An Exploration in Proportional Theory

Within the Realm of Order

AN ARCHITECTURAL THESIS

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Proportions which delight the eye can in fact be learned through analysis and application to design problems and should not be approached in a purely intuitive manner. It is this fact that intrigued me about the book Dynamic Symmetry A Primer written by Christine Herter. This book documents the work and basic concepts of Jay Hambidge, an artist who is said to have rediscovered the system of proportion employed by the Greeks in their architecture.

It is agreed that the Greeks were an advanced culture who were highly knowledgeable in areas of science, philosophy, and the arts. The Greeks being also a highly literate people used the written word to document their theories and ideals, however for some unknown reason they chose not to express in writing their theories in the one area which is of most interest to those of use who study and practice architecture. The Greeks are acknowledged to have become masters of the art of building, and the Parthenon is viewed by many to be the greatest achievement in the history of architecture.

At the core of any successful theory of architecture throughout history is a theory of proportion. And if we agree that the Greeks were in fact masters, how exciting it is to think that we may learn about the techniques they employed and perhaps apply them in our architecture. This is what intrigued me about the work of Jay Hambidge who spent much of his life analyzing Greek architecture and artifacts. In his work he puts forth a series of concrete geometrical relationships which the Greeks used to determine the shapes and sizes of objects. I found it interesting that relationships which are known to please the eye could be approached analytically, and that the process of creating desired relationships did not have to be purely intuitive. I was excited to learn that perhaps I could employ those relationships in my own design process, and that I could eliminate the haphazard trial and error method of creating pieces which are complementary to each other and to a whole.

Correct proportion is in fact essential to a successful work of architecture, however it is something that is dealt with in later stages of the design process. It is an element of refinement, whereas other factors determine the overall organization and composition of a building. At this broader level is where the notions of order begin to come into play and
continue through a successful work. I am concerned as well with this realm of order and its ability to hold together a piece of architecture or to destroy it. I am particularly intrigued by the notion of collisions and the visual articulation and complexity they can create.

It was with these precepts that I looked to a project that would allow a level of freedom so that I could explore these areas. An interest in water edge architecture combined with these issues is what led me to choose a sailing club as an architectural project through which I could pursue my interests of study.

It is with this preface that I present my positions and their application to the Prairie Creek Sailing Club.
ON ORDER

Excitement. This seems to be a word of most importance when speaking of life. Striving for success -- opening new doors -- quest for adventure. These are ideals which add excitement to an individual's life and make it an adventure in itself. And an integral part of every individual's life is an architecture. The home, workplace, as well as public space, all create an environment which serves as the aesthetic framework for an individual. These environments should in fact create and hold feelings of adventure and excitement, therefore making positive contributions to the quality of life.

Yet there must be order - "the degree and kind of lawfulness governing the relations among the parts of an entity." ¹ It seems to be the case that too often order opposes excitement. They seem to be at opposite ends of the pole when creating architecture. Order can in fact restrain and create monotonous repetition if a designer so allows it. The key to innovation is to create excitement within a new or established order. It should in fact be a simple and as pure an order as possible. It should answer all the questions yet it should allow for variety and excitement. I believe however that this order does not and perhaps should not necessarily become at first apparent to the participant. This element of uncertainty is what generates excitement, and it lets the individual travel through the process of understanding. Things he does not immediately understand creates interest and inquisition therefore occupying his mind and keeping him content. However, the system must be spatially pure so as to not create confusion as the participant moves through a series of spaces. That is to say that the excitement should be discovered through the mind and eyes in visual articulation and complexity -- "the multiplicity of the relationships among the parts of an entity."²
"Order and complexity are antagonistic, in that order tends to reduce complexity while complexity tends to reduce order. To create order requires not only rearrangement but in most cases also the elimination of what does not fit the principles determining the order, on the other hand, when one increases the complexity of an object, order will be harder to achieve.

Order and complexity cannot exist without each other. Complexity without order provides confusion; order without complexity produces boredom."³

1. Arnheim, p. 123.
3. Arnheim, p. 128.
Correct proportion is a means in which to achieve order. Other higher ordering principles must be considered first in the design process. But when those are accounted for, the conscious application of proportional theory is an integral aspect of a successful work of architecture. Fitness and custom as well as hierarchy, rhythm, repetition, etc., all determine the form which the building is to take. With these principles serving as the basis, proportion defines the pieces so as to create unified order throughout the whole. The objective is in fact to achieve harmony and beauty.

"I shall define beauty to be a harmony of all the parts in whatsoever subject it appears, fitted together with such proportion and connection that nothing could be added, diminished or altered, but for the worse."\(^4\)

When we consider proportion we are speaking about the art of determining shapes and sizes of objects which are pleasing to the eye. There are indeed other factors which determine the shapes and sizes of objects such as availability of manufactured building parts and the required sizes of pieces due to functional and structural needs. "However, it is the common experience of designers in practice that when all requirements of fitness have been met, a good deal of choice usually remains between proportions which appear pleasant and those which appear unpleasant."\(^5\) Latitude does exist and it should be utilized in the adjusting and refinement of forms so they are complementary to each other and to the whole.

There are certain relationships which the eye can recognize and discern. The key to a theory of proportion in architecture is the relationship of objects having the same shape, or in similarity of shape.
"In a building where all the parts are of different shapes, the visual effect is one of greatest possible disorder, indeed chaos. Order can be introduced by the repetition of similar shapes, and the highest degree of order results when comparatively few shapes are used, repeated as often as possible. This sort of order is clearly one which the eye can recognize. It is not a purely mathematical order which has to be understood before it can be appreciated. We can therefore define the object of architectural proportion as the creation of visible order by the repetition of similar shapes."6

Once that we have determined what we are attempting to achieve, it now becomes a question of the means we are to use. Throughout history the question has been debated as to whether or not correct proportion should be achieved through the conscious application of a system of proportion or through pure innovation. The latter relies on experience and the trial and error method of adjusting compositions. Mondrian would seem to favor this.

"In all art it is the artist's task to make forms and colors living and capable of arousing emotion. If he makes art into an "algebraic equation" that is no argument against the art; it only proves that he is not an artist."7

It is true that an artist can become entangled with intricate mathematics if he so allows that to happen. An artist however can use simple concepts in the formation of a work which is pleasing to the eye. These concepts can serve as a direct approach to learning correct proportion rather than relying on years of haphazard searching for delightful compositions. Once we have established what is the key to proportion it is purely a matter of applying what we have learned.

"But once what we do intuitively what can be described and compared with nonintuitive ways of doing the same things, we cannot go on accepting the intuitive method innocently."8

Alexander speaks of "preserving innocence", the designer's inability to make decisions which leads him to excuses.
"The modern designer relies more and more on his position as an 'artist', on catchwords, personal idiom, and intuition — for all these relieve him of some of the burden of decision, and make his cognitive problems manageable."

5. Scholfield, p. 4.
SYSTEMS

We can effectively apply systems of proportions which aid us in the design process. Proportional systems can be classified in two ways, either by the methods used to put them into effect or by their mathematical characteristics. The methods used to put them into effect are termed either geometrical or analytical. The difference lies mainly in how a designer perceives and utilizes proportional systems. If a designer constructs a root 2 rectangle, for instance, by tracing the diagonal of a square and striking an arc to form the longer side, he is using a geometrical method. He is more concerned with the shape itself than with its actual dimensions. If however the designer scales each side of the rectangle so that it has the correct ratio he is using an analytical method. He is concerned with the shape of rectangle but the emphasis is placed on its linear dimensions.

The second way proportional systems are classified is by the mathematical ratios which they employ, either commensurable or incommensurable. Commensurable systems are those which are based on whole numbers, that is the shapes formed have simple whole number ratios such as 1:2, 2:3, 3:4, etc. Systems which are based on irrational numbers such as the golden section ratio are considered incommensurable systems. Geometrical systems are usually incommensurable in that they are based on irrational numbers and therefore easier to deal with in a geometrical fashion. The geometrical incommensurable system is said to be what the Greeks employed
in their architecture. Analytical systems however are found to be for the most part commensurable. This method based on whole numbers can easily be dealt with mathematically, and it is the system which architects during the Renaissance utilized.

\[ \begin{array}{c}
1 \\
2 \\
\hline
\end{array} \quad \begin{array}{c}
1 \\
1.618 \\
\hline
\end{array} \\
\text{COMMENSURABLE} \quad \text{INCOMMENSURABLE}
\]

The difference in characteristics of certain shapes lies in their practicality as a tool for design and in their flexibility. For architectural purposes an analytical system is more advantageous than a geometrical system in that the designer is very much concerned with actual linear dimensions, the scale being used as the tool. With regards to flexibility, an incommensurable system which is based on forms composed of irrational numbers have a much greater degree of flexibility greater than those shapes which are based on commensurable ratios. The ideal condition therefore is to combine practicality and maximum flexibility or more directly an analytical incommensurable system. This in fact has been achieved by Hambidge is his system of dynamic symmetry.

Before we discuss Hambidge's system in further detail it is necessary to discuss the nature of certain shapes and their supposed inherent beauty. The golden section rectangle for instance has been discussed throughout history as to whether or not it is in itself a form which is pleasing to the eye. The golden section is in fact regarded as a superior shape but it is not due to any inherent beauty in the particular form. Rather it is regarded as important in that it can be used as the basis for a system that has enormous flexibility, and it is the laws of mathematics which determine this.
"It is clear that no individual rectangles are in themselves either outstandingly beautiful or ugly. The secret of proportion seems to lie, not in the shapes themselves, but in the relationships between them. It would otherwise be difficult to explain how, for instance, the buildings of Palladio and those of Corbusier can both be well proportioned, when they embody quite different systems of shapes."

DYNAMIC SYMMETRY

As I have stated, Hambidge's systems of dynamic symmetry is a combination of the main advantages of systems of the past. He has combined the practicality of the Renaissance along with the flexibility of the Greeks. Symmetria, symmetry in the Greek sense, means the due proportion of the several parts of the body to each other. Dynamis, the Greek word for dynamic, means power. He further defines dynamic symmetry as it opposes static symmetry.

"Static Symmetry is apparent in nature in certain crystal forms, radiolaria, diatosis, flowers, and seed pods, and has been used consciously in art at several periods. The principle of Dynamic Symmetry is manifest in shell growth and in leaf distribution in plants. A study of the basis of design in art shows that this active symmetry was known to but two peoples, the Egyptians and the Greeks."

11
The Greeks were the masters of dynamic symmetry, however they approached it geometrically in that the powers of arithmetic were not available to them. They used string, points, and levels when composing building parts, and it was this idealistic method which allowed them to proportion each part with relationship to each other and most significantly to the whole. The powers of arithmetic are available to us however, and it is this notion which led Hambidge to believe that modern day designers could apply these same geometric principles with a more practical approach. Not only is this system practical but in being incommensurable it allows for enormous flexibility. It is here that the ideas of order which I have previously spoke become apparent. Through the flexibility of the system we can achieve complexity within an established order and we can create a pleasing composition with variety and excitement.

The Elements

THE SQUARE AND DIAGONAL
The square is the form which serves as the basis for constructing the elements of dynamic symmetry of which the root rectangles are of most importance. The root rectangles are formed by another integral element in the system which is the diagonal. The root 2 rectangle is formed by taking the diagonal of a square and striking an arc downwards to form the longer side. This longer side is equivalent to the square root of 2 in keeping with pythagoreum's theory, the sides of the square being equal to one making the diagonal equal to $1^2 + 1^2 = \sqrt{2}$. The root 3 rectangle is then formed by taking the diagonal of the root 2 rectangle and striking an arc to form the longer side as well, $1^2 + 2^2 = \sqrt{3}$. This process can be continued to infinity, however the root 5 rectangle is usually the last in the series with which we are concerned.
Once the basis for the system has been established in the root rectangles, it will become apparent that the key to flexibility for their use in design lies in the subdivision of these rectangles. Various compositions can be achieved with the manipulation of forms within these rectangles, and this system can be used to establish relationships between parts of a work as well as to the whole.

![Two GS Rectangles Within Root 5](image1)

The root 5 rectangle is of special interest within the elements of dynamic symmetry. It is the most versatile of all the root rectangles, the reason for this being that the golden section rectangle is contained within its boundaries. It is these two geometrical forms which the Greeks used in the design and construction of the Parthenon. Although the Greeks mastered and successfully applied the system of dynamic symmetry, it is said by some critics to be somewhat clumsy in today's practice due to its fractional number basis.

![Root 5 Rectangle Subdivided](image2)
An answer to this problem may be a geometric progression known as the Fibonacci series (1, 1, 2, 3, 5, 8, 13, 21,...). This series is a whole number progression which approximates the ratio of the golden section rectangle. As the series progresses, successive pairs of numbers are added together to form the next number, and their ratios become closer to that of the GS ratio as the progression continues.

As I have already stated the art of proportioning can be approached in two ways, either intuitively or by the conscious application of a system. I have taken the position that a purely intuitive approach is not valid when it is known that there does exist concrete relationships which delight the eye and serve as a means to an aesthetic end. Taking this one step further, perhaps these systems should be viewed as a means of training the eye in correct proportion but that ultimately it should become unnecessary to obtain these relationships through measurement and calculation.

"The geometry of design first of all must be in his eye, in fact his mental eye, and if later he consciously applies design formulas to his realized vision, it must be as a simple and straightforward means of clarifying and correcting his intention. It cannot be stressed too strongly that mathematical formulas never help a designer to create. They serve but two useful ends: first, to train the designer's mental and physical eye in right proportions, and second, to test and correct the proportions his eye has established."12

10. Scholfield, p. 5.
11. Hambridge, Scholfield, p. 117.
12. Teague, p. 142.
There are several reasons which led me to select a sailing club as an architectural project through which to demonstrate my studies. The pleasures of sailing immediately evoke feelings of excitement and it sets the tone for adventure. An individual who sets out to use a club of this nature is in a light-hearted mood and therefore can appreciate intrigue and discovery. The water is an active ingredient which adds to the spirit of movement and carries with it a certain mystique.

A waterfront is typically an active place as on the shores of New York and Boston. When one visits these places feelings of excitement and complexity are understandable. One experiences among other things sailing vessels, bridges, and markets. One searches for the order amongst the vast visually articulated surroundings. The water edge is of special importance in that it is an area of confrontation which accentuates the notions of stability and routine vs. instability and excitement.

Although admittedly a sailing club is at a much different level than that of Boston Harbor, it still has the potential to enhance life in a way similar to this waterfront area. The site I have chosen assures that the context will play a completely different role in that it is a much more serene and contemplative setting. Therefore the building itself must generate its own complexity amidst the existing natural order.

THE SITE

The site for the Prairie Creek Sailing Club is the Prairie Creek Reservoir which is located southeast of Muncie, Indiana. It is a rural setting surrounded by farmland and by the farm vernacular architecture. I have located the building in a central position on the site where it sits atop a plateau which gradually builds from the waters edge. This elevation allows for a panoramic view of the water body and surroundings.

In the initial stages of development I distinguished three points of importance. The first of these is the formal entry to the club which serves as the point of origin for two paths. The first of these paths, the Garden Promenade, travels west towards the large body of water. This axis allows the participants to wander down on to the natural landscape and to
the shore. The path terminates as a tower adjacent to the water edge which serves as a weather and signal flag indicator. The second path, the Dock Promenade, originates at the formal entry and continues towards the water inlet and dock area connecting the club with the major area of activity. The line which connects the tower and the culmination of this connecting axis forms the dock area.

It is this second axis and its angle juxtaposed begins the first axis which becomes the organizer of the structure. This angle is that of the diagonal of the root 5 rectangle, and it is this geometry which serves as the basis for organization and compositional ideas.
THE ARCHITECTURE

The two paths which serve as the organizers and means of circulation occur on both the dock and mezzanine levels of the Club. The Dock Promenade is enclosed by an arcade constructed of wood timber and diagonal tie rods with glass as infill. The Garden Promenade is defined both by two parallel 3' high walls slicing through the space as well as by the two major trusses above. Each of these paths carry the main mechanical supply/return lines in the structure above. These lines originate on this upper level from the two shafts which frame the formal entry of the Club.

The dock level of the building contains functions which serve the immediate needs of the members who are sailing as well as the support spaces for the Club itself. This level acts as a base which serves to order the complexity which occurs above. The mezzanine level contains the three major gathering spaces -- Library/Trophy Room, Formal Dining Space, Informal Dining space -- as well as the Administrative offices.

As has been stated the angle of the dock promenade in relation to the perpendicular of the garden promenade is that of the diagonal of the root 5 rectangle. This came about as a result of the plateau positioning of the building and its relationship to the dock area. The root 5 rectangle therefore became the configuration of the dock level which serves as the base on which the mezzanine level rests. It is on this upper level where the lines of division in the root 5 rectangle allow for complexity to occur. The composition is achieved by the collisions of three major rectangular forms which house the three major gathering spaces.

The dock level serves to order the structure as well and allows for complexity above. The main structural steel columns originate in the concrete base and continue up through the mezzanine level on the line of the wood Dock Promenade. From these columns hang major steel trusses which support the roof structure of the upper level. What this accomplishes is a layer of free space slicing horizontally through this level. This allows for an unobstructed panoramic view of the water body from any point in each of the three major gathering spaces. The tension wires above are connected to the trusses on the grid of the root 5 rectangle. This orders their existence yet the complexity achieved is visually exciting and articulated.
TRUSS/ROOF DETAIL VIEW
CONCLUSION
The spirit or mood of a particular individual who is or soon to be a participant in an environment has great impact on how he perceives that environment. His mood is most likely predetermined before he enters an environment due to his anticipation of reaching that space. If for instance an individual's destination is a courthouse or library it could be said that the person is in a more serious or rational state of mind. If however he is anticipating a visit to a sailing club he is in a more high-spirited or irrational state of mind. The notions must be considered in the design of an environment so that the participant is comfortable in his surroundings.

"Different states of mind call for different degrees of order and complexity and, in consequence, produce different interpretations of nature."13

A participant in a courthouse is there for a specific purpose and his rational state of mind searches for a greater degree of order than complexity. He is not inclined to observe or partake in complex relationships unless they in fact obstruct his purpose and create confusion.

"I call a shape or relation visually rational when the eye can understand it as being formed according to some principle, such as straightness, symmetry, constant curvature, etc., and thus can use it as one of the models by which the mind unifies nature."14

A participant in a sailing club can in fact enjoy discovery and visual excitement. Formal principles can be discarded in favor or innovation, and the notions of complexity can play a much larger role in the design of an environment.

I believe that my thesis has been an overall success, however I don't think the reason lies in the finding of solutions which I originally set out to achieve. I believe that when I began this work that there were two contradicting notions contained within my mind. The first was a desire to search for rationality and reason which is evidenced by the subject I selected to study, that of proportional systems. The second notion was that of order vs. complexity and my desire to experiment and achieve rich
complexities within established orders. It was this second notion that led me to select a building type which allowed freedom of spirit so that I could explore these complexities. Perhaps a more appropriate architectural project would have been an environment which generates rational states of mind so as to facilitate the study of proportional systems, a highly rational approach to aesthetics.

But then again, maybe not.

APPENDIX
Program

ARCHITECTURAL THESIS
THOMAS P. KERWIN
"AN EXPLORATION IN PROPORTIONAL THEORY WITHIN THE REALM OF ORDER"

PROGRAM REQUIREMENTS FOR ARCHITECTURAL SOLUTION
PROJECT: PRAIRIE CREEK SAILING CLUB

PROGRAM

Administrative

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<tr>
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</tr>
<tr>
<td>Vice-Commodore Office</td>
<td>150 Sq. Ft.</td>
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<tr>
<td>Secretary/Treasurer/Receptionist</td>
<td>300 Sq. Ft.</td>
</tr>
<tr>
<td>Meeting Room</td>
<td>200 Sq. Ft.</td>
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<tr>
<td></td>
<td>850 Sq. Ft.</td>
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Club Facilities

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<tr>
<td>Coat Storage</td>
<td>80</td>
</tr>
<tr>
<td>Library/Lounge/Trophy Room</td>
<td>600</td>
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<tr>
<td>Banquet Room/Conference/</td>
<td></td>
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<tr>
<td>Ballroom/Formal Dining Storage</td>
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</tr>
<tr>
<td>Informal Dining/Concession Kitchen</td>
<td>1000</td>
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<tr>
<td>Kitchen</td>
<td>1000</td>
</tr>
<tr>
<td>Men's Locker Room</td>
<td>1200</td>
</tr>
<tr>
<td>Women's Locker Room</td>
<td>1200</td>
</tr>
<tr>
<td>First Aid Room</td>
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<tr>
<td>Restrooms</td>
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<tr>
<td>Vending Machine Room</td>
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<td>Laundry Room</td>
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Storage

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<tr>
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<td>BSU Equipment Storage</td>
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Service

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<tr>
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<tr>
<td>Circulation (10%)</td>
<td>1550</td>
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<td></td>
<td>3380</td>
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Total = 19,010 Sq. Ft.
The following is a synopsis of The Theory of Proportion in Architecture written by P.H. Scholfield, a text which gives an excellent account of proportional concepts and their use throughout history. This book served as the beginning of my study, and I found it very helpful as both a source of background information and as an indicator of further areas of study from which to branch out.

INTRODUCTION

It is evident that designers of the past were indeed conscious of and utilized systems of proportion, Scholfield stating that "research into the architecture of these periods shows constantly recurring mathematical relationships which could not have occurred by chance." The author is interested in those relationships with regards to the object of designing in a way that is pleasing to the eye. There are indeed other factors which determine shapes and sizes of objects such as availability of manufactured building parts and the required sizes of pieces due to functional and structural needs. Scholfield however states that "the first thing to be
made clear is that we are concerned only with visual proportion, with the relationships of the shapes and sizes of objects which please the eye. Our theory must therefore be founded firmly on known facts about the kind of proportional relationships which are significant to the eye."

It is generally agreed that functional and structural requirements must be taken into account first when determining the sizes of objects. However it is obvious that if one concentrates only on how a building is to stand up, that work of architecture will not necessarily be pleasing to the eye. The use of the building and the amount of space that is required for each space is also a factor when determining sizes. Both structure and utility constitute "fitness" and it is obvious that a successful theory of proportion cannot rely on "fitness" alone.

"In fact, however, it is the common experience of designers in practice that when all requirements of fitness have been met, a good deal of choice usually remains between proportions which appear pleasant and those which appear unpleasant." Latitude does exist and it should be utilized in the adjusting and refinement of pieces so that they are complementary to each other and the whole."

It is first necessary to determine what in fact are the relationships which are known to please the eye. Once we determine what these are we can employ them in our designs as often as possible. An extremely important point to be made is that the key does not lie with any particular form in itself and that different shapes have been used successfully in different periods.

"It is clear that no individual rectangles are in themselves either outstandingly beautiful or ugly. The secret of proportion seems to lie, not in the shapes themselves, but in the relationships between them. It would otherwise be difficult to explain how, for instance, the buildings of Palladio and those of Corbusier can both be well proportioned, when they embody quite different systems of shapes."
Scholfield distinguishes three relationships which are significant to the eye.

1. objects having the same shape
2. objects having the same size as well as the same shape
3. objects having the same size but different shapes

He sees as the key to the theory of proportion in architecture the first relationship, that being the similarity of shapes.

"In a building where all the parts are of different shapes, the visual effect is one of greatest possible disorder, indeed chaos. Order can be introduced by the repetition of similar shapes, and the highest degree of order results when comparatively few shapes are used, repeated as often as possible. This sort of order is clearly one which the eye can recognize. It is not a purely mathematical order which has to be understood before it can be appreciated. We can therefore define the object of architectural proportion as the creation of visible order by the repetition of similar shapes."

The problem begins when the shapes of smaller objects are chosen in some fashion for some reason. These smaller pieces compose larger pieces which then make up even larger pieces as well. All these pieces add together to become a whole composition, but there is no guarantee that those larger pieces will come together in the same shape as the smaller ones. The advantages and disadvantages of any individual shape therefore are due to their ability and to what degree they can be divided into smaller similar shapes. It is mathematics which determines which shapes are more suitable for this than others. When the linear dimensions of shapes are studied in geometrical progressions it is found that the additive properties of shapes based on whole numbers are far less flexible than those based on irrational numbers. The golden section rectangle is an example of a shape based on an irrational number. Whether or not it is significant in itself has been a topic of discussion throughout history, but its use as the basis for a system of proportion cannot be disputed.

The problem lies in the fact that systems based on irrational numbers become complex and difficult to deal with on an architectural level.
"A system of proportion in practice must not only lead to the flexible repetition of clearly defined shapes, but it must also be easy to use. Above all a designer must be able to use it without intricate mathematical calculations, just as the composer can use the scales of music without considering the mathematical relationships underlying them."

An answer to this problem may be a geometric progression known as the Fibonacci series (1, 1, 2, 3, 5, 8, 13, 21, ...). This series is a whole number progression which approximates the ratio of the golden section rectangle. As the series progresses, successive pairs of numbers are added together to form the next number, and their ratios become closer to that of the GS ratio as the progression continues.

Scholfield states that systems of proportion can be classified in two different ways, either by the "practical method which is used to put them into effect or by the type of mathematical relationships which they embody." The practical methods used to put these systems into effect are classified as either geometrical or analytical. The difference lies mainly in how a designer perceives and utilizes proportional systems. If a designer constructs a root 2 rectangle, for instance, by taking the diagonal of a square and striking an arc to form the longer side, he is using a geometrical method. He is more concerned with the shape itself than with its actual dimensions. If however the designer scales each side of the rectangle so that it has the correct ratio he is using an analytical method. He is concerned with the shape of the rectangle but the emphasis is placed on its linear dimensions.

The second way Scholfield classifies proportional systems is by the type of mathematical relationships they embody, these being commensurable and incommensurable systems. Commensurable systems are those which are based on whole numbers. Systems which are based on irrational numbers such as the golden section ratio are considered incommensurable systems. As stated earlier the Fibonacci series is a whole number approximation of the GS ratio therefore it would be considered commensurable when used as the basis of a system.

Geometrical system are usually incommensurable in that they are based on irrational numbers and therefore easier to deal with in a geometrical
fashion. The geometrical incommensurable system is said to be what the Greeks employed in their architecture. Analytical systems however are found to me for the most part commensurable. This method based on whole numbers can easily be dealt with mathematically, and it is the system which architects during the Renaissance utilized. It would seem that perhaps the most desirable combination would be an analytical system which is incommensurable. This system ideally would have enormous flexibility as well as being easy to use for architectural purposes. This in fact has been achieved, Corbusier's Modulor being an example.

Direct literary evidence on theories of proportion is limited. The Greeks who are said to have mastered architectural proportion left no writings on the subject. Vitruvius being the first to document proportional ideas attempted unsuccessfully to understand and identify the methods employed by the Greeks. Not until the Renaissance was the subject taken up again in written form, the architects of this period relying on the writing of Vitruvius who they believed understood Greek theory. Since the Renaissance the study of proportion has been extensive, and it was not until after the collapse of the Renaissance that the Greek theory of proportion was rediscovered. The following is a summary of proportional theory during the various periods beginning with Vitruvius and continuing with Renaissance theory in that they are substantiated with direct literary evidence. The work of the Greeks will be discussed in detail along with modern day theory, that being the period in which it was rediscovered.

**VITRUVIUS**

Vitruvius never developed a comprehensive theory of proportion, and as we stated earlier attempted unsuccessfully to interpret the work of the Greeks. Scholfield further points out that Vitruvius never really came to an understanding of how to in fact study proportion. "It is clear that many of Vitruvius' frequent references to proportion had nothing to do with its aesthetic aspect which can in itself be approached from two quite different points of view." Vitruvius spoke of proportion in terms of firmness and commodity and did not make the distinction between the two approaches of which Scholfield speaks. The first of these is the intuitive approach in which the designers rely on experience and the trial and error technique of determining shapes and sizes of objects. The second approach is the use of a system of proportion by the designer.
Vitruvius was the first to discuss the proportions of the human figure and its relationships to architecture stating "that the proportions of a temple want to be like those of a well-formed human figure." He opened the door for numerous debates later in history as to the merit of systems based on the human figure. His system was in fact important in that he defined pieces of the body as submultiples of the whole and it was this feature that serves as the basis for Vitruvius' theory. "What he did consciously or not, is to ensure a proportional relationship between the sizes of the smaller parts and those of the larger." The system of proportion developed by Vitruvius was analytical and commensurable. He attempted to interpret the systems of the Greeks but it is obvious that he never came to understand the geometrical and incommensurable system which the Greeks employed. Scholfield however attributed much of the misunderstanding of Vitruvius to misinterpretation.

"Finally, our investigation seems to show that Vitruvius, at first sight so muddled on the subject of proportion, was in many ways far ahead of his later interpreters and commentators. For instance, although he has often been accused of doing so, he never tried to establish the false analogy between musical harmony and architectural proportion which exposed Renaissance theory to so much destructive criticism. Commensurability is treated merely as a means to an end, while a mature theory of proportion, derived from the Greeks, seems to hide not very far away behind statements which appear at times to be almost incoherent."

RENAISSANCE

Very little was written on the subject of proportion after Vitruvius until the work of Alberti who began the extensive study which has taken place to the present. The Renaissance took as its starting point the writings of Vitruvius which were difficult to interpret and incoherent. Therefore, throughout the Renaissance no furthering of proportional ideas took place, but instead what occurred was a revival of the same discussions until the collapse of the Renaissance theory of architecture. What did come from Vitruvius were "the basic ideas of the importance of proportion as a sense of beauty, of its being concerned with the relationship of the parts to each other and to the whole, and of its being subject to reason and rules rather than intuition." The Renaissance also continued the dialogue of
Vitruvius in the areas of both the proportions of the human figure and its relationships to architecture as well as in the area of music. Renaissance architects, however, misinterpreted the writings of Vitruvius with regards to music. Vitruvius' concern was with acoustics in theatre design and not with the application of musical theory to a theory of proportion as Renaissance architects attempted.

Alberti was one of the first to take up this renewed interest in proportion in the early Renaissance, and he was also intrigued by notions of beauty as well. Scholfield states that there are certain aesthetic principles which serve as the background for the study of proportion with regards to the nature of beauty. The first of these is "that art is concerned with creating things which are beautiful." The second is "that objective standards of beauty are possible, and that our sense of beauty is not entirely arbitrary and unpredictable, but is aroused by certain real qualities in external objects which we can learn to understand and reproduce." The early Renaissance architects along with Alberti believed that there did exist definite objective rules which contributed to an objects beauty as opposed to the subjective or purely intuitive. Alberti writes:

"I shall define beauty to be a harmony of all the parts in whatsoever subject it appears, fitted together with such proportion and connexion that nothing could be added, diminished or altered, but for the worse..."

Renaissance architects followed Vitruvius in the sense that they were chiefly concerned with systems of proportion that were both analytical and commensurable. Vitruvius however did deal with incommensurable numbers on a small scale, using the root 2 rectangle as the configuration of houses. Irrational numbers did find their way into the Renaissance as well, Palladio advocating the use of the root 2 rectangle for the shapes of rooms. Alberti speaks of the use of irrational numbers when simple proportions such as 1:2, 1:3, 3:4, 2:3, etc. are not appropriate.

"The explanation of this contradiction seems to lie in the fact that the Renaissance never really produced a consistent theory of proportion. It might have done so if the critical and experimental attitude of the fifteenth century had continued.
throughout the sixteenth century in Italy, but it faded out before the recommendations of Vitruvius and the results of experiment had been digested sufficiently to produce a unified theory. All that survived was on the one hand the uncritical acceptance of the authority of Vitruvius, and on the other hand a rather narrow doctrine of commensurable proportions."

Musical Analogy

Renaissance architects were most intrigued with what they thought were important relationships between music and architecture. Alberti suggested that the proportions of notes on the musical scale would just as they please the ears be appropriate as the proportions for a building. Alberti writes:

"The rule of these proportions is best gathered from those things in which we find nature herself to most compleat and admirable; and indeed I am every day more and more convinced of the truth of Pythagoras's saying, that nature is sure to act consistently and with a constant analogy in all her operations: From whence I conclude the same numbers, by means of which the agreement of sounds affects our ears with delight, are the very same which please our eyes and our mind."

What Alberti and other Renaissance architects were advocating was the use of simply commensurable ratios in architecture just as they are used in music. This rational for proportional theory as well as the Renaissance theory of architecture eventually collapsed in the nineteenth century. This is not to say however that the use of simple commensurable ratios do not have some merit but that the reasons do not lie in the Renaissance musical analogy theory. Scholfield writes:

"In music simply commensurable ratios please the mind without its being conscious of them as ratios at all. In architecture wide variations in ratio are tolerated without discomfort, although the mind can in this case form an estimate of the value of the ratios concerned. The cases are quite different, and no analogy is possible."
The use of simply commensurable proportions can in fact lead to a successful work, however it has to be realized that they are not beautiful in themselves but that they must be regarded as a means to an end. The problem lies in the fact that commensurable numbers lack the flexibility that can be achieved with incommensurable numbers, this fact being due to mathematics.

Alberti and Palladio applied their theory to architecture however they were not totally successful. They began at the level of the individual room and proportioned each of these according to their system, however when the rooms were brought together to form a completed building the ratio of the whole was not the same as the ratios of the individual spaces. Not only were the theories of Alberti and Palladio discredited but the Renaissance theory as a whole was disallowed. Scholfield writes:

"In the end they denied that proportion could make an independent contribution to the beauty of objects at all. The importance of proportion was explained as being due to the varying and subjective influences of custom, knowledge of fitness, or a combination of both. The conclusions to be drawn were that proportion was a varying and subjective factor in design, determined by the taste of the individual designer, and that no rules could be established for its control."

Although his ideas were not realized to be innovative at the time, Barca was an instrumental figure in that he defined how a theory of proportion should actually work in practice. His three aspects of architecture were fitness, custom, and proportion. He stated that when fitness and custom were both accounted for that proportion came into play and refined a work of architecture. He divided proportion into two rules:

1. He advocated the use of simply commensurable ratios. Although he limited himself in this respect he acknowledged that these ratios should be treated merely as a means to an end and that those ratios in themselves were not the key.

2. The most important of these rules was that the objective should be in the use of repetition of ratios or more directly the repetition of similar shapes.
Barca's ideas however were not realized to be innovative until later in the nineteenth century. "Meanwhile the collapse of the theory of proportion left architectural theory in a state of confusion whose results are still apparent. In one way, however, this disintegration was not without disadvantages. It provided the theory of architecture with a kind of pupal phase in which all its parts could be taken to pieces and reexamined."

MODERN DAY THEORY

With the collapse of the Renaissance architects of the nineteenth century found themselves in an extremely transitional period. It became apparent that two alternatives existed with regards to proportion, an element that would have great impact on any new architectural approach. The first alternative and most prevalent was to rely on the intuitive approach to proportion, and it is this path which most architects have chosen today. The second alternative was to develop a new theory of proportion, and those who were interested in this approach looked to Greek and Gothic architecture for possible answers. It was this renewed interest in history that led to the Greek and Gothic revivals.

This period was especially important in that proportion was studied geometrically rather than analytically as was the case with Vitruvius and Renaissance architects. This geometric approach opened the door for incommensurable ratios which in turn led to the study of incommensurable ratios analytically. Gothic architecture was studied geometrically because of the obvious interest of architects of this period in geometry. Greek architecture however was originally studied analytically because the writings of Vitruvius were still thought to have some merit, that is until the work of an artist named Jay Hambidge was presented. Scholfield writes:

"One of the great merits of Hambidge's contribution to the archaeology of proportion was to be that, although he believed Greek designs to have been carried out geometrically, he also saw that this was no obstacle to making use of the enormous advantages which arithmetic offered for analysis."

The golden section rectangle has been discussed throughout history as to whether it has merit as a design tool. Fechner, the founder of experimental
aesthetics, found that when the GS rectangle was placed among nine other rectangles it was the most popular choice when the participant was asked to pick between the ten. The results of Fechner’s experiments however were not very strong, and studies of this kind led Hambidge to conclude that the GS rectangle in itself is not superior to any other form. It however is superior as a “mathematical means to an aesthetic end” and Hambidge suggested that experiments could be geared in this direction. Whereas in the past analytical systems had dealt strictly with commensurable ratios, Hambidge and Corbusier concentrated on the study of forms such as the GS rectangle with an analytical approach. Scholfield writes:

"What is new about the more recent systems developed by Hambidge and Corbusier is that they successfully combine the advantages of analytical systems with the use of incommensurable ratios. Hambidge’s system shows some features of a transitional stage of development from the geometrical to the analytical. His analytical system is based on the manipulation of incommensurable ratios expressed in ordinary decimal notation, and although well within the mathematical powers of the average architect, it is in practice somewhat clumsy. Corbusier has developed an analytical system which does away with the need for any calculations, at the cost of restricting the user to a fixed scale of dimensions, which, when used consistently, seems to give a much less flexible system than Hambidge’s."

Scholfield continues and clarifies further this new approach:

"Analytical systems are those in which the problem of proportion is reduced from two or three dimensions to one. The manipulation of geometrical shapes is replaced by the manipulation of linear dimensions. The repetition of similar shapes, the end in view, recedes into the background, and attention is directed to the means to that end, the proportional relationships between linear dimensions."

Hambidge

Hambidge’s work can be divided into two parts, the first of which is his analysis of Greek vases and architecture. He devoted much more of his time
to the study of vases because they were more readily available and it was easier and less time consuming to determine the proportional theme on which they were based. He documents as well the proportional system on which the Parthenon is based, and he does this in great detail. The second aspect of Hambidge's work was the development of a workable proportional system that would be easy for a designer to use. He has given the name Dynamic Symmetry to the system which the Greeks utilized and to his own workable system.

Hambidge defines the term Dynamic Symmetry as a system based on incommensurable ratios as opposed to Static Symmetry which is based on commensurable ratios. He also defines the terms as they apply to nature, writing:

"Static Symmetry is apparent in nature in certain crystal forms, radiolaria, diatosis, flowers, and seed pods, and has been used consciously in art at several periods. The principle of Dynamic Symmetry is manifest in shell growth and in leaf distribution in plants. A study of the basis of design in art shows that this active symmetry was known to but two peoples, the Egyptians and the Greeks."

Hambidge found evidence of the use of static symmetry early in Greek history, but as the culture became more advanced the use of dynamic symmetry became predominant. This advanced proportional system was completely lost in the Greco-Roman period and remained as such until the work of Hambidge. Once rediscovered it was his desire to develop it as a design tool. He taught his students to use both geometrical and arithmetical methods when constructing forms for use in design. Some felt, however, that the system developed by Hambidge was much too complicated and favored the system developed by Corbusier which was said to be easier to use but less flexible.

Corbusier

Whereas Hambidge approached the development of his system both geometrically and analytically Corbusier's Modulor is an entirely analytical method of proportion. This system consists of two scales, red and blue, which are based on the GS ratio geometric progression, the blue scale intervals being twice those of the red. This system was designed not
only as a tool of proportional design but also as a standard for the mass production of building components. This was important in that the tendency in designing mass produced components was to totally ignore proportion in favor of economy and function.

There are two important aspects which Corbusier stressed with regards to his system. The first was its relation to the human figure. Corbusier used as the basis for his system measurements of a 6' man in various positions, therefore making his system useful in furniture design. The second feature that Corbusier stressed was the system's easy transition from the European metric to the foot/inch system. It was a pleasant surprise to Corbusier to find that the metric figures could be very closely approximated using half-inch intervals in the U.S. system. The Modulor however is said to have the same drawbacks as the method utilized by Palladio in that the parts of the design adhere to the system, but when added together the whole fails to relate to the parts.

CONCLUSION

We have seen that Vitruvius advocated a proportional system that was analytical and commensurable, and that the Renaissance which took its cue from Vitruvius employed a similar system. With the collapse of the Renaissance came new research into proportional theory however no comprehensible workable theory has of yet been invented or discovered. Both Greek and Gothic theories which were determined to be geometrical and incommensurable opened the door for the work of Hambidge and Corbusier. Although each of these systems has its disadvantages they are successful in that they combine analytical systems with incommensurable ratios.

Direction in the development of a comprehensive proportional theory may lie in the perfection of an analytical system, in simplifying a geometrical system, or with the Fibonacci series. Scholfield writes:

"But a more detailed investigation of these possibilities would fall outside the scope of the present discussion. We set ourselves the task of stating a theory of proportion which would explain the history of proportion in terms, not of metaphysical systems, but of simple relationships of form which are apparent
to the eye. We cannot proceed any further here with the next step. This is the application of the theory of proportion to the perfection of practical systems, whose aim is to aid the designer in his task of bringing delight to the human eye."
Bibliography


