:

```

1 Function isUniqueSpider(G)
   | Data: G: a gracefully labeled graph
   | Result: If G is a spider (and not a candidate for double counting), return true.
   | Else, return false.
2 if G is not a spider then
3   | return false
4 if G has odd order and its branch label is  $(\text{order}(G) - 1)/2$  then
5   | return G.hasEdge(0,  $\text{order}(G) - 2$ )
6 return true

```

This function serves two purposes. First, it verifies that the completed graph is a spider. (This check is necessary because the last step of the process could create a cycle instead of connecting the final two components.) Second, it prevents double-counting of a specific type of graceful spider. We intentionally only generate spiders with root labels less than $n/2$. For even spiders, this poses no problem, as the complement of a spider with $2m$ nodes and root label $m - 1$ yields a complementary spider with root label m (which is not less than $2m/2$). For spiders of order $2m + 1$, the maximum root label is m , whose complement is *also* m . To prevent double-counting these spiders, we make use of the fact that every graceful spider must have a leg labeled $q - 1$, and this leg can only be edge $(0, q - 1)$ or $(1, q)$. Furthermore, these two possibilities are complementary, so we need only detect one to derive the other. Therefore, in order to avoid double-counting, when we encounter a potentially problematic spider, we only accept it if it has edge $(0, q - 1)$.

4.2 Performance

We conclude with some notes on the performance of this algorithm. The asymptotic performance of this algorithm sits somewhere between exponential and factorial growth. The true brute-force algorithm has factorial time complexity, and although we realize significant reduction in computation by pruning dead

computational branches, the possibility remains that the algorithm still exhibits factorial time complexity in the worst case. Due to the fact that there is a lower bound of $(5/3)^{n-2}$ on the number of graceful labelings of spiders of order n , no algorithm can generate all graceful labelings of spiders in less than exponential time. While our algorithm could likely be made marginally more efficient, the problem of generating all graceful labelings of spiders remains an intractable problem for large spiders even in the best case.

An implementation of this algorithm in Python was able to generate all possible graceful labelings for spiders of order 16 or less in approximately eight hours (running single-threaded on an i7-2600 at 3.4GHz). This was on the order of tens of millions of graceful spiders per hour, with an observed amortized runtime that was approximately constant per generated spider. The generated data occupied approximately 10 GB of disk space (when stored as uncompressed plaintext), and the spiders of order 17 are estimated to take up approximately 40 GB. Further discussion of this data can be found in the Observations section.

5 Observations and Related Results

Equipped with an enumeration of all possible graceful spiders of order 16 or less, we are able to investigate these low-order spiders for interesting properties. This section is dedicated to observations and some related conjectures resulting from our investigation into this enumerated dataset. (For brevity, we refer to spiders of order 16 or less as “small spiders” in the remainder of this section.)

5.1 Distinct Labeling Counts

One of the most natural questions to ask upon beginning to investigate this dataset is simply how many graceful spiders there are. As noted previously, the number of spiders of order n is bounded on the lower end by $(5/3)^{n-2}$, and on the upper end by $(n-1)!$ While both of these bounds are relatively large, it is unclear whether we should expect the true asymptotic growth of distinct graceful labelings of spiders to be exponential or factorial in the number of vertices. We present the labeling counts for both graceful and α -graceful graphs:

n	Graceful Graphs		α -Graceful Graphs	
	$count(n)$	$\frac{count(n)}{count(n-1)}$	$count(n)$	$\frac{count(n)}{count(n-1)}$
4	2		2	
5	8	4	6	3
6	22	2.75	12	2
7	102	4.64	28	2.33
8	416	4.08	78	2.79
9	1,484	3.57	156	2.00
10	5,350	3.61	320	2.05
11	22,182	4.15	718	2.24
12	97,334	4.39	1,672	2.33
13	440,460	4.53	4,068	2.43
14	2,070,400	4.70	9,128	2.24
15	10,529,380	5.09	20,832	2.28
16	56,302,746	5.35	53,504	2.57

Based on the mostly increasing nature of $\frac{count(n)}{count(n-1)}$ for graceful spider labelings and the relatively constant nature of $\frac{count(n)}{count(n-1)}$ for α -graceful spider labelings, we offer the two following conjectures:

Conjecture 5.1.1. *The number of graceful labelings for spiders of order n is asymptotically super-exponential in n . That is, $\lim_{n \rightarrow \infty} \frac{c^n}{g(n)} = 0$, where $g(n)$ is the number of distinct graceful labelings for spiders of order n .*

Conjecture 5.1.2. *The number of α -graceful labelings for spiders of order n is asymptotically exponential in n .*

Of course, attempting to extrapolate patterns in low-ordered data into asymptotic bounds is an inherently risky endeavor, but such initial estimates should serve as starting points for future investigation.

5.2 Branch Labels

Potentially one of the most interesting and useful features of this dataset is also one of the most readily observable. Every small spider has at least one graceful labeling with a zero-root. This leads us naturally to the following conjecture:

Conjecture 5.2.1 (Spider Zero-Branch Conjecture). *All spiders are graceful with a zero-branch.*

This conjecture is stronger than the conjecture that all spiders are graceful, making it not immediately helpful toward that end. A counterexample to this conjecture, however, could provide insights into the graceful nature of spiders.

In addition to the presence of a zero-branch, we also note that all small spiders have more zero-branches than any other m -branch (except, of course, when m is the complement of 0, in which case the counts are necessarily equal). This leads us to suspect the following:

Conjecture 5.2.2 (Spider Movable-Zero Conjecture). *Every graceful spider has a zero-branch.*

At first glance, this may seem like weaker conjecture than the Spider Zero-Branch Conjecture, but in actuality:

Theorem 5.2.3. *The Spider Movable-Zero Conjecture is equivalent to the Spider Zero-Branch Conjecture.*

Proof. Assume that the Spider Movable-Zero Conjecture (SMZC) is true. By induction, we show that all spiders are zero-branch graceful. All spiders with three legs are already known to be graceful. As the base case, By SMZC, all spiders with three legs are also zero-branch graceful. As the inductive case, we show that if all m -legged spiders are zero-branch graceful, then all $(m + 1)$ -legged spiders must be graceful (and by SMZC, zero-branch graceful). Construct a graceful labeling for any $(m + 1)$ -legged spider by first removing an arbitrary leg to yield an m -legged spider, labeling this m -legged spider with a zero-branch labeling (which is guaranteed to exist), and then adding the leg back on by Theorem 3.3.1 (with one of the two new legs being of length 0). Hence, if all m -legged spiders are zero-rooted graceful, then all $(m + 1)$ -legged spiders are graceful (and by SMZC, also zero-branch graceful). By induction on the number of legs, SMZC therefore implies that all spiders are graceful with a zero-branch. Hence, SMZC implies SZBC.

That SZBC implies SMZC should be obvious. If all spiders are graceful with zero-branches, then all graceful spiders have zero-branches.

Hence, the two conjectures are equivalent. □

In light of this equivalence, further research into methods for “rotating” a given labeling so that the branch node has label 0 might prove fruitful. Similarly, the discovery of classes of spiders that permit such label rotation would immediately be extensible by the methods discussed previously for building spiders from other zero-branched spiders.

We also make observations regarding the potential branch labels of a graceful spider:

Our first observation is that spiders with “many” “short” legs tend to have significantly more forbidden branch labels. While we provide no formal specification for what qualifies a spider as having many short legs, we observed in the data that spiders with duplicate 1-legs were significantly more likely to have forbidden labels than their counterparts with fewer short legs. On average, a small spider with multiple 1- legs had 4.9 forbidden branch labels, while small spiders with no such duplicates averaged at only 2.0 forbidden branch labels.

In partial explanation of this phenomenon, consider a spider with n 1-legs, and no legs of any other length. The only possible ways to gracefully label this spider are to assign either 0 or n to its branch, and then assign the remaining labels to the leg vertices. More generally:

Theorem 5.2.4. *For any spider S with order n and i legs. If $i > n/2$, then $\{n - i, \dots, i - 1\}$ are forbidden labels (under graceful labeling) for the branch node of S .*

Proof. Assume S , n , and i are as described above, with branch vertex w . Let m be a desired label for w in the range $\{n - i, \dots, i - 1\}$. Since S has i legs, w has i adjacent edges, which must each have unique induced labels. The number of possible distinct induced edge labels that can be adjacent to w is $\max(m, (n - 1) - m)$.

Since $n - i \leq m \leq i - 1$, and the outer boundaries of this range yield the greatest maximum values, $\max(m, (n - 1) - m) \leq \max(n - i, i - 1)$. (This holds for either outer bound.) Since $i > n/2$ by definition, $n - i < n/2$, and $\max(n - i, i - 1) < i$. Hence, the maximum number of unique induced edge labels adjacent to w is necessarily less than i . That is, there are not enough available edge labels to gracefully label S using m as the branch label. □

By a similar argument to the above, the following can also be shown:

Theorem 5.2.5. *For any graph G with q edges and any vertex w with degree $\delta > (q + 1)/2$, the values $\{q - \delta + 1, \dots, \delta - 1\}$ are forbidden labels of w under graceful labeling.*

Our second observation related to freedom of root labels is that spiders with all distinct leg lengths are significantly more root-flexible than spiders with duplicate leg lengths. Call a spider whose branch can have any label $\{0, \dots, q\}$ a *branch-rotatable* spider. For all small spiders, $\frac{127}{612}$ spiders (20.8%) are branch-rotatable. Considering only spiders with all-unique leg lengths, $\frac{63}{72}$ (87.7%) are branch-rotatable. For spiders

with duplicate leg lengths, $\frac{64}{540}$ (11.9%) are branch-rotatable. While this observation is likely somewhat linked to the previous one, the two seem distinct enough to each warrant mentioning.

A few spiders are of particular interest in relation to this observation. There are only four small spiders with more than two legs of equal length which are still branch-rotatable. These spiders are:

Branch-Rotatable Spiders with 3+ Duplicate Legs

S(10, 1, 1, 1) S(11, 1, 1, 1) S(9, 2, 2, 2) S(12, 1, 1, 1)

Similarly, only 9 spiders with no duplicate leg lengths are *not* branch-rotatable. These spiders, and their disallowed labels are:

Spider	Forbidden Branch Labels
S(3, 2, 1)	3
S(4, 3, 2, 1)	5
S(5, 3, 2, 1)	5, 6
S(6, 3, 2, 1)	6
S(5, 4, 2, 1)	6
S(5, 4, 3)	6
S(5, 4, 3, 1)	6, 7
S(5, 4, 3, 2)	6, 7, 8
S(5, 4, 3, 2, 1)	6, 7, 8, 9

While we do not attempt to make any generalizations upon either of these lists of exceptions, the relatively small size of each of them indicates that further investigation may be warranted.

5.3 Leaf Labels

Aside from the branch labels, the labels that leaves can take on are also of interest, as these are the only other vertices that can be appended to with the potential to retain a graph's spider-ness. Again, the zero-leaves are the most immediately interesting because they are comparatively easy to work with.

Small spiders display an interesting property of extremely flexible leaf nodes. Our data shows that the following conjecture holds for small spiders, and we suspect that it might hold for the full class of spiders:

Conjecture 5.3.1. *For any spider S of order n and any m such that $0 \leq m < n$, there exists a graceful labeling of S such that a leaf vertex of S has label m .*

This does not specify that *any* leaf can receive any label, but rather that any label can be assigned to *some* leaf.

Call a spider “leaf-rotatable” if any leaf can be assigned any (valid) label and still admit graceful labeling. Most small spiders are leaf-rotatable, with the exceptions being:

Spider	Leg Length	Forbidden Leaf Labels
$S(3, 1, 1, 1)$	3	3
$S(3, 1, 1, 1, 1, 1)$	3	3, 5
$S(3, 1, 1, 1, 1, 1, 1, 1)$	3	3, 5, 7
$S(3, 1, 1, 1, 1, 1, 1, 1, 1)$	3	3, 8
$S(3, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	3	5, 7
$S(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	3	5, 7, 9

This fact lends itself to multiple conjectures:

Conjecture 5.3.2. *A spider that is not $S(3, 1, 1, \dots)$ is leaf-rotatable.*¹⁰

Conjecture 5.3.3. *A spider $S(3, 1, 1, \dots)$ with an odd number of 1-legs is not leaf-rotatable. Specifically, the leaf vertex of the 3-leg has forbidden labels. The 1-legs have no forbidden labels.*¹¹

Conjecture 5.3.4 (Spider Zero-Leaf Conjecture). *For a given spider S , there exists a graceful labeling of S such the label 0 can be assigned to any leaf vertex in S .*

All of these conjectures hold for small spiders. Conjecture 5.3.3 is comparatively limited in scope, and seems like a readily attackable problem. Conjecture 5.3.2 implies that all spiders are graceful (since the only exceptions are caterpillars, which are well-known to be graceful). Conjecture 5.3.4 also directly implies that all spiders are graceful. This indicates that proofs of these conjectures may be difficult to construct, however, as always, counterexamples would provide valuable insights into the class of spiders. Additionally:

Theorem 5.3.5. *The Spider Zero-Leaf Conjecture is equivalent to the Spider Zero-Branch Conjecture.*

Proof. Assume the Spider Zero-Leaf Conjecture is true. For any Spider S , construct S' by adding an extra vertex w and attaching it to the branch of S . By SZLC, a graceful labeling of S' exists with w having label 0. Since the only way to induce label q in a spider is by connecting label 0 with label q , the branch node of S' must be a q -vertex. By complement, this means that the branch can be labeled 0, with w having label q . Since q is the maximum label, and only connected to the branch, we can remove it while maintaining the graceful nature of the graph. The removal of w recovers S from S' , with its branch node now labeled 0. Therefore SZLC implies SZBC.

¹⁰This leaves out spiders $S(3, 1, 1, \dots)$ with an even number of 1-legs. Several of these have shown to be leaf-rotatable, with one counterexample.

¹¹This conjecture leaves out $S(3, 1, 1, \dots)$ with 8 1-legs from the generated data. This particular spider breaks the surrounding pattern, and it's unclear based only on a single outlier datapoint what might be driving this deviation.

Conversely, assume the Spider Zero-Branch Conjecture is true. For any spider S , construct graph G by removing an arbitrary leg of length m . Let q be the number of edges in G , and let w be the vertex in G that was the branch vertex in S . If G is a spider, w is a zero vertex by SMZC. If G is *not* a spider, then it is a path, which is known to be zero-rotatable, and w is still a zero vertex. Reconstruct S as follows:

Gracefully label G such that w is labeled 0. Add $\lfloor m/2 \rfloor$ to each label in G . Construct a path with the labels (in order) $\{\lfloor m/2 \rfloor + q + 1, \lfloor m/2 \rfloor - 1, \lfloor m/2 \rfloor + q + 2, \lfloor m/2 \rfloor - 2, \dots\}$ of length m . Attach this path to w by the node labeled $\lfloor m/2 \rfloor$ to yield a gracefully labeled spider isomorphic to S . The leaf vertex in the newly appended leg will be either 0, or $q + m$ (which is equivalent to 0 by complement). Hence SZBC implies SZLC. \square

We close the discussion on leaf labels with an observation about the existence of zero-vertex leaves. The existence of zero-leaves in a given spider is directly tied to the existence of zero-branches in smaller spiders.

Theorem 5.3.6. *For a given Spider S , let r be the branch vertex of S , let w be a leaf vertex in S , and let G be the graph obtained by removing the branch containing w from S . The number of possible graceful labelings of S such that w is labeled 0 is exactly the number of graceful labelings of G in which r is labeled 0.¹²*

Proof. Let S be a spider with branch r and leaf w as described above. Suppose there is a graceful labeling of S such that w has label 0. Since w is labeled 0, and has only one adjacent vertex, that vertex must have label q . Create a new graceful graph by taking the complement labeling of S , and removing w , exposing a new leaf with label 0. Iterate this process until r is labeled 0, yielding graph G . Because the vertices removed by this process always have the same set of labels for a given S and w , if two labelings of S are distinct, then this process will yield two distinct labelings of G .

The process is also reversible. For any labeling of G that has zero as a root, we can construct a graceful labeling of S by iteratively appending maximally labeled vertices and taking the complement until S is retrieved.

Hence, there is a bijective mapping from the graceful labelings of G where r has label zero to the graceful labelings of S where w has label 0. \square

Since in general, smaller spiders are easier to gracefully label than larger ones, this relationship indicates that searching for zero-branch graceful labelings is likely to be more productive than searching for zero-leaf labelings, since the same number of solutions exist in a smaller space of potential solutions. Since

¹²Roughly speaking, the number of zero-leaf labelings of a spider is equal to the number of zero-roots labelings of all of the (non-isomorphic) spiders obtained by removing a leg. This captures the spirit of the theorem, but fails to account for the fact that removing a leg from a 3-legged spider yields a path rather than a smaller spider.

theoretically, both of these labeling methods are equivalent, we suggest that any future computational efforts involving spiders focus on labelings with zero-branches.

5.4 Rotatability

This section of our observations is concerned with the rotatability of labels within spiders. A graph is “ k -rotatable” if any vertex can receive the label k under graceful labeling. If a graph is “ubiquitously rotatable” if it is k -rotatable for any $0 \leq k \leq q$. In general, small spiders are highly rotatable. Our observations on small spiders yield the following conjectures:

Conjecture 5.4.1. *Every spider is k -rotatable for some k .*

Conjecture 5.4.2. *Every graceful spider is k -rotatable for some k .*

Conjecture 5.4.1 immediately implies that all spiders are graceful, but counterexamples could prove enlightening. Conjecture 5.4.2 does not directly imply that all spiders are graceful. The possibility remains that these two conjectures are equivalent, but we see no obvious arguments for or against this possibility. Therefore, we pose the following as an open problem:

Conjecture 5.4.3. *Conjectures 5.4.1 and 5.4.2 are equivalent.*

Interestingly, the vast majority of small spiders are zero-rotatable. The exceptions are all members of $S(3, 1, 1, \dots)$:

Spiders which are not zero-rotatable

$S(3, 1, 1)$	$S(3, 1, 1, 1, 1)$	$S(3, 1, 1, 1, 1, 1, 1, 1)$
$S(3, 1, 1, 1, 1, 1, 1, 1, 1)$	$S(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$	$S(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$

Interestingly, even though this family of spiders has shown up in previous observations, 0 has never been a forbidden label for branches or leaves in this family. This leads us to the following:

Conjecture 5.4.4 (Zero-Rotatable Spider Conjecture). *If a spider is not in the family $S(3, 1, 1, \dots)$, then it is zero-rotatable*

The Zero-Rotatable Spider Conjecture also relates to the Spider Zero-Branch Conjecture:

Theorem 5.4.5. *The Zero-Rotatable Spider Conjecture implies the Spider Zero-Branch Conjecture (and the equivalent Spider Zero-Leaf Conjecture).*

Proof. Assuming the Zero-Rotatable Spider Conjecture holds, all spiders *not* members of $S(3, 1, 1, \dots)$ have a zero-branch (since a zero-rotatable spider must, by definition, have a zero-branch labeling). It is sufficient, then, to prove that the family $S(3, 1, 1, \dots)$ always has a zero-branch. Construct $S \in S(3, 1, 1, \dots)$ as follows:

Begin with P_5 , labeled 4-0-3-1-2. Append as many 1-legs as desired to the node with label 0, labeling these nodes 5, 6, ..., respectively. The result is a zero-branch graceful labeling of S .

Hence, the Zero-Rotatable Spider Conjecture implies the Spider Zero-Branch Conjecture. □

We provide a further specification of the offending vertex of $S(3, 1, 1, \dots)$ that causes it not to be zero-rotatable:

Theorem 5.4.6. *Let S be a spider in the family $S(3, 1, 1, \dots)$, and w be the vertex on the 3-leg that is adjacent to the branch vertex. S is zero-rotatable if and only if w is a zero-vertex.*

Proof. Let S be a spider in the family $S(3, 1, 1, \dots)$ with m 1-legs.

The previous proof gives a method for gracefully labeling S such that the branch node is labeled 0. The complement of this labeling yields a labeling with 0 on a 1-leg leaf.

Alternatively, we can construct S as follows: Construct a star with central label $(m + 2)$ and m 1-legs labeled $2, \dots, (m + 1)$. Append P_3 to the star's branch. Starting from the vertex next to the branch, label these new vertices $1, (m + 3), 0$. This construction demonstrates that the leaf of the 3-leg of S is always a zero-vertex. By the complement of this construction, the vertex adjacent to the leaf of the 3-leg is also a zero vertex.

The only vertex in S that has not been confirmed to necessarily be a zero-vertex is the vertex on the 3-leg, adjacent to the branch vertex (w from the statement of the theorem). Therefore, if w is a zero-vertex, S is zero-rotatable, and if S is *not* zero-rotatable, then w must not be a zero-vertex. □

As a final discussion point relating to rotatability, we make an observation related to spiders with all-unique leg lengths. Nearly all small spiders with all-unique leg lengths are zero-rotatable (*under α -labeling*). Only two spiders with all-unique leg lengths break this trend:

**Spiders with all-unique leg lengths
which are not alpha-zero-rotatable**

S(5, 4, 3, 2) S(5, 4, 3, 2, 1)

Even these exceptional spiders are *nearly* alpha-zero-rotatable, with the only vertices that are not zero-vertices under α -labeling are the branch vertex (in both cases), and the leaf of the 1-leg (in the case of the second spider). Using methods described previously, these alpha-zero-rotatable spiders could prove to be

extremely useful as building blocks for larger graceful (or α -graceful) spiders. Because of this vast potential for usefulness, further investigation into the properties of the class of spiders with all-unique leg lengths.

5.5 Alpha-ness

We conclude our observations with a note on the nature of small spiders related to α -labeling.

First, we note that small spiders are overall, very permissibile of α -labelings. Of the 612 distinct spiders of order 16 or less, only 89 do not permit an α -labeling, while the remaining 523 *do* permit α -labeling.

Interestingly, nearly all spiders that *do not* permit α labelings have two or more 2-legs. The exceptions to this rule are:

**Spiders with fewer than two 2-legs
which do not permit α -labeling**

S(3, 3, 3, 3, 2) S(3, 3, 3, 3, 2, 1) S(3, 3, 3, 3, 3)

Extracting a rule from this limited set of outlier data is difficult at best, but we provide the following conjecture in the hope of stimulating additional investigation:

Conjecture 5.5.1. *All spiders which do not permit α -labeling have either two or more 2-legs, or four or more 3-legs.*

It is our full expectation that this conjecture *does not* hold, but it should serve as a starting point for additional investigation into the alpha-nature of spiders.

Setting aside the exceptions, the fact that most non-alpha spiders have multiple 2-legs serves to drive home an important point. In our work above, we noted the particular trouble that is introduced by the fact that P_5 has no α -labeling with the central node labeled 0. Attaching P_5 by its central node to a spider adds two additional 2-legs to the original spider. It is our intuition that this special property of P_5 is directly related to the lack of α -labelings for so many spiders with multiple 2-legs.

Although it is not immediately obvious how one might go about proving this relationship, it appears to be one of the key complexities behind gracefully labeling spiders, and we strongly suggest investigation of this relationship as a future research direction.

6 Conclusion

We conclude with a brief summary of our results and suggestions for future investigation.

6.1 Summary

Throughout this thesis we have presented numerous results including:

- A general method for constructing graceful graphs by identifying the zero-vertices of two graceful graphs, provided at least one of them is α -graceful
- A general method for connecting two α -graceful graphs by an edge without disturbing the labels of the connected vertices
- Extensions upon known classes of graceful spiders into larger graceful classes of spiders, including near-symmetric spiders, diameter 5 spiders, “Superdock” spiders, and caterpillar spiders
- A method for constructing k -branched α -graceful spiders from smaller k -branched α -graceful spiders
- A relatively efficient algorithm for generating all possible graceful labelings for spiders of a given order
- Several observations, conjectures, and results related to the data generated by the above algorithm including:
 - Two conjectures about the number of graceful labelings for spiders
 - A conjecture that all spiders have zero-vertex branches
 - Conjectures about the the freedom of leaf labels for graceful spiders
 - Conjectures about the k -rotatability of spiders
 - A conjecture about which spiders are α -graceful
 - Multiple theorems relating several of the above conjectures
 - A theorem about forbidden branch labels in certain spiders
 - Notes on the impact of duplicate leg lengths on the branch-rotatability of spiders
 - Notes on properties of spiders with all-unique leg lengths
 - Notes on several peculiar properties of the family of spiders $S(3, 1, 1, \dots)$

6.2 Future Directions

While the results that we have presented are somewhat varied, this means that there are multiple possibilities for related future work. Here we present several suggestions for follow-up investigations.

Conjectures: We have presented several conjectures that hold for small spiders, and imply that all spiders are graceful. While the gracefulness of the entire class of spiders has evaded proof for many years, these new conjectures may pose additional angles from which to approach the problem. In particular, any general method for modifying a gracefully labeled spider to move the label 0 onto the branch vertex would immediately prove all spiders to be graceful.

Alternatively, the discovery of a counterexample or class of counterexamples to these conjectures would likely lead to additional insights into the graceful nature of spiders.

Special Classes of Spiders: We have noted a few subclasses of spiders that have peculiar properties. The two classes that seem to be the most immediately interesting are spiders with all-unique leg lengths and the class $S(3, 1, 1, \dots)$. Further investigation into the properties of these particular spiders could provide insights into the unique behaviors that they exhibit relative to other spiders.

Computational Investigation: The algorithm that we have presented for enumerating all possible graceful labelings for spiders of order n can be extended in multiple ways.

It would be relatively simple to modify the algorithm to only output α -labelings. Since there are orders of magnitude fewer α -graceful labelings than general graceful labelings for spiders, this could allow for results of significantly higher order than what we were able to generate using the general approach.

Alternatively, the algorithm could be modified to generate other classes of graphs, such as unicyclic graphs, trees of maximum degree three, lobsters etc. This would allow similar experimental investigations into additional classes of graphs that have yet to be shown graceful.¹³

Additionally, new programs could be written that leverage existing results for graceful spiders into an efficient verification program for graceful spiders. One such example might combine the computational and theoretical components of this work to verify the gracefulfulness of larger orders of spiders. As a starting point, it would be easy to show that any spider with the property that the removal of its two longest legs yields a graph of order 16 or less is graceful. Applying this rule alone could likely prove with minimal computation that the majority of spiders under orders as high as 40 are graceful.¹⁴ Making use of additional results to avoid unnecessary computation could lead to extreme efficiency gains for verifying graceful spiders, with the potential to greatly exceed the current computational result that all spiders of degree 35 or less are graceful.

¹³While it is certain that our algorithm can be made more efficient both theoretically and by implementation in a more efficient language, the exponential nature of the problem indicates to us that extending our results by more than a few orders would be prohibitively expensive computationally. Therefore, we recommend against directly extending our results in favor of applying similar methods to different problems.

¹⁴For reference, nearly half (46.8%) of spiders order 16 or less have order 5 or less after the removal of their two longest legs.

7 References

- [1] R. E. Aldred, J. Širáň, and M. Širáň, “A note on the number of graceful labellings of paths,” *Discrete Mathematics*, vol. 261, no. 1-3, pp. 27–30, 2003.
- [2] M. Alfalayleh, L. Brankovic, H. Giggins, and M. Z. Islam, “Towards the graceful tree conjecture: a survey,” *Proceedings of AWOCA2004, Ballina, Australia*, 2004.
- [3] J. Bagga, A. Heinz, and M. M. Majumder, “An algorithm for graceful labelings of cycles,” *Congressus Numerantium*, vol. 186, p. 57, 2007.
- [4] P. Bahls, S. Lake, and A. Wertheim, “Gracefulness of families of spiders,” *Involve, a Journal of Mathematics*, vol. 3, no. 3, pp. 241–247, 2010.
- [5] R. Cattell, “Graceful labellings of paths,” *Discrete Mathematics*, vol. 307, no. 24, pp. 3161–3176, 2007.
- [6] W. Fang, “A computational approach to the graceful tree conjecture,” *arXiv preprint arXiv:1003.3045*, 2010.
- [7] J. A. Gallian, “A dynamic survey of graph labeling,” *Electronic Journal of Combinatorics*, 2015.
- [8] P. Hrnčiar and A. Haviar, “All trees of diameter five are graceful,” *Discrete Mathematics*, vol. 233, no. 1-3, pp. 133–150, 2001.
- [9] C. Huang, A. Kotzig, and A. Rosa, “Further results on tree labellings,” *Util. math.*, 21c, pp. 31–48, 1982.
- [10] A. Kotzig and G. Ringle, “On certain vertex valuations of finite graphs,” *Utilitas Math*, vol. 4, pp. 261–290, 1973.
- [11] A. Pastel and H. Raynaud, “Numerotation gracieuse des oliviers,” in *Colloq. Grenoble, Publications Université de Grenoble*, vol. 55, 1978, pp. 218–223.
- [12] S. Poljak and M. Sûra, “An algorithm for graceful labeling of a class of symmetrical trees,” *Ars Combin*, vol. 14, pp. 57–66, 1982.
- [13] G. Ringel, “Problem 25, theory of graphs and its applications (proc. int. symp. smolenice 1963),” *Czech. Acad. Sci., Prague*, 1963.
- [14] A. Rosa, “On certain valuations of the vertices of a graph,” in *Theory of Graphs (Internat. Symposium, Rome, July 1966)*, 1966, pp. 349–355.
- [15] —, “Labelling snakes,” *Ars Combin*, vol. 3, pp. 67–74, 1977.
- [16] R. Stanton and C. Zarnke, “Labeling of balanced trees,” in *Proc. 4th Southeast Conf. Combin., Graph Theory, Comput*, 1973, pp. 479–495.
- [17] M. C. Superdock, “The graceful tree conjecture: a class of graceful diameter-6 trees,” *arXiv preprint arXiv:1403.1564*, 2014.

Appendices

A Graceful Spider Counts

As noted in the above thesis, we have generated every possible graceful labeling of a spider with 16 vertices or fewer. The number of unique graceful labelings on these spiders totals just under 70 million, making it infeasible to list them all here. We do, however, provide several aggregate-level summaries of this data here in the hopes that they may prove useful.

While we have made use of this rich data set in several ways, we are also certain that a vast number of insights remain hidden just below the surface of the data, still waiting to be uncovered.

A.1 Graceful Spiders by Branch Label

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1)	1	-	-	1												
(2, 1, 1)	2	1	-	1	2											
(1, 1, 1, 1)	1	-	-	-	1											
(2, 2, 1)	3	1	-	-	1	3										
(3, 1, 1)	2	1	-	-	1	2										
(2, 1, 1, 1)	2	1	-	-	1	2										
(1, 1, 1, 1, 1)	1	-	-	-	-	1										
(3, 2, 1)	11	6	1	-	1	6	11									
(4, 1, 1)	7	2	2	-	2	2	7									
(2, 2, 1, 1)	6	2	-	-	-	2	6									
(2, 2, 2)	4	1	-	-	-	1	4									
(3, 1, 1, 1)	3	1	-	-	-	1	3									
(2, 1, 1, 1, 1)	3	1	-	-	-	1	3									
(1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	1									
(4, 2, 1)	31	10	6	5	5	6	10	31								
(3, 2, 1, 1)	26	9	1	-	-	1	9	26								
(3, 2, 2)	14	8	1	-	-	1	8	14								
(5, 1, 1)	8	7	5	3	3	5	7	8								
(3, 3, 1)	10	9	1	-	-	1	9	10								
(2, 2, 2, 1)	16	3	-	-	-	-	3	16								
(4, 1, 1, 1)	10	3	2	-	-	2	3	10								
(2, 2, 1, 1, 1)	9	2	-	-	-	-	2	9								
(3, 1, 1, 1, 1)	3	1	-	-	-	-	1	3								
(2, 1, 1, 1, 1, 1)	3	1	-	-	-	-	1	3								
(1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	1								
(4, 3, 1)	58	25	9	3	2	3	9	25	58							
(3, 2, 2, 1)	72	21	1	-	-	-	1	21	72							
(5, 2, 1)	42	23	13	10	8	10	13	23	42							
(4, 2, 1, 1)	70	13	5	3	-	3	5	13	70							
(3, 3, 2)	34	22	4	-	-	-	4	22	34							
(3, 2, 1, 1, 1)	40	11	1	-	-	-	1	11	40							
(4, 2, 2)	30	13	3	3	-	3	3	13	30							
(2, 2, 2, 1, 1)	36	6	-	-	-	-	-	6	36							
(3, 3, 1, 1)	28	13	1	-	-	-	1	13	28							
(6, 1, 1)	14	10	8	5	2	5	8	10	14							
(5, 1, 1, 1)	16	8	5	-	-	-	5	8	16							
(4, 1, 1, 1, 1)	14	2	2	-	-	-	2	2	14							
(2, 2, 1, 1, 1, 1)	14	3	-	-	-	-	-	3	14							
(2, 2, 2, 2)	7	4	-	-	-	-	-	4	7							
(3, 1, 1, 1, 1, 1)	4	1	-	-	-	-	-	1	4							
(2, 1, 1, 1, 1, 1, 1)	4	1	-	-	-	-	-	1	4							
(1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	1							
(3, 2, 2, 1, 1)	204	47	2	-	-	-	-	2	47	204						
(3, 3, 2, 1)	167	57	6	-	-	-	-	6	57	167						
(5, 3, 1)	118	58	22	17	7	7	17	22	58	118						
(4, 3, 2)	140	54	14	9	2	2	9	14	54	140						
(4, 2, 2, 1)	140	53	10	3	-	-	3	10	53	140						
(6, 2, 1)	122	37	20	10	10	10	10	20	37	122						
(4, 3, 1, 1)	139	39	9	2	-	-	2	9	39	139						

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 2, 1, 1)	123	37	15	6	-	-	6	15	37	123						
(4, 2, 1, 1, 1)	133	21	8	3	-	-	3	8	21	133						
(4, 4, 1)	62	26	11	4	2	2	4	11	26	62						
(5, 2, 2)	57	25	8	6	4	4	6	8	25	57						
(7, 1, 1)	45	19	13	10	9	9	10	13	19	45						
(2, 2, 2, 1, 1, 1)	76	9	-	-	-	-	-	-	9	76						
(3, 2, 2, 2)	54	20	1	-	-	-	-	1	20	54						
(3, 2, 1, 1, 1, 1)	57	14	1	-	-	-	-	1	14	57						
(3, 3, 1, 1, 1)	39	17	1	-	-	-	-	1	17	39						
(2, 2, 2, 2, 1)	32	16	-	-	-	-	-	-	16	32						
(6, 1, 1, 1)	25	7	7	3	-	-	3	7	7	25						
(3, 3, 3)	21	17	4	-	-	-	-	4	17	21						
(5, 1, 1, 1, 1)	21	9	5	-	-	-	-	5	9	21						
(4, 1, 1, 1, 1, 1)	17	3	2	-	-	-	-	2	3	17						
(2, 2, 1, 1, 1, 1, 1)	17	3	-	-	-	-	-	-	3	17						
(3, 1, 1, 1, 1, 1, 1)	5	1	-	-	-	-	-	-	1	5						
(2, 1, 1, 1, 1, 1, 1, 1)	4	1	-	-	-	-	-	-	1	4						
(1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	1						
(4, 3, 2, 1)	825	275	51	17	1	-	1	17	51	275	825					
(4, 2, 2, 1, 1)	566	125	15	6	-	-	-	6	15	125	566					
(3, 3, 2, 1, 1)	511	148	9	-	-	-	-	-	9	148	511					
(5, 3, 2)	349	139	37	29	11	4	11	29	37	139	349					
(5, 2, 2, 1)	397	109	29	13	2	-	2	13	29	109	397					
(5, 3, 1, 1)	377	110	31	16	6	-	6	16	31	110	377					
(5, 4, 1)	267	149	53	35	17	10	17	35	53	149	267					
(7, 2, 1)	295	92	50	35	28	26	28	35	50	92	295					
(3, 2, 2, 1, 1, 1)	427	77	2	-	-	-	-	-	2	77	427					
(6, 3, 1)	255	134	47	22	20	18	20	22	47	134	255					
(3, 2, 2, 2, 1)	367	107	3	-	-	-	-	-	3	107	367					
(6, 2, 1, 1)	328	76	35	22	8	-	8	22	35	76	328					
(5, 2, 1, 1, 1)	290	65	24	6	-	-	-	6	24	65	290					
(4, 3, 1, 1, 1)	267	57	11	2	-	-	-	2	11	57	267					
(3, 3, 2, 2)	232	81	8	-	-	-	-	-	8	81	232					
(6, 2, 2)	192	54	28	6	6	-	6	6	28	54	192					
(4, 4, 2)	194	56	17	5	2	-	2	5	17	56	194					
(4, 3, 3)	164	84	13	9	1	-	1	9	13	84	164					
(4, 4, 1, 1)	184	44	21	5	1	-	1	5	21	44	184					
(8, 1, 1)	80	63	46	31	19	18	19	31	46	63	80					
(4, 2, 1, 1, 1, 1)	203	24	7	3	-	-	-	3	7	24	203					
(3, 3, 3, 1)	158	55	9	-	-	-	-	-	9	55	158					
(4, 2, 2, 2)	165	46	8	3	-	-	-	3	8	46	165					
(2, 2, 2, 2, 1, 1)	139	36	-	-	-	-	-	-	-	36	139					
(7, 1, 1, 1)	89	21	18	8	4	-	4	8	18	21	89					
(2, 2, 2, 1, 1, 1, 1)	120	14	-	-	-	-	-	-	-	14	120					
(3, 2, 1, 1, 1, 1, 1)	81	16	1	-	-	-	-	-	1	16	81					
(3, 3, 1, 1, 1, 1)	75	22	1	-	-	-	-	-	1	22	75					
(6, 1, 1, 1, 1)	36	10	8	3	-	-	-	3	8	10	36					
(5, 1, 1, 1, 1, 1)	37	11	5	-	-	-	-	-	5	11	37					
(2, 2, 1, 1, 1, 1, 1, 1)	25	4	-	-	-	-	-	-	-	4	25					
(4, 1, 1, 1, 1, 1, 1, 1)	23	2	2	-	-	-	-	-	2	2	23					
(2, 2, 2, 2, 2)	18	7	-	-	-	-	-	-	-	7	18					
(3, 1, 1, 1, 1, 1, 1, 1)	5	1	-	-	-	-	-	-	-	1	5					
(2, 1, 1, 1, 1, 1, 1, 1, 1)	5	1	-	-	-	-	-	-	-	1	5					

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	1					
(5, 3, 2, 1)	2716	746	179	74	19	-	-	19	74	179	746	2716				
(4, 3, 2, 1, 1)	2824	665	84	22	1	-	-	1	22	84	665	2824				
(3, 3, 2, 2, 1)	1677	426	21	-	-	-	-	-	-	21	426	1677				
(4, 3, 2, 2)	1555	436	54	24	1	-	-	1	24	54	436	1555				
(5, 2, 2, 1, 1)	1458	309	56	21	2	-	-	2	21	56	309	1458				
(6, 2, 2, 1)	1334	325	124	30	14	-	-	14	30	124	325	1334				
(4, 3, 3, 1)	1217	431	65	20	1	-	-	1	20	65	431	1217				
(4, 4, 2, 1)	1216	383	89	25	3	-	-	3	25	89	383	1216				
(5, 4, 2)	979	373	151	78	29	17	17	29	78	151	373	979				
(6, 3, 2)	920	403	118	75	45	20	20	45	75	118	403	920				
(3, 2, 2, 2, 1, 1)	1240	286	6	-	-	-	-	-	-	6	286	1240				
(7, 2, 1, 1)	1032	275	114	45	37	11	11	37	45	114	275	1032				
(5, 4, 1, 1)	979	338	120	41	17	5	5	17	41	120	338	979				
(8, 2, 1)	632	263	231	130	91	113	113	91	130	231	263	632				
(4, 2, 2, 1, 1, 1)	1185	217	27	6	-	-	-	-	6	27	217	1185				
(3, 3, 2, 1, 1, 1)	1108	236	11	-	-	-	-	-	-	11	236	1108				
(7, 3, 1)	708	346	124	78	52	38	38	52	78	124	346	708				
(6, 3, 1, 1)	900	278	92	37	25	8	8	25	37	92	278	900				
(4, 2, 2, 2, 1)	1007	232	36	9	-	-	-	-	9	36	232	1007				
(6, 4, 1)	605	308	168	94	58	39	39	58	94	168	308	605				
(5, 3, 1, 1, 1)	868	199	46	16	-	-	-	-	16	46	199	868				
(6, 2, 1, 1, 1)	774	133	39	18	6	-	-	6	18	39	133	774				
(3, 2, 2, 1, 1, 1, 1)	778	114	3	-	-	-	-	-	-	3	114	778				
(7, 2, 2)	546	161	77	39	23	19	19	23	39	77	161	546				
(5, 3, 3)	489	237	60	22	8	-	-	8	22	60	237	489				
(4, 4, 3)	458	226	57	16	4	-	-	4	16	57	226	458				
(3, 3, 3, 2)	490	164	22	-	-	-	-	-	-	22	164	490				
(5, 5, 1)	336	164	73	59	22	14	14	22	59	73	164	336				
(3, 3, 3, 1, 1)	478	152	13	-	-	-	-	-	-	13	152	478				
(5, 2, 2, 2)	474	115	29	13	2	-	-	2	13	29	115	474				
(5, 2, 1, 1, 1, 1)	494	81	26	6	-	-	-	-	6	26	81	494				
(4, 3, 1, 1, 1, 1)	456	85	11	2	-	-	-	-	2	11	85	456				
(4, 4, 1, 1, 1)	411	68	25	7	1	-	-	1	7	25	68	411				
(9, 1, 1)	148	123	98	69	50	22	22	50	69	98	123	148				
(2, 2, 2, 2, 1, 1, 1)	303	76	-	-	-	-	-	-	-	-	76	303				
(3, 2, 2, 2, 2)	287	83	4	-	-	-	-	-	-	4	83	287				
(8, 1, 1, 1)	189	76	58	28	23	-	-	23	28	58	76	189				
(4, 2, 1, 1, 1, 1, 1)	320	35	10	3	-	-	-	-	3	10	35	320				
(7, 1, 1, 1, 1)	164	36	21	8	-	-	-	-	8	21	36	164				
(2, 2, 2, 1, 1, 1, 1, 1)	190	17	-	-	-	-	-	-	-	-	17	190				
(2, 2, 2, 2, 2, 1)	113	32	-	-	-	-	-	-	-	-	32	113				
(3, 2, 1, 1, 1, 1, 1, 1)	106	20	1	-	-	-	-	-	-	1	20	106				
(3, 3, 1, 1, 1, 1, 1, 1)	94	25	1	-	-	-	-	-	-	1	25	94				
(6, 1, 1, 1, 1, 1)	57	10	7	3	-	-	-	-	3	7	10	57				
(5, 1, 1, 1, 1, 1, 1)	40	10	5	-	-	-	-	-	-	5	10	40				
(2, 2, 1, 1, 1, 1, 1, 1, 1)	30	4	-	-	-	-	-	-	-	-	4	30				
(4, 1, 1, 1, 1, 1, 1, 1, 1)	28	3	2	-	-	-	-	-	-	2	3	28				
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	5	1	-	-	-	-	-	-	-	-	1	5				
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	5	1	-	-	-	-	-	-	-	-	1	5				
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	1				
(4, 3, 2, 2, 1)	10560	2338	251	56	1	-	-	-	1	56	251	2338	10560			
(5, 3, 2, 1, 1)	9252	2004	311	98	13	-	-	-	13	98	311	2004	9252			

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(6, 3, 2, 1)	7378	2120	537	205	79	18	-	18	79	205	537	2120	7378			
(5, 4, 2, 1)	7462	2006	542	207	47	14	-	14	47	207	542	2006	7462			
(4, 3, 2, 1, 1, 1)	7069	1278	128	31	1	-	-	-	1	31	128	1278	7069			
(3, 3, 2, 2, 1, 1)	6193	1313	47	-	-	-	-	-	-	-	47	1313	6193			
(5, 3, 2, 2)	4886	1181	198	91	19	-	-	-	19	91	198	1181	4886			
(4, 3, 3, 2)	4669	1429	183	50	4	-	-	-	4	50	183	1429	4669			
(4, 3, 3, 1, 1)	4443	1159	113	31	1	-	-	-	1	31	113	1159	4443			
(6, 2, 2, 1, 1)	4610	838	239	45	12	-	-	-	12	45	239	838	4610			
(4, 4, 2, 1, 1)	4454	869	199	36	3	-	-	-	3	36	199	869	4454			
(5, 3, 3, 1)	3802	1208	207	82	14	-	-	-	14	82	207	1208	3802			
(7, 2, 2, 1)	3811	873	285	107	59	16	-	16	59	107	285	873	3811			
(4, 4, 3, 1)	3541	1220	237	63	8	-	-	-	8	63	237	1220	3541			
(4, 2, 2, 2, 1, 1)	4113	817	96	18	-	-	-	-	-	18	96	817	4113			
(3, 3, 3, 2, 1)	3578	935	57	-	-	-	-	-	-	-	57	935	3578			
(6, 4, 2)	2619	1079	455	209	98	51	102	51	98	209	455	1079	2619			
(5, 2, 2, 1, 1, 1)	3759	640	101	28	2	-	-	-	2	28	101	640	3759			
(5, 4, 3)	2571	1197	332	154	51	4	-	4	51	154	332	1197	2571			
(7, 3, 2)	2575	968	348	196	123	78	42	78	123	196	348	968	2575			
(5, 2, 2, 2, 1)	3419	694	113	32	2	-	-	-	2	32	113	694	3419			
(8, 2, 1, 1)	2697	774	413	157	94	53	26	53	94	157	413	774	2697			
(3, 2, 2, 2, 1, 1, 1)	3517	641	9	-	-	-	-	-	-	-	9	641	3517			
(7, 3, 1, 1)	2676	887	286	114	60	28	26	28	60	114	286	887	2676			
(6, 4, 1, 1)	2600	875	328	107	48	35	6	35	48	107	328	875	2600			
(8, 3, 1)	1719	833	410	224	177	136	104	136	177	224	410	833	1719			
(7, 4, 1)	1742	880	399	222	149	113	74	113	149	222	399	880	1742			
(9, 2, 1)	1518	607	509	353	236	180	114	180	236	353	509	607	1518			
(6, 5, 1)	1648	845	423	272	142	95	68	95	142	272	423	845	1648			
(7, 2, 1, 1, 1)	2473	395	140	67	28	-	-	-	28	67	140	395	2473			
(5, 4, 1, 1, 1)	2319	537	149	52	14	-	-	-	14	52	149	537	2319			
(4, 4, 2, 2)	2259	661	84	39	2	-	-	-	2	39	84	661	2259			
(6, 3, 1, 1, 1)	2301	536	104	41	18	-	-	-	18	41	104	536	2301			
(4, 2, 2, 1, 1, 1, 1)	2505	343	31	9	-	-	-	-	-	9	31	343	2505			
(3, 2, 2, 2, 2, 1)	2184	485	16	-	-	-	-	-	-	-	16	485	2184			
(3, 3, 2, 1, 1, 1, 1)	2171	390	14	-	-	-	-	-	-	-	14	390	2171			
(3, 3, 2, 2, 2)	1945	511	20	-	-	-	-	-	-	-	20	511	1945			
(8, 2, 2)	1327	390	268	127	68	49	90	49	68	127	268	390	1327			
(6, 3, 3)	1265	630	170	91	39	10	-	10	39	91	170	630	1265			
(5, 5, 2)	1300	483	194	116	51	16	18	16	51	116	194	483	1300			
(6, 2, 2, 2)	1563	379	85	34	10	-	-	-	10	34	85	379	1563			
(5, 5, 1, 1)	1268	438	158	78	20	6	10	6	20	78	158	438	1268			
(5, 3, 1, 1, 1, 1)	1572	266	53	16	-	-	-	-	-	16	53	266	1572			
(6, 2, 1, 1, 1, 1)	1409	191	54	25	6	-	-	-	6	25	54	191	1409			
(3, 3, 3, 1, 1, 1)	1174	262	17	-	-	-	-	-	-	-	17	262	1174			
(3, 2, 2, 1, 1, 1, 1, 1)	1264	162	3	-	-	-	-	-	-	-	3	162	1264			
(10, 1, 1)	370	289	222	175	134	102	128	102	134	175	222	289	370			
(4, 2, 2, 2, 2)	919	224	15	12	-	-	-	-	-	12	15	224	919			
(5, 2, 1, 1, 1, 1, 1)	782	122	36	6	-	-	-	-	-	6	36	122	782			
(9, 1, 1, 1)	439	207	147	77	50	24	-	24	50	77	147	207	439			
(4, 4, 1, 1, 1, 1)	754	107	33	5	1	-	-	-	1	5	33	107	754			
(4, 3, 1, 1, 1, 1, 1)	704	107	14	2	-	-	-	-	-	2	14	107	704			
(2, 2, 2, 2, 1, 1, 1, 1)	699	120	-	-	-	-	-	-	-	-	-	120	699			
(4, 4, 4)	422	200	67	26	5	-	-	-	5	26	67	200	422			
(2, 2, 2, 2, 2, 1, 1)	559	139	-	-	-	-	-	-	-	-	-	139	559			

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(8, 1, 1, 1, 1)	367	119	65	24	15	-	-	-	15	24	65	119	367			
(3, 3, 3, 3)	404	155	17	-	-	-	-	-	-	-	17	155	404			
(4, 2, 1, 1, 1, 1, 1, 1)	451	40	9	3	-	-	-	-	-	3	9	40	451			
(2, 2, 2, 1, 1, 1, 1, 1, 1)	286	25	-	-	-	-	-	-	-	-	-	25	286			
(7, 1, 1, 1, 1, 1)	238	31	22	8	-	-	-	-	-	8	22	31	238			
(3, 3, 1, 1, 1, 1, 1, 1)	126	30	1	-	-	-	-	-	-	-	1	30	126			
(3, 2, 1, 1, 1, 1, 1, 1, 1)	132	21	1	-	-	-	-	-	-	-	1	21	132			
(2, 2, 2, 2, 2, 2)	95	18	-	-	-	-	-	-	-	-	-	18	95			
(6, 1, 1, 1, 1, 1, 1)	74	11	8	3	-	-	-	-	-	3	8	11	74			
(5, 1, 1, 1, 1, 1, 1, 1)	51	12	5	-	-	-	-	-	-	-	5	12	51			
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	38	5	-	-	-	-	-	-	-	-	-	5	38			
(4, 1, 1, 1, 1, 1, 1, 1, 1)	33	2	2	-	-	-	-	-	-	-	2	2	33			
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	7	1	-	-	-	-	-	-	-	-	-	1	7			
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	6	1	-	-	-	-	-	-	-	-	-	1	6			
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	-	1			
(4, 3, 2, 2, 1, 1)	41913	7991	699	128	2	-	-	-	-	2	128	699	7991	41913		
(5, 3, 2, 2, 1)	35965	8037	988	312	31	-	-	-	-	31	312	988	8037	35965		
(4, 3, 3, 2, 1)	35283	8598	860	160	6	-	-	-	-	6	160	860	8598	35283		
(5, 4, 2, 1, 1)	29665	6352	1440	387	64	8	-	-	8	64	387	1440	6352	29665		
(6, 3, 2, 1, 1)	29128	6216	1181	375	111	13	-	-	13	111	375	1181	6216	29128		
(5, 4, 3, 1)	21573	6971	1495	447	97	10	-	-	10	97	447	1495	6971	21573		
(5, 3, 2, 1, 1, 1)	25363	4395	610	154	13	-	-	-	-	13	154	610	4395	25363		
(7, 3, 2, 1)	21475	5766	1430	522	289	94	27	27	94	289	522	1430	5766	21475		
(6, 4, 2, 1)	21328	5494	1555	624	198	61	38	38	61	198	624	1555	5494	21328		
(4, 4, 2, 2, 1)	17796	3784	564	164	6	-	-	-	-	6	164	564	3784	17796		
(3, 3, 2, 2, 1, 1, 1)	16854	2896	77	-	-	-	-	-	-	-	-	77	2896	16854		
(5, 3, 3, 1, 1)	15210	3545	496	138	14	-	-	-	-	14	138	496	3545	15210		
(7, 2, 2, 1, 1)	15116	2741	788	215	86	15	-	-	15	86	215	788	2741	15116		
(3, 3, 2, 2, 2, 1)	15314	3489	107	-	-	-	-	-	-	-	-	107	3489	15314		
(4, 4, 3, 2)	13831	4085	679	161	13	-	-	-	-	13	161	679	4085	13831		
(5, 4, 2, 2)	13974	3613	760	264	26	4	-	-	4	26	264	760	3613	13974		
(5, 3, 3, 2)	13942	3811	557	209	44	-	-	-	-	44	209	557	3811	13942		
(6, 3, 2, 2)	13774	3573	634	228	62	5	-	-	5	62	228	634	3573	13774		
(4, 4, 3, 1, 1)	13861	3370	508	104	6	-	-	-	-	6	104	508	3370	13861		
(3, 3, 3, 2, 1, 1)	14484	2940	148	-	-	-	-	-	-	-	-	148	2940	14484		
(5, 2, 2, 2, 1, 1)	14552	2518	375	67	4	-	-	-	-	4	67	375	2518	14552		
(4, 3, 2, 2, 2)	13428	3332	231	56	1	-	-	-	-	1	56	231	3332	13428		
(4, 3, 2, 1, 1, 1, 1)	14490	2144	189	36	1	-	-	-	-	1	36	189	2144	14490		
(6, 3, 3, 1)	10713	3497	684	233	77	13	-	-	13	77	233	684	3497	10713		
(6, 2, 2, 2, 1)	12222	2395	372	156	27	-	-	-	-	27	156	372	2395	12222		
(5, 5, 2, 1)	10967	2787	848	379	94	26	9	9	26	94	379	848	2787	10967		
(8, 2, 2, 1)	11003	2323	908	415	186	74	64	64	74	186	415	908	2323	11003		
(6, 2, 2, 1, 1, 1)	12345	1913	424	66	20	-	-	-	-	20	66	424	1913	12345		
(4, 3, 3, 1, 1, 1)	11692	2407	199	41	1	-	-	-	-	1	41	199	2407	11692		
(4, 4, 2, 1, 1, 1)	11875	2042	338	52	4	-	-	-	-	4	52	338	2042	11875		
(4, 2, 2, 2, 1, 1, 1)	11012	1818	164	27	-	-	-	-	-	-	27	164	1818	11012		
(6, 4, 3)	7466	3366	1043	453	164	64	28	28	64	164	453	1043	3366	7466		
(7, 4, 2)	7536	2835	1088	490	271	155	95	95	155	271	490	1088	2835	7536		
(8, 3, 2)	7245	2582	1045	515	335	244	156	156	244	335	515	1045	2582	7245		
(6, 5, 2)	7260	2603	1096	554	231	99	75	75	99	231	554	1096	2603	7260		
(8, 3, 1, 1)	7367	2473	863	317	190	105	45	45	105	190	317	863	2473	7367		
(6, 5, 1, 1)	7302	2364	888	349	166	77	29	29	77	166	349	888	2364	7302		
(7, 4, 1, 1)	7243	2387	832	284	177	93	28	28	93	177	284	832	2387	7243		

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 2, 2, 2, 2, 1, 1)	9108	1709	36	-	-	-	-	-	-	-	-	36	1709	9108		
(9, 2, 1, 1)	6952	1782	995	501	281	183	48	48	183	281	501	995	1782	6952		
(8, 2, 1, 1, 1)	7180	1381	651	218	118	63	-	-	63	118	218	651	1381	7180		
(4, 2, 2, 2, 2, 1)	7978	1440	111	48	-	-	-	-	-	-	48	111	1440	7978		
(7, 5, 1)	4550	2246	1163	720	424	228	157	157	228	424	720	1163	2246	4550		
(8, 4, 1)	4517	2385	1165	538	384	246	207	207	246	384	538	1165	2385	4517		
(7, 3, 1, 1, 1)	7125	1616	331	128	57	15	-	-	15	57	128	331	1616	7125		
(9, 3, 1)	4220	2182	1128	765	438	261	236	236	261	438	765	1128	2182	4220		
(5, 2, 2, 1, 1, 1, 1)	7948	1079	147	36	2	-	-	-	-	2	36	147	1079	7948		
(6, 4, 1, 1, 1)	6571	1545	466	131	54	30	-	-	30	54	131	466	1545	6571		
(10, 2, 1)	3745	1568	1230	723	583	367	336	336	367	583	723	1230	1568	3745		
(3, 3, 3, 2, 2)	6770	1689	81	-	-	-	-	-	-	-	-	81	1689	6770		
(3, 2, 2, 2, 1, 1, 1, 1)	7354	1136	14	-	-	-	-	-	-	-	-	14	1136	7354		
(5, 4, 4)	3958	1826	552	232	67	10	-	-	10	67	232	552	1826	3958		
(4, 3, 3, 3)	4615	1500	182	40	4	-	-	-	-	4	40	182	1500	4615		
(5, 5, 3)	3738	1556	485	324	117	20	11	11	20	117	324	485	1556	3738		
(7, 3, 3)	3669	1584	467	264	136	67	33	33	67	136	264	467	1584	3669		
(7, 2, 2, 2)	4612	1128	269	100	35	3	-	-	3	35	100	269	1128	4612		
(7, 2, 1, 1, 1, 1)	5084	683	203	76	28	-	-	-	-	28	76	203	683	5084		
(5, 4, 1, 1, 1, 1)	4825	899	230	58	14	-	-	-	-	14	58	230	899	4825		
(9, 2, 2)	3411	1051	600	325	202	136	86	86	136	202	325	600	1051	3411		
(6, 3, 1, 1, 1, 1)	4552	798	144	48	18	-	-	-	-	18	48	144	798	4552		
(4, 4, 4, 1)	3456	1079	242	84	9	-	-	-	-	9	84	242	1079	3456		
(4, 2, 2, 1, 1, 1, 1, 1)	4205	530	51	9	-	-	-	-	-	-	9	51	530	4205		
(5, 5, 1, 1, 1)	3492	836	233	87	19	7	-	-	7	19	87	233	836	3492		
(6, 6, 1)	2139	1150	599	311	182	108	79	79	108	182	311	599	1150	2139		
(5, 2, 2, 2, 2)	3566	725	73	28	2	-	-	-	-	2	28	73	725	3566		
(3, 3, 2, 1, 1, 1, 1, 1)	3575	540	16	-	-	-	-	-	-	-	16	540	3575			
(3, 3, 3, 3, 1)	3095	852	55	-	-	-	-	-	-	-	55	852	3095			
(11, 1, 1)	815	736	633	506	313	261	177	177	261	313	506	633	736	815		
(5, 3, 1, 1, 1, 1, 1)	2557	369	67	16	-	-	-	-	-	-	16	67	369	2557		
(3, 3, 3, 1, 1, 1, 1)	2343	446	22	-	-	-	-	-	-	-	22	446	2343			
(6, 2, 1, 1, 1, 1, 1)	2351	269	62	21	6	-	-	-	-	6	21	62	269	2351		
(10, 1, 1, 1)	1165	510	325	161	102	41	24	24	41	102	161	325	510	1165		
(3, 2, 2, 1, 1, 1, 1, 1, 1)	1925	214	4	-	-	-	-	-	-	-	4	214	1925			
(3, 2, 2, 2, 2, 2)	1695	370	7	-	-	-	-	-	-	-	7	370	1695			
(2, 2, 2, 2, 2, 1, 1, 1)	1521	303	-	-	-	-	-	-	-	-	-	303	1521			
(4, 4, 1, 1, 1, 1, 1, 1)	1242	143	43	7	1	-	-	-	-	1	7	43	143	1242		
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1)	1215	190	-	-	-	-	-	-	-	-	-	-	190	1215		
(9, 1, 1, 1, 1)	862	242	130	62	46	3	-	-	3	46	62	130	242	862		
(5, 2, 1, 1, 1, 1, 1, 1)	1143	143	36	6	-	-	-	-	-	-	6	36	143	1143		
(4, 3, 1, 1, 1, 1, 1, 1, 1)	1003	123	13	2	-	-	-	-	-	-	2	13	123	1003		
(8, 1, 1, 1, 1, 1)	580	131	79	25	15	-	-	-	-	15	25	79	131	580		
(2, 2, 2, 2, 2, 2, 2, 1)	713	113	-	-	-	-	-	-	-	-	-	-	113	713		
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1)	620	50	12	3	-	-	-	-	-	-	3	12	50	620		
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	401	30	-	-	-	-	-	-	-	-	-	-	30	401		
(7, 1, 1, 1, 1, 1, 1, 1)	340	34	24	8	-	-	-	-	-	-	8	24	34	340		
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	161	24	1	-	-	-	-	-	-	-	-	1	24	161		
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1)	147	35	1	-	-	-	-	-	-	-	-	1	35	147		
(6, 1, 1, 1, 1, 1, 1, 1, 1)	95	8	7	3	-	-	-	-	-	-	3	7	8	95		
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1)	55	14	5	-	-	-	-	-	-	-	5	14	55			
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	44	5	-	-	-	-	-	-	-	-	-	-	5	44		
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	39	3	2	-	-	-	-	-	-	-	-	-	3	39		

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	7	1	-	-	-	-	-	-	-	-	-	-	1	7		
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	6	1	-	-	-	-	-	-	-	-	-	-	1	6		
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	-	-	1		
(5, 3, 2, 2, 1, 1)	156696	27096	2779	657	49	-	-	-	-	-	49	657	2779	27096	156696	
(4, 3, 3, 2, 1, 1)	151897	29882	2209	379	9	-	-	-	-	-	9	379	2209	29882	151897	
(5, 3, 3, 2, 1)	126291	28490	3046	913	100	-	-	-	-	-	100	913	3046	28490	126291	
(5, 4, 2, 2, 1)	126234	24926	4205	1168	129	6	-	-	-	6	129	1168	4205	24926	126234	
(6, 3, 2, 2, 1)	125411	25682	3530	1130	269	11	-	-	-	11	269	1130	3530	25682	125411	
(4, 4, 3, 2, 1)	122190	27764	3485	822	36	-	-	-	-	-	36	822	3485	27764	122190	
(4, 3, 2, 2, 2, 1)	117805	22441	1323	307	3	-	-	-	-	-	3	307	1323	22441	117805	
(4, 3, 2, 2, 1, 1, 1)	121230	18695	1338	213	2	-	-	-	-	-	2	213	1338	18695	121230	
(5, 4, 3, 1, 1)	97715	23008	3866	936	129	5	-	-	-	5	129	936	3866	23008	97715	
(5, 4, 3, 2)	92898	25938	4597	1437	247	23	-	-	-	23	247	1437	4597	25938	92898	
(7, 3, 2, 1, 1)	97646	19701	3953	1353	488	119	5	-	5	119	488	1353	3953	19701	97646	
(6, 4, 2, 1, 1)	94780	19677	4601	1238	336	93	30	-	30	93	336	1238	4601	19677	94780	
(5, 4, 2, 1, 1, 1)	85684	13666	2529	544	70	8	-	-	-	8	70	544	2529	13666	85684	
(6, 3, 2, 1, 1, 1)	84350	14484	2207	534	124	11	-	-	-	11	124	534	2207	14484	84350	
(6, 4, 3, 1)	69480	21949	4873	1646	445	96	23	10	23	96	445	1646	4873	21949	69480	
(6, 5, 2, 1)	69111	17825	5382	2200	684	209	85	82	85	209	684	2200	5382	17825	69111	
(7, 4, 2, 1)	69224	17834	5381	1776	785	290	106	82	106	290	785	1776	5381	17834	69224	
(8, 3, 2, 1)	68621	17761	5131	1837	957	416	167	138	167	416	957	1837	5131	17761	68621	
(4, 3, 3, 2, 2)	72520	17491	1268	234	8	-	-	-	-	-	8	234	1268	17491	72520	
(4, 4, 2, 2, 1, 1)	75940	12897	1461	359	9	-	-	-	-	-	9	359	1461	12897	75940	
(3, 3, 2, 2, 2, 1, 1)	67655	12414	286	-	-	-	-	-	-	-	-	-	286	12414	67655	
(3, 3, 3, 2, 2, 1)	58224	12475	426	-	-	-	-	-	-	-	-	-	426	12475	58224	
(5, 3, 2, 1, 1, 1, 1)	55510	7686	830	188	13	-	-	-	-	-	13	188	830	7686	55510	
(6, 3, 3, 1, 1)	49309	11652	1793	486	134	8	-	-	-	8	134	486	1793	11652	49309	
(5, 5, 2, 1, 1)	50142	9621	2434	770	142	35	1	-	1	35	142	770	2434	9621	50142	
(6, 3, 3, 2)	47118	12852	2112	638	225	37	-	-	-	37	225	638	2112	12852	47118	
(5, 3, 2, 2, 2)	49793	11016	942	285	36	-	-	-	-	-	36	285	942	11016	49793	
(8, 2, 2, 1, 1)	49325	8696	2763	784	333	99	36	-	36	99	333	784	2763	8696	49325	
(6, 2, 2, 2, 1, 1)	51798	8330	1221	302	42	-	-	-	-	-	42	302	1221	8330	51798	
(6, 4, 2, 2)	46683	11169	2558	992	205	35	30	-	30	35	205	992	2558	11169	46683	
(7, 3, 2, 2)	46415	11217	2270	770	325	72	5	-	5	72	325	770	2270	11217	46415	
(4, 3, 3, 3, 1)	41878	10819	938	148	9	-	-	-	-	-	9	148	938	10819	41878	
(5, 3, 3, 1, 1, 1)	43646	7963	861	196	14	-	-	-	-	-	14	196	861	7963	43646	
(7, 2, 2, 2, 1)	42637	7420	1468	484	163	13	-	-	-	13	163	484	1468	7420	42637	
(7, 2, 2, 1, 1, 1)	43490	6204	1322	341	109	11	-	-	-	11	109	341	1322	6204	43490	
(5, 4, 4, 1)	35255	10925	2573	855	172	13	-	-	-	13	172	855	2573	10925	35255	
(5, 2, 2, 2, 1, 1, 1)	42631	6026	662	117	4	-	-	-	-	-	4	117	662	6026	42631	
(3, 3, 3, 2, 1, 1, 1)	41935	6981	236	-	-	-	-	-	-	-	-	-	236	6981	41935	
(4, 4, 3, 1, 1, 1)	40287	7603	943	157	9	-	-	-	-	-	9	157	943	7603	40287	
(5, 5, 3, 1)	34637	10578	2408	1099	209	38	8	-	8	38	209	1099	2408	10578	34637	
(7, 3, 3, 1)	34471	10355	2268	949	321	113	26	4	26	113	321	949	2268	10355	34471	
(9, 2, 2, 1)	33792	6888	3057	1396	658	294	106	80	106	294	658	1396	3057	6888	33792	
(3, 3, 2, 2, 1, 1, 1, 1)	38671	5733	114	-	-	-	-	-	-	-	-	-	114	5733	38671	
(4, 2, 2, 2, 2, 1, 1)	34721	5645	313	108	-	-	-	-	-	-	-	-	108	313	5645	34721
(7, 4, 3)	22579	9903	3128	1377	677	295	146	116	146	295	677	1377	3128	9903	22579	
(6, 5, 3)	22591	9392	3154	1655	642	218	92	88	92	218	642	1655	3154	9392	22591	
(7, 5, 2)	21492	7991	3245	1644	775	415	235	164	235	415	775	1644	3245	7991	21492	
(5, 2, 2, 2, 2, 1)	29991	5051	457	161	6	-	-	-	-	-	6	161	457	5051	29991	
(8, 4, 2)	20867	8009	3193	1413	842	501	419	354	419	501	842	1413	3193	8009	20867	
(9, 3, 2)	20518	7483	3094	1669	1086	627	419	330	419	627	1086	1669	3094	7483	20518	

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(8, 4, 1, 1)	22118	7769	2604	919	513	313	177	94	177	313	513	919	2604	7769	22118	
(7, 5, 1, 1)	22410	7061	2722	1258	525	302	125	36	125	302	525	1258	2722	7061	22410	
(9, 3, 1, 1)	21575	6901	2752	1372	588	372	157	44	157	372	588	1372	2752	6901	21575	
(3, 2, 2, 2, 2, 1, 1, 1)	28073	4648	76	-	-	-	-	-	-	-	-	-	76	4648	28073	
(5, 5, 2, 2)	24521	5664	1257	523	78	9	1	-	1	9	78	523	1257	5664	24521	
(4, 4, 3, 3)	23228	7442	1055	233	12	-	-	-	-	-	12	233	1055	7442	23228	
(10, 2, 1, 1)	20112	5433	3007	1369	874	450	302	102	302	450	874	1369	3007	5433	20112	
(4, 4, 2, 2, 2)	25360	5241	584	143	6	-	-	-	-	-	6	143	584	5241	25360	
(6, 2, 2, 1, 1, 1, 1)	26634	3388	631	92	18	-	-	-	-	-	18	92	631	3388	26634	
(4, 2, 2, 2, 1, 1, 1, 1)	26640	3639	292	42	-	-	-	-	-	-	42	292	3639	26640		
(8, 3, 1, 1, 1)	22777	5242	1416	462	227	118	20	-	20	118	227	462	1416	5242	22777	
(4, 3, 3, 1, 1, 1, 1)	25591	4220	282	57	1	-	-	-	-	-	1	57	282	4220	25591	
(4, 3, 2, 1, 1, 1, 1, 1)	26525	3318	244	45	1	-	-	-	-	-	1	45	244	3318	26525	
(4, 4, 2, 1, 1, 1, 1, 1)	25916	3313	562	66	4	-	-	-	-	-	4	66	562	3313	25916	
(6, 5, 1, 1, 1)	22285	5207	1576	505	174	79	13	-	13	79	174	505	1576	5207	22285	
(7, 4, 1, 1, 1)	21915	5065	1526	429	203	91	11	-	11	91	203	429	1526	5065	21915	
(9, 2, 1, 1, 1)	21646	4232	1939	764	332	165	28	-	28	165	332	764	1939	4232	21646	
(8, 5, 1)	12751	6313	3331	1889	1167	703	538	426	538	703	1167	1889	3331	6313	12751	
(9, 4, 1)	12166	6671	3336	1909	1095	710	478	422	478	710	1095	1909	3336	6671	12166	
(7, 6, 1)	12613	6486	3311	1799	1060	623	403	334	403	623	1060	1799	3311	6486	12613	
(10, 3, 1)	11698	5635	3252	2111	1377	784	599	400	599	784	1377	2111	3252	5635	11698	
(11, 2, 1)	9980	4568	3455	2033	1423	1095	767	788	767	1095	1423	2033	3455	4568	9980	
(5, 3, 3, 3)	15493	4703	676	219	27	-	-	-	-	-	27	219	676	4703	15493	
(8, 2, 2, 2)	16189	3175	881	383	142	76	36	-	36	76	142	383	881	3175	16189	
(4, 4, 4, 2)	15460	4250	846	194	13	-	-	-	-	-	13	194	846	4250	15460	
(4, 4, 4, 1, 1)	15562	3725	707	144	14	-	-	-	-	-	14	144	707	3725	15562	
(8, 2, 1, 1, 1, 1)	16147	2416	926	243	116	49	-	-	-	49	116	243	926	2416	16147	
(5, 5, 4)	11728	5087	1650	817	230	56	9	-	9	56	230	817	1650	5087	11728	
(3, 3, 2, 2, 2, 2)	15678	3506	83	-	-	-	-	-	-	-	-	-	83	3506	15678	
(6, 4, 4)	11108	5355	1586	683	215	71	35	14	35	71	215	683	1586	5355	11108	
(8, 3, 3)	11349	4381	1361	726	470	238	135	162	135	238	470	726	1361	4381	11349	
(7, 3, 1, 1, 1, 1)	15302	2707	512	159	54	-	-	-	-	-	54	159	512	2707	15302	
(6, 4, 1, 1, 1, 1)	14768	2741	706	147	61	24	-	-	-	24	61	147	706	2741	14768	
(3, 2, 2, 2, 2, 2, 1)	14973	2814	32	-	-	-	-	-	-	-	-	-	32	2814	14973	
(6, 6, 2)	10473	3854	1637	839	340	161	131	114	131	161	340	839	1637	3854	10473	
(5, 2, 2, 1, 1, 1, 1, 1)	15136	1679	211	43	2	-	-	-	-	-	2	43	211	1679	15136	
(6, 6, 1, 1)	10907	3675	1374	498	222	137	71	32	71	137	222	498	1374	3675	10907	
(3, 2, 2, 2, 1, 1, 1, 1, 1)	14717	1894	17	-	-	-	-	-	-	-	-	-	17	1894	14717	
(3, 3, 3, 3, 1, 1)	13423	2764	152	-	-	-	-	-	-	-	-	-	152	2764	13423	
(10, 2, 2)	9420	2777	1746	926	581	411	284	306	284	411	581	926	1746	2777	9420	
(6, 2, 2, 2, 2)	12924	2478	311	114	24	-	-	-	-	-	24	114	311	2478	12924	
(3, 3, 3, 3, 2)	12454	3181	164	-	-	-	-	-	-	-	-	-	164	3181	12454	
(7, 2, 1, 1, 1, 1, 1)	8843	938	231	90	28	-	-	-	-	-	28	90	231	938	8843	
(5, 4, 1, 1, 1, 1, 1)	8547	1205	284	60	14	-	-	-	-	-	14	60	284	1205	8547	
(6, 3, 1, 1, 1, 1, 1)	8133	1132	172	45	18	-	-	-	-	-	18	45	172	1132	8133	
(12, 1, 1)	2107	1906	1634	1222	956	589	585	722	585	589	956	1222	1634	1906	2107	
(5, 5, 1, 1, 1, 1)	7513	1353	366	100	18	-	-	-	-	-	18	100	366	1353	7513	
(4, 2, 2, 1, 1, 1, 1, 1, 1)	7224	727	56	12	-	-	-	-	-	-	-	12	56	727	7224	
(4, 2, 2, 2, 2, 2)	6619	1227	56	21	-	-	-	-	-	-	-	21	56	1227	6619	
(11, 1, 1, 1)	3161	1607	972	533	272	254	115	88	115	254	272	533	972	1607	3161	
(3, 3, 2, 1, 1, 1, 1, 1, 1)	5612	766	20	-	-	-	-	-	-	-	-	-	20	766	5612	
(3, 3, 3, 1, 1, 1, 1, 1)	4062	649	25	-	-	-	-	-	-	-	-	-	25	649	4062	
(2, 2, 2, 2, 2, 1, 1, 1, 1)	3990	699	-	-	-	-	-	-	-	-	-	-	-	699	3990	

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label																	
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15		
(10, 1, 1, 1, 1)	2775	837	381	207	114	36	-	-	-	-	36	114	207	381	837	2775		
(5, 3, 1, 1, 1, 1, 1, 1)	3739	449	78	16	-	-	-	-	-	-	-	16	78	449	3739			
(6, 2, 1, 1, 1, 1, 1, 1)	3549	346	75	28	6	-	-	-	-	-	6	28	75	346	3549			
(2, 2, 2, 2, 2, 2, 1, 1)	3213	559	-	-	-	-	-	-	-	-	-	-	-	559	3213			
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1)	2730	276	4	-	-	-	-	-	-	-	-	-	4	276	2730			
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1)	2214	286	-	-	-	-	-	-	-	-	-	-	-	286	2214			
(9, 1, 1, 1, 1, 1)	1545	380	198	74	40	-	-	-	-	-	40	74	198	380	1545			
(4, 4, 1, 1, 1, 1, 1, 1)	1906	170	49	5	1	-	-	-	-	-	1	5	49	170	1906			
(5, 2, 1, 1, 1, 1, 1, 1, 1)	1636	177	46	6	-	-	-	-	-	-	-	6	46	177	1636			
(4, 3, 1, 1, 1, 1, 1, 1, 1)	1383	152	16	2	-	-	-	-	-	-	-	2	16	152	1383			
(8, 1, 1, 1, 1, 1, 1)	906	156	86	24	15	-	-	-	-	-	15	24	86	156	906			
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1)	820	54	11	3	-	-	-	-	-	-	-	3	11	54	820			
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	542	38	-	-	-	-	-	-	-	-	-	-	-	38	542			
(2, 2, 2, 2, 2, 2, 2)	429	95	-	-	-	-	-	-	-	-	-	-	-	95	429			
(7, 1, 1, 1, 1, 1, 1, 1)	422	41	26	8	-	-	-	-	-	-	-	8	26	41	422			
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1)	219	38	1	-	-	-	-	-	-	-	-	-	1	38	219			
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	203	27	1	-	-	-	-	-	-	-	-	-	1	27	203			
(6, 1, 1, 1, 1, 1, 1, 1, 1, 1)	120	14	8	3	-	-	-	-	-	-	-	3	8	14	120			
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1)	75	14	5	-	-	-	-	-	-	-	-	-	5	14	75			
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	56	6	-	-	-	-	-	-	-	-	-	-	-	6	56			
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	45	2	2	-	-	-	-	-	-	-	-	-	2	2	45			
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	7	1	-	-	-	-	-	-	-	-	-	-	-	1	7			
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	7	1	-	-	-	-	-	-	-	-	-	-	-	1	7			
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	-	-	-	1			
(5, 4, 3, 2, 1)	896469	194242	26818	6545	724	39	-	-	-	-	39	724	6545	26818	194242	896469		
(4, 3, 3, 2, 2, 1)	653347	127455	7750	1215	21	-	-	-	-	-	-	21	1215	7750	127455	653347		
(5, 3, 3, 2, 1, 1)	566999	103398	9272	2073	150	-	-	-	-	-	-	150	2073	9272	103398	566999		
(5, 4, 2, 2, 1, 1)	571535	93417	12655	2980	214	14	-	-	-	-	14	214	2980	12655	93417	571535		
(6, 3, 2, 2, 1, 1)	568099	94212	10539	2622	490	18	-	-	-	-	18	490	2622	10539	94212	568099		
(4, 4, 3, 2, 1, 1)	556809	100826	10318	1970	62	-	-	-	-	-	-	62	1970	10318	100826	556809		
(4, 3, 2, 2, 2, 1, 1)	532665	87067	4706	852	6	-	-	-	-	-	-	6	852	4706	87067	532665		
(6, 3, 3, 2, 1)	452939	97365	12088	3062	742	57	-	-	-	-	57	742	3062	12088	97365	452939		
(6, 4, 2, 2, 1)	455035	83854	14450	4115	769	130	21	-	-	-	21	130	769	4115	14450	83854	455035	
(5, 3, 2, 2, 1, 1, 1)	476708	71215	6288	1184	61	-	-	-	-	-	-	61	1184	6288	71215	476708		
(7, 3, 2, 2, 1)	444652	84229	13417	3632	1274	195	-	-	-	-	195	1274	3632	13417	84229	444652		
(4, 3, 3, 2, 1, 1, 1)	459758	75207	4729	625	11	-	-	-	-	-	-	11	625	4729	75207	459758		
(5, 3, 2, 2, 2, 1)	441526	79318	5963	1478	89	-	-	-	-	-	-	89	1478	5963	79318	441526		
(6, 4, 3, 2)	344888	93137	16878	5383	1198	245	72	6	-	-	6	72	245	1198	5383	16878	344888	
(6, 4, 3, 1, 1)	336136	79557	14098	3553	737	122	21	-	-	-	21	122	737	3553	14098	79557	336136	
(6, 5, 2, 1, 1)	340791	66332	15962	4541	1106	280	90	-	-	-	90	280	1106	4541	15962	66332	340791	
(8, 3, 2, 1, 1)	340624	66529	14980	4303	1774	572	156	-	-	-	156	572	1774	4303	14980	66529	340624	
(7, 4, 2, 1, 1)	339880	66189	15221	4085	1363	369	98	-	-	-	98	369	1363	4085	15221	66189	339880	
(5, 4, 3, 1, 1, 1)	298827	54820	7749	1429	145	5	-	-	-	-	5	145	1429	7749	54820	298827		
(7, 3, 2, 1, 1, 1)	298618	47227	7355	1967	626	91	-	-	-	-	91	626	1967	7355	47227	298618		
(6, 4, 2, 1, 1, 1)	296911	45764	8680	2010	414	94	21	-	-	-	21	94	414	2010	8680	45764	296911	
(5, 3, 3, 2, 2)	279349	60173	5174	1353	157	-	-	-	-	-	-	157	1353	5174	60173	279349		
(6, 5, 3, 1)	244791	72270	18181	6874	1883	435	141	41	-	-	41	141	435	1883	6874	18181	72270	244791
(4, 4, 3, 2, 2)	277407	59406	6047	1241	43	-	-	-	-	-	-	43	1241	6047	59406	277407		
(7, 4, 3, 1)	239669	73534	17378	5806	2077	685	192	55	55	192	685	2077	5806	17378	73534	239669		
(4, 3, 2, 2, 1, 1, 1, 1)	289062	38971	2397	313	3	-	-	-	-	-	-	3	313	2397	38971	289062		
(8, 4, 2, 1)	237301	62126	18205	6221	2855	1279	649	357	357	649	1279	2855	6221	18205	62126	237301		
(7, 5, 2, 1)	234536	60756	19227	7276	2685	1006	409	155	155	409	1006	2685	7276	19227	60756	234536		
(9, 3, 2, 1)	228876	57764	18473	7848	3647	1812	778	329	329	778	1812	3647	7848	18473	57764	228876		

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 3, 2, 2, 1, 1)	261943	47242	1313	-	-	-	-	-	-	-	-	-	-	1313	47242	261943
(4, 4, 3, 3, 1)	221423	55338	6152	1143	54	-	-	-	-	-	-	54	1143	6152	55338	221423
(5, 5, 2, 2, 1)	224409	41928	7462	2398	281	40	-	-	-	-	40	281	2398	7462	41928	224409
(4, 4, 2, 2, 1, 1, 1)	235318	33540	3600	646	15	-	-	-	-	-	-	15	646	3600	33540	235318
(4, 4, 2, 2, 2, 1)	224145	39146	3606	666	24	-	-	-	-	-	-	24	666	3606	39146	224145
(3, 3, 2, 2, 2, 1, 1, 1)	214350	33782	641	-	-	-	-	-	-	-	-	-	-	641	33782	214350
(5, 4, 3, 3)	172353	52379	8094	2335	330	17	-	-	-	-	17	330	2335	8094	52379	172353
(4, 3, 3, 3, 2)	184563	44761	3251	458	22	-	-	-	-	-	-	22	458	3251	44761	184563
(5, 4, 4, 2)	172698	47512	9057	2578	322	27	-	-	-	-	27	322	2578	9057	47512	172698
(5, 5, 3, 2)	173352	45823	8793	3245	549	75	8	-	-	-	8	75	549	3245	8793	45823
(5, 4, 2, 1, 1, 1, 1)	197099	27192	4478	746	85	8	-	-	-	-	8	85	746	4478	27192	197099
(4, 3, 3, 3, 1, 1)	187656	38564	2839	373	13	-	-	-	-	-	-	13	373	2839	38564	187656
(6, 3, 2, 2, 2)	188020	35553	4027	1120	209	15	-	-	-	-	15	209	1120	4027	35553	188020
(7, 3, 3, 2)	171220	45435	7797	2593	1077	283	24	-	-	24	283	1077	2593	7797	45435	171220
(6, 5, 2, 2)	173805	39118	9685	3631	830	218	76	24	24	76	218	830	3631	9685	39118	173805
(5, 4, 2, 2, 2)	184740	36461	4485	1265	107	10	-	-	-	-	10	107	1265	4485	36461	184740
(7, 2, 2, 2, 1, 1)	192465	28433	4300	1160	293	16	-	-	-	-	16	293	1160	4300	28433	192465
(6, 3, 2, 1, 1, 1, 1)	195052	27061	3461	696	156	11	-	-	-	-	11	156	696	3461	27061	195052
(8, 3, 2, 2)	173625	38312	8416	3078	1341	537	201	70	70	201	537	1341	3078	8416	38312	173625
(7, 4, 2, 2)	170526	39763	9131	2721	1041	312	86	34	34	86	312	1041	2721	9131	39763	170526
(5, 4, 4, 1, 1)	171344	40023	7399	1818	251	17	-	-	-	-	17	251	1818	7399	40023	171344
(5, 5, 3, 1, 1)	172361	38542	7147	2284	319	48	2	-	-	2	48	319	2284	7147	38542	172361
(7, 3, 3, 1, 1)	169951	37717	6243	1949	575	141	10	-	-	10	141	575	1949	6243	37717	169951
(9, 2, 2, 1, 1)	167823	26594	8724	3131	1222	409	124	-	-	124	409	1222	3131	8724	26594	167823
(5, 3, 3, 3, 1)	151859	35684	3781	1030	113	-	-	-	-	-	-	113	1030	3781	35684	151859
(4, 4, 4, 2, 1)	150650	31923	4682	1033	66	-	-	-	-	-	-	66	1033	4682	31923	150650
(6, 2, 2, 2, 1, 1, 1)	162121	21497	2568	626	73	-	-	-	-	-	-	73	626	2568	21497	162121
(6, 3, 3, 1, 1, 1)	152224	28233	3441	715	159	8	-	-	-	-	8	159	715	3441	28233	152224
(8, 2, 2, 2, 1)	151478	24042	4978	1639	612	201	27	-	-	27	201	612	1639	4978	24042	151478
(8, 2, 2, 1, 1, 1)	154241	20744	5304	1416	483	120	27	-	-	27	120	483	1416	5304	20744	154241
(5, 5, 2, 1, 1, 1)	151977	23655	4737	1235	181	27	-	-	-	-	27	181	1235	4737	23655	151977
(5, 5, 4, 1)	123313	37818	9398	3358	662	96	12	-	-	12	96	662	3358	9398	37818	123313
(6, 4, 4, 1)	122010	38726	9297	2929	705	180	42	2	2	42	180	705	2929	9297	38726	122010
(8, 3, 3, 1)	121890	35025	8133	3001	1384	560	227	66	66	227	560	1384	3001	8133	35025	121890
(3, 3, 2, 2, 2, 2, 1)	142975	26565	485	-	-	-	-	-	-	-	-	-	-	485	26565	142975
(6, 6, 2, 1)	118793	30395	9494	3838	1179	444	223	130	130	223	444	1179	3838	9494	30395	118793
(5, 2, 2, 2, 2, 1, 1)	136587	21110	1541	402	12	-	-	-	-	-	-	12	402	1541	21110	136587
(10, 2, 2, 1)	111850	22232	9827	4052	2239	1001	505	232	232	505	1001	2239	4052	9827	22232	111850
(4, 3, 2, 2, 2, 2)	124569	24180	1182	230	4	-	-	-	-	-	-	4	230	1182	24180	124569
(3, 3, 3, 3, 2, 1)	110855	23087	935	-	-	-	-	-	-	-	-	-	-	935	23087	110855
(6, 2, 2, 2, 2, 1)	113865	18550	1649	434	104	-	-	-	-	-	-	104	434	1649	18550	113865
(4, 2, 2, 2, 2, 1, 1, 1)	110902	15254	888	228	-	-	-	-	-	-	-	-	228	888	15254	110902
(6, 5, 4)	71616	31819	10912	4877	1754	510	206	81	81	206	510	1754	4877	10912	31819	71616
(5, 3, 2, 1, 1, 1, 1, 1)	106504	12634	1279	236	13	-	-	-	-	-	-	13	236	1279	12634	106504
(8, 4, 3)	71185	29284	9477	4388	2347	1114	741	594	594	741	1114	2347	4388	9477	29284	71185
(7, 5, 3)	70296	28646	9950	5214	2375	1016	482	267	267	482	1016	2375	5214	9950	28646	70296
(5, 3, 3, 1, 1, 1, 1)	101387	14757	1329	258	14	-	-	-	-	-	-	14	258	1329	14757	101387
(5, 2, 2, 2, 1, 1, 1, 1)	101976	12827	1303	186	6	-	-	-	-	-	-	6	186	1303	12827	101976
(7, 2, 2, 1, 1, 1, 1)	100215	12228	2252	466	134	11	-	-	-	-	11	134	466	2252	12228	100215
(8, 5, 1, 1)	73519	24710	9414	3759	1878	981	548	256	256	548	981	1878	3759	9414	24710	73519
(9, 4, 1, 1)	70468	25093	9511	4297	1927	1010	575	321	321	575	1010	1927	4297	9511	25093	70468
(3, 3, 3, 2, 1, 1, 1, 1)	98757	14027	390	-	-	-	-	-	-	-	-	-	-	390	14027	98757
(7, 6, 1, 1)	71957	24444	9270	3791	1762	909	435	232	232	435	909	1762	3791	9270	24444	71957

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(8, 5, 2)	65818	23948	10878	5278	2733	1610	1130	882	882	1130	1610	2733	5278	10878	23948	65818
(7, 6, 2)	64540	24638	10726	5508	2765	1384	853	699	699	853	1384	2765	5508	10726	24638	64540
(9, 4, 2)	63673	24872	10337	5045	2939	1735	1233	1033	1033	1233	1735	2939	5045	10337	24872	63673
(10, 3, 1, 1)	69733	22243	8780	4149	2172	1172	593	316	316	593	1172	2172	4149	8780	22243	69733
(10, 3, 2)	63402	21658	9580	5457	3415	1994	1430	1177	1177	1430	1994	3415	5457	9580	21658	63402
(4, 4, 3, 1, 1, 1, 1)	91909	13913	1504	223	7	-	-	-	-	-	-	7	223	1504	13913	91909
(11, 2, 1, 1)	63426	18228	10206	5025	2728	1516	757	453	453	757	1516	2728	5025	10206	18228	63426
(8, 4, 1, 1, 1)	74644	18528	5278	1499	653	283	144	-	-	144	283	653	1499	5278	18528	74644
(7, 5, 1, 1, 1)	75717	17368	5082	1742	617	286	39	-	-	39	286	617	1742	5082	17368	75717
(3, 3, 3, 2, 2, 2)	82172	17440	511	-	-	-	-	-	-	-	-	-	511	17440	82172	
(9, 3, 1, 1, 1)	72851	16927	5080	1957	744	377	76	-	-	76	377	744	1957	5080	16927	72851
(10, 2, 1, 1, 1)	72187	13466	5809	2061	956	459	213	-	-	213	459	956	2061	5809	13466	72187
(3, 3, 2, 2, 1, 1, 1, 1)	75590	9530	162	-	-	-	-	-	-	-	-	-	162	9530	75590	
(3, 2, 2, 2, 2, 2, 1, 1)	71207	11799	139	-	-	-	-	-	-	-	-	-	139	11799	71207	
(3, 2, 2, 2, 2, 1, 1, 1, 1)	69838	9952	120	-	-	-	-	-	-	-	-	-	120	9952	69838	
(9, 5, 1)	35178	18007	9937	6268	3520	2542	1673	1475	1475	1673	2542	3520	6268	9937	18007	35178
(4, 4, 4, 3)	56937	18150	2702	658	47	-	-	-	-	-	-	47	658	2702	18150	56937
(8, 6, 1)	36096	18959	9960	5333	3561	2097	1421	1064	1064	1421	2097	3561	5333	9960	18959	36096
(6, 3, 3, 3)	57495	17192	2568	790	237	23	-	-	-	-	23	237	790	2568	17192	57495
(10, 4, 1)	35071	18719	9539	5125	3668	2311	1645	1239	1239	1645	2311	3668	5125	9539	18719	35071
(11, 3, 1)	33358	15529	9215	6359	4157	2960	2007	1624	1624	2007	2960	4157	6359	9215	15529	33358
(9, 2, 2, 2)	56482	10320	3391	1260	634	318	118	42	42	118	318	634	1260	3391	10320	56482
(4, 2, 2, 2, 2, 2, 1)	59034	10262	413	96	-	-	-	-	-	-	-	96	413	10262	59034	
(12, 2, 1)	26268	12513	9950	6232	5230	3756	2684	2213	2213	2684	3756	5230	6232	9950	12513	26268
(8, 3, 1, 1, 1, 1)	53990	9566	2197	529	236	89	-	-	-	-	89	236	529	2197	9566	53990
(6, 5, 1, 1, 1, 1)	52854	9118	2344	554	172	62	-	-	-	-	62	172	554	2344	9118	52854
(7, 4, 1, 1, 1, 1)	51986	8833	2103	493	190	70	-	-	-	-	70	190	493	2103	8833	51986
(9, 2, 1, 1, 1, 1)	52003	7215	2591	868	371	149	-	-	-	-	149	371	868	2591	7215	52003
(7, 4, 4)	35319	16130	4944	2056	993	393	171	77	77	171	393	993	2056	4944	16130	35319
(6, 6, 3)	35349	14748	5235	2716	1098	369	200	173	173	200	369	1098	2716	5235	14748	35349
(4, 2, 2, 2, 1, 1, 1, 1, 1)	52424	6326	456	51	-	-	-	-	-	-	-	51	456	6326	52424	
(6, 2, 2, 1, 1, 1, 1, 1, 1)	51774	5557	892	109	26	-	-	-	-	-	-	26	109	892	5557	51774
(4, 4, 4, 1, 1, 1)	48304	8370	1310	217	15	-	-	-	-	-	-	15	217	1310	8370	48304
(9, 3, 3)	34259	13155	4475	2602	1549	926	499	391	391	499	926	1549	2602	4475	13155	34259
(4, 4, 2, 1, 1, 1, 1, 1)	49757	5882	799	88	5	-	-	-	-	-	-	5	88	799	5882	49757
(7, 2, 2, 2, 2)	45610	8114	1144	280	120	17	-	-	-	-	17	120	280	1144	8114	45610
(4, 3, 3, 1, 1, 1, 1, 1)	47447	6597	399	64	1	-	-	-	-	-	-	1	64	399	6597	47447
(11, 2, 2)	27501	8260	5517	3022	1900	1320	984	819	819	984	1320	1900	3022	5517	8260	27501
(6, 6, 1, 1, 1)	36460	8729	2754	823	273	116	51	-	-	51	116	273	823	2754	8729	36460
(3, 3, 3, 3, 1, 1, 1)	41209	6828	262	-	-	-	-	-	-	-	-	-	262	6828	41209	
(4, 3, 2, 1, 1, 1, 1, 1, 1)	43084	4672	316	53	1	-	-	-	-	-	-	1	53	316	4672	43084
(7, 7, 1)	17490	9460	4830	2793	1825	1116	671	554	554	671	1116	1825	2793	4830	9460	17490
(8, 2, 1, 1, 1, 1, 1)	29938	3515	1223	290	135	49	-	-	-	-	49	135	290	1223	3515	29938
(7, 3, 1, 1, 1, 1, 1)	28794	3725	578	182	54	-	-	-	-	-	-	54	182	578	3725	28794
(6, 4, 1, 1, 1, 1, 1)	27113	3881	864	156	56	24	-	-	-	-	24	56	156	864	3881	27113
(5, 2, 2, 2, 2, 2)	22863	4801	289	93	8	-	-	-	-	-	-	8	93	289	4801	22863
(5, 2, 2, 1, 1, 1, 1, 1, 1)	25090	2424	284	51	2	-	-	-	-	-	-	2	51	284	2424	25090
(3, 2, 2, 2, 1, 1, 1, 1, 1, 1)	24822	2866	25	-	-	-	-	-	-	-	-	-	25	2866	24822	
(13, 1, 1)	5018	4734	4148	3297	2451	2139	1663	1726	1726	1663	2139	2451	3297	4148	4734	5018
(12, 1, 1, 1)	9347	4986	2902	1583	1142	783	484	211	211	484	783	1142	1583	2902	4986	9347
(5, 5, 5)	12208	5053	1979	1066	287	72	20	-	-	20	72	287	1066	1979	5053	12208
(5, 5, 1, 1, 1, 1, 1)	14395	2029	473	123	18	-	-	-	-	-	-	18	123	473	2029	14395
(7, 2, 1, 1, 1, 1, 1, 1)	14297	1342	299	101	28	-	-	-	-	-	-	28	101	299	1342	14297

Table 1: Number of graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 4, 1, 1, 1, 1, 1, 1)	13743	1676	369	62	14	-	-	-	-	-	-	14	62	369	1676	13743
(6, 3, 1, 1, 1, 1, 1, 1)	12602	1486	205	53	18	-	-	-	-	-	-	18	53	205	1486	12602
(11, 1, 1, 1, 1)	8613	2970	1433	687	313	230	66	-	-	66	230	313	687	1433	2970	8613
(2, 2, 2, 2, 2, 2, 1, 1, 1)	10621	1521	-	-	-	-	-	-	-	-	-	-	-	-	1521	10621
(3, 2, 2, 2, 2, 2, 2)	9986	2070	18	-	-	-	-	-	-	-	-	-	-	18	2070	9986
(3, 3, 3, 3, 3)	9307	2457	155	-	-	-	-	-	-	-	-	-	-	155	2457	9307
(4, 2, 2, 1, 1, 1, 1, 1, 1, 1)	10579	987	77	12	-	-	-	-	-	-	-	-	12	77	987	10579
(2, 2, 2, 2, 2, 1, 1, 1, 1, 1)	8301	1215	-	-	-	-	-	-	-	-	-	-	-	-	1215	8301
(3, 3, 2, 1, 1, 1, 1, 1, 1, 1)	8137	956	21	-	-	-	-	-	-	-	-	-	-	21	956	8137
(3, 3, 3, 1, 1, 1, 1, 1, 1)	6691	929	30	-	-	-	-	-	-	-	-	-	-	30	929	6691
(10, 1, 1, 1, 1, 1)	5619	1169	497	212	88	26	-	-	-	-	26	88	212	497	1169	5619
(5, 3, 1, 1, 1, 1, 1, 1, 1)	5433	612	88	16	-	-	-	-	-	-	-	-	16	88	612	5433
(6, 2, 1, 1, 1, 1, 1, 1, 1)	5152	419	80	24	6	-	-	-	-	-	-	6	24	80	419	5152
(2, 2, 2, 2, 2, 2, 2, 1)	3664	713	-	-	-	-	-	-	-	-	-	-	-	-	713	3664
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	3782	339	5	-	-	-	-	-	-	-	-	-	-	5	339	3782
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	3387	401	-	-	-	-	-	-	-	-	-	-	-	-	401	3387
(4, 4, 1, 1, 1, 1, 1, 1, 1)	2876	229	60	7	1	-	-	-	-	-	-	1	7	60	229	2876
(9, 1, 1, 1, 1, 1, 1)	2429	370	168	76	40	-	-	-	-	-	-	40	76	168	370	2429
(5, 2, 1, 1, 1, 1, 1, 1, 1, 1)	2163	201	49	6	-	-	-	-	-	-	-	-	6	49	201	2163
(4, 3, 1, 1, 1, 1, 1, 1, 1, 1)	1786	182	16	2	-	-	-	-	-	-	-	-	2	16	182	1786
(8, 1, 1, 1, 1, 1, 1, 1)	1226	167	97	25	15	-	-	-	-	-	-	15	25	97	167	1226
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1)	1041	70	14	3	-	-	-	-	-	-	-	-	3	14	70	1041
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	713	44	-	-	-	-	-	-	-	-	-	-	-	-	44	713
(7, 1, 1, 1, 1, 1, 1, 1, 1)	589	48	29	8	-	-	-	-	-	-	-	-	8	29	48	589
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1)	237	42	1	-	-	-	-	-	-	-	-	-	-	1	42	237
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	234	30	1	-	-	-	-	-	-	-	-	-	-	1	30	234
(6, 1, 1, 1, 1, 1, 1, 1, 1, 1)	142	9	7	3	-	-	-	-	-	-	-	-	3	7	9	142
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1)	79	14	5	-	-	-	-	-	-	-	-	-	-	5	14	79
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	62	6	-	-	-	-	-	-	-	-	-	-	-	-	6	62
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	53	3	2	-	-	-	-	-	-	-	-	-	-	2	3	53
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	8	1	-	-	-	-	-	-	-	-	-	-	-	-	1	8
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	7	1	-	-	-	-	-	-	-	-	-	-	-	-	1	7
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1

A.2 Graceful Spiders by Leaf Label

Table 2: Number of graceful labelings for spiders order 16 or less, by leaf label
(Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1)	1	2	2	1												
(2, 1, 1)	2	5	4	5	2											
(1, 1, 1, 1)	1	2	2	2	1											
(3, 1, 1)	3	2	4	4	2	3										
(2, 2, 1)	2	5	5	5	5	2										
(2, 1, 1, 1)	3	4	5	5	4	3										
(1, 1, 1, 1, 1)	1	2	2	2	2	1										
(4, 1, 1)	6	7	12	16	12	7	6									
(3, 2, 1)	9	15	21	18	21	15	9									
(3, 1, 1, 1)	3	5	5	6	5	5	3									
(2, 2, 2)	1	4	8	4	8	4	1									
(2, 2, 1, 1)	5	10	11	12	11	10	5									
(2, 1, 1, 1, 1)	3	7	6	8	6	7	3									
(1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	1									
(5, 1, 1)	6	12	24	27	27	24	12	6								
(4, 2, 1)	13	37	50	56	56	50	37	13								
(4, 1, 1, 1)	8	14	15	23	23	15	14	8								
(3, 3, 1)	5	11	24	20	20	24	11	5								
(3, 2, 2)	7	14	19	29	29	19	14	7								
(3, 2, 1, 1)	15	40	43	46	46	43	40	15								
(3, 1, 1, 1, 1)	4	4	6	6	6	6	4	4								
(2, 2, 2, 1)	7	20	24	25	25	24	20	7								
(2, 2, 1, 1, 1)	8	15	16	16	16	15	8									
(2, 1, 1, 1, 1, 1)	4	6	7	7	7	6	4									
(1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	1									
(6, 1, 1)	6	16	32	41	38	41	32	16	6							
(5, 2, 1)	13	52	73	89	98	89	73	52	13							
(5, 1, 1, 1)	9	22	29	36	40	36	29	22	9							
(4, 3, 1)	14	46	74	100	108	100	74	46	14							
(4, 2, 2)	8	31	39	47	44	47	39	31	8							
(4, 2, 1, 1)	40	80	85	102	114	102	85	80	40							
(4, 1, 1, 1, 1)	11	20	21	24	28	24	21	20	11							
(3, 3, 2)	9	34	45	62	60	62	45	34	9							
(3, 3, 1, 1)	12	36	48	46	52	46	48	36	12							
(3, 2, 2, 1)	28	76	102	116	108	116	102	76	28							
(3, 2, 1, 1, 1)	31	55	73	68	66	68	73	55	31							
(3, 1, 1, 1, 1, 1)	4	7	7	9	6	9	7	7	4							
(2, 2, 2, 2)	4	8	12	14	12	14	12	8	4							
(2, 2, 2, 1, 1)	22	48	56	52	64	52	56	48	22							
(2, 2, 1, 1, 1, 1)	12	25	25	27	26	27	25	25	12							
(2, 1, 1, 1, 1, 1, 1)	4	9	8	10	8	10	9	8	4							
(1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	1							
(7, 1, 1)	13	41	66	89	79	79	89	66	41	13						
(6, 2, 1)	26	94	139	157	181	181	157	139	94	26						
(6, 1, 1, 1)	15	21	41	45	46	46	45	41	21	15						
(5, 3, 1)	25	97	149	200	195	195	200	149	97	25						
(5, 2, 2)	8	36	71	88	97	97	88	71	36	8						
(5, 2, 1, 1)	52	142	172	167	191	191	167	172	142	52						
(5, 1, 1, 1, 1)	17	26	37	45	50	50	45	37	26	17						
(4, 4, 1)	14	35	64	97	105	105	97	64	35	14						
(4, 3, 2)	20	92	151	190	204	204	190	151	92	20						
(4, 3, 1, 1)	67	124	164	189	212	212	189	164	124	67						
(4, 2, 2, 1)	64	131	198	215	216	216	215	198	131	64						
(4, 2, 1, 1, 1)	82	169	178	189	207	207	189	178	169	82						
(4, 1, 1, 1, 1, 1)	15	24	27	35	31	31	35	27	24	15						
(3, 3, 3)	3	18	34	31	40	40	31	34	18	3						
(3, 3, 2, 1)	55	178	205	238	244	244	238	205	178	55						
(3, 3, 1, 1, 1)	31	50	71	62	71	71	62	71	50	31						
(3, 2, 2, 2)	18	50	76	72	84	84	72	76	50	18						
(3, 2, 2, 1, 1)	104	237	283	326	315	315	326	283	237	104						
(3, 2, 1, 1, 1, 1)	46	87	96	105	98	98	105	96	87	46						
(3, 1, 1, 1, 1, 1, 1)	5	8	10	9	10	10	9	10	8	5						
(2, 2, 2, 2, 1)	23	38	63	56	60	60	56	63	38	23						
(2, 2, 2, 1, 1, 1)	45	110	117	115	123	123	115	117	110	45						
(2, 2, 1, 1, 1, 1, 1)	17	29	31	32	31	31	32	31	29	17						
(2, 1, 1, 1, 1, 1, 1, 1)	5	8	9	9	9	9	9	9	8	5						
(1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	1						

Table 2: Number of graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for each leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(8, 1, 1)	19	56	147	194	221	214	221	194	147	56	19					
(7, 2, 1)	57	197	302	362	415	412	415	362	302	197	57					
(7, 1, 1, 1)	46	69	97	145	139	128	139	145	97	69	46					
(6, 3, 1)	51	137	293	356	415	418	415	356	293	137	51					
(6, 2, 2)	21	97	164	191	265	240	265	191	164	97	21					
(6, 2, 1, 1)	138	284	390	408	433	446	433	408	390	284	138					
(6, 1, 1, 1, 1)	26	35	49	72	70	66	70	72	49	35	26					
(5, 4, 1)	51	164	295	387	449	464	449	387	295	164	51					
(5, 3, 2)	31	154	362	405	503	492	503	405	362	154	31					
(5, 3, 1, 1)	128	313	420	491	532	552	532	491	420	313	128					
(5, 2, 2, 1)	102	351	451	515	507	548	507	515	451	351	102					
(5, 2, 1, 1, 1)	141	341	382	422	398	482	398	422	382	341	141					
(5, 1, 1, 1, 1, 1)	22	54	56	75	73	76	73	75	56	54	22					
(4, 4, 2)	16	78	168	193	257	220	257	193	168	78	16					
(4, 4, 1, 1)	69	139	174	236	272	260	272	236	174	139	69					
(4, 3, 3)	10	67	175	206	242	226	242	206	175	67	10					
(4, 3, 2, 1)	240	724	919	1056	1160	1154	1160	1056	919	724	240					
(4, 3, 1, 1, 1)	152	272	320	353	383	410	383	353	320	272	152					
(4, 2, 2, 2)	34	130	186	225	204	218	204	225	186	130	34					
(4, 2, 2, 1, 1)	216	650	718	794	765	834	765	794	718	650	216					
(4, 2, 1, 1, 1, 1)	150	259	281	285	294	306	294	285	281	259	150					
(4, 1, 1, 1, 1, 1, 1)	18	36	33	42	41	38	41	42	33	36	18					
(3, 3, 3, 1)	31	124	194	219	223	194	223	219	194	124	31					
(3, 3, 2, 2)	48	191	254	295	341	310	341	295	254	191	48					
(3, 3, 2, 1, 1)	221	559	660	758	747	790	747	758	660	559	221					
(3, 3, 1, 1, 1, 1)	42	116	119	123	125	126	125	123	119	116	42					
(3, 2, 2, 2, 1)	142	393	483	545	532	580	532	545	483	393	142					
(3, 2, 2, 1, 1, 1)	253	532	612	659	667	626	667	659	612	532	253					
(3, 2, 1, 1, 1, 1, 1)	64	127	139	141	145	140	145	141	139	127	64					
(3, 1, 1, 1, 1, 1, 1, 1)	6	8	9	10	9	12	9	10	8	6						
(2, 2, 2, 2, 2)	7	21	24	32	26	30	26	32	24	21	7					
(2, 2, 2, 2, 1, 1)	68	186	224	227	227	236	227	227	224	186	68					
(2, 2, 2, 1, 1, 1, 1)	90	172	195	185	197	198	197	185	195	172	90					
(2, 2, 1, 1, 1, 1, 1, 1)	21	47	45	50	45	48	45	50	47	21						
(2, 1, 1, 1, 1, 1, 1, 1, 1)	5	11	10	12	10	12	10	12	11	5						
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	1						
(9, 1, 1)	34	93	234	306	431	432	432	431	306	234	93	34				
(8, 2, 1)	91	372	659	994	1087	1177	1177	1087	994	659	372	91				
(8, 1, 1, 1)	81	159	226	316	348	366	366	348	316	226	159	81				
(7, 3, 1)	100	318	630	872	1054	1064	1064	1054	872	630	318	100				
(7, 2, 2)	45	276	385	565	673	651	673	565	385	276	45					
(7, 2, 1, 1)	342	805	1020	1233	1321	1335	1335	1321	1233	1020	805	342				
(7, 1, 1, 1, 1)	90	140	191	234	243	247	247	243	234	191	140	90				
(6, 4, 1)	99	284	561	827	960	1085	1085	960	827	561	284	99				
(6, 3, 2)	70	387	722	987	1240	1337	1337	1240	987	722	387	70				
(6, 3, 1, 1)	271	598	889	1108	1232	1262	1262	1232	1108	889	598	271				
(6, 2, 2, 1)	317	943	1274	1495	1585	1694	1694	1585	1495	1274	943	317				
(6, 2, 1, 1, 1)	355	731	896	928	962	978	978	962	928	896	731	355				
(6, 1, 1, 1, 1, 1)	37	59	81	83	100	102	102	100	83	81	37					
(5, 5, 1)	48	147	309	448	527	525	525	527	448	309	147	48				
(5, 4, 2)	55	392	769	1091	1263	1311	1311	1263	1091	769	392	55				
(5, 4, 1, 1)	282	692	975	1248	1347	1456	1456	1347	1248	975	692	282				
(5, 3, 3)	21	176	381	551	651	668	668	651	551	381	176	21				
(5, 3, 2, 1)	520	1828	2603	3109	3342	3534	3534	3342	3109	2603	1828	520				
(5, 3, 1, 1, 1)	396	741	995	1134	1159	1220	1220	1159	1134	995	741	396				
(5, 2, 2, 2)	61	306	446	536	601	582	582	601	536	446	61					
(5, 2, 2, 1, 1)	526	1425	1690	1796	1870	1923	1923	1870	1796	1690	1425	526				
(5, 2, 1, 1, 1, 1)	314	592	641	687	700	708	708	700	687	641	592	314				
(5, 1, 1, 1, 1, 1, 1)	38	50	63	73	80	81	81	80	73	63	50	38				
(4, 4, 3)	16	159	368	482	591	667	667	591	482	368	159	16				
(4, 4, 2, 1)	287	801	1184	1379	1580	1633	1633	1580	1379	1184	801	287				
(4, 4, 1, 1, 1)	194	328	424	486	561	567	567	561	486	424	328	194				
(4, 3, 3, 1)	232	766	1159	1476	1628	1675	1675	1628	1476	1159	766	232				
(4, 3, 2, 2)	184	976	1522	1733	1929	1936	1936	1929	1733	1522	976	184				
(4, 3, 2, 1, 1)	1060	2620	3240	3557	3658	3845	3845	3658	3557	3240	2620	1060				
(4, 3, 1, 1, 1, 1)	284	502	598	604	669	667	667	604	598	502	284					
(4, 2, 2, 2, 1)	321	932	1180	1296	1340	1351	1351	1340	1296	1180	932	321				
(4, 2, 2, 1, 1, 1)	708	1291	1584	1660	1663	1704	1704	1663	1660	1584	1291	708				
(4, 2, 1, 1, 1, 1, 1)	223	446	459	476	489	483	483	489	476	459	223					
(4, 1, 1, 1, 1, 1, 1, 1)	24	42	45	52	50	51	51	50	45	42	24					
(3, 3, 3, 2)	55	312	477	578	622	600	600	622	578	477	312	55				

Table 2: Number of graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for each leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(4, 3, 2, 1, 1, 1, 1, 1, 1)	28085	51710	56218	57569	59112	59655	59972	60813	60813	59972	59655	59112	57569	56218	51710	28085
(4, 3, 1, 1, 1, 1, 1, 1, 1, 1)	1421	2377	2592	2616	2688	2659	2747	2760	2760	2747	2659	2688	2616	2592	2377	1421
(4, 2, 2, 2, 2, 2, 1)	14710	50054	62614	66982	71136	73141	74972	75026	75026	74972	73141	71136	66982	62614	50054	14710
(4, 2, 2, 2, 2, 1, 1, 1)	46036	118287	132569	138813	141784	145493	147182	148012	148012	147182	145493	141784	138813	132569	118287	46036
(4, 2, 2, 2, 1, 1, 1, 1, 1)	31035	64202	70360	71712	73078	73808	74313	74805	74805	74313	73808	73078	71712	70360	64202	31035
(4, 2, 2, 1, 1, 1, 1, 1, 1, 1)	7874	14356	15403	15498	15785	15893	15864	15877	15877	15864	15893	15785	15498	15403	14356	7874
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	864	1580	1626	1664	1659	1670	1672	1672	1673	1673	1672	1670	1659	1664	1626	864
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	46	88	87	95	95	94	95	96	96	95	95	94	95	87	88	46
(3, 3, 3, 3, 3)	404	3865	6261	8010	9089	10235	10753	10978	10978	10753	10235	9089	8010	6261	3865	404
(3, 3, 3, 3, 2, 1)	19127	70763	97684	110631	120043	126738	131782	132494	132494	131782	126738	120043	110631	97684	70763	19127
(3, 3, 3, 3, 1, 1, 1)	14597	33673	41564	46285	48351	50888	51357	51378	51378	51357	50888	48351	46285	41564	33673	14597
(3, 3, 3, 2, 2, 2)	8715	52561	74087	81944	90718	95460	98669	98584	98584	98669	95460	90718	81944	74087	52561	8715
(3, 3, 3, 2, 2, 1, 1)	78901	222308	278005	295171	313460	321715	330801	333125	333125	330801	321715	313460	295171	278005	222308	78901
(3, 3, 3, 2, 1, 1, 1, 1)	46449	100565	117278	122206	127204	129329	130825	131536	131536	130825	129329	127204	122206	117278	100565	46449
(3, 3, 3, 1, 1, 1, 1, 1, 1)	4188	8124	9321	9045	9582	9444	9640	9506	9506	9640	9444	9582	9045	9321	8124	4188
(3, 3, 2, 2, 2, 2, 1)	33176	121630	152468	163512	173103	179201	183104	183981	183981	183104	179201	173103	163512	152468	121630	33176
(3, 3, 2, 2, 2, 1, 1, 1)	88026	222177	261674	269905	281528	284012	290144	292718	292718	290144	284012	281528	269905	261674	222177	88026
(3, 3, 2, 2, 1, 1, 1, 1, 1)	43510	91689	101337	103200	105779	106042	107658	108323	108323	107658	106042	105779	103200	101337	91689	43510
(3, 3, 2, 1, 1, 1, 1, 1, 1, 1)	5891	11336	12137	12171	12356	12328	12434	12487	12487	12434	12328	12356	12171	12137	11336	5891
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	226	352	423	416	428	386	436	413	413	436	386	428	416	423	352	226
(3, 2, 2, 2, 2, 2, 2)	1790	8229	11088	11571	12673	12934	12925	13308	13308	12925	12934	12673	11571	11088	8229	1790
(3, 2, 2, 2, 2, 2, 1, 1)	24640	72833	87522	91241	94983	96813	98588	98540	98540	98588	96813	94983	91241	87522	72833	24640
(3, 2, 2, 2, 2, 1, 1, 1, 1)	36126	84756	95704	97499	99891	100536	102170	102508	102508	102170	100536	99891	97499	95704	84756	36126
(3, 2, 2, 2, 1, 1, 1, 1, 1, 1)	16928	33871	37038	37176	37795	37692	38302	38328	38328	38302	37692	37795	37176	37038	33871	16928
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	2929	5794	5990	6119	6081	6160	6149	6164	6164	6149	6160	6081	6119	5990	5794	2929
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	216	390	416	438	431	430	433	426	426	433	430	431	438	416	390	216
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	8	15	16	14	15	17	15	17	17	15	17	15	16	14	15	8
(2, 2, 2, 2, 2, 2, 2, 1)	1142	3548	4632	4788	5157	5247	5239	5263	5263	5239	5247	5157	4788	4632	3548	1142
(2, 2, 2, 2, 2, 2, 1, 1, 1)	4734	12726	14636	14860	15258	15703	15664	15697	15697	15664	15703	15258	14860	14636	12726	4734
(2, 2, 2, 2, 2, 1, 1, 1, 1, 1)	5205	11487	12813	12820	13104	13178	13230	13323	13323	13230	13178	13104	12820	12813	11487	5205
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	2615	5123	5639	5548	5712	5636	5677	5718	5718	5677	5636	5712	5548	5639	5123	2615
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	586	1184	1221	1207	1216	1223	1219	1228	1228	1219	1223	1216	1207	1221	1184	586
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	62	112	118	120	119	117	119	117	117	119	117	119	120	118	112	62
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	8	14	15	15	15	15	15	15	15	15	15	15	15	15	14	8
(13, 1, 1)	401	1616	4949	7450	10924	15055	17415	17718	17718	17415	15055	10924	7450	4949	1616	401
(12, 2, 1)	1618	7414	15025	23877	31568	38848	42964	45224	45224	42964	38848	31568	23877	15025	7414	1618
(12, 1, 1, 1)	2108	4292	8241	9863	12834	15306	16522	16586	16586	16522	15306	12834	9863	8241	4292	2108
(11, 3, 1)	2291	6463	15462	24294	34170	43621	48930	50396	50396	48930	43621	34170	24294	15462	6463	2291
(11, 2, 2)	649	7626	10404	17392	21987	27531	31090	31290	31290	31090	27531	21987	17392	10404	7626	649
(11, 2, 1, 1)	10797	31370	44420	53958	61161	66732	69619	71299	71299	69619	66732	61161	53958	44420	31370	10797
(11, 1, 1, 1, 1)	3162	4955	7806	8989	10560	11665	12160	12263	12263	12160	11665	8989	7806	4955	3162	3162
(10, 4, 1)	2514	6515	15863	27295	35687	44151	48305	51621	51621	48305	44151	35687	27295	15863	6515	2514
(10, 3, 2)	1216	10618	21406	37109	51249	61898	68633	72210	72210	68633	61898	51249	37109	21406	10618	1216
(10, 3, 1, 1)	12070	25853	41012	54148	65830	75765	80377	81577	81577	80377	75765	65830	54148	41012	25853	12070
(10, 2, 2, 1)	13168	52888	66759	81317	89722	96797	102746	104355	104355	102746	96797	89722	81317	66759	52888	13168
(10, 2, 1, 1, 1)	21279	46468	57972	63686	68683	71549	72812	73306	73306	72812	71549	68683	63686	57972	46468	21279
(10, 1, 1, 1, 1, 1)	2776	3705	5193	5728	6538	6979	7420	7327	7327	7420	6979	6538	5728	5193	3705	2776
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1

A.3 Rotatability of Graceful Spiders by Branch Label

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1)	Yes	-	-	Yes												
(2, 1, 1)	Yes	-	-	-	Yes											
(1, 1, 1, 1)	Yes	-	-	-	Yes											
(3, 1, 1)	-	Yes	-	-	Yes	-										
(2, 2, 1)	Yes	Yes	-	-	Yes	Yes										
(2, 1, 1, 1)	Yes	Yes	-	-	Yes	Yes										
(1, 1, 1, 1, 1)	Yes	-	-	-	-	Yes										
(4, 1, 1)	Yes	Yes	Yes	-	Yes	Yes	Yes									
(3, 2, 1)	Yes	Yes	Yes	-	Yes	Yes	Yes									
(3, 1, 1, 1)	Yes	Yes	-	-	-	Yes	Yes	Yes								
(2, 2, 2)	Yes	Yes	-	-	-	Yes	Yes									
(2, 2, 1, 1)	Yes	Yes	-	-	-	Yes	Yes									
(2, 1, 1, 1, 1)	Yes	-	-	-	-	-	Yes									
(1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	Yes								
(5, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes							
(4, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes						
(4, 1, 1, 1)	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes							
(3, 3, 1)	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes							
(3, 2, 2)	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes							
(3, 2, 1, 1)	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes							
(3, 1, 1, 1, 1)	-	Yes	-	-	-	-	Yes	-	-							
(2, 2, 2, 1)	Yes	Yes	-	-	-	-	-	Yes	Yes							
(2, 2, 1, 1, 1)	Yes	Yes	-	-	-	-	-	Yes	Yes							
(2, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	Yes	Yes							
(1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	Yes							
(6, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes						
(5, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes						
(5, 1, 1, 1)	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes						
(4, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes						
(4, 2, 2)	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes						
(4, 2, 1, 1)	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes						
(4, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes						
(3, 3, 2)	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes						
(3, 3, 1, 1)	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes						
(3, 2, 2, 1)	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes						
(3, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes						
(3, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes						
(2, 2, 2, 2)	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes						
(2, 2, 2, 1, 1)	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes						
(2, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes						
(2, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	Yes	Yes						
(1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	Yes						
(7, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes					
(6, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes					
(6, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes					
(5, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes					
(5, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes					
(5, 2, 1, 1)	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes					
(5, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes					

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(4, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes					
(4, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes					
(4, 3, 1, 1)	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes					
(4, 2, 2, 1)	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes					
(4, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes					
(4, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes					
(3, 3, 3)	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes					
(3, 3, 2, 1)	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes					
(3, 3, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes					
(3, 2, 2, 2)	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes					
(3, 2, 2, 1, 1)	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes					
(3, 2, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes					
(3, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes					
(2, 2, 2, 2, 1)	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes					
(2, 2, 2, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes					
(2, 2, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes					
(2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes					
(1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	Yes					
(8, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes				
(7, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(7, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(6, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(6, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(6, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(6, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes			
(5, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(5, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(5, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(5, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(5, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes			
(5, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes			
(4, 4, 2)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(4, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(4, 3, 3)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(4, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes			
(4, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes			
(4, 2, 2, 2)	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes			
(4, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes			
(4, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes			
(4, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes			
(3, 3, 3, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			
(3, 3, 2, 2)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			
(3, 3, 2, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			
(3, 3, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			
(3, 2, 2, 2, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			
(3, 2, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			
(3, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			
(3, 1, 1, 1, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	Yes	-	-			
(2, 2, 2, 2, 2)	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes			
(2, 2, 2, 2, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes			
(2, 2, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes			
(2, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(2, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes		

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	Yes				
(9, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(8, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(8, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(7, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(7, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(7, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(7, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes			Yes
(6, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(6, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(6, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(6, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(6, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(6, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes			Yes
(5, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(5, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(5, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(5, 3, 3)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(5, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(5, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes			Yes
(5, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(5, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(5, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes			Yes
(5, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(4, 4, 3)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(4, 4, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(4, 4, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(4, 3, 3, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(4, 3, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(4, 3, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes			Yes
(4, 3, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes			Yes
(4, 2, 2, 2, 1)	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes			Yes
(4, 2, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes			Yes
(4, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes			Yes
(4, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 3, 3, 2)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 3, 3, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 3, 2, 2, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 3, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 3, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 2, 2, 2, 2)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 2, 2, 2, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 2, 2, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes			Yes
(3, 1, 1, 1, 1, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	Yes	-	-			Yes
(2, 2, 2, 2, 2, 1)	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes			Yes
(2, 2, 2, 2, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes			Yes
(2, 2, 2, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes			Yes
(2, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes			Yes
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes			Yes
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes			Yes
(9, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		Yes
(9, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes		Yes

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(8, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 3, 3, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 3, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 3, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 3, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 2, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 2, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(4, 4, 4)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 4, 3, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 4, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 4, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 4, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 3, 3, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 3, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 3, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 3, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 3, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 2, 2, 2, 2)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 2, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 2, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(4, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(3, 3, 3, 3)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(3, 3, 3, 2, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(3, 3, 3, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(3, 3, 2, 2, 2)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(3, 3, 2, 2, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(3, 3, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(3, 2, 2, 2, 2, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(3, 2, 2, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(3, 2, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(3, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes			
(2, 2, 2, 2, 2, 2)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(2, 2, 2, 2, 2, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(2, 2, 2, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(2, 2, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes		
(10, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	
(9, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 6, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 4)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 4, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes		
(5, 4, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes		
(5, 4, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes		
(5, 4, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(5, 3, 3, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(5, 3, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(5, 3, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(5, 3, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(5, 3, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes		
(5, 2, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(5, 2, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(5, 2, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(5, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes		
(5, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(4, 4, 4, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 4, 3, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 4, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 4, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 4, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 4, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 3, 3, 3)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 3, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 3, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 3, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 3, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 3, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes		
(4, 3, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes		
(4, 2, 2, 2, 2, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes		
(4, 2, 2, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes		
(4, 2, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes		
(4, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes		
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 3, 3, 3, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 3, 3, 2, 2)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 3, 3, 2, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 3, 3, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 3, 2, 2, 2, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 3, 2, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 3, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 3, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 2, 2, 2, 2, 2)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 2, 2, 2, 2, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 2, 2, 2, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 2, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-		
(2, 2, 2, 2, 2, 2, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 2, 2, 2, 2, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 2, 2, 2, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(11, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(10, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes		
(10, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	
(9, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(8, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 6, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(6, 6, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 6, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 4)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 2, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	
(3, 3, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	
(3, 2, 2, 2, 2, 2, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	
(3, 2, 2, 2, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(3, 2, 2, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	
(2, 2, 2, 2, 2, 2, 2, 2)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 2, 2, 2, 2, 2, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(2, 2, 2, 2, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	
(12, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(9, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(9, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(8, 6, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 5, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 5, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(8, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(7, 7, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 6, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 6, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(7, 5, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 4)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(7, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(6, 6, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 6, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 6, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 4)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(6, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(6, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(5, 5, 5)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(5, 4, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes

Table 3: n -rotatability of spiders order 16 or less

Spider	n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 2, 2, 2, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 3, 2, 2, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 3, 2, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 3, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 2, 2, 2, 2, 2, 2)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 2, 2, 2, 2, 2, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 2, 2, 2, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 2, 2, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(2, 2, 2, 2, 2, 2, 2, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(2, 2, 2, 2, 2, 2, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(2, 2, 2, 2, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(13, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(12, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(12, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 2, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 2, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes

A.4 Alpha-Graceful Spiders by Branch Label

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(2, 1, 1)	1	1	-	1	1											
(1, 1, 1)	1	-	-	1												
(1, 1, 1, 1)	1	-	-	-	1											
(3, 1, 1)	1	1	-	-	1	1										
(2, 2, 1)	-	1	-	-	1	-										
(2, 1, 1, 1)	1	1	-	-	1	1										
(1, 1, 1, 1, 1)	1	-	-	-	-	1										
(4, 1, 1)	2	-	2	-	2	-	2									
(3, 2, 1)	1	2	1	-	1	2	1									
(3, 1, 1, 1)	1	1	-	-	-	1	1									
(2, 2, 2)	-	-	-	-	-	-	-									
(2, 2, 1, 1)	-	1	-	-	-	1	-									
(2, 1, 1, 1, 1)	1	1	-	-	-	1	1									
(1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	1									
(5, 1, 1)	3	2	3	-	-	3	2	3								
(4, 2, 1)	3	2	2	3	3	2	2	3								
(4, 1, 1, 1)	2	-	2	-	-	2	-	2								
(3, 3, 1)	1	3	1	-	-	1	3	1								
(3, 2, 2)	-	1	1	-	-	1	1	-								
(3, 2, 1, 1)	1	2	1	-	-	1	2	1								
(3, 1, 1, 1, 1)	1	1	-	-	-	-	1	1								
(2, 2, 2, 1)	-	-	-	-	-	-	-	-								
(2, 2, 1, 1, 1)	-	1	-	-	-	-	1	-								
(2, 1, 1, 1, 1, 1)	1	1	-	-	-	-	1	1								
(1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	1								
(6, 1, 1)	2	1	1	2	-	2	1	1	2							
(5, 2, 1)	4	3	3	4	-	4	3	3	4							
(5, 1, 1, 1)	3	2	3	-	-	-	3	2	3							
(4, 3, 1)	2	2	2	2	-	2	2	2	2							
(4, 2, 2)	-	3	-	3	-	3	-	3	-							
(4, 2, 1, 1)	3	2	2	3	-	3	2	2	3							
(4, 1, 1, 1, 1)	2	-	2	-	-	-	2	-	2							
(3, 3, 2)	-	2	2	-	-	-	2	2	-							
(3, 3, 1, 1)	1	4	1	-	-	-	1	4	1							
(3, 2, 2, 1)	-	1	1	-	-	-	1	1	-							
(3, 2, 1, 1, 1)	1	2	1	-	-	-	1	2	1							
(3, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	1	1							
(2, 2, 2, 2)	-	-	-	-	-	-	-	-	-							
(2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-							
(2, 2, 1, 1, 1, 1)	-	1	-	-	-	-	-	1	-							
(2, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	1	1							
(1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	1							
(7, 1, 1)	5	2	2	5	-	-	5	2	2	5						
(6, 2, 1)	6	1	-	1	6	6	1	-	1	6						
(6, 1, 1, 1)	2	1	1	2	-	-	2	1	1	2						
(5, 3, 1)	7	6	6	7	-	-	7	6	6	7						
(5, 2, 2)	2	1	-	1	2	2	1	-	1	2						
(5, 2, 1, 1)	4	3	3	4	-	-	4	3	3	4						
(5, 1, 1, 1, 1)	3	2	3	-	-	-	-	3	2	3						

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(4, 4, 1)	1	-	3	-	1	1	-	3	-	1						
(4, 3, 2)	1	4	2	4	1	1	4	2	4	1						
(4, 3, 1, 1)	2	2	2	2	-	-	2	2	2	2						
(4, 2, 2, 1)	-	3	-	3	-	-	3	-	3	-						
(4, 2, 1, 1, 1)	3	2	2	3	-	-	3	2	2	3						
(4, 1, 1, 1, 1, 1)	2	-	2	-	-	-	-	2	-	2						
(3, 3, 3)	-	2	2	-	-	-	-	2	2	-						
(3, 3, 2, 1)	-	2	2	-	-	-	-	2	2	-						
(3, 3, 1, 1, 1)	1	5	1	-	-	-	-	1	5	1						
(3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-						
(3, 2, 2, 1, 1)	-	1	1	-	-	-	-	1	1	-						
(3, 2, 1, 1, 1, 1)	1	2	1	-	-	-	-	1	2	1						
(3, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	1	1						
(2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-						
(2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-						
(2, 2, 1, 1, 1, 1, 1)	-	1	-	-	-	-	-	-	1	-						
(2, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	1	1						
(1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	1						
(8, 1, 1)	11	2	2	2	11	-	11	2	2	2	11					
(7, 2, 1)	15	5	-	5	15	-	15	5	-	5	15					
(7, 1, 1, 1)	5	2	2	5	-	-	-	5	2	2	5					
(6, 3, 1)	10	5	2	5	10	-	10	5	2	5	10					
(6, 2, 2)	-	3	-	-	3	-	-	3	-	-	3					
(6, 2, 1, 1)	6	2	-	2	6	-	6	2	-	2	6					
(6, 1, 1, 1, 1)	2	1	1	2	-	-	-	2	1	1	2					
(5, 4, 1)	9	5	8	5	9	-	9	5	8	5	9					
(5, 3, 2)	5	4	2	4	5	-	5	4	2	4	5					
(5, 3, 1, 1)	7	7	7	7	-	-	-	7	7	7	7					
(5, 2, 2, 1)	2	4	-	4	2	-	2	4	-	4	2					
(5, 2, 1, 1, 1)	4	3	3	4	-	-	-	4	3	3	4					
(5, 1, 1, 1, 1, 1)	3	2	3	-	-	-	-	3	2	3						
(4, 4, 2)	-	1	3	3	1	-	1	3	3	1	-					
(4, 4, 1, 1)	1	-	4	-	1	-	1	-	4	-	1					
(4, 3, 3)	1	3	2	3	1	-	1	3	2	3	1					
(4, 3, 2, 1)	1	5	4	5	1	-	1	5	4	5	1					
(4, 3, 1, 1, 1)	2	2	2	2	-	-	-	2	2	2	2					
(4, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-					
(4, 2, 2, 1, 1)	-	3	-	3	-	-	-	3	-	3	-					
(4, 2, 1, 1, 1, 1)	3	2	2	3	-	-	-	3	2	2	3					
(4, 1, 1, 1, 1, 1, 1)	2	-	2	-	-	-	-	-	2	-	2					
(3, 3, 3, 1)	-	3	3	-	-	-	-	-	3	3	-					
(3, 3, 2, 2)	-	-	1	-	-	-	-	-	1	-	-					
(3, 3, 2, 1, 1)	-	2	2	-	-	-	-	-	2	2	-					
(3, 3, 1, 1, 1, 1)	1	6	1	-	-	-	-	-	1	6	1					
(3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-					
(3, 2, 2, 1, 1, 1)	-	1	1	-	-	-	-	-	1	1	-					
(3, 2, 1, 1, 1, 1, 1)	1	2	1	-	-	-	-	-	1	2	1					
(3, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	1	1					
(2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-					
(2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-					
(2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-					
(2, 2, 1, 1, 1, 1, 1, 1)	-	1	-	-	-	-	-	-	-	1	-					
(2, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	1	1					

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	1					
(9, 1, 1)	11	5	10	5	11	-	-	11	5	10	5	11				
(8, 2, 1)	29	8	4	4	8	29	29	8	4	4	8	29				
(8, 1, 1, 1)	12	2	2	2	12	-	-	12	2	2	2	12				
(7, 3, 1)	15	12	6	12	15	-	-	15	12	6	12	15				
(7, 2, 2)	3	9	1	1	9	3	3	9	1	1	9	3				
(7, 2, 1, 1)	19	7	-	7	19	-	-	19	7	-	7	19				
(7, 1, 1, 1, 1)	5	2	2	5	-	-	-	5	2	2	5					
(6, 4, 1)	14	4	7	7	4	14	14	4	7	7	4	14				
(6, 3, 2)	5	9	1	1	9	5	5	9	1	1	9	5				
(6, 3, 1, 1)	13	6	2	6	13	-	-	13	6	2	6	13				
(6, 2, 2, 1)	-	6	-	-	6	-	-	6	-	-	6	-				
(6, 2, 1, 1, 1)	6	2	-	2	6	-	-	6	2	-	2	6				
(6, 1, 1, 1, 1, 1)	2	1	1	2	-	-	-	2	1	1	2					
(5, 5, 1)	8	7	8	7	8	-	-	8	7	8	7	8				
(5, 4, 2)	4	6	6	6	6	4	4	6	6	6	6	4				
(5, 4, 1, 1)	10	5	12	5	10	-	-	10	5	12	5	10				
(5, 3, 3)	3	3	4	3	3	-	-	3	3	4	3	3				
(5, 3, 2, 1)	8	13	6	13	8	-	-	8	13	6	13	8				
(5, 3, 1, 1, 1)	7	8	8	7	-	-	-	7	8	8	7					
(5, 2, 2, 2)	-	2	-	-	2	-	-	2	-	-	2	-				
(5, 2, 2, 1, 1)	2	4	-	4	2	-	-	2	4	-	4	2				
(5, 2, 1, 1, 1, 1)	4	3	3	4	-	-	-	4	3	3	4					
(5, 1, 1, 1, 1, 1, 1)	3	2	3	-	-	-	-	3	2	3						
(4, 4, 3)	-	1	2	2	1	-	-	1	2	2	1	-				
(4, 4, 2, 1)	-	1	6	6	1	-	-	1	6	6	1	-				
(4, 4, 1, 1, 1)	1	-	4	-	1	-	-	1	-	4	-	1				
(4, 3, 3, 1)	1	6	4	6	1	-	-	1	6	4	6	1				
(4, 3, 2, 2)	-	1	3	3	1	-	-	-	1	3	3	1				
(4, 3, 2, 1, 1)	1	5	4	5	1	-	-	1	5	4	5	1				
(4, 3, 1, 1, 1, 1)	2	2	2	2	-	-	-	2	2	2	2					
(4, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(4, 2, 2, 1, 1, 1)	-	3	-	3	-	-	-	-	3	-	3	-				
(4, 2, 1, 1, 1, 1, 1)	3	2	2	3	-	-	-	3	2	2	3					
(4, 1, 1, 1, 1, 1, 1, 1)	2	-	2	-	-	-	-	2	-	2	-	2				
(3, 3, 3, 2)	-	-	2	-	-	-	-	-	-	2	-	-				
(3, 3, 3, 1, 1)	-	4	4	-	-	-	-	-	4	4	-	-				
(3, 3, 2, 2, 1)	-	-	1	-	-	-	-	-	-	1	-	-				
(3, 3, 2, 1, 1, 1)	-	2	2	-	-	-	-	-	2	2	-	-				
(3, 3, 1, 1, 1, 1, 1)	1	7	1	-	-	-	-	1	7	1						
(3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 2, 2, 1, 1, 1, 1)	-	1	1	-	-	-	-	-	1	1	-	-				
(3, 2, 1, 1, 1, 1, 1, 1)	1	2	1	-	-	-	-	-	1	2	1					
(3, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	1	1						
(2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 1, 1, 1, 1, 1, 1, 1)	-	1	-	-	-	-	-	-	1	-	-	-				
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	1	1						
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	1				
(9, 2, 1)	48	20	13	13	20	48	-	48	20	13	13	20	48			
(9, 1, 1, 1)	19	5	10	5	19	-	-	19	5	10	5	19				

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(8, 3, 1)	37	21	10	10	21	37	-	37	21	10	10	21	37			
(8, 2, 2)	27	10	4	-	4	10	54	10	4	-	4	10	27			
(8, 2, 1, 1)	34	13	1	1	13	34	-	34	13	1	1	13	34			
(8, 1, 1, 1, 1)	12	2	2	2	12	-	-	-	12	2	2	2	12			
(7, 4, 1)	29	8	13	13	8	29	-	29	8	13	13	8	29			
(7, 3, 2)	14	19	5	5	19	14	-	14	19	5	5	19	14			
(7, 3, 1, 1)	31	15	6	15	31	-	-	-	31	15	6	15	31			
(7, 2, 2, 1)	9	15	-	-	15	9	-	9	15	-	-	15	9			
(7, 2, 1, 1, 1)	19	7	-	7	19	-	-	-	19	7	-	7	19			
(7, 1, 1, 1, 1, 1)	5	2	2	5	-	-	-	-	5	2	2	5				
(6, 5, 1)	22	14	21	21	14	22	-	22	14	21	21	14	22			
(6, 4, 2)	21	7	10	6	10	7	42	7	10	6	10	7	21			
(6, 4, 1, 1)	21	4	5	5	4	21	-	21	4	5	5	4	21			
(6, 3, 3)	2	11	5	5	11	2	-	2	11	5	5	11	2			
(6, 3, 2, 1)	9	16	1	1	16	9	-	9	16	1	1	16	9			
(6, 3, 1, 1, 1)	13	6	2	6	13	-	-	-	13	6	2	6	13			
(6, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(6, 2, 2, 1, 1)	-	6	-	-	6	-	-	-	6	-	-	6	-			
(6, 2, 1, 1, 1, 1)	6	2	-	2	6	-	-	-	6	2	-	2	6			
(6, 1, 1, 1, 1, 1, 1)	2	1	1	2	-	-	-	-	2	1	1	2				
(5, 5, 2)	3	11	5	5	11	3	-	3	11	5	5	11	3			
(5, 5, 1, 1)	11	7	14	7	11	-	-	-	11	7	14	7	11			
(5, 4, 3)	2	12	14	14	12	2	-	2	12	14	14	12	2			
(5, 4, 2, 1)	6	9	14	14	9	6	-	6	9	14	14	9	6			
(5, 4, 1, 1, 1)	10	5	12	5	10	-	-	-	10	5	12	5	10			
(5, 3, 3, 1)	9	13	10	13	9	-	-	-	9	13	10	13	9			
(5, 3, 2, 2)	-	7	1	1	7	-	-	-	7	1	1	7	-			
(5, 3, 2, 1, 1)	8	14	6	14	8	-	-	-	8	14	6	14	8			
(5, 3, 1, 1, 1, 1)	7	9	9	7	-	-	-	-	7	9	9	7				
(5, 2, 2, 2, 1)	-	2	-	-	2	-	-	-	2	-	-	2	-			
(5, 2, 2, 1, 1, 1)	2	4	-	4	2	-	-	-	2	4	-	4	2			
(5, 2, 1, 1, 1, 1, 1)	4	3	3	4	-	-	-	-	4	3	3	4				
(5, 1, 1, 1, 1, 1, 1, 1)	3	2	3	-	-	-	-	-	3	2	3					
(4, 4, 4)	-	-	2	-	2	-	-	-	2	-	2	-	-			
(4, 4, 3, 1)	-	1	3	3	1	-	-	-	1	3	3	1	-			
(4, 4, 2, 2)	-	-	-	9	-	-	-	-	-	9	-	-	-			
(4, 4, 2, 1, 1)	-	1	6	6	1	-	-	-	1	6	6	1	-			
(4, 4, 1, 1, 1, 1)	1	-	4	-	1	-	-	-	1	-	4	-	1			
(4, 3, 3, 2)	-	2	4	4	2	-	-	-	2	4	4	2	-			
(4, 3, 3, 1, 1)	1	8	4	8	1	-	-	-	1	8	4	8	1			
(4, 3, 2, 2, 1)	-	1	3	3	1	-	-	-	1	3	3	1	-			
(4, 3, 2, 1, 1, 1)	1	5	4	5	1	-	-	-	1	5	4	5	1			
(4, 3, 1, 1, 1, 1, 1)	2	2	2	2	-	-	-	-	2	2	2	2				
(4, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 2, 2, 1, 1, 1, 1)	-	3	-	3	-	-	-	-	-	3	-	3	-			
(4, 2, 1, 1, 1, 1, 1, 1)	3	2	2	3	-	-	-	-	3	2	2	3				
(4, 1, 1, 1, 1, 1, 1, 1, 1)	2	-	2	-	-	-	-	-	2	-	2	-	2			
(3, 3, 3, 3)	-	-	2	-	-	-	-	-	-	2	-	-	-			
(3, 3, 3, 2, 1)	-	-	2	-	-	-	-	-	-	2	-	-	-			
(3, 3, 3, 1, 1, 1)	-	5	5	-	-	-	-	-	-	5	5	-	-			
(3, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(3, 3, 2, 2, 1, 1)	-	-	1	-	-	-	-	-	-	1	-	-	-			

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 1, 1, 1, 1)	-	2	2	-	-	-	-	-	-	-	2	2	-	-	-	-
(3, 3, 1, 1, 1, 1, 1, 1)	1	8	1	-	-	-	-	-	-	-	1	8	1	-	-	-
(3, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1)	-	1	1	-	-	-	-	-	-	-	1	1	-	-	-	-
(3, 2, 1, 1, 1, 1, 1, 1, 1)	1	2	1	-	-	-	-	-	-	-	1	2	1	-	-	-
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	-	-	1	1	-	-	-
(2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	-	1	-	-	-	-	-	-	-	-	-	1	-	-	-	-
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	-	-	1	1	-	-	-
(10, 1, 1)	15	6	10	10	6	15	-	15	6	10	10	6	15	-	-	-
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	-	-	1	-	-
(9, 3, 1)	69	56	20	20	56	69	-	-	69	56	20	20	56	69	-	-
(9, 2, 2)	18	18	3	6	3	18	18	18	18	3	6	3	18	18	-	-
(9, 2, 1, 1)	67	24	9	9	24	67	-	-	67	24	9	9	24	67	-	-
(9, 1, 1, 1, 1)	19	5	10	5	19	-	-	-	19	5	10	5	19	-	-	-
(8, 4, 1)	54	16	19	20	19	16	54	54	16	19	20	19	16	54	-	-
(8, 3, 2)	33	27	9	8	9	27	33	33	27	9	8	9	27	33	-	-
(8, 3, 1, 1)	46	23	6	6	23	46	-	-	46	23	6	6	23	46	-	-
(8, 2, 2, 1)	27	29	-	-	-	29	27	27	29	-	-	-	29	27	-	-
(8, 2, 1, 1, 1)	34	14	1	1	14	34	-	-	34	14	1	1	14	34	-	-
(8, 1, 1, 1, 1, 1)	12	2	2	2	12	-	-	-	12	2	2	2	12	-	-	-
(7, 5, 1)	55	43	25	25	43	55	-	-	55	43	25	25	43	55	-	-
(7, 4, 2)	29	21	18	16	18	21	29	29	21	18	16	18	21	29	-	-
(7, 4, 1, 1)	43	8	15	15	8	43	-	-	43	8	15	15	8	43	-	-
(7, 3, 3)	14	24	9	9	24	14	-	-	14	24	9	9	24	14	-	-
(7, 3, 2, 1)	34	36	5	5	36	34	-	-	34	36	5	5	36	34	-	-
(7, 3, 1, 1, 1)	31	16	6	16	31	-	-	-	31	16	6	16	31	-	-	-
(7, 2, 2, 2)	-	3	-	-	-	3	-	-	3	-	-	-	3	-	-	-
(7, 2, 2, 1, 1)	9	19	-	-	19	9	-	-	9	19	-	-	19	9	-	-
(7, 2, 1, 1, 1, 1)	19	7	-	7	19	-	-	-	19	7	-	7	19	-	-	-
(7, 1, 1, 1, 1, 1, 1)	5	2	2	5	-	-	-	-	5	2	2	5	-	-	-	-
(6, 6, 1)	18	11	8	4	8	11	18	18	11	8	4	8	11	18	-	-
(6, 5, 2)	23	11	17	26	17	11	23	23	11	17	26	17	11	23	-	-
(6, 5, 1, 1)	31	10	16	16	10	31	-	-	31	10	16	16	10	31	-	-
(6, 4, 3)	10	13	19	20	19	13	10	10	13	19	20	19	13	10	-	-
(6, 4, 2, 1)	21	14	9	6	9	14	21	21	14	9	6	9	14	21	-	-
(6, 4, 1, 1, 1)	21	4	6	6	4	21	-	-	21	4	6	6	4	21	-	-
(6, 3, 3, 1)	6	18	5	5	18	6	-	-	6	18	5	5	18	6	-	-
(6, 3, 2, 2)	-	5	3	-	3	5	-	-	5	3	-	3	5	-	-	-
(6, 3, 2, 1, 1)	9	19	2	2	19	9	-	-	9	19	2	2	19	9	-	-
(6, 3, 1, 1, 1, 1)	13	6	2	6	13	-	-	-	13	6	2	6	13	-	-	-
(6, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 1, 1, 1)	-	6	-	-	6	-	-	-	6	-	-	6	-	-	-	-
(6, 2, 1, 1, 1, 1, 1)	6	2	-	2	6	-	-	-	6	2	-	2	6	-	-	-
(6, 1, 1, 1, 1, 1, 1, 1)	2	1	1	2	-	-	-	-	2	1	1	2	-	-	-	-
(5, 5, 3)	3	18	8	8	18	3	-	-	3	18	8	8	18	3	-	-
(5, 5, 2, 1)	8	14	11	11	14	8	-	-	8	14	11	11	14	8	-	-
(5, 5, 1, 1, 1)	11	7	16	7	11	-	-	-	11	7	16	7	11	-	-	-
(5, 4, 4)	-	-	11	20	11	-	-	-	-	11	20	11	-	-	-	-

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 4, 3, 1)	5	16	23	23	16	5	-	-	5	16	23	23	16	5		
(5, 4, 2, 2)	-	4	2	6	2	4	-	-	4	2	6	2	4	-		
(5, 4, 2, 1, 1)	6	10	17	17	10	6	-	-	6	10	17	17	10	6		
(5, 4, 1, 1, 1, 1)	10	5	12	5	10	-	-	-	-	10	5	12	5	10		
(5, 3, 3, 2)	-	10	4	4	10	-	-	-	-	10	4	4	10	-		
(5, 3, 3, 1, 1)	9	18	12	18	9	-	-	-	-	9	18	12	18	9		
(5, 3, 2, 2, 1)	-	10	4	4	10	-	-	-	-	10	4	4	10	-		
(5, 3, 2, 1, 1, 1)	8	15	6	15	8	-	-	-	-	8	15	6	15	8		
(5, 3, 1, 1, 1, 1, 1)	7	10	10	7	-	-	-	-	-	-	7	10	10	7		
(5, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(5, 2, 2, 2, 1, 1)	-	2	-	-	2	-	-	-	-	2	-	-	-	2		
(5, 2, 2, 1, 1, 1, 1)	2	4	-	4	2	-	-	-	-	2	4	-	4	2		
(5, 2, 1, 1, 1, 1, 1, 1)	4	3	3	4	-	-	-	-	-	-	4	3	3	4		
(5, 1, 1, 1, 1, 1, 1, 1, 1)	3	2	3	-	-	-	-	-	-	-	3	2	3	-		
(4, 4, 4, 1)	-	-	2	-	2	-	-	-	-	2	-	2	-	-		
(4, 4, 3, 2)	-	-	2	6	2	-	-	-	-	2	6	2	-	-		
(4, 4, 3, 1, 1)	-	1	4	4	1	-	-	-	-	1	4	4	1	-		
(4, 4, 2, 2, 1)	-	-	-	9	-	-	-	-	-	-	9	-	-	-		
(4, 4, 2, 1, 1, 1)	-	1	6	6	1	-	-	-	-	1	6	6	1	-		
(4, 4, 1, 1, 1, 1, 1)	1	-	4	-	1	-	-	-	-	1	-	4	-	1		
(4, 3, 3, 3)	-	2	3	3	2	-	-	-	-	2	3	3	2	-		
(4, 3, 3, 2, 1)	-	2	5	5	2	-	-	-	-	2	5	5	2	-		
(4, 3, 3, 1, 1, 1)	1	10	4	10	1	-	-	-	-	1	10	4	10	1		
(4, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(4, 3, 2, 2, 1, 1)	-	1	3	3	1	-	-	-	-	1	3	3	1	-		
(4, 3, 2, 1, 1, 1, 1)	1	5	4	5	1	-	-	-	-	1	5	4	5	1		
(4, 3, 1, 1, 1, 1, 1, 1)	2	2	2	2	-	-	-	-	-	-	2	2	2	2		
(4, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(4, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(4, 2, 2, 1, 1, 1, 1, 1)	-	3	-	3	-	-	-	-	-	-	3	-	3	-		
(4, 2, 1, 1, 1, 1, 1, 1, 1)	3	2	2	3	-	-	-	-	-	-	3	2	2	3		
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	-	2	-	-	-	-	-	-	-	-	2	-	2		
(3, 3, 3, 3, 1)	-	-	3	-	-	-	-	-	-	-	-	3	-	-		
(3, 3, 3, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 3, 3, 2, 1, 1)	-	-	2	-	-	-	-	-	-	-	-	2	-	-		
(3, 3, 3, 1, 1, 1, 1)	-	6	6	-	-	-	-	-	-	-	-	6	6	-		
(3, 3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 3, 2, 2, 1, 1, 1)	-	-	1	-	-	-	-	-	-	-	-	1	-	-		
(3, 3, 2, 1, 1, 1, 1, 1)	-	2	2	-	-	-	-	-	-	-	-	2	2	-		
(3, 3, 1, 1, 1, 1, 1, 1, 1)	1	9	1	-	-	-	-	-	-	-	-	1	9	1		
(3, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 2, 2, 1, 1, 1, 1, 1, 1)	-	1	1	-	-	-	-	-	-	-	-	1	1	-		
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	1	-	-	-	-	-	-	-	-	1	2	1		
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	-	-	-	1	1		
(2, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(2, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(2, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	1	-	-	-	-	-	-	-	-	-	-	1	-		
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	-	-	-	1	1		
(11, 1, 1)	44	22	18	18	22	44	-	-	44	22	18	18	22	44		

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(10, 2, 1)	100	28	17	12	17	28	100	100	28	17	12	17	28	100		
(10, 1, 1, 1)	23	2	5	5	2	23	-	-	23	2	5	5	2	23		
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	-	-	-	1	
(9, 4, 1)	90	36	30	46	30	36	90	-	90	36	30	46	30	36	90	
(9, 3, 2)	89	61	16	10	16	61	89	-	89	61	16	10	16	61	89	
(9, 3, 1, 1)	118	65	18	18	65	118	-	-	-	118	65	18	18	65	118	
(9, 2, 2, 1)	44	51	1	14	1	51	44	-	44	51	1	14	1	51	44	
(9, 2, 1, 1, 1)	80	32	9	9	32	80	-	-	-	80	32	9	9	32	80	
(9, 1, 1, 1, 1, 1)	19	5	10	5	19	-	-	-	-	-	19	5	10	5	19	
(8, 5, 1)	100	54	41	32	41	54	100	-	100	54	41	32	41	54	100	
(8, 4, 2)	66	24	28	13	13	28	24	132	24	28	13	13	28	24	66	
(8, 4, 1, 1)	66	11	24	10	24	11	66	-	66	11	24	10	24	11	66	
(8, 3, 3)	33	29	13	2	13	29	33	-	33	29	13	2	13	29	33	
(8, 3, 2, 1)	72	69	11	8	11	69	72	-	72	69	11	8	11	69	72	
(8, 3, 1, 1, 1)	53	24	6	6	24	53	-	-	-	53	24	6	6	24	53	
(8, 2, 2, 2)	-	27	-	-	-	-	27	-	27	-	-	-	-	-	27	
(8, 2, 2, 1, 1)	27	34	-	-	-	34	27	-	27	34	-	-	-	-	34	27
(8, 2, 1, 1, 1, 1)	34	14	1	1	14	34	-	-	-	34	14	1	1	14	34	
(8, 1, 1, 1, 1, 1, 1)	12	2	2	2	12	-	-	-	-	-	12	2	2	2	12	
(7, 6, 1)	77	52	44	30	44	52	77	-	77	52	44	30	44	52	77	
(7, 5, 2)	45	36	33	38	33	36	45	-	45	36	33	38	33	36	45	
(7, 5, 1, 1)	82	46	33	33	46	82	-	-	-	82	46	33	33	46	82	
(7, 4, 3)	34	42	26	44	26	42	34	-	34	42	26	44	26	42	34	
(7, 4, 2, 1)	52	33	25	36	25	33	52	-	52	33	25	36	25	33	52	
(7, 4, 1, 1, 1)	48	8	17	17	8	48	-	-	-	48	8	17	17	8	48	
(7, 3, 3, 1)	28	43	12	12	43	28	-	-	-	28	43	12	12	43	28	
(7, 3, 2, 2)	-	17	9	-	9	17	-	-	-	17	9	-	9	17	-	
(7, 3, 2, 1, 1)	46	59	7	7	59	46	-	-	-	46	59	7	7	59	46	
(7, 3, 1, 1, 1, 1)	31	17	6	17	31	-	-	-	-	-	31	17	6	17	31	
(7, 2, 2, 2, 1)	-	9	-	-	-	9	-	-	-	9	-	-	-	9	-	
(7, 2, 2, 1, 1, 1)	9	19	-	-	19	9	-	-	-	9	19	-	-	19	9	
(7, 2, 1, 1, 1, 1, 1)	19	7	-	7	19	-	-	-	-	-	19	7	-	7	19	
(7, 1, 1, 1, 1, 1, 1, 1)	5	2	2	5	-	-	-	-	-	-	-	5	2	2	5	
(6, 6, 2)	24	6	5	5	5	5	6	48	6	5	5	5	5	6	24	
(6, 6, 1, 1)	23	5	5	9	5	5	23	-	23	5	5	9	5	5	23	
(6, 5, 3)	18	28	42	28	42	28	18	-	18	28	42	28	42	28	18	
(6, 5, 2, 1)	40	24	18	30	18	24	40	-	40	24	18	30	18	24	40	
(6, 5, 1, 1, 1)	38	10	18	18	10	38	-	-	-	38	10	18	18	10	38	
(6, 4, 4)	-	-	15	11	11	15	-	-	-	15	11	11	15	-	-	
(6, 4, 3, 1)	11	18	25	24	25	18	11	-	11	18	25	24	25	18	11	
(6, 4, 2, 2)	-	21	-	9	9	-	21	-	21	-	9	9	-	21	-	
(6, 4, 2, 1, 1)	21	21	12	12	12	21	21	-	21	21	12	12	12	21	21	
(6, 4, 1, 1, 1, 1)	21	4	6	6	4	21	-	-	-	21	4	6	6	4	21	
(6, 3, 3, 2)	-	7	9	-	9	7	-	-	-	7	9	-	9	7	-	
(6, 3, 3, 1, 1)	8	24	6	6	24	8	-	-	-	8	24	6	6	24	8	
(6, 3, 2, 2, 1)	-	9	6	-	6	9	-	-	-	9	6	-	6	9	-	
(6, 3, 2, 1, 1, 1)	9	19	2	2	19	9	-	-	-	9	19	2	2	19	9	
(6, 3, 1, 1, 1, 1, 1)	13	6	2	6	13	-	-	-	-	-	13	6	2	6	13	
(6, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(6, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(6, 2, 2, 1, 1, 1, 1)	-	6	-	-	6	-	-	-	-	-	6	-	-	6	-	
(6, 2, 1, 1, 1, 1, 1, 1)	6	2	-	2	6	-	-	-	-	-	6	2	-	2	6	
(6, 1, 1, 1, 1, 1, 1, 1, 1)	2	1	1	2	-	-	-	-	-	-	-	2	1	1	2	

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 5, 4)	3	10	17	20	17	10	3	-	3	10	17	20	17	10	3	
(5, 5, 3, 1)	9	31	22	22	31	9	-	-	-	9	31	22	22	31	9	
(5, 5, 2, 2)	-	3	2	7	2	3	-	-	-	3	2	7	2	3	-	
(5, 5, 2, 1, 1)	12	17	20	20	17	12	-	-	-	12	17	20	20	17	12	
(5, 5, 1, 1, 1, 1)	11	7	18	7	11	-	-	-	-	-	11	7	18	7	11	
(5, 4, 4, 1)	-	-	21	18	21	-	-	-	-	-	21	18	21	-	-	
(5, 4, 3, 2)	-	6	12	20	12	6	-	-	-	6	12	20	12	6	-	
(5, 4, 3, 1, 1)	5	18	33	33	18	5	-	-	-	5	18	33	33	18	5	
(5, 4, 2, 2, 1)	-	6	4	24	4	6	-	-	-	6	4	24	4	6	-	
(5, 4, 2, 1, 1, 1)	6	10	17	17	10	6	-	-	-	6	10	17	17	10	6	
(5, 4, 1, 1, 1, 1, 1)	10	5	12	5	10	-	-	-	-	-	10	5	12	5	10	
(5, 3, 3, 3)	-	6	3	3	6	-	-	-	-	-	6	3	3	6	-	
(5, 3, 3, 2, 1)	-	21	13	13	21	-	-	-	-	-	21	13	13	21	-	
(5, 3, 3, 1, 1, 1)	9	23	14	23	9	-	-	-	-	-	9	23	14	23	9	
(5, 3, 2, 2, 2)	-	-	2	-	2	-	-	-	-	-	2	-	2	-	-	
(5, 3, 2, 2, 1, 1)	-	10	4	4	10	-	-	-	-	-	10	4	4	10	-	
(5, 3, 2, 1, 1, 1, 1)	8	16	6	16	8	-	-	-	-	-	8	16	6	16	8	
(5, 3, 1, 1, 1, 1, 1, 1)	7	11	11	7	-	-	-	-	-	-	-	7	11	11	7	
(5, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(5, 2, 2, 2, 1, 1, 1)	-	2	-	-	2	-	-	-	-	-	2	-	-	2	-	
(5, 2, 2, 1, 1, 1, 1, 1)	2	4	-	4	2	-	-	-	-	-	2	4	-	4	2	
(5, 2, 1, 1, 1, 1, 1, 1, 1)	4	3	3	4	-	-	-	-	-	-	-	4	3	3	4	
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	2	3	-	-	-	-	-	-	-	-	-	3	2	3	
(4, 4, 4, 2)	-	-	-	3	3	-	-	-	-	-	3	3	-	-	-	
(4, 4, 4, 1, 1)	-	-	2	-	2	-	-	-	-	-	2	-	2	-	-	
(4, 4, 3, 3)	-	-	2	6	2	-	-	-	-	-	2	6	2	-	-	
(4, 4, 3, 2, 1)	-	-	3	12	3	-	-	-	-	-	3	12	3	-	-	
(4, 4, 3, 1, 1, 1)	-	1	4	4	1	-	-	-	-	-	1	4	4	1	-	
(4, 4, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 4, 2, 2, 1, 1)	-	-	-	9	-	-	-	-	-	-	-	9	-	-	-	
(4, 4, 2, 1, 1, 1, 1)	-	1	6	6	1	-	-	-	-	-	1	6	6	1	-	
(4, 4, 1, 1, 1, 1, 1, 1)	1	-	4	-	1	-	-	-	-	-	1	-	4	-	1	
(4, 3, 3, 3, 1)	-	3	6	6	3	-	-	-	-	-	3	6	6	3	-	
(4, 3, 3, 2, 2)	-	-	1	-	1	-	-	-	-	-	1	-	1	-	-	
(4, 3, 3, 2, 1, 1)	-	2	5	5	2	-	-	-	-	-	2	5	5	2	-	
(4, 3, 3, 1, 1, 1, 1)	1	12	4	12	1	-	-	-	-	-	1	12	4	12	1	
(4, 3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 3, 2, 2, 1, 1, 1)	-	1	3	3	1	-	-	-	-	-	1	3	3	1	-	
(4, 3, 2, 1, 1, 1, 1, 1)	1	5	4	5	1	-	-	-	-	-	1	5	4	5	1	
(4, 3, 1, 1, 1, 1, 1, 1, 1)	2	2	2	2	-	-	-	-	-	-	2	2	2	2	-	
(4, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 2, 2, 1, 1, 1, 1, 1, 1)	-	3	-	3	-	-	-	-	-	-	-	3	-	3	-	
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1)	3	2	2	3	-	-	-	-	-	-	-	3	2	2	3	
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	-	2	-	-	-	-	-	-	-	-	-	2	-	2	
(3, 3, 3, 3, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(3, 3, 3, 3, 1, 1)	-	-	4	-	-	-	-	-	-	-	-	-	4	-	-	
(3, 3, 3, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(3, 3, 3, 2, 1, 1, 1)	-	-	2	-	-	-	-	-	-	-	-	-	2	-	-	
(3, 3, 3, 1, 1, 1, 1, 1)	-	7	7	-	-	-	-	-	-	-	-	-	7	7	-	
(3, 3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(3, 3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 2, 1, 1, 1, 1)	-	-	1	-	-	-	-	-	-	-	-	-	1	-	-	-
(3, 3, 2, 1, 1, 1, 1, 1, 1)	-	2	2	-	-	-	-	-	-	-	-	-	2	2	-	-
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1)	1	10	1	-	-	-	-	-	-	-	-	-	1	10	1	-
(3, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	1	1	-	-	-	-	-	-	-	-	-	1	1	-	-
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	1	-	-	-	-	-	-	-	-	-	1	2	1	-
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	-	-	-	-	1	1	-
(2, 2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	1	-	-	-	-	-	-	-	-	-	-	-	1	-	-
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	-	-	-	-	1	1	-
(12, 1, 1)	52	43	37	8	37	43	52	-	52	43	37	8	37	43	52	-
(11, 2, 1)	144	43	26	42	26	43	144	-	144	43	26	42	26	43	144	-
(11, 1, 1, 1)	60	10	15	15	10	60	-	-	60	10	15	15	10	60	-	-
(10, 3, 1)	112	92	28	16	28	92	112	-	112	92	28	16	28	92	112	-
(10, 2, 2)	59	25	9	2	2	9	25	118	25	9	2	2	9	25	59	-
(10, 2, 1, 1)	129	24	9	12	9	24	129	-	129	24	9	12	9	24	129	-
(10, 1, 1, 1, 1)	24	2	5	5	2	24	-	-	24	2	5	5	2	24	-	-
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1
(9, 5, 1)	260	125	99	152	99	125	260	-	260	125	99	152	99	125	260	-
(9, 4, 2)	146	81	52	45	45	52	81	146	146	81	52	45	45	52	81	146
(9, 4, 1, 1)	130	44	43	26	43	44	130	-	130	44	43	26	43	44	130	-
(9, 3, 3)	81	63	26	16	26	63	81	-	81	63	26	16	26	63	81	-
(9, 3, 2, 1)	214	164	31	20	31	164	214	-	214	164	31	20	31	164	214	-
(9, 3, 1, 1, 1)	163	76	18	18	76	163	-	-	163	76	18	18	76	163	-	-
(9, 2, 2, 2)	12	18	-	2	2	-	18	12	12	18	-	2	2	-	18	12
(9, 2, 2, 1, 1)	66	70	-	10	-	70	66	-	66	70	-	10	-	70	66	-
(9, 2, 1, 1, 1, 1)	80	32	9	9	32	80	-	-	80	32	9	9	32	80	-	-
(9, 1, 1, 1, 1, 1, 1)	19	5	10	5	19	-	-	-	19	5	10	5	19	-	-	-
(8, 6, 1)	160	92	73	46	46	73	92	160	160	92	73	46	46	73	92	160
(8, 5, 2)	135	102	49	61	61	49	102	135	135	102	49	61	61	49	102	135
(8, 5, 1, 1)	153	69	54	34	54	69	153	-	153	69	54	34	54	69	153	-
(8, 4, 3)	93	72	52	31	31	52	72	93	93	72	52	31	31	52	72	93
(8, 4, 2, 1)	137	54	51	25	25	51	54	137	137	54	51	25	25	51	54	137
(8, 4, 1, 1, 1)	80	11	26	10	26	11	80	-	80	11	26	10	26	11	80	-
(8, 3, 3, 1)	74	77	23	6	23	77	74	-	74	77	23	6	23	77	74	-
(8, 3, 2, 2)	20	60	10	-	-	10	60	20	20	60	10	-	-	10	60	20
(8, 3, 2, 1, 1)	94	83	13	8	13	83	94	-	94	83	13	8	13	83	94	-
(8, 3, 1, 1, 1, 1)	53	24	6	6	24	53	-	-	53	24	6	6	24	53	-	-
(8, 2, 2, 2, 1)	-	27	-	-	-	-	27	-	-	27	-	-	-	-	27	-
(8, 2, 2, 1, 1, 1)	27	34	-	-	-	34	27	-	27	34	-	-	-	34	27	-
(8, 2, 1, 1, 1, 1, 1)	34	14	1	1	14	34	-	-	34	14	1	1	14	34	-	-
(8, 1, 1, 1, 1, 1, 1, 1, 1)	12	2	2	2	12	-	-	-	12	2	2	2	12	-	-	-
(7, 7, 1)	102	54	55	47	55	54	102	-	102	54	55	47	55	54	102	-
(7, 6, 2)	96	58	34	49	49	34	58	96	96	58	34	49	49	34	58	96
(7, 6, 1, 1)	120	59	47	28	47	59	120	-	120	59	47	28	47	59	120	-
(7, 5, 3)	76	84	61	82	61	84	76	-	76	84	61	82	61	84	76	-
(7, 5, 2, 1)	116	84	65	66	65	84	116	-	116	84	65	66	65	84	116	-

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(7, 5, 1, 1, 1)	120	46	38	38	46	120	-	-	-	-	120	46	38	38	46	120
(7, 4, 4)	18	10	30	26	26	30	10	18	18	10	30	26	26	30	10	18
(7, 4, 3, 1)	73	72	45	54	45	72	73	-	-	73	72	45	54	45	72	73
(7, 4, 2, 2)	9	29	11	30	30	11	29	9	9	29	11	30	30	11	29	9
(7, 4, 2, 1, 1)	62	47	34	44	34	47	62	-	-	62	47	34	44	34	47	62
(7, 4, 1, 1, 1, 1)	48	8	17	17	8	48	-	-	-	-	48	8	17	17	8	48
(7, 3, 3, 2)	7	41	19	-	19	41	7	-	-	7	41	19	-	19	41	7
(7, 3, 3, 1, 1)	56	77	15	15	77	56	-	-	-	-	56	77	15	15	77	56
(7, 3, 2, 2, 1)	-	43	15	-	15	43	-	-	-	-	43	15	-	15	43	-
(7, 3, 2, 1, 1, 1)	46	62	7	7	62	46	-	-	-	-	46	62	7	7	62	46
(7, 3, 1, 1, 1, 1, 1)	31	18	6	18	31	-	-	-	-	-	31	18	6	18	31	-
(7, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(7, 2, 2, 2, 1, 1)	-	9	-	-	-	9	-	-	-	-	9	-	-	-	9	-
(7, 2, 2, 1, 1, 1, 1)	9	19	-	-	19	9	-	-	-	-	9	19	-	-	19	9
(7, 2, 1, 1, 1, 1, 1, 1)	19	7	-	7	19	-	-	-	-	-	19	7	-	7	19	-
(7, 1, 1, 1, 1, 1, 1, 1, 1)	5	2	2	5	-	-	-	-	-	-	-	5	2	2	5	-
(6, 6, 3)	24	17	23	21	21	23	17	24	24	17	23	21	21	23	17	24
(6, 6, 2, 1)	51	18	6	17	17	6	18	51	51	18	6	17	17	6	18	51
(6, 6, 1, 1, 1)	29	5	5	4	5	5	29	-	-	29	5	5	4	5	5	29
(6, 5, 4)	20	23	51	68	68	51	23	20	20	23	51	68	68	51	23	20
(6, 5, 3, 1)	39	60	69	78	69	60	39	-	-	39	60	69	78	69	60	39
(6, 5, 2, 2)	8	23	-	8	8	-	23	8	8	23	-	8	8	-	23	8
(6, 5, 2, 1, 1)	51	33	20	30	20	33	51	-	-	51	33	20	30	20	33	51
(6, 5, 1, 1, 1, 1)	38	10	18	18	10	38	-	-	-	-	38	10	18	18	10	38
(6, 4, 4, 1)	-	-	32	11	11	32	-	-	-	-	32	11	11	32	-	-
(6, 4, 3, 2)	3	31	16	25	25	16	31	3	3	31	16	25	25	16	31	3
(6, 4, 3, 1, 1)	14	25	28	24	28	25	14	-	-	14	25	28	24	28	25	14
(6, 4, 2, 2, 1)	-	21	-	18	18	-	21	-	-	21	-	18	18	-	21	-
(6, 4, 2, 1, 1, 1)	21	21	12	12	12	21	21	-	-	21	21	12	12	12	21	21
(6, 4, 1, 1, 1, 1, 1)	21	4	6	6	4	21	-	-	-	-	21	4	6	6	4	21
(6, 3, 3, 3)	-	7	11	-	11	7	-	-	-	-	7	11	-	11	7	-
(6, 3, 3, 2, 1)	-	15	16	-	16	15	-	-	-	-	15	16	-	16	15	-
(6, 3, 3, 1, 1, 1)	8	27	6	6	27	8	-	-	-	-	8	27	6	6	27	8
(6, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 3, 2, 2, 1, 1)	-	9	6	-	6	9	-	-	-	-	9	6	-	6	9	-
(6, 3, 2, 1, 1, 1, 1)	9	19	2	2	19	9	-	-	-	-	9	19	2	2	19	9
(6, 3, 1, 1, 1, 1, 1, 1)	13	6	2	6	13	-	-	-	-	-	13	6	2	6	13	-
(6, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 1, 1, 1, 1, 1)	-	6	-	-	6	-	-	-	-	-	-	6	-	-	6	-
(6, 2, 1, 1, 1, 1, 1, 1, 1)	6	2	-	2	6	-	-	-	-	-	-	2	-	2	6	-
(6, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	1	1	2	-	-	-	-	-	-	-	2	1	1	2	-
(5, 5, 5)	3	5	13	28	13	5	3	-	-	3	5	13	28	13	5	3
(5, 5, 4, 1)	6	13	45	46	45	13	6	-	-	6	13	45	46	45	13	6
(5, 5, 3, 2)	3	9	19	14	19	9	3	-	-	3	9	19	14	19	9	3
(5, 5, 3, 1, 1)	17	39	38	38	39	17	-	-	-	-	17	39	38	38	39	17
(5, 5, 2, 2, 1)	-	8	6	17	6	8	-	-	-	-	8	6	17	6	8	-
(5, 5, 2, 1, 1, 1)	12	17	23	23	17	12	-	-	-	-	12	17	23	23	17	12
(5, 5, 1, 1, 1, 1, 1)	11	7	20	7	11	-	-	-	-	-	11	7	20	7	11	-
(5, 4, 4, 2)	-	-	4	20	20	4	-	-	-	-	4	20	20	4	-	-
(5, 4, 4, 1, 1)	-	-	22	18	22	-	-	-	-	-	-	22	18	22	-	-
(5, 4, 3, 3)	-	5	19	12	19	5	-	-	-	-	5	19	12	19	5	-
(5, 4, 3, 2, 1)	-	11	25	64	25	11	-	-	-	-	11	25	64	25	11	-

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 4, 3, 1, 1, 1)	5	19	36	36	19	5	-	-	-	-	5	19	36	36	19	5
(5, 4, 2, 2, 2)	-	-	-	6	6	-	-	-	-	-	-	6	6	-	-	-
(5, 4, 2, 2, 1, 1)	-	6	4	24	4	6	-	-	-	-	6	4	24	4	6	-
(5, 4, 2, 1, 1, 1, 1)	6	10	17	17	10	6	-	-	-	-	6	10	17	17	10	6
(5, 4, 1, 1, 1, 1, 1, 1)	10	5	12	5	10	-	-	-	-	-	-	10	5	12	5	10
(5, 3, 3, 3, 1)	-	18	13	13	18	-	-	-	-	-	-	18	13	13	18	-
(5, 3, 3, 2, 2)	-	-	7	-	7	-	-	-	-	-	-	7	-	7	-	-
(5, 3, 3, 2, 1, 1)	-	23	14	14	23	-	-	-	-	-	-	23	14	14	23	-
(5, 3, 3, 1, 1, 1, 1)	9	28	16	28	9	-	-	-	-	-	-	9	28	16	28	9
(5, 3, 2, 2, 2, 1)	-	-	2	-	2	-	-	-	-	-	-	2	-	2	-	-
(5, 3, 2, 2, 1, 1, 1)	-	10	4	4	10	-	-	-	-	-	-	10	4	4	10	-
(5, 3, 2, 1, 1, 1, 1, 1)	8	17	6	17	8	-	-	-	-	-	-	8	17	6	17	8
(5, 3, 1, 1, 1, 1, 1, 1, 1)	7	12	12	7	-	-	-	-	-	-	-	7	12	12	7	-
(5, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 2, 1, 1, 1, 1)	-	2	-	-	2	-	-	-	-	-	-	2	-	-	2	-
(5, 2, 2, 1, 1, 1, 1, 1, 1)	2	4	-	4	2	-	-	-	-	-	-	2	4	-	4	2
(5, 2, 1, 1, 1, 1, 1, 1, 1, 1)	4	3	3	4	-	-	-	-	-	-	-	4	3	3	4	-
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	2	3	-	-	-	-	-	-	-	-	3	2	3	-	-
(4, 4, 4, 3)	-	-	-	1	1	-	-	-	-	-	-	1	1	-	-	-
(4, 4, 4, 2, 1)	-	-	-	3	3	-	-	-	-	-	-	3	3	-	-	-
(4, 4, 4, 1, 1, 1)	-	-	2	-	2	-	-	-	-	-	-	2	-	2	-	-
(4, 4, 3, 3, 1)	-	-	3	12	3	-	-	-	-	-	-	3	12	3	-	-
(4, 4, 3, 2, 2)	-	-	-	3	3	-	-	-	-	-	-	3	3	-	-	-
(4, 4, 3, 2, 1, 1)	-	-	3	12	3	-	-	-	-	-	-	3	12	3	-	-
(4, 4, 3, 1, 1, 1, 1)	-	1	4	4	1	-	-	-	-	-	-	1	4	4	1	-
(4, 4, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 2, 1, 1, 1)	-	-	-	9	-	-	-	-	-	-	-	-	9	-	-	-
(4, 4, 2, 1, 1, 1, 1, 1)	-	1	6	6	1	-	-	-	-	-	-	1	6	6	1	-
(4, 4, 1, 1, 1, 1, 1, 1, 1)	1	-	4	-	1	-	-	-	-	-	-	1	-	4	-	1
(4, 3, 3, 3, 2)	-	-	2	-	2	-	-	-	-	-	-	2	-	2	-	-
(4, 3, 3, 3, 1, 1)	-	4	8	8	4	-	-	-	-	-	-	4	8	8	4	-
(4, 3, 3, 2, 2, 1)	-	-	1	-	1	-	-	-	-	-	-	1	-	1	-	-
(4, 3, 3, 2, 1, 1, 1)	-	2	5	5	2	-	-	-	-	-	-	2	5	5	2	-
(4, 3, 3, 1, 1, 1, 1, 1)	1	14	4	14	1	-	-	-	-	-	-	1	14	4	14	1
(4, 3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 2, 1, 1, 1, 1)	-	1	3	3	1	-	-	-	-	-	-	1	3	3	1	-
(4, 3, 2, 1, 1, 1, 1, 1, 1)	1	5	4	5	1	-	-	-	-	-	-	1	5	4	5	1
(4, 3, 1, 1, 1, 1, 1, 1, 1, 1)	2	2	2	2	-	-	-	-	-	-	-	2	2	2	2	-
(4, 2, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	3	-	3	-	-	-	-	-	-	-	-	3	-	3	-
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	2	2	3	-	-	-	-	-	-	-	3	2	2	3	-
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	-	2	-	-	-	-	-	-	-	-	2	-	2	-	2
(3, 3, 3, 3, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 1, 1, 1)	-	-	5	-	-	-	-	-	-	-	-	-	5	-	-	-
(3, 3, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 1, 1, 1, 1)	-	-	2	-	-	-	-	-	-	-	-	-	2	-	-	-
(3, 3, 3, 1, 1, 1, 1, 1, 1)	-	8	8	-	-	-	-	-	-	-	-	-	8	8	-	-

Table 4: Number of α -graceful labelings for spiders order 16 or less, by branch label

Spider	Branch Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 1, 1, 1, 1, 1)	-	-	1	-	-	-	-	-	-	-	-	-	-	1	-	-
(3, 3, 2, 1, 1, 1, 1, 1, 1, 1)	-	2	2	-	-	-	-	-	-	-	-	-	-	2	2	-
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	11	1	-	-	-	-	-	-	-	-	-	-	1	11	1
(3, 2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	-	1	1	-	-	-	-	-	-	-	-	-	-	1	1	-
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	1	-	-	-	-	-	-	-	-	-	-	1	2	1
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	-	-	-	-	-	1	1
(2, 2, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	1	-	-	-	-	-	-	-	-	-	-	-	-	1	-
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	-	-	-	-	-	-	-	-	-	-	-	-	1	1
(13, 1, 1)	135	64	77	48	77	64	135	-	-	135	64	77	48	77	64	135
(12, 2, 1)	409	98	60	55	55	60	98	409	409	98	60	55	55	60	98	409
(12, 1, 1, 1)	86	18	26	10	26	18	86	-	-	86	18	26	10	26	18	86
(11, 3, 1)	324	163	74	92	74	163	324	-	-	324	163	74	92	74	163	324
(11, 2, 2)	154	68	21	26	26	21	68	154	154	68	21	26	26	21	68	154
(11, 2, 1, 1)	248	85	37	30	37	85	248	-	-	248	85	37	30	37	85	248
(11, 1, 1, 1, 1)	70	10	15	15	10	70	-	-	-	70	10	15	15	10	70	-
(10, 4, 1)	178	95	41	35	35	41	95	178	178	95	41	35	35	41	95	178
(10, 3, 2)	181	110	32	13	13	32	110	181	181	110	32	13	13	32	110	181
(10, 3, 1, 1)	169	104	29	14	29	104	169	-	-	169	104	29	14	29	104	169
(10, 2, 2, 1)	105	100	7	8	8	7	100	105	105	100	7	8	8	7	100	105
(10, 2, 1, 1, 1)	137	32	5	8	5	32	137	-	-	137	32	5	8	5	32	137
(10, 1, 1, 1, 1, 1)	24	2	5	5	2	24	-	-	-	24	2	5	5	2	24	-
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1

A.5 Alpha-Graceful Spiders by Leaf Label

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
(Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Spider	-	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1)	1	2	2	1												
(2, 1, 1)	2	3	2	3	2											
(1, 1, 1, 1)	1	2	2	2	1											
(3, 1, 1)	2	1	3	3	1	2										
(2, 2, 1)	1	1	1	1	1	1										
(2, 1, 1, 1)	2	3	3	3	3	2										
(1, 1, 1, 1, 1)	1	2	2	2	2	1										
(4, 1, 1)	3	4	3	4	3	4	3									
(3, 2, 1)	3	4	3	4	3	4	3									
(3, 1, 1, 1)	2	1	4	2	4	1	2									
(2, 2, 2)	-	-	-	-	-	-	-									
(2, 2, 1, 1)	1	1	2	-	2	1	1									
(2, 1, 1, 1, 1)	2	3	3	4	3	3	2									
(1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	1									
(5, 1, 1)	4	5	6	9	9	6	5	4								
(4, 2, 1)	6	9	9	6	6	9	9	6								
(4, 1, 1, 1)	3	5	3	5	5	3	5	3								
(3, 3, 1)	2	3	5	5	5	5	3	2								
(3, 2, 2)	1	2	2	1	1	2	2	1								
(3, 2, 1, 1)	3	5	3	5	5	3	5	3								
(3, 1, 1, 1, 1)	2	1	4	3	3	4	1	2								
(2, 2, 2, 1)	-	-	-	-	-	-	-	-								
(2, 2, 1, 1, 1)	1	1	2	1	1	2	1	1								
(2, 1, 1, 1, 1, 1)	2	3	3	4	4	3	3	2								
(1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	1								
(6, 1, 1)	2	5	6	4	2	4	6	5	2							
(5, 2, 1)	5	11	14	10	4	10	14	11	5							
(5, 1, 1, 1)	4	7	5	9	14	9	5	7	4							
(4, 3, 1)	4	5	9	3	6	3	9	5	4							
(4, 2, 2)	3	3	6	3	6	3	6	3	3							
(4, 2, 1, 1)	6	13	9	9	6	9	9	13	6							
(4, 1, 1, 1, 1)	3	5	4	5	6	5	4	5	3							
(3, 3, 2)	2	3	3	3	2	3	3	3	2							
(3, 3, 1, 1)	2	5	5	7	10	7	5	5	2							
(3, 2, 2, 1)	1	3	1	3	-	3	1	3	1							
(3, 2, 1, 1, 1)	3	5	4	5	6	5	4	5	3							
(3, 1, 1, 1, 1, 1)	2	1	4	3	4	3	4	1	2							
(2, 2, 2, 2)	-	-	-	-	-	-	-	-	-							
(2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-							
(2, 2, 1, 1, 1, 1)	1	1	2	1	2	1	2	1	1							
(2, 1, 1, 1, 1, 1, 1)	2	3	3	4	4	4	3	3	2							
(1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	1							
(7, 1, 1)	4	9	8	8	13	13	8	8	9	4						
(6, 2, 1)	5	13	6	13	5	5	13	6	13	5						
(6, 1, 1, 1)	3	4	7	6	4	4	6	7	4	3						
(5, 3, 1)	6	15	18	18	21	21	18	18	15	6						
(5, 2, 2)	1	5	6	5	1	1	5	6	5	1						

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 2, 1, 1)	8	11	13	12	12	12	12	13	11	8						
(5, 1, 1, 1, 1)	4	7	7	8	14	14	8	7	7	4						
(4, 4, 1)	3	4	1	4	3	3	4	1	4	3						
(4, 3, 2)	5	8	10	8	5	5	8	10	8	5						
(4, 3, 1, 1)	5	5	9	7	6	6	7	9	5	5						
(4, 2, 2, 1)	3	6	6	6	3	3	6	6	6	3						
(4, 2, 1, 1, 1)	6	13	13	9	9	9	9	13	13	6						
(4, 1, 1, 1, 1, 1)	3	5	4	6	6	6	6	4	5	3						
(3, 3, 3)	1	3	2	2	4	4	2	2	3	1						
(3, 3, 2, 1)	2	4	3	3	4	4	3	3	4	2						
(3, 3, 1, 1, 1)	2	6	7	7	13	13	7	7	6	2						
(3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-						
(3, 2, 2, 1, 1)	1	3	2	2	2	2	2	2	3	1						
(3, 2, 1, 1, 1, 1)	3	5	4	6	6	6	6	4	5	3						
(3, 1, 1, 1, 1, 1, 1)	2	1	4	3	4	4	3	4	1	2						
(2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-						
(2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-						
(2, 2, 1, 1, 1, 1, 1)	1	1	2	1	2	2	1	2	1	1						
(2, 1, 1, 1, 1, 1, 1, 1)	2	3	3	4	4	4	4	3	3	2						
(1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	1						
(8, 1, 1)	7	17	23	21	10	12	10	21	23	17	7					
(7, 2, 1)	13	25	30	23	19	20	19	23	30	25	13					
(7, 1, 1, 1)	6	9	7	12	13	18	13	12	7	9	6					
(6, 3, 1)	8	19	27	24	11	14	11	24	27	19	8					
(6, 2, 2)	3	3	3	3	3	6	3	3	3	3						
(6, 2, 1, 1)	9	13	14	15	7	12	7	15	14	13	9					
(6, 1, 1, 1, 1)	3	5	6	7	6	6	6	7	6	5	3					
(5, 4, 1)	11	23	25	25	16	16	16	25	25	23	11					
(5, 3, 2)	4	15	14	13	12	4	12	13	14	15	4					
(5, 3, 1, 1)	11	14	19	24	30	28	30	24	19	14	11					
(5, 2, 2, 1)	6	4	20	10	6	4	6	10	20	4	6					
(5, 2, 1, 1, 1)	8	14	13	11	14	20	14	11	13	14	8					
(5, 1, 1, 1, 1, 1)	4	7	7	10	13	14	13	10	7	7	4					
(4, 4, 2)	4	4	4	4	4	8	4	4	4	4						
(4, 4, 1, 1)	3	7	2	6	5	2	5	6	2	7	3					
(4, 3, 3)	2	7	7	8	5	2	5	8	7	7	2					
(4, 3, 2, 1)	7	12	18	16	7	8	7	16	18	12	7					
(4, 3, 1, 1, 1)	5	6	9	7	10	6	10	7	9	6	5					
(4, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-						
(4, 2, 2, 1, 1)	3	6	9	6	6	-	6	6	9	6	3					
(4, 2, 1, 1, 1, 1)	6	13	13	13	9	12	9	13	13	13	6					
(4, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	6	7	6	4	5	3					
(3, 3, 3, 1)	1	5	4	4	5	10	5	4	4	5	1					
(3, 3, 2, 2)	-	2	-	1	1	-	1	1	-	2	-					
(3, 3, 2, 1, 1)	2	4	4	3	4	6	4	3	4	4	2					
(3, 3, 1, 1, 1, 1)	2	7	8	9	14	16	14	9	8	7	2					
(3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-						
(3, 2, 2, 1, 1, 1)	1	3	2	3	1	4	1	3	2	3	1					
(3, 2, 1, 1, 1, 1, 1)	3	5	4	6	7	6	7	6	4	5	3					
(3, 1, 1, 1, 1, 1, 1, 1)	2	1	4	3	4	4	4	3	4	1	2					
(2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-						
(2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-						

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1)	1	1	2	1	2	2	2	1	2	1	1					
(2, 1, 1, 1, 1, 1, 1, 1, 1)	2	3	3	4	4	4	4	4	3	3	2					
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	2	1					
(9, 1, 1)	7	17	28	22	28	24	24	28	22	28	17	7				
(8, 2, 1)	17	48	58	58	48	17	17	48	58	58	48	17				
(8, 1, 1, 1)	12	19	24	29	15	21	21	15	29	24	19	12				
(7, 3, 1)	11	27	35	34	35	38	38	35	34	35	27	11				
(7, 2, 2)	9	12	18	18	12	9	9	12	18	18	12	9				
(7, 2, 1, 1)	21	38	39	41	28	41	41	28	41	39	38	21				
(7, 1, 1, 1, 1)	6	11	7	11	17	18	18	17	11	7	11	6				
(6, 4, 1)	10	26	39	39	26	10	10	26	39	39	26	10				
(6, 3, 2)	10	16	19	19	16	10	10	16	19	19	16	10				
(6, 3, 1, 1)	13	27	36	34	23	27	27	23	34	36	27	13				
(6, 2, 2, 1)	6	6	12	12	6	6	6	6	12	12	6	6				
(6, 2, 1, 1, 1)	9	17	13	19	8	14	14	8	19	13	17	9				
(6, 1, 1, 1, 1, 1)	3	5	7	6	7	8	8	7	6	7	5	3				
(5, 5, 1)	8	14	23	24	23	22	22	23	24	23	14	8				
(5, 4, 2)	10	15	23	23	15	10	10	15	23	23	15	10				
(5, 4, 1, 1)	14	34	32	31	25	32	32	25	31	32	34	14				
(5, 3, 3)	3	10	8	6	8	13	13	8	6	8	10	3				
(5, 3, 2, 1)	17	26	41	47	36	25	25	36	47	41	26	17				
(5, 3, 1, 1, 1)	11	21	17	26	37	38	38	37	26	17	21	11				
(5, 2, 2, 2)	2	-	6	6	-	2	2	-	6	6	-	2				
(5, 2, 2, 1, 1)	6	9	16	12	8	9	9	8	12	16	9	6				
(5, 2, 1, 1, 1, 1)	8	14	16	11	13	22	22	13	11	16	14	8				
(5, 1, 1, 1, 1, 1, 1)	4	7	7	10	15	13	13	15	10	7	7	4				
(4, 4, 3)	2	3	4	4	3	2	2	3	4	4	3	2				
(4, 4, 2, 1)	4	14	10	10	14	4	4	14	10	10	14	4				
(4, 4, 1, 1, 1)	3	7	3	8	5	4	4	5	8	3	7	3				
(4, 3, 3, 1)	4	13	15	18	12	10	10	12	18	15	13	4				
(4, 3, 2, 2)	1	9	6	6	9	1	1	9	6	6	9	1				
(4, 3, 2, 1, 1)	7	14	17	22	10	10	10	10	22	17	14	7				
(4, 3, 1, 1, 1, 1)	5	6	10	7	10	10	10	10	7	10	6	5				
(4, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(4, 2, 2, 1, 1, 1)	3	6	9	9	6	3	3	6	9	9	6	3				
(4, 2, 1, 1, 1, 1, 1)	6	13	13	13	13	12	12	13	13	13	13	6				
(4, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	7	7	6	4	5	3				
(3, 3, 3, 2)	-	3	1	1	1	2	2	1	1	1	3	-				
(3, 3, 3, 1, 1)	1	7	5	7	8	12	12	8	7	5	7	1				
(3, 3, 2, 2, 1)	-	2	-	2	-	1	1	-	2	-	2	-				
(3, 3, 2, 1, 1, 1)	2	4	4	4	4	6	6	4	4	4	4	2				
(3, 3, 1, 1, 1, 1, 1)	2	8	9	10	17	17	17	10	9	8	2					
(3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 2, 2, 1, 1, 1, 1)	1	3	2	3	2	3	3	2	3	2	3	1				
(3, 2, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	7	7	6	4	5	3				
(3, 1, 1, 1, 1, 1, 1, 1, 1)	2	1	4	3	4	4	4	4	3	4	1	2				
(2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 1, 1, 1, 1, 1, 1, 1)	1	1	2	1	2	2	2	2	1	2	1	1				

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	3	3	4	4	4	4	4	4	3	3	2				
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	2	2	1				
(9, 2, 1)	22	71	100	149	77	51	32	51	77	149	100	71	22			
(9, 1, 1, 1)	12	25	38	41	41	50	50	50	41	41	38	25	12			
(8, 3, 1)	21	56	88	123	65	35	40	35	65	123	88	56	21			
(8, 2, 2)	10	35	41	74	41	35	20	35	41	74	41	35	10			
(8, 2, 1, 1)	41	60	75	79	65	35	58	35	65	79	75	60	41			
(8, 1, 1, 1, 1)	13	23	24	29	21	26	28	26	21	29	24	23	13			
(7, 4, 1)	18	46	62	78	49	32	30	32	49	78	62	46	18			
(7, 3, 2)	17	40	44	40	47	30	20	30	47	40	44	40	17			
(7, 3, 1, 1)	21	43	62	70	70	83	86	83	70	70	62	43	21			
(7, 2, 2, 1)	18	30	42	35	29	35	6	35	29	35	42	30	18			
(7, 2, 1, 1, 1)	25	42	42	42	34	46	58	46	34	42	42	42	25			
(7, 1, 1, 1, 1, 1)	6	11	9	11	16	22	18	22	16	11	9	11	6			
(6, 5, 1)	13	51	76	95	64	33	20	33	64	95	76	51	13			
(6, 4, 2)	13	35	38	74	38	35	26	35	38	74	38	35	13			
(6, 4, 1, 1)	18	40	54	51	40	23	28	23	40	51	54	40	18			
(6, 3, 3)	7	16	24	25	19	11	12	11	19	25	24	16	7			
(6, 3, 2, 1)	22	28	43	47	34	23	22	23	34	47	43	28	22			
(6, 3, 1, 1, 1)	16	28	39	36	25	40	32	40	25	36	39	28	16			
(6, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(6, 2, 2, 1, 1)	6	9	12	15	6	12	-	12	6	15	12	9	6			
(6, 2, 1, 1, 1, 1)	9	17	17	18	12	15	16	15	12	18	17	17	9			
(6, 1, 1, 1, 1, 1, 1)	3	5	7	7	6	9	10	9	6	7	7	5	3			
(5, 5, 2)	6	12	24	40	18	13	2	13	18	40	24	12	6			
(5, 5, 1, 1)	11	24	36	29	29	47	48	47	29	29	36	24	11			
(5, 4, 3)	9	25	34	49	26	18	14	18	26	49	34	25	9			
(5, 4, 2, 1)	20	36	50	48	39	32	14	32	39	48	50	36	20			
(5, 4, 1, 1, 1)	15	36	35	37	27	37	46	37	27	37	35	36	15			
(5, 3, 3, 1)	11	25	34	38	38	45	50	45	38	38	34	25	11			
(5, 3, 2, 2)	7	3	18	18	9	7	4	7	9	18	18	3	7			
(5, 3, 2, 1, 1)	20	35	37	51	46	38	46	38	46	51	37	35	20			
(5, 3, 1, 1, 1, 1)	11	23	23	25	39	48	46	48	39	25	23	23	11			
(5, 2, 2, 2, 1)	2	2	4	8	-	4	-	4	-	8	4	2	2			
(5, 2, 2, 1, 1, 1)	6	9	21	8	10	11	14	11	10	8	21	9	6			
(5, 2, 1, 1, 1, 1, 1)	8	14	16	14	13	21	24	21	13	14	16	14	8			
(5, 1, 1, 1, 1, 1, 1, 1)	4	7	7	10	15	15	12	15	10	7	7	4				
(4, 4, 4)	1	2	1	4	1	2	2	2	1	4	1	2	1			
(4, 4, 3, 1)	3	5	7	4	9	3	2	3	9	4	7	5	3			
(4, 4, 2, 2)	-	9	9	-	9	9	-	9	9	-	9	9	-			
(4, 4, 2, 1, 1)	4	14	11	19	14	8	-	8	14	19	11	14	4			
(4, 4, 1, 1, 1, 1)	3	7	3	9	7	4	6	4	7	9	3	7	3			
(4, 3, 3, 2)	2	9	11	10	10	5	2	5	10	10	11	9	2			
(4, 3, 3, 1, 1)	4	18	17	26	19	16	20	16	19	26	17	18	4			
(4, 3, 2, 2, 1)	1	10	5	13	6	5	-	5	6	13	5	10	1			
(4, 3, 2, 1, 1, 1)	7	14	19	21	16	13	12	13	16	21	19	14	7			
(4, 3, 1, 1, 1, 1, 1)	5	6	10	8	10	10	14	10	10	8	10	6	5			
(4, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 2, 2, 1, 1, 1, 1)	3	6	9	9	9	3	6	3	9	9	9	6	3			
(4, 2, 1, 1, 1, 1, 1, 1)	6	13	13	13	13	16	12	16	13	13	13	13	6			
(4, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	7	7	6	4	5	3			

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 3, 3)	-	2	1	1	1	1	4	1	1	1	1	2	-	-	-	-
(3, 3, 3, 2, 1)	-	3	1	2	1	1	4	1	1	2	1	3	-	-	-	-
(3, 3, 3, 1, 1, 1)	1	9	6	9	11	18	12	18	11	9	6	9	1	-	-	-
(3, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 1, 1)	-	2	-	2	1	-	2	-	1	2	-	2	-	-	-	-
(3, 3, 2, 1, 1, 1, 1)	2	4	4	4	5	6	6	6	5	4	4	4	2	-	-	-
(3, 3, 1, 1, 1, 1, 1, 1)	2	9	10	11	19	20	18	20	19	11	10	9	2	-	-	-
(3, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1)	1	3	2	3	2	4	2	4	2	3	2	3	1	-	-	-
(3, 2, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	7	7	6	4	5	3	-	-	-
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	1	4	3	4	4	4	4	4	3	4	1	2	-	-	-
(2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	2	1	2	2	2	2	2	1	2	1	1	-	-	-
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	3	3	4	4	4	4	4	4	4	3	3	2	-	-	-
(10, 1, 1)	13	24	41	47	37	12	24	12	37	47	41	24	13	-	-	-
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	2	2	2	1	-	-	-
(9, 3, 1)	27	101	139	168	168	139	128	128	139	168	168	139	101	27	-	-
(9, 2, 2)	15	32	48	62	48	32	15	15	32	48	62	48	32	15	-	-
(9, 2, 1, 1)	60	97	142	154	137	100	110	110	100	137	154	142	97	60	-	-
(9, 1, 1, 1, 1)	20	22	38	43	44	63	60	60	63	44	43	38	22	20	-	-
(8, 4, 1)	32	91	127	94	127	91	32	32	91	127	94	127	91	32	-	-
(8, 3, 2)	25	60	77	114	77	60	25	25	60	77	114	77	60	25	-	-
(8, 3, 1, 1)	49	65	111	124	100	63	88	88	63	100	124	111	65	49	-	-
(8, 2, 2, 1)	56	41	76	102	76	41	56	56	41	76	102	76	41	56	-	-
(8, 2, 1, 1, 1)	47	79	77	90	79	42	76	76	42	79	90	77	79	47	-	-
(8, 1, 1, 1, 1, 1)	13	24	28	29	21	32	33	33	32	21	29	28	24	13	-	-
(7, 5, 1)	24	94	131	120	120	131	118	118	131	120	120	131	94	24	-	-
(7, 4, 2)	25	57	84	124	84	57	25	25	57	84	124	84	57	25	-	-
(7, 4, 1, 1)	36	71	91	103	90	59	78	78	59	90	103	91	71	36	-	-
(7, 3, 3)	8	39	45	49	49	45	47	47	45	49	49	45	39	8	-	-
(7, 3, 2, 1)	45	79	106	99	101	92	78	78	92	101	99	106	79	45	-	-
(7, 3, 1, 1, 1)	37	36	65	78	68	115	101	101	115	68	78	65	36	37	-	-
(7, 2, 2, 2)	3	-	7	4	7	-	3	3	-	7	4	7	-	3	-	-
(7, 2, 2, 1, 1)	28	33	51	55	38	43	32	32	43	38	55	51	33	28	-	-
(7, 2, 1, 1, 1, 1)	25	46	46	45	35	52	63	63	52	35	46	46	45	25	-	-
(7, 1, 1, 1, 1, 1, 1, 1)	6	11	9	13	16	21	22	22	21	16	13	9	11	6	-	-
(6, 6, 1)	7	36	53	42	53	36	7	7	36	53	42	53	36	7	-	-
(6, 5, 2)	13	50	73	112	73	50	13	13	50	73	112	73	50	13	-	-
(6, 5, 1, 1)	27	56	90	89	75	59	60	60	59	75	89	90	56	27	-	-
(6, 4, 3)	14	39	61	84	61	39	14	14	39	61	84	61	39	14	-	-
(6, 4, 2, 1)	44	44	52	96	52	44	44	44	52	96	52	44	44	44	-	-
(6, 4, 1, 1, 1)	25	42	60	64	50	28	41	41	28	50	64	60	42	25	-	-
(6, 3, 3, 1)	13	28	44	49	41	27	30	30	27	41	49	44	28	13	-	-
(6, 3, 2, 2)	5	9	9	18	9	9	5	5	9	9	18	9	9	5	-	-
(6, 3, 2, 1, 1)	29	35	54	61	49	39	33	33	39	49	61	54	35	29	-	-
(6, 3, 1, 1, 1, 1)	16	31	40	39	27	42	45	45	42	27	39	40	31	16	-	-
(6, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 1, 1, 1)	6	9	15	15	9	12	6	6	12	9	15	15	9	6	-	-

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(6, 2, 1, 1, 1, 1, 1)	9	17	17	22	11	19	17	17	19	11	22	17	17	9		
(6, 1, 1, 1, 1, 1, 1, 1)	3	5	7	7	7	8	11	11	8	7	7	7	5	3		
(5, 5, 3)	7	21	26	33	33	26	28	28	26	33	33	26	21	7		
(5, 5, 2, 1)	15	32	46	55	48	38	30	30	38	48	55	46	32	15		
(5, 5, 1, 1, 1)	14	25	39	36	32	50	64	64	50	32	36	39	25	14		
(5, 4, 4)	7	19	22	30	22	19	7	7	19	22	30	22	19	7		
(5, 4, 3, 1)	20	42	59	77	70	44	40	40	44	70	77	59	42	20		
(5, 4, 2, 2)	6	6	16	16	16	6	6	6	6	16	16	16	6	6		
(5, 4, 2, 1, 1)	23	47	62	65	53	44	36	36	44	53	65	62	47	23		
(5, 4, 1, 1, 1, 1, 1)	15	37	37	40	33	39	51	51	39	33	40	37	37	15		
(5, 3, 3, 2)	8	12	20	24	19	15	14	14	15	19	24	20	12	8		
(5, 3, 3, 1, 1)	17	30	45	50	46	67	75	75	67	46	50	45	30	17		
(5, 3, 2, 2, 1)	10	14	27	35	23	19	12	12	19	23	35	27	14	10		
(5, 3, 2, 1, 1, 1)	20	40	45	49	49	49	60	60	49	49	49	45	40	20		
(5, 3, 1, 1, 1, 1, 1)	11	25	24	32	38	52	56	56	52	38	32	24	25	11		
(5, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(5, 2, 2, 2, 1, 1)	2	2	6	6	2	4	2	2	4	2	6	6	2	2		
(5, 2, 2, 1, 1, 1, 1)	6	9	21	13	6	13	16	16	13	6	13	21	9	6		
(5, 2, 1, 1, 1, 1, 1, 1)	8	14	16	14	16	21	23	23	21	16	14	16	14	8		
(5, 1, 1, 1, 1, 1, 1, 1, 1)	4	7	7	10	15	15	14	14	15	15	10	7	7	4		
(4, 4, 4, 1)	1	4	1	4	1	4	1	1	4	1	4	1	4	1		
(4, 4, 3, 2)	1	9	7	6	7	9	1	1	9	7	6	7	9	1		
(4, 4, 3, 1, 1)	3	7	10	8	10	9	3	3	9	10	8	10	7	3		
(4, 4, 2, 2, 1)	-	9	9	9	9	9	-	-	9	9	9	9	9	-		
(4, 4, 2, 1, 1, 1)	4	14	11	20	23	8	4	4	8	23	20	11	14	4		
(4, 4, 1, 1, 1, 1, 1)	3	7	3	9	8	6	6	6	6	8	9	3	7	3		
(4, 3, 3, 3)	1	7	5	9	10	3	5	5	3	10	9	5	7	1		
(4, 3, 3, 2, 1)	2	12	13	15	14	7	7	7	7	14	15	13	12	2		
(4, 3, 3, 1, 1, 1)	4	22	20	32	25	25	28	28	25	25	32	20	22	4		
(4, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(4, 3, 2, 2, 1, 1)	1	10	6	12	13	2	4	4	2	13	12	6	10	1		
(4, 3, 2, 1, 1, 1, 1)	7	14	19	23	15	19	15	15	19	15	23	19	14	7		
(4, 3, 1, 1, 1, 1, 1, 1)	5	6	10	8	11	10	14	14	10	11	8	10	6	5		
(4, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(4, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(4, 2, 2, 1, 1, 1, 1, 1)	3	6	9	9	9	6	6	6	6	9	9	9	6	3		
(4, 2, 1, 1, 1, 1, 1, 1, 1)	6	13	13	13	13	16	16	16	16	13	13	13	13	6		
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	8	7	7	6	4	5	3		
(3, 3, 3, 3, 1)	-	3	1	2	3	2	4	4	2	3	2	1	3	-		
(3, 3, 3, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 3, 3, 2, 1, 1)	-	3	1	2	2	1	3	3	1	2	2	1	3	-		
(3, 3, 3, 1, 1, 1, 1)	1	11	7	11	13	23	18	18	23	13	11	7	11	1		
(3, 3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 3, 2, 2, 1, 1, 1)	-	2	-	2	1	1	1	1	1	2	-	2	-	-		
(3, 3, 2, 1, 1, 1, 1, 1)	2	4	4	4	5	7	6	6	7	5	4	4	4	2		
(3, 3, 1, 1, 1, 1, 1, 1, 1)	2	10	11	12	21	22	21	21	22	12	11	10	2	2		
(3, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-		
(3, 2, 2, 1, 1, 1, 1, 1, 1)	1	3	2	3	2	4	3	3	4	2	3	2	3	1		
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	8	7	7	6	4	5	3		
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	1	4	3	4	4	4	4	4	4	3	4	1	2		

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(2, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	2	1	2	2	2	2	2	2	1	2	1	1		
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	3	3	4	4	4	4	4	4	4	4	3	3	2		
(11, 1, 1)	19	58	96	79	79	96	77	77	96	79	79	96	58	19		
(10, 2, 1)	38	138	169	216	169	138	38	38	138	169	216	169	138	38		
(10, 1, 1, 1)	16	39	39	40	41	26	39	39	26	41	40	39	39	16		
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	2	2	2	2	1		
(9, 4, 1)	46	106	214	211	231	147	79	80	79	147	231	211	214	106	46	
(9, 3, 2)	36	119	206	183	190	188	88	32	88	188	190	183	206	119	36	
(9, 3, 1, 1)	81	149	216	248	220	248	297	298	297	248	220	248	216	149	81	
(9, 2, 2, 1)	66	93	128	167	155	100	97	36	97	100	155	167	128	93	66	
(9, 2, 1, 1, 1)	87	142	158	208	175	174	173	186	173	174	175	208	158	142	87	
(9, 1, 1, 1, 1, 1)	20	30	35	43	46	66	73	70	73	66	46	43	35	30	20	
(8, 5, 1)	46	147	223	279	255	169	104	86	104	169	255	279	223	147	46	
(8, 4, 2)	37	107	127	122	122	127	107	74	107	127	122	122	127	107	37	
(8, 4, 1, 1)	67	126	141	127	144	121	68	108	68	121	144	127	141	126	67	
(8, 3, 3)	15	45	92	99	93	64	34	28	34	64	93	99	92	45	15	
(8, 3, 2, 1)	100	153	189	246	240	155	103	124	103	155	240	246	189	153	100	
(8, 3, 1, 1, 1)	59	93	124	148	114	99	132	122	132	99	114	148	124	93	59	
(8, 2, 2, 2)	27	-	27	54	54	27	-	54	-	27	54	54	27	-	27	
(8, 2, 2, 1, 1)	61	87	86	118	95	46	90	54	90	46	95	118	86	87	61	
(8, 2, 1, 1, 1, 1)	47	85	95	90	87	55	81	96	81	55	87	90	95	85	47	
(8, 1, 1, 1, 1, 1, 1)	13	24	29	33	21	32	39	38	39	32	21	33	29	24	13	
(7, 6, 1)	27	120	221	255	232	157	92	48	92	157	232	255	221	120	27	
(7, 5, 2)	28	89	164	138	156	143	70	20	70	143	156	138	164	89	28	
(7, 5, 1, 1)	63	132	176	191	164	191	239	264	239	191	164	191	176	132	63	
(7, 4, 3)	21	68	141	176	162	116	46	28	46	116	162	176	141	68	21	
(7, 4, 2, 1)	76	143	152	183	193	134	111	64	111	134	193	183	152	143	76	
(7, 4, 1, 1, 1)	50	87	101	124	111	92	109	112	109	92	111	124	101	87	50	
(7, 3, 3, 1)	30	72	88	107	70	107	118	144	118	107	70	107	88	72	30	
(7, 3, 2, 2)	17	24	43	26	36	31	28	6	28	31	36	26	43	24	17	
(7, 3, 2, 1, 1)	85	113	142	182	192	178	161	134	161	178	192	182	142	113	85	
(7, 3, 1, 1, 1, 1)	37	53	59	81	76	113	134	118	134	113	76	81	59	53	37	
(7, 2, 2, 2, 1)	9	3	17	23	20	6	12	-	12	6	20	23	17	3	9	
(7, 2, 2, 1, 1, 1)	28	43	50	56	46	48	32	66	32	48	46	56	50	43	28	
(7, 2, 1, 1, 1, 1, 1)	25	46	50	49	38	53	69	68	69	53	38	49	50	46	25	
(7, 1, 1, 1, 1, 1, 1, 1)	6	11	9	13	18	21	21	26	21	21	18	13	9	11	6	
(6, 6, 2)	9	36	37	38	38	37	36	18	36	37	38	38	37	36	9	
(6, 6, 1, 1)	20	41	63	38	52	48	20	36	20	48	52	38	63	41	20	
(6, 5, 3)	17	61	115	147	124	91	42	30	42	91	124	147	115	61	17	
(6, 5, 2, 1)	55	95	116	160	155	104	62	58	62	104	155	160	116	95	55	
(6, 5, 1, 1, 1)	36	76	101	118	97	85	98	98	85	97	118	101	76	36		
(6, 4, 4)	8	20	24	26	26	24	20	16	20	24	26	26	24	20	8	
(6, 4, 3, 1)	36	53	83	129	102	72	29	48	29	72	102	129	83	53	36	
(6, 4, 2, 2)	21	18	30	51	51	30	18	42	18	30	51	51	30	18	21	
(6, 4, 2, 1, 1)	51	81	84	133	96	63	71	42	71	63	96	133	84	81	51	
(6, 4, 1, 1, 1, 1)	25	49	60	69	60	37	45	54	45	37	60	69	60	49	25	
(6, 3, 3, 2)	7	22	18	24	21	19	12	10	12	19	21	24	18	22	7	
(6, 3, 3, 1, 1)	20	36	61	69	68	47	57	44	57	47	68	69	61	36	20	

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(6, 3, 2, 2, 1)	9	20	18	41	29	13	20	-	20	13	29	41	18	20	9	
(6, 3, 2, 1, 1, 1)	29	42	56	65	51	50	42	50	42	50	51	65	56	42	29	
(6, 3, 1, 1, 1, 1, 1)	16	31	43	40	30	44	47	58	47	44	30	40	43	31	16	
(6, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 1, 1, 1, 1)	6	9	15	18	9	15	6	12	6	15	9	18	15	9	6	
(6, 2, 1, 1, 1, 1, 1, 1)	9	17	17	22	15	18	21	18	21	18	15	22	17	17	9	
(6, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	7	7	7	9	10	12	10	9	7	7	7	5	3	
(5, 5, 4)	11	20	51	45	54	33	20	12	20	33	54	45	51	20	11	
(5, 5, 3, 1)	18	54	63	78	70	78	81	108	81	78	70	78	63	54	18	
(5, 5, 2, 2)	5	2	14	15	22	4	4	4	4	4	22	15	14	2	5	
(5, 5, 2, 1, 1)	23	49	78	93	66	80	66	70	66	80	66	93	78	49	23	
(5, 5, 1, 1, 1, 1)	14	29	40	40	39	53	69	80	69	53	39	40	40	29	14	
(5, 4, 4, 1)	10	36	33	53	44	42	21	2	21	42	44	53	33	36	10	
(5, 4, 3, 2)	12	33	34	41	48	29	22	10	22	29	48	41	34	33	12	
(5, 4, 3, 1, 1)	24	59	83	100	103	81	74	72	74	81	103	100	83	59	24	
(5, 4, 2, 2, 1)	8	23	49	38	58	27	17	-	17	27	58	38	49	23	8	
(5, 4, 2, 1, 1, 1)	23	50	66	72	58	54	43	60	43	54	58	72	66	50	23	
(5, 4, 1, 1, 1, 1, 1)	15	37	38	42	36	45	53	56	53	45	36	42	38	37	15	
(5, 3, 3, 3)	3	9	11	6	14	6	14	18	14	6	14	6	11	9	3	
(5, 3, 3, 2, 1)	17	32	49	63	59	56	41	46	41	56	59	63	49	32	17	
(5, 3, 3, 1, 1, 1)	17	44	51	63	59	76	107	102	107	76	59	63	51	44	17	
(5, 3, 2, 2, 2)	-	4	2	6	4	2	2	-	2	2	4	6	2	4	-	
(5, 3, 2, 2, 1, 1)	10	17	29	35	22	27	15	26	15	27	22	35	29	17	10	
(5, 3, 2, 1, 1, 1, 1)	20	42	49	59	46	52	73	74	73	52	46	59	42	42	20	
(5, 3, 1, 1, 1, 1, 1, 1)	11	27	25	34	45	53	59	68	59	53	45	34	25	27	11	
(5, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(5, 2, 2, 2, 1, 1, 1)	2	2	6	8	-	6	2	4	2	6	-	8	6	2	2	
(5, 2, 2, 1, 1, 1, 1, 1)	6	9	21	13	11	9	18	18	18	9	11	13	21	9	6	
(5, 2, 1, 1, 1, 1, 1, 1, 1)	8	14	16	14	16	24	23	22	23	24	16	14	16	14	8	
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1)	4	7	7	10	15	15	14	16	14	15	10	7	7	4		
(4, 4, 4, 2)	-	6	3	3	3	3	6	-	6	3	3	3	3	6	-	
(4, 4, 4, 1, 1)	1	4	1	6	2	4	2	-	2	4	2	6	1	4	1	
(4, 4, 3, 3)	1	7	5	8	8	9	2	-	2	9	8	8	5	7	1	
(4, 4, 3, 2, 1)	1	14	18	15	20	19	3	-	3	19	20	15	18	14	1	
(4, 4, 3, 1, 1, 1)	3	7	10	10	11	9	8	4	8	9	11	10	10	7	3	
(4, 4, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 4, 2, 2, 1, 1)	-	9	9	9	18	9	-	-	-	9	18	9	9	9	-	
(4, 4, 2, 1, 1, 1, 1)	4	14	11	20	24	17	4	8	4	17	24	20	11	14	4	
(4, 4, 1, 1, 1, 1, 1, 1, 1)	3	7	3	9	8	7	8	6	8	7	8	9	3	7	3	
(4, 3, 3, 3, 1)	1	13	11	18	17	14	8	16	8	14	17	18	11	13	1	
(4, 3, 3, 2, 2)	-	2	1	3	2	1	1	-	1	1	2	3	1	2	-	
(4, 3, 3, 2, 1, 1)	2	12	14	16	16	10	8	12	8	10	16	16	14	12	2	
(4, 3, 3, 1, 1, 1, 1)	4	26	22	39	29	31	41	36	41	31	29	39	22	26	4	
(4, 3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 3, 2, 2, 1, 1, 1)	1	10	6	13	12	9	1	8	1	9	12	13	6	10	1	
(4, 3, 2, 1, 1, 1, 1, 1)	7	14	19	23	17	18	21	18	21	18	17	23	19	14	7	
(4, 3, 1, 1, 1, 1, 1, 1, 1)	5	6	10	8	11	11	14	14	14	11	11	8	10	6	5	
(4, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
(4, 2, 2, 1, 1, 1, 1, 1, 1)	3	6	9	9	9	6	9	6	9	6	9	9	6	9	3	

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1)	6	13	13	13	13	16	16	20	16	16	13	13	13	13	6	
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	8	8	7	7	6	4	5	3	
(3, 3, 3, 3, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 1, 1)	-	4	1	3	4	4	7	2	7	4	4	3	1	4	-	-
(3, 3, 3, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 1, 1, 1)	-	3	1	2	2	2	3	2	3	2	2	2	1	3	-	-
(3, 3, 3, 1, 1, 1, 1, 1)	1	13	8	13	15	27	22	26	22	27	15	13	8	13	1	
(3, 3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 1, 1, 1, 1)	-	2	-	2	1	1	2	-	2	1	1	2	-	2	-	-
(3, 3, 2, 1, 1, 1, 1, 1, 1)	2	4	4	4	5	7	7	6	7	7	5	4	4	4	2	
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	11	12	13	23	24	23	24	23	24	23	13	12	11	2	
(3, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	1	3	2	3	2	4	3	4	3	4	2	3	2	3	1	
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	8	8	7	7	6	4	5	3	
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	1	4	3	4	4	4	4	4	4	4	3	4	1	2	
(2, 2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	3	2	3	2	4	3	4	3	4	2	3	2	3	1	
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	8	8	7	7	6	4	5	3	
(2, 2, 2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	2	1	2	2	2	2	2	2	2	1	2	1	1	
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	3	3	4	4	4	4	4	4	4	4	3	3	2		
(12, 1, 1)	25	70	166	191	170	123	47	48	47	123	170	191	166	70	25	
(11, 2, 1)	62	173	252	263	272	210	128	88	128	210	272	263	252	173	62	
(11, 1, 1, 1)	45	64	102	91	76	91	147	128	147	91	76	91	102	64	45	
(10, 3, 1)	47	156	268	321	306	177	119	92	119	177	306	321	268	156	47	
(10, 2, 2)	25	82	95	83	83	95	82	50	82	95	83	83	95	82	25	
(10, 2, 1, 1)	116	197	192	249	203	194	93	200	93	194	203	249	192	197	116	
(10, 1, 1, 1, 1)	24	36	48	47	44	34	57	40	57	34	44	47	48	36	24	
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	2	2	2	2	2	1	
(9, 5, 1)	88	238	453	617	568	617	453	326	326	453	617	568	617	453	238	88
(9, 4, 2)	56	204	307	405	405	307	204	56	56	204	307	405	405	307	204	56
(9, 4, 1, 1)	103	192	277	261	375	238	169	225	225	169	238	375	261	277	192	103
(9, 3, 3)	21	80	170	175	176	175	170	101	101	170	175	176	175	170	80	21
(9, 3, 2, 1)	207	360	424	544	558	547	359	353	353	359	547	558	544	424	360	207
(9, 3, 1, 1, 1)	138	197	270	328	352	352	466	467	467	466	352	352	328	270	197	138
(9, 2, 2, 2)	18	24	46	40	40	46	24	18	18	24	46	40	40	46	24	18
(9, 2, 2, 1, 1)	111	161	198	228	213	195	164	140	140	164	195	213	228	198	161	111
(9, 2, 1, 1, 1, 1)	100	156	169	203	200	170	231	223	223	231	170	200	203	169	156	100
(9, 1, 1, 1, 1, 1, 1, 1)	20	30	43	40	46	68	76	83	83	76	68	46	40	43	30	20
(8, 6, 1)	81	234	361	437	437	361	234	81	81	234	361	437	437	361	234	81
(8, 5, 2)	48	198	354	441	441	354	198	48	48	198	354	441	441	354	198	48
(8, 5, 1, 1)	114	254	350	382	425	320	214	285	285	214	320	425	382	350	254	114
(8, 4, 3)	39	133	266	306	306	266	133	39	39	133	266	306	306	266	133	39
(8, 4, 2, 1)	152	311	288	317	317	288	311	152	152	311	288	317	317	288	311	152
(8, 4, 1, 1, 1)	80	178	189	162	182	176	95	158	158	95	176	182	162	189	178	80
(8, 3, 3, 1)	71	128	207	278	244	203	122	163	163	122	203	244	278	207	128	71
(8, 3, 2, 2)	60	64	98	138	138	98	64	60	60	64	98	138	138	98	64	60
(8, 3, 2, 1, 1)	153	221	287	303	314	269	178	215	215	178	269	314	303	287	221	153

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(8, 3, 1, 1, 1, 1)	66	95	139	152	128	97	167	152	152	167	97	128	152	139	95	66
(8, 2, 2, 2, 1)	27	27	27	54	54	27	27	27	27	27	27	54	54	27	27	27
(8, 2, 2, 1, 1, 1)	61	92	127	118	101	55	90	88	88	90	55	101	118	127	92	61
(8, 2, 1, 1, 1, 1, 1)	47	85	101	108	87	63	94	101	101	94	63	87	108	101	85	47
(8, 1, 1, 1, 1, 1, 1, 1)	13	24	29	34	25	32	39	44	44	39	32	25	34	29	24	13
(7, 7, 1)	36	111	201	242	227	242	201	147	147	201	242	227	242	201	111	36
(7, 6, 2)	35	148	241	287	287	241	148	35	35	148	241	287	287	241	148	35
(7, 6, 1, 1)	84	189	283	334	374	274	176	206	206	176	274	374	334	283	189	84
(7, 5, 3)	27	135	217	284	246	284	217	162	162	217	284	246	284	217	135	27
(7, 5, 2, 1)	119	259	348	369	389	369	304	227	227	304	369	389	369	348	259	119
(7, 5, 1, 1, 1)	90	185	224	244	277	277	334	409	409	334	277	277	244	224	185	90
(7, 4, 4)	14	52	83	103	103	83	52	14	14	52	83	103	103	83	52	14
(7, 4, 3, 1)	80	162	226	345	305	298	154	166	166	154	298	305	345	226	162	80
(7, 4, 2, 2)	32	71	98	115	115	98	71	32	32	71	98	115	115	98	71	32
(7, 4, 2, 1, 1)	117	202	223	261	270	248	169	160	160	169	248	270	261	223	202	117
(7, 4, 1, 1, 1, 1)	55	96	103	129	123	97	142	131	131	142	97	123	129	103	96	55
(7, 3, 3, 2)	28	55	86	74	97	70	73	53	53	73	70	97	74	86	55	28
(7, 3, 3, 1, 1)	60	127	149	188	216	216	248	276	276	248	216	216	188	149	127	60
(7, 3, 2, 2, 1)	43	50	80	111	100	83	70	43	43	70	83	100	111	80	50	43
(7, 3, 2, 1, 1, 1)	97	141	137	184	215	199	213	194	194	213	199	215	184	137	141	97
(7, 3, 1, 1, 1, 1, 1)	37	54	77	75	78	123	132	152	152	132	123	78	75	77	54	37
(7, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(7, 2, 2, 2, 1, 1)	9	9	14	21	27	7	12	9	9	12	7	27	21	14	9	9
(7, 2, 2, 1, 1, 1, 1)	28	43	60	55	47	56	37	66	66	37	56	47	55	60	43	28
(7, 2, 1, 1, 1, 1, 1, 1)	25	46	50	53	42	56	70	74	74	70	56	42	53	50	46	25
(7, 1, 1, 1, 1, 1, 1, 1, 1)	6	11	9	13	18	23	21	25	25	21	23	18	13	9	11	6
(6, 6, 3)	12	52	92	99	99	92	52	12	12	52	92	99	99	92	52	12
(6, 6, 2, 1)	48	97	115	108	108	115	97	48	48	97	115	108	108	115	97	48
(6, 6, 1, 1, 1)	25	60	65	56	61	61	28	54	54	28	61	61	56	65	60	25
(6, 5, 4)	22	100	152	212	212	152	100	22	22	100	152	212	212	152	100	22
(6, 5, 3, 1)	57	137	248	336	313	263	162	140	140	162	263	313	336	248	137	57
(6, 5, 2, 2)	25	22	49	60	60	49	22	25	25	22	49	60	60	49	22	25
(6, 5, 2, 1, 1)	81	139	168	203	190	179	111	119	119	111	179	190	203	168	139	81
(6, 5, 1, 1, 1, 1)	43	78	105	116	117	93	120	120	120	93	117	116	105	78	43	43
(6, 4, 4, 1)	15	36	37	84	84	37	36	15	15	36	37	84	84	37	36	15
(6, 4, 3, 2)	37	61	85	117	117	85	61	37	37	61	85	117	117	85	61	37
(6, 4, 3, 1, 1)	47	73	110	165	149	113	68	65	65	68	113	149	165	110	73	47
(6, 4, 2, 2, 1)	21	48	48	78	78	48	48	21	21	48	48	78	78	48	48	21
(6, 4, 2, 1, 1, 1)	51	88	105	145	101	87	74	69	69	74	87	101	145	105	88	51
(6, 4, 1, 1, 1, 1, 1)	25	49	67	69	65	47	54	58	58	54	47	65	69	67	49	25
(6, 3, 3, 3)	2	20	21	23	30	18	13	17	17	13	18	20	21	20	2	2
(6, 3, 3, 2, 1)	15	35	41	66	55	44	26	28	28	26	44	55	66	41	35	15
(6, 3, 3, 1, 1, 1)	22	43	66	76	79	64	73	69	69	73	64	79	76	66	43	22
(6, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 3, 2, 2, 1, 1)	9	24	19	42	32	25	14	15	15	14	25	32	42	19	24	9
(6, 3, 2, 1, 1, 1, 1)	29	42	63	67	55	52	53	59	59	53	52	55	67	63	42	29
(6, 3, 1, 1, 1, 1, 1, 1)	16	31	43	43	31	47	49	60	60	49	47	31	43	43	31	16
(6, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 1, 1, 1, 1, 1)	6	9	15	18	12	15	9	12	12	9	15	12	18	15	9	6
(6, 2, 1, 1, 1, 1, 1, 1, 1)	9	17	17	22	15	22	20	22	22	20	22	15	22	17	17	9
(6, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	7	7	7	9	11	11	11	11	9	7	7	7	5	3

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 5, 5)	5	7	27	43	46	43	27	12	12	27	43	46	43	27	7	5
(5, 5, 4, 1)	20	78	105	126	107	117	82	61	61	82	117	107	126	105	78	20
(5, 5, 3, 2)	11	30	41	59	57	50	27	29	29	27	50	57	59	41	30	11
(5, 5, 3, 1, 1)	27	82	115	123	123	123	150	197	197	150	123	123	123	115	82	27
(5, 5, 2, 2, 1)	10	13	42	46	44	36	21	13	13	21	36	44	46	42	13	10
(5, 5, 2, 1, 1, 1)	27	55	80	104	87	83	94	94	94	83	87	104	80	55	27	27
(5, 5, 1, 1, 1, 1, 1)	14	30	44	42	43	60	73	86	86	73	60	43	42	44	30	14
(5, 4, 4, 2)	4	29	42	21	21	42	29	4	4	29	42	21	21	42	29	4
(5, 4, 4, 1, 1)	11	39	39	59	54	59	33	16	16	33	59	54	59	39	39	11
(5, 4, 3, 3)	6	27	36	46	40	34	27	24	24	27	34	40	46	36	27	6
(5, 4, 3, 2, 1)	20	76	101	119	145	123	61	35	35	61	123	145	119	101	76	20
(5, 4, 3, 1, 1, 1)	24	68	86	121	114	95	111	101	101	111	95	114	121	86	68	24
(5, 4, 2, 2, 2)	-	6	12	12	12	12	6	-	-	6	12	12	12	12	6	-
(5, 4, 2, 2, 1, 1)	8	25	51	49	50	47	23	11	11	23	47	50	49	51	25	8
(5, 4, 2, 1, 1, 1, 1)	23	50	69	76	65	59	53	67	67	53	59	65	76	69	50	23
(5, 4, 1, 1, 1, 1, 1, 1)	15	37	38	43	38	48	58	58	58	48	38	43	38	37	15	15
(5, 3, 3, 3, 1)	9	26	37	38	45	45	47	63	63	47	45	45	38	37	26	9
(5, 3, 3, 2, 2)	-	12	9	14	14	9	5	7	7	5	9	14	14	9	12	-
(5, 3, 3, 2, 1, 1)	17	42	53	75	63	62	65	67	67	65	62	63	75	53	42	17
(5, 3, 3, 1, 1, 1, 1)	17	52	66	71	69	97	121	137	137	121	97	69	71	66	52	17
(5, 3, 2, 2, 2, 1)	-	4	2	8	2	6	-	2	2	-	6	2	8	2	4	-
(5, 3, 2, 2, 1, 1, 1)	10	17	32	37	22	26	23	29	29	23	26	22	37	32	17	10
(5, 3, 2, 1, 1, 1, 1, 1)	20	44	50	65	55	49	77	88	88	77	49	55	65	50	44	20
(5, 3, 1, 1, 1, 1, 1, 1, 1)	11	29	26	36	47	62	59	72	72	59	62	47	36	26	29	11
(5, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 2, 1, 1, 1, 1)	2	2	6	8	2	4	4	4	4	4	4	2	8	6	2	2
(5, 2, 2, 1, 1, 1, 1, 1, 1)	6	9	21	13	11	14	14	20	20	14	14	11	13	21	9	6
(5, 2, 1, 1, 1, 1, 1, 1, 1, 1)	8	14	16	14	16	24	26	22	22	26	24	16	14	16	14	8
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	4	7	7	10	15	15	14	16	16	14	15	15	10	7	7	4
(4, 4, 4, 3)	-	2	1	1	1	1	2	-	-	2	1	1	1	1	2	-
(4, 4, 4, 2, 1)	-	6	3	6	6	3	6	-	-	6	3	6	6	3	6	-
(4, 4, 4, 1, 1, 1)	1	4	1	6	4	5	2	1	1	2	5	4	6	1	4	1
(4, 4, 3, 3, 1)	1	14	9	14	20	24	6	2	2	6	24	20	14	9	14	1
(4, 4, 3, 2, 2)	-	3	6	6	6	6	3	-	-	3	6	6	6	6	3	-
(4, 4, 3, 2, 1, 1)	1	14	18	20	19	26	8	2	2	8	26	19	20	18	14	1
(4, 4, 3, 1, 1, 1, 1)	3	7	10	10	13	10	8	9	9	8	10	13	10	10	7	3
(4, 4, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 2, 1, 1, 1)	-	9	9	9	18	18	-	-	-	18	18	9	9	9	9	-
(4, 4, 2, 1, 1, 1, 1, 1)	4	14	11	20	24	18	13	8	8	13	18	24	20	11	14	4
(4, 4, 1, 1, 1, 1, 1, 1, 1, 1)	3	7	3	9	8	7	9	8	8	9	7	8	9	3	7	3
(4, 3, 3, 3, 2)	-	3	3	4	4	3	1	2	2	1	3	4	4	3	3	-
(4, 3, 3, 3, 1, 1)	1	18	13	26	26	19	22	19	19	22	19	26	26	13	18	1
(4, 3, 3, 2, 2, 1)	-	2	1	4	1	3	-	1	1	-	3	1	4	1	2	-
(4, 3, 3, 2, 1, 1, 1)	2	12	14	17	17	12	11	13	13	11	12	17	17	14	12	2
(4, 3, 3, 1, 1, 1, 1, 1)	4	30	24	45	34	35	49	51	51	49	35	34	45	24	30	4
(4, 3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 2, 1, 1, 1, 1)	1	10	6	13	13	8	8	5	5	8	8	13	13	6	10	1
(4, 3, 2, 1, 1, 1, 1, 1, 1)	7	14	19	23	17	20	20	24	24	20	20	17	23	19	14	7
(4, 3, 1, 1, 1, 1, 1, 1, 1, 1)	5	6	10	8	11	11	15	14	14	15	11	11	8	10	6	5
(4, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 5: Number of α -graceful labelings for spiders order 16 or less, by leaf label
 (Note that spiders are counted once for *each* leg.)

Spider	Leaf Label															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(4, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 1, 1, 1, 1, 1, 1, 1)	3	6	9	9	9	6	9	9	9	9	6	9	9	9	6	3
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	6	13	13	13	13	16	16	20	20	16	16	13	13	13	13	6
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	8	8	8	7	7	6	4	5	3
(3, 3, 3, 3, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 1, 1, 1)	-	5	1	4	5	5	10	5	5	10	5	5	4	1	5	-
(3, 3, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 1, 1, 1, 1)	-	3	1	2	2	2	4	2	2	4	2	2	2	1	3	-
(3, 3, 3, 1, 1, 1, 1, 1, 1)	1	15	9	15	17	31	25	31	31	25	31	17	15	9	15	1
(3, 3, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 1, 1, 1, 1, 1)	-	2	-	2	1	1	2	1	1	2	1	1	2	-	2	-
(3, 3, 2, 1, 1, 1, 1, 1, 1, 1)	2	4	4	4	5	7	7	7	7	7	5	4	4	4	4	2
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	12	13	14	25	26	25	26	26	25	26	25	14	13	12	2
(3, 2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	1	3	2	3	2	4	3	4	4	3	4	2	3	2	3	1
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	3	5	4	6	7	7	8	8	8	8	7	7	6	4	5	3
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	1	4	3	4	4	4	4	4	4	4	4	3	4	1	2
(2, 2, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	1	2	1	2	2	2	2	2	2	2	1	2	1	1	1
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	2	3	3	4	4	4	4	4	4	4	4	4	4	3	3	2
(13, 1, 1)	50	125	251	335	278	335	251	175	175	251	335	278	335	251	125	50
(12, 2, 1)	115	422	607	722	722	607	422	115	115	422	607	722	722	607	422	115
(12, 1, 1, 1)	53	116	150	194	190	153	93	131	131	93	153	190	194	150	116	53
(11, 3, 1)	106	287	459	695	548	695	459	393	393	459	695	548	695	459	287	106
(11, 2, 2)	43	183	235	346	346	235	183	43	43	183	235	346	346	235	183	43
(11, 2, 1, 1)	189	352	424	497	465	443	331	379	379	331	443	465	497	424	352	189
(11, 1, 1, 1, 1)	61	84	105	106	119	119	167	189	189	167	119	119	106	105	84	61
(10, 4, 1)	97	221	392	337	337	392	221	97	97	221	392	337	337	392	221	97
(10, 3, 2)	60	225	348	375	375	348	225	60	60	225	348	375	375	348	225	60
(10, 3, 1, 1)	128	247	372	428	449	319	226	303	303	226	319	449	428	372	247	128
(10, 2, 2, 1)	159	197	239	285	285	239	197	159	159	197	239	285	285	239	197	159
(10, 2, 1, 1, 1)	153	258	226	284	228	244	124	263	263	124	244	228	284	226	258	153
(10, 1, 1, 1, 1, 1)	25	43	43	55	50	35	65	56	56	65	35	50	55	43	43	25
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1

A.6 Alpha-Rotatability of Graceful Spiders by Branch Label

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1)	Yes	-	-	Yes												
(2, 1, 1)	Yes	-	-	-	Yes											
(1, 1, 1, 1)	Yes	-	-	-	Yes											
(3, 1, 1)	-	-	-	-	-	-										
(2, 2, 1)	-	-	-	-	-	-										
(2, 1, 1, 1)	Yes	-	-	-	-	Yes										
(1, 1, 1, 1, 1)	Yes	-	-	-	-	Yes										
(4, 1, 1)	-	-	-	-	-	-	-									
(3, 2, 1)	Yes	-	Yes	-	Yes	-	Yes									
(3, 1, 1, 1)	-	-	-	-	-	-	-									
(2, 2, 2)	-	-	-	-	-	-	-									
(2, 2, 1, 1)	-	-	-	-	-	-	-									
(2, 1, 1, 1, 1)	Yes	-	-	-	-	-	Yes									
(1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	Yes									
(5, 1, 1)	Yes	Yes	-	-	-	-	Yes	Yes								
(4, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes							
(4, 1, 1, 1)	-	-	-	-	-	-	-	-								
(3, 3, 1)	Yes	Yes	-	-	-	-	Yes	Yes								
(3, 2, 2)	-	-	-	-	-	-	-	-								
(3, 2, 1, 1)	Yes	-	-	-	-	-	-	Yes								
(3, 1, 1, 1, 1)	-	-	-	-	-	-	-	-								
(2, 2, 2, 1)	-	-	-	-	-	-	-	-								
(2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-								
(2, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	Yes								
(1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	Yes								
(6, 1, 1)	Yes	-	-	-	-	-	-	-	Yes							
(5, 2, 1)	Yes	Yes	-	Yes	-	Yes	-	Yes	Yes							
(5, 1, 1, 1)	Yes	-	-	-	-	-	-	-	Yes							
(4, 3, 1)	Yes	-	-	-	-	-	-	-	-	Yes						
(4, 2, 2)	-	-	-	-	-	-	-	-	-	-						
(4, 2, 1, 1)	Yes	Yes	-	Yes	-	Yes	-	Yes	Yes	Yes						
(4, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-						
(3, 3, 2)	-	-	-	-	-	-	-	-	-	-						
(3, 3, 1, 1)	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes							
(3, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-						
(3, 2, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	Yes						
(3, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-						
(2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-						
(2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-						
(2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-						
(2, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	Yes						
(1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	Yes						
(7, 1, 1)	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes					
(6, 2, 1)	Yes	Yes	-	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes					
(6, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes					
(5, 3, 1)	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes					
(5, 2, 2)	-	-	-	-	-	-	-	-	-	-	-					
(5, 2, 1, 1)	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes					
(5, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	Yes					

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(4, 4, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2)	Yes	Yes	-	Yes	Yes	Yes	Yes	-	Yes	Yes	-	-	-	-	-	-
(4, 3, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	-	-	-	-	-
(4, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3)	-	Yes	Yes	-	-	-	-	-	Yes	Yes	-	-	-	-	-	-
(3, 3, 2, 1)	-	-	Yes	-	-	-	-	-	Yes	-	-	-	-	-	-	-
(3, 3, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	-	-	-	-	-
(3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-	-
(3, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(8, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 1)	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	Yes	Yes
(7, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 3, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 1, 1)	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	Yes	Yes
(6, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(5, 4, 1)	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 3, 2)	Yes	Yes	-	-	Yes	-	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 3, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 2, 2, 1)	-	Yes	-	-	Yes	-	Yes	-	-	-	Yes	-	Yes	-	-	-
(5, 2, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(5, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(4, 4, 2)	-	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	-	-	Yes	-	-	-
(4, 4, 1, 1)	-	-	-	-	Yes	-	Yes	-	-	-	-	-	-	-	-	-
(4, 3, 3)	Yes	Yes	-	-	Yes	-	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(4, 3, 2, 1)	Yes	Yes	-	-	Yes	-	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(4, 3, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(4, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 1)	-	Yes	Yes	-	-	-	-	-	-	Yes	Yes	-	-	-	-	-
(3, 3, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(3, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-	-

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	Yes					
(9, 1, 1)	Yes	Yes	Yes	-	Yes	-	-	Yes	-	Yes	Yes	Yes				
(8, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes				
(8, 1, 1, 1)	Yes	Yes	Yes	-	Yes	-	-	Yes	-	Yes	Yes	Yes				
(7, 3, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes				
(7, 2, 2)	-	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	-				
(7, 2, 1, 1)	Yes	Yes	-	Yes	Yes	-	-	Yes	Yes	-	Yes	Yes				
(7, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	Yes	-	Yes				
(6, 4, 1)	Yes	Yes	-	-	Yes	Yes	Yes	Yes	-	-	Yes	Yes				
(6, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes				
(6, 3, 1, 1)	Yes	Yes	Yes	-	Yes	-	-	Yes	-	Yes	Yes	Yes				
(6, 2, 2, 1)	-	Yes	-	-	Yes	-	-	Yes	-	-	Yes	-				
(6, 2, 1, 1, 1)	Yes	-	-	-	Yes	-	-	Yes	-	-	-	Yes				
(6, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes			
(5, 5, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes				
(5, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes				
(5, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes				
(5, 3, 3)	Yes	Yes	Yes	-	Yes	-	-	Yes	-	Yes	Yes	Yes				
(5, 3, 2, 1)	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes				
(5, 3, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes				
(5, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-				
(5, 2, 2, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	Yes				
(5, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes			
(5, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes			
(4, 4, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 4, 2, 1)	-	Yes	-	-	Yes	-	-	Yes	-	-	Yes	-				
(4, 4, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(4, 3, 3, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes			
(4, 3, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-				
(4, 3, 2, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes			
(4, 3, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(4, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(4, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(4, 2, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes			
(4, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 3, 3, 2)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 3, 3, 1, 1)	-	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes				
(3, 3, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 3, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 3, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes			
(3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(3, 2, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes			
(3, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-				
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes			
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes			
(9, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes		
(9, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes		

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(8, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes			
(8, 2, 2)	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-			
(8, 2, 1, 1)	Yes	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	Yes			
(8, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(7, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes			
(7, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes			
(7, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes			
(7, 2, 2, 1)	-	Yes	-	-	Yes	Yes	-	Yes	Yes	-	-	Yes	-			
(7, 2, 1, 1, 1)	Yes	Yes	-	Yes	Yes	-	-	-	Yes	Yes	-	Yes	Yes			
(7, 1, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	Yes	-	Yes			
(6, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes			
(6, 4, 2)	Yes	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	Yes			
(6, 4, 1, 1)	Yes	Yes	-	-	Yes	Yes	-	Yes	Yes	-	-	Yes	Yes			
(6, 3, 3)	Yes	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	Yes			
(6, 3, 2, 1)	Yes	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	Yes			
(6, 3, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(6, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(6, 2, 2, 1, 1)	-	-	-	-	Yes	-	-	-	Yes	-	-	-	-			
(6, 2, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-		Yes	
(6, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-		-	Yes
(5, 5, 2)	Yes	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	Yes			
(5, 5, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes			
(5, 4, 3)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes			
(5, 4, 2, 1)	Yes	Yes	Yes	-	Yes	Yes	-	Yes	Yes	-	Yes	Yes	Yes			
(5, 4, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes			
(5, 3, 3, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes			
(5, 3, 2, 2)	-	Yes	-	-	-	-	-	-	-	-	-	Yes	-			
(5, 3, 2, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(5, 3, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes		
(5, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(5, 2, 2, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-		
(5, 2, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(5, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes		
(4, 4, 4)	-	-	Yes	-	Yes	-	-	-	Yes	-	Yes	-	-			
(4, 4, 3, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 4, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 4, 2, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-		
(4, 4, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 3, 3, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 3, 3, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(4, 3, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 3, 2, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(4, 3, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(4, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes		
(4, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(3, 3, 3, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(3, 3, 3, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(3, 3, 3, 1, 1, 1)	-	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	-			
(3, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-			
(3, 3, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-			

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	-	-
(3, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(10, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(9, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 2)	-	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-
(9, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 2, 2, 1)	-	Yes	-	-	-	Yes	-	-	Yes	-	-	-	-	Yes	-	-
(8, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 1, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	-	-	Yes	-	Yes	-
(7, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(7, 2, 2, 1, 1)	-	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-	-	-	Yes	-	-
(7, 2, 1, 1, 1, 1)	Yes	Yes	-	Yes	-	-	-	-	-	-	-	Yes	-	Yes	-	Yes
(7, 1, 1, 1, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	-	-	Yes	-	Yes	-
(6, 6, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 3)	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes
(6, 4, 2, 1)	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes
(6, 4, 1, 1, 1)	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-	-	-	Yes	Yes	Yes
(6, 3, 3, 1)	Yes	Yes	Yes	-	-	Yes	-	-	Yes	-	-	Yes	Yes	Yes	Yes	Yes
(6, 3, 2, 2)	-	-	Yes	-	Yes	-	-	-	-	Yes	-	Yes	-	-	-	-
(6, 3, 2, 1, 1)	Yes	Yes	Yes	-	-	Yes	-	-	Yes	-	-	Yes	Yes	Yes	Yes	Yes
(6, 3, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(6, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(6, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(5, 5, 3)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 4)	-	-	Yes	-	Yes	-	-	-	-	Yes	-	Yes	-	-	-	-

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 4, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	-	-
(5, 4, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 4, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 3, 3, 2)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(5, 3, 3, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(5, 3, 2, 2, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(5, 3, 2, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(5, 3, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(5, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(5, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(5, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(4, 4, 4, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(4, 4, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(4, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(4, 3, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 1, 1, 1, 1)	-	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	-	-	-
(3, 3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(3, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(11, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(10, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(9, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 2, 1)	-	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-
(9, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(8, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 1, 1, 1)	Yes	Yes	Yes	-	Yes	-	-	-	-	-	Yes	-	Yes	Yes	Yes	Yes
(8, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(8, 2, 2, 1, 1)	-	Yes	-	-	-	Yes	-	-	-	Yes	-	-	-	-	Yes	-
(8, 2, 1, 1, 1, 1)	Yes	Yes	Yes	-	Yes	-	-	-	-	-	Yes	-	Yes	Yes	Yes	Yes
(8, 1, 1, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	-	-	Yes	-	Yes	-
(7, 6, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 1, 1, 1)	Yes	Yes	Yes	Yes	-	Yes	-	-	-	Yes	-	Yes	Yes	Yes	Yes	Yes
(7, 3, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2, 2)	-	Yes	Yes	-	Yes	Yes	-	-	-	Yes	Yes	-	Yes	Yes	-	-
(7, 3, 2, 1, 1)	Yes	Yes	Yes	Yes	-	Yes	-	-	-	Yes	-	Yes	Yes	Yes	Yes	Yes
(7, 3, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(7, 2, 2, 1, 1, 1)	-	Yes	-	-	-	Yes	-	-	-	Yes	-	-	-	-	Yes	-
(7, 2, 1, 1, 1, 1, 1)	Yes	Yes	-	Yes	-	-	-	-	-	-	-	Yes	-	Yes	Yes	Yes
(7, 1, 1, 1, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	-	-	Yes	-	Yes	-
(6, 6, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 6, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 4)	-	-	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	-	-	-
(6, 4, 3, 1)	Yes	Yes	Yes	-	Yes	Yes	Yes	-	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes
(6, 4, 2, 2)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(6, 4, 2, 1, 1)	Yes	Yes	Yes	-	Yes	Yes	Yes	-	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes
(6, 4, 1, 1, 1, 1)	Yes	Yes	-	-	-	Yes	-	-	-	Yes	-	-	-	-	Yes	Yes
(6, 3, 3, 2)	-	-	Yes	-	-	Yes	-	-	-	Yes	-	-	-	Yes	-	-
(6, 3, 3, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(6, 3, 2, 2, 1)	-	Yes	Yes	-	-	Yes	-	-	-	Yes	-	-	-	Yes	Yes	-
(6, 3, 2, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-	Yes
(6, 3, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(6, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(6, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 5, 4)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 2, 2)	-	Yes	Yes	-	-	Yes	-	-	-	Yes	-	-	Yes	Yes	Yes	-
(5, 5, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 4, 1)	-	-	Yes	-	Yes	-	-	-	-	-	Yes	-	Yes	-	-	-
(5, 4, 3, 2)	-	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	-
(5, 4, 3, 1, 1)	Yes	Yes	Yes	Yes	-	Yes	-	-	-	Yes	-	Yes	Yes	Yes	Yes	Yes
(5, 4, 2, 2, 1)	-	Yes	-	-	-	Yes	-	-	-	Yes	-	-	-	Yes	-	-
(5, 4, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	-	Yes	-	-	-	Yes	-	Yes	Yes	Yes	Yes	Yes
(5, 4, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(5, 3, 3, 3)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(5, 3, 3, 2, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(5, 3, 3, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(5, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 3, 2, 2, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(5, 3, 2, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(5, 3, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(5, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 1, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(5, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(4, 4, 4, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 4, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(4, 4, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 3, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	-
(4, 3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	-
(4, 3, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	-
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 1, 1, 1, 1, 1, 1)	-	Yes	Yes	-	-	-	-	-	-	-	-	-	Yes	Yes	-	-
(3, 3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes	-
(3, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(12, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 2, 2)	-	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-
(10, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(9, 5, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 4, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 4, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(9, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(9, 2, 2, 2)	-	Yes	-	Yes	Yes	-	Yes	-	-	Yes	-	Yes	Yes	-	Yes	-
(9, 2, 2, 1, 1)	-	Yes	-	Yes	-	Yes	Yes	-	-	Yes	Yes	-	Yes	-	Yes	-
(9, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(9, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(8, 6, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 5, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 5, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 4, 1, 1, 1)	Yes	Yes	Yes	Yes	-	Yes	Yes	-	-	Yes	Yes	-	Yes	Yes	Yes	Yes
(8, 3, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 2, 2)	-	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-
(8, 3, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(8, 3, 1, 1, 1, 1)	Yes	Yes	Yes	-	Yes	Yes	-	-	-	-	Yes	Yes	-	Yes	Yes	Yes
(8, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(8, 2, 2, 1, 1, 1)	-	Yes	-	-	-	Yes	-	-	-	-	Yes	-	-	-	Yes	-
(8, 2, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	Yes	Yes	-	-	-	-	Yes	Yes	-	Yes	Yes	Yes
(8, 1, 1, 1, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-	Yes
(7, 7, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 6, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 6, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 5, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(7, 5, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 4)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 2, 2)	-	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-
(7, 4, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 4, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(7, 3, 3, 2)	-	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-
(7, 3, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(7, 3, 2, 2, 1)	-	Yes	Yes	-	-	Yes	-	-	-	Yes	-	-	-	Yes	Yes	-
(7, 3, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(7, 3, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(7, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(7, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(7, 2, 2, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(7, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	Yes	-	-	-	-	-	-	-	-	Yes	-	Yes	Yes
(7, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-	Yes
(6, 6, 3)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 6, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 6, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 4)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 2, 2)	-	Yes	-	Yes	Yes	-	Yes	-	-	Yes	-	Yes	Yes	-	Yes	-
(6, 5, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 5, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(6, 4, 4, 1)	-	-	Yes	-	-	Yes	-	-	-	Yes	-	-	Yes	-	-	-
(6, 4, 3, 2)	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes
(6, 4, 3, 1, 1)	Yes	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	Yes
(6, 4, 2, 2, 1)	-	Yes	-	-	-	-	Yes	-	-	Yes	-	-	-	-	Yes	-
(6, 4, 2, 1, 1, 1)	Yes	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	-	-	Yes	Yes	Yes
(6, 4, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(6, 3, 3, 3)	-	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	-
(6, 3, 3, 2, 1)	-	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(6, 3, 3, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(6, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 3, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 3, 2, 1, 1, 1, 1)	Yes	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-	Yes
(6, 3, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(6, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(6, 2, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(6, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(5, 5, 5)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 3, 2)	Yes	Yes	Yes	-	-	-	Yes	-	-	Yes	-	-	-	Yes	Yes	Yes
(5, 5, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 2, 2, 1)	-	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	-
(5, 5, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 5, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	-	-	-	-	-	-	Yes	Yes	Yes	Yes	Yes
(5, 4, 4, 2)	-	-	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	-	-
(5, 4, 4, 1, 1)	-	-	Yes	-	-	-	-	-	-	-	-	-	-	Yes	-	-
(5, 4, 3, 3)	-	Yes	Yes	Yes	-	Yes	-	-	-	-	Yes	-	Yes	Yes	Yes	-
(5, 4, 3, 2, 1)	-	Yes	Yes	Yes	-	Yes	-	-	-	-	Yes	-	Yes	Yes	Yes	-

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(5, 4, 3, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes
(5, 4, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 4, 2, 2, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(5, 4, 2, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(5, 4, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	-	-	-	-	-	-	-	-	Yes	Yes	Yes	Yes
(5, 3, 3, 3, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(5, 3, 3, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 3, 3, 2, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(5, 3, 3, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(5, 3, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 3, 2, 2, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(5, 3, 2, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(5, 3, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	Yes
(5, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(5, 2, 2, 1, 1, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(5, 2, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(5, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(4, 4, 4, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 4, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 4, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 3, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 3, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 4, 2, 1, 1, 1, 1, 1)	-	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	-
(4, 4, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 3, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 3, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 3, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(4, 3, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 3, 2, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(4, 3, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(4, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(4, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 3)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 3, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 3, 1, 1, 1, 1, 1, 1)	-	Yes	Yes	-	-	-	-	-	-	-	-	-	-	Yes	Yes	-

Table 6: α n -rotatability of spiders order 16 or less

Spider	α n -rotatable?															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(3, 3, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	Yes	-	-	-	-	-	-	-	-	-	-	-	-	Yes	Yes
(3, 2, 2, 2, 2, 2, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 2, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 2, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 2, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 2, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 2, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 2, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
(2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes
(13, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(12, 2, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(12, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 3, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 2, 2)	-	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	-
(11, 2, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(11, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 4, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 3, 2)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 3, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 2, 2, 1)	-	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	-
(10, 2, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(10, 1, 1, 1, 1, 1)	Yes	Yes	Yes	Yes	Yes	Yes	-	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)	Yes	-	-	-	-	-	-	-	-	-	-	-	-	-	-	Yes

B Location of Source Code

A Python 2.7 implementation of our graceful spider enumerator, along with a simple graph class, and some other related utilities can be found online at <https://github.com/bjpatterson/graceful-spiders>. The code is provided under an MIT license, and can be freely used, copied, modified, etc.