DO FIFTH GRADE CHILDREN UNDERSTAND
EQUIVALENT FRACTIONS?

SENIOR HONORS THESIS

by

JOYCE ANDERSON

DR. REBECCA NELSON, ADVISOR

MAY 19, 1983
Fractions play an important role in the school mathematics program. As early as first and second grade, teachers work with students naming fractions. By the time a pupil enters the fourth, fifth, and sixth grade, he is learning to add, subtract, multiply, and divide fractions. However, before these operational tasks may be effectively utilized, a student needs to demonstrate an understanding of equivalent fractions.

Teachers often evaluate competency in the area of equivalent fractions by paper and pencil methods. If a student can correctly complete a ditto with at least 75 to 100% accuracy, they deem that the child understands equivalent fractions and quickly move on to addition of fractions. However, does the student really comprehend? Is he ready to begin operational tasks based upon a paper and pencil assessment? What is the relationship between what students can demonstrate using paper and pencil methods and what they can show using manipulatives when dealing with equivalent fraction problems?

Few studies have addressed this particular problem. Related studies in fraction work indicate that many students use rules to complete operational exercises without understanding them (i.e., Hase-mann, 1981; Post, 1981). The children mechanically use the rules they have learned; sometimes with success, but often with nonsensical results. They rarely can explain why the rule is so and not otherwise. Does the same thing apply in equivalent fraction work? Do the students merely use "prescribed rules" to complete workbook exercises without understanding what they mean? This study was designed to gather some initial data relative to this question.
I obtained permission to test four fifth grade classrooms in four different schools in a major Midwest town. The socioeconomic backgrounds of the schools ranged from lower to upper socioeconomic class. Most of the schools were homogeneous due to the fact that this town is basically a homogeneous city. In all, I tested 77 students: 38 of whom were boys and 39 of whom were girls.

The test consisted of two sections. First, each child was asked to complete a ditto page (Figure 1) with ten equivalent fractions on it. Attempting to guide the child's thinking along the correct path, the first two problems utilized pictorial diagrams. The last eight problems became the source for our data collection of children's understanding of paper and pencil algorithms. No verbal clues were given to the students at this particular point. They were on their own to figure out and complete the exercise.

In the second section, the pupils were asked to show two fractions and two equivalent fractions using manipulatives. First, six 12" color coded circular regions were set before the child. One of the colored regions remained uncut representing one whole. The other color coded regions were cut into thirds, fourths, sixths, nineths, and twelfths respectfully (Figure 2). Each child was asked to show the fraction $\frac{4}{5}$ using any of the pieces laid before him. After this task was accomplished, the problem $\frac{2}{3} = \frac{6}{9}$ was presented. Each student first demonstrated their conception of $\frac{2}{3}$ on the whole circular region. Next another whole circular region was placed beside the first. Then the pupils were asked, "Two thirds is the same as six what? Can you show me?"
Give equal fractions for shaded area.

\[
\frac{1}{3} = \frac{\square}{12}
\]

Write each missing numerator or denominator.

\[
\frac{1}{2} = \frac{\square}{8} \quad \frac{2}{3} = \frac{6}{\square} \quad \frac{8}{12} = \frac{4}{\square} \quad \frac{1}{3} = \frac{\square}{9}
\]

\[
\frac{9}{12} = \frac{3}{\square} \quad \frac{9}{9} = \frac{\square}{3} \quad \frac{2}{8} = \frac{1}{\square} \quad \frac{6}{12} = \frac{\square}{6}
\]

Figure 1
Next six 8" x 12" color coded rectangular regions were set before the child. Like the circular regions, one remained uncut while the other five were cut into thirds, fourths, sixths, nineths, and twelfths (Figure 3). Each child was asked to show the fraction \( \frac{3}{6} \) using any of the pieces laid before him. Following his completion of this task, the problem \( \frac{2}{5} = \frac{?}{12} \) was given. Once more the process used with circular regions was repeated. The student first demonstrated \( \frac{2}{5} \), another whole rectangular region was placed beside the first, and then he was asked, "Two sixths is the same as how many twelfths? Can you show me?"

The guiding questions used were minimal. If a child attempted to fill the area with one color of pieces and said, "This doesn't work." The question posed was, "Do any of the other pieces work?" In the case of the rectangular region, if a child had problems placing the nineths pieces on the whole, the question, "Is there another way to put the same pieces on the whole?" was asked. If the child looked worried or upset, I would ask, "What is wrong? Tell me." The child usually proceeded to tell me what he wanted, and we continued.

The results of the testing were organized and tallied. If a child completed six out of the eight written exercises (excluding the first two problems), it was deemed that the child could perform equivalent fractions using paper and pencil methods. If the student completed less than six of the eight exercises correctly, it was assumed that he did not have an understanding of equivalent fractions based on the ditto.
In utilizing the manipulative information gathered, the results where students showed the individual fractions were disregarded. This step aided the student in becoming familiar with the material. If the pupil could demonstrate both equivalent fractions (using the circular region and the rectangular region), he was assessed as understanding equivalent fractions. If the student showed one out of the two equivalences correctly, it was concluded that he had a shaky idea of what equivalent fractions are. However, if he missed both equivalences with the manipulatives, it was assumed that he lacked an understanding of equivalent fractions.

The results of cross matching the findings of both sections of the test are shown below.

<table>
<thead>
<tr>
<th>Manipulatives</th>
<th>Yes</th>
<th>Shaky</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Both Correct</td>
<td>1 Correct</td>
<td>Both Wrong</td>
</tr>
<tr>
<td>Paper &amp; Pencil</td>
<td>9</td>
<td>5</td>
<td>9</td>
</tr>
<tr>
<td>0-5 Correct (6-8 Correct)</td>
<td>6</td>
<td>10</td>
<td>38</td>
</tr>
</tbody>
</table>

The findings of this study indicate that only nine children out of 77 students tested have grasped the concept of equivalent fractions by the end of the fifth grade. These pupils appear to be ready to work with addition and subtraction of fractions. But what about the 38
children who could not demonstrate knowledge of equivalent fractions by either method? When I interviewed the teachers, all the pupils were adding and subtracting fractions. How can an individual add or subtract two fractions before they can identify the families of fractions? Many of these children could not even show a fraction. They had misconstrued ideas of what a fraction truly was.

The five students who could perform pencil and paper equivalence yet were shaky when it came to showing it, need more work with equivalent fractions. I believe they need some hands on experiences to solidify the knowledge they already have.

It is interesting that six children understood and ten had a shaky idea of what equivalent fractions were, yet could not complete the ditto correctly. This indicates that they have reached a level of maturity where they are ready to move on to performing in the abstract but have not yet mastered this concept at the abstract level. Perhaps tying concrete work directly with abstract work would aid these students as they progress.

The last category indicates that pencil and paper work does not help nine of the children to understand fractions. They are skimming by the teachers who believe that they understand. Apparently they have wandered upon a prescribed rule which aids them in completing ditto exercises; yet, they do not understand why the rule is so. And when asked to illustrate it, they are at a loss.

I observed numerous students grapple with the idea of a fraction. They would use unequal pieces to fill the whole unit. Many went completely outside of the unit to show me a portion of it. Some stud-
ents demonstrated what they thought to be ratios instead of showing me a fractional part of a whole. They indicated with all the questions asked during the testing and all the incorrect examples that few children whom we are teaching have developed an understanding of equivalent fractions.

It seems apparent that more effort needs to be made in helping children internalize the concept of equivalent fractions. Previous studies and instructional strategies indicate that we need to utilize various types of manipulative materials as we teach, and we need to emphasize the development of understanding prior to symbolic work.

This pilot research project has shed some light upon what our students actually understand about equivalent fractions. I would like to see the study repeated in a different locality with a larger sample. At that time, I would suggest changing one of the fractions demonstrated on the rectangular region, using the fraction \( \frac{3}{4} \) instead of \( \frac{2}{3} \) because of the difficulty in placing the pieces on the whole unit. Another study which could be performed would be to find out what fifth grade students understand about the naming of fractions. I discovered numerous children having difficulty with this skill while testing. There is a great need for further research in the whole area of fraction work.
REFERENCES


