Manipulative Versus Traditional Teaching for Mathematics Concepts: Instruction-Testing Match

An Honors Thesis (HONRS 499)

by

Maile A. Keagle and Angela J. Brummett

Thesis Advisor
Dr. Betty E. Gridley

Ball State University
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In recent years, there has been an emphasis in increasing mathematics achievement through making it more meaningful for students. The trend is towards understanding mathematical processes in place of rote practice, rote memorization of rules, use of work sheets, and teaching by telling as a primary method. In order that students might better understand processes, attention has been increasingly given to use of manipulative materials or physical models, cooperative work among students, content integration, and use of calculators and computers. Students are also being encouraged to discuss mathematics, write about mathematics, discover properties on their own, and to feel comfortable to question and justify what they are being taught. However, many of these techniques have been promoted with limited empirical research to support their use. While use of manipulatives in teaching mathematics has been promoted, many of the variables related to their use have not been explored.

Indeed, much of the early research on manipulatives simply compared the use of manipulatives with nonuse. In the early 1970's studies were conducted to explore manipulative use versus nonuse in teaching several mathematical concepts, including: fractions (Bisio, 1971; Bledsoe, 1974; Brown, 1973; Wallace, 1974); integers (Coltharp, 1979); geometry (Bring, 1972; Smith, 1974); multiplication and division (Babb, 1976; Nichols, 1972; Trask, 1973); problem solving (Bolduc, 1970; Nickel, 1971); remedial mathematics (Dunlap et al., 1971; Tobin, 1974); rational numbers (Carney, 1973); and several topics at once (Cook, 1968; Davidson, 1973; Earhart, 1964; Weber, 1970).
The results of these studies have been contradictory with some favoring the use of manipulatives (e.g., Babb, 1976; Bledsoe, 1974; Bolduc, 1970; Bring, 1972; Brown, 1973; Cook, 1968; Earhart, 1964; Nichols, 1972; Nickel, 1971; Tobin, 1974; Wallace, 1974). Others found no significant differences between the two teaching methods (e.g., Anderson, 1958; Bisio, 1971; Coltharp, 1969; Davidson, 1973; Dunlap, 1971; Macy, 1957; Tobin, 1974; Trask, 1973; Weber, 1970). A small number of studies had findings that favored not using manipulatives (e.g., Carney, 1973; Smith, 1974).

One example of an early study which seemed to be particularly methodologically sound was conducted by Fennema (1972). She randomly assigned 95 subjects ages seven and eight to eight groups. Four groups were taught using a concrete method, and four were taught using the symbolic method. Fennema examined the concept of multiplication defined as union of equivalent disjoint sets.

Children in the concrete method were taught using Cuisenaire rods as a model, and those in the symbolic method were taught using only a symbolic model. Subjects attended fourteen instructional sessions in which all eight groups used the same work sheets, problems, and drill games. Subjects were also given the same three tests; Recall, Transfer I, and Transfer II.

The Recall Test assessed the principles that the subjects were to have learned in the instructional sessions. Both Transfer Tests involved assessing the children on their ability to use what they had learned in the instructional sessions to solve problems that they had never seen before. Transfer Test I allowed the subjects in the Concrete treatment to use
the Cuisenaire rods, and the subjects in the Symbolic treatment to use pencil and paper. To solve the problems in Transfer Test II all eight groups used counters.

Fennema found that subjects scored equally well on the Recall test. However, subjects who participated in the symbolic method performed at higher levels on the Transfer tests than the subjects who participated in the concrete method.

Fennema attributes the results to the fact that the subjects had participated in a program the year before that emphasized the use of concrete objects in solving problems.

Another concern involved the use of counters on Transfer Test II. Cuisenaire rods are a length representation and the counters represent the set idea used most commonly when teaching subtraction and addition. Therefore, the change of the concrete material being used can cause great confusion in the subject.

For several years, there was a lack of research dealing with hands-on learning and manipulatives. Many researchers turned to the subjects of computer-aided learning and active game learning. In the late 1980's attention turned back to the use of concrete materials in the classroom. For example, Harrison, Brindley and Bye (1989) conducted a study that addressed the Piagetian cognitive levels of students and how well they learn fractions and ratios with the concrete method of teaching. The researchers randomly assigned seventh graders with comparable mean scores on Stanford Intermediate Level II, Mathematics Form B to experimental and control groups. The researchers determined the cognitive ability of these students on ratios and fractions through administration of several different tests. They compared these tests scores with those of both older and
younger children to determine what the cutting points were for the cognitive levels. These scores were used to place the subjects in either the concrete operational, transitional, or formal operational stage.

The control group was then taught fractions and ratios using a Grade 7 textbook and teacher's guide. These lessons consisted of teacher demonstrations and seat work, the children used no concrete materials. The experimental group was taught using both simple and complex manipulatives. Students in this group were encouraged to carry out their own investigations, record their actions and the outcomes. These lessons took place from October through December.

When tested using the General Mathematics Test, there was a significant difference between the groups with the experimental group scoring higher. The researchers also found a significant difference favoring the concrete method of instruction in more positive attitudes toward fractions and ratios, and decreased levels of anxiety toward these topics.

Also, in 1989, The National Council of Teachers of Mathematics published The Curriculum and Evaluation Standards for School Mathematics, better known as "The Standards". This report cited the need for changes in the areas of teaching methods, curriculum, and assessment of mathematics for all grades. The report focused on identifying areas where attention should be increased and decreased. Overall, it called for more attention on "actively involving students individually and in groups in exploring, conjecturing, analyzing, and applying mathematics in both a mathematical and a real-
world context." It also called for more use of concrete materials. The report also placed emphasis on communication in mathematics learning, and suggested that students discuss, write, read, and listen to mathematical ideas.

In mathematics instruction, the report discouraged the use of fill-in-the-blank work sheets; answering questions that require only yes, no, or a number as responses; practicing routine, one-step problems; and memorizing procedures without understanding. It also discouraged testing for the sole purpose of assigning grades.

Following the publication of this report, Stemple and Mitchell (1991) addressed the issue of how the suggested changes in mathematics instruction would affect the procedure for the assessment of mathematics achievement. For instance, how does one know that children are solving problems using the skills that a concrete teaching method facilitates?

Some ways the article suggests are: a) encouraging students to write about math, b) keeping portfolios of student work, and c) asking open-ended questions on tests. All of these methods move away from the traditional "only one right answer" method of testing, and have been termed "performance assessment". Students are encouraged to write out all of their work on these types of tests so that the teacher can observe their thought processes. They are also encouraged to use calculators and other tools.

There are several states that have started administering performance-based assessment tests on a large scale. As of Spring 1992, sixteen states had statewide writing tests, and at least nine states were administering tests with open-ended mathematics questions. Several obstacles have been encountered with this large-scale testing. Two of
these include cost of administration, and reliability of the tests. Other questions have been raised about the consistency of the scorers (Wagner, 1992). Problems have also been encountered when students who have not been taught using the suggestions in "The Standards" are tested with a performance based method.

An example of this is the Michigan Educational Assessment Program, a test used to monitor the achievement of students statewide. Each year fourth, seventh, and tenth graders take the mathematics portion. In 1991, this section was changed to a performance based format. The majority of the students received failing scores (Michigan Department of Education, 1991). This is attributed to the fact that these students had not been taught by the methods suggested in "The Standards". These results raise the question, can students taught by one method accurately show their ability when the test is based on another teaching method.
PURPOSE STATEMENT AND RESEARCH QUESTIONS

The purpose of the present study was to examine the effects of two different methods of teaching fractions on the performance of children when being assessed with both traditional questions that require only an answer, and more performance based questions that encourage the use of manipulatives and require either a pictorial or written explanation of the procedure that was used to solve the problem.

Specifically, the following questions were addressed:

1. Are there differences in the achievement on fraction problems between children taught using the traditional method, and those taught using the hands-on method with manipulatives?

2. Are there differences in how the two groups perform on the types of problems that only ask for an answer, and those requiring a description of the problem solving process?

3. Are there differences in the groups’ attitudes toward math, specifically fractions, between the two groups that vary between pre and post testing?
METHOD

Subjects

The subjects for the study were fourth grade students in the three classrooms at a rural elementary school in Indiana. A total of fifty-seven subjects, 23 males and 34 females, were used. Each classroom contained nineteen students. These students had been assigned to the classrooms by the principal at the beginning of the academic year.

Operational Definitions of Terms

There are several terms used throughout this paper that need to be clarified. These include manipulatives, Cuisenaire rods, concrete teaching method, and traditional teaching method.

Manipulatives are any objects used by students that enhance the learning process.

Cuisenaire rods are one type of manipulatives used in mathematics. (See Appendix A) These are sometimes also called centimeter rods. They come in ten different sizes and colors ranging from one to ten centimeters in length. The rods are used by the teacher and students to demonstrate the relationship between whole objects and their fractional pieces.

The concrete teaching method is based on the use of manipulatives as the major teaching tool. When designing the concrete lessons for this study the experimenters used "The Standards" (National Council of Teachers of Mathematics, 1989) as their guideline. Therefore, the lessons utilized group work, hands-on exploration, and explanation of the
procedures used to solve the problems.

Unlike the concrete method, the traditional teaching method utilizes rote memorization, the importance of the correct answer and not correct process, teacher demonstration, and work sheets. The lessons used in this study were designed specifically to reflect the methods that were discouraged by "The Standards".

Instrumentation

Attitude Survey. (See Appendix B). The purpose of this survey was to determine the student's attitudes about the subject of mathematics and their textbook. A total of eight questions were asked. Three questions focused on how the students felt about doing mathematics at certain times. One question each asked about attitudes towards mathematics for the following: homework, the mathematics text book, working with a friend, working on fractions, and learning new things in mathematics.

This survey used a Likert-type Scale. Four pictures of Garfield the Cat were used to represent the attitudes. First was a "big grin" Garfield which represent strong enjoyment. The second Garfield had a simple closed mouth smile and that represented enjoyment. The third Garfield had a simple frown and represented dislike. The last Garfield had hands on hips and an angry glare. This last one represented strong dislike.

Test of Knowledge. (See Appendix C). A test, developed by the experimenters, was used to assess knowledge of the students at the conclusion of the teaching experiment. Two types of questions were developed to reflect the two types of teaching. The test included a total of twenty-two questions. One question was not included in scoring
because of an error by the examiners. Four questions dealt with looking at a picture and identifying the fraction represented. Two questions dealt with adding fractions with like denominators, one in the pictorial format and the other in the abstract format. Four questions asked the students to compare a list of fractions written in the abstract format and one asked for an explanation. One question asked the student to draw two different models of the same fraction. There were two questions dealing with subtracting fractions with like denominators. Both were written in the abstract format but one required the students to draw a model of the problem and solution. The addition of fractions with unlike denominators was represented with four questions, three written in the abstract format and one of those required the students to draw a model of the problem and solution. The other problem was given in the pictorial format and the students were to show their answer by shading in a blank rectangle with the correct numbers of parts. Two problems written in the abstract format tested the students' knowledge of subtracting fractions with unlike denominators. Two questions were story problems for which the students were asked to find and explain their answers. There was no time limit and students in the experimental group were allowed to use the Cuisenaire rods.

Procedure

Three classrooms of fourth grade students were involved in the study. Classroom I was taught three lessons on fractions using the traditional teaching method. They were also given a test to assess their knowledge of fractions at the end of the three days. Classroom II was taught three lessons on fractions using the concrete teaching method.
Copies of all lesson plans are included in Appendix D. These lessons were taught exactly one week later than Classroom I, on the same days at the same time. On the day that Classroom II received the test, Classroom III, that received no treatment also received the test. During the lessons, the same experimenter taught all the lessons, and the other assisted by passing out papers and assisting the children while they were doing seat work. The fraction lessons were taught to each classroom in the following ways:

Classroom I

**Day I.** The objectives for this day were for the students to complete the attitude survey, to be introduced to the experimenters, and for the students to be able to recognize and name fractions. The experimenters first introduced themselves and told the students that they were in the classroom because they were interested in the way students learn fractions. The students were also told that the work done for the experimenters would not affect their grades and they were asked to try their best. The students were then given the attitude survey. The experimenters instructed Classroom I on how to complete the survey and asked them to honestly fill out the survey. The students were then shown circles on the chalkboard with different sized pie-shaped pieces representing various fractions. (See Appendix E) Classroom I participated in a discussion on how fractions are used, where they had experienced fractions before, and how fractions are named. After ten to fifteen minutes, the students were able to name the fractions shown by the experimenters. For review, the students were given a work sheet on which to work individually at their seats. (See Appendix F)
Day II. The objective of the second lesson was for the students to be able compare fractions and equivalent fractions, and to be able to add and subtract fractions with like denominators. The lesson began with a review of naming fractions. The experimenters then lead a discussion on unequal fractions centered around the question, "Would you have more pizza if you had one-half or one-third of the pizza?" The experimenter used the blue circle to show equal and unequal fractions on the board. The students were asked to raise their hands and participate in the discussion. The students were asked to complete two work sheets on these topics individually at their seats. (See Appendix G) The experimenter then lead a discussion and demonstration on the addition and subtraction of fractions with like denominators. The students were called upon to do practice problems at the board. With time left over from the lesson, the students were divided into five groups and played some card games. (See Appendix H) These cards showed fractions both pictorially and numerically. The students were required to match the pictures with the numbers in order to play such games as Go Fish and Memory.

Day III. The objectives of this day were to review addition and subtraction of fractions with like denominators and introduce the addition and subtraction of fractions with unlike denominators. The lesson began with the review. The experimenter then demonstrated addition and subtraction of unlike denominators with the use of blue circles and orange pie pieces. The lesson was interrupted by a convocation. After the convocation, the experimenters did further demonstrations. There was no time for the students to practice these types of problems.
Day IV. The students were given the attitude survey again. The test was then given and no time limit was placed on the students. The students were given a pencil as a reward for participating in the study.

Classroom II

Day I. The objectives of this day were to introduce the students to the experimenters, the Cuisenaire rods, complete an attitude survey, and recognize and name fractions. The experimenters first introduced themselves and explained their purpose for being in the classroom in the same manner as was done in Classroom I. The students were given the attitude survey and instructed on how to complete the survey as was done in Classroom I. The lesson began with a discussion on how fractions are used, where the students have had experiences with fractions before, and how fractions are named. Blue circles and orange pie pieces were used. After ten to fifteen minutes, the students were able to name the fractions shown by the experimenters. Each student received Cuisenaire rods and a work sheet that helped familiarize themselves with the rods. (See Appendix I) The experimenter used overhead projector Cuisenaire rods throughout all the lessons. As a group, the class went over the work sheet. For the rest of the time, the students modeled various fractions that were presented in the numerical form by the experimenter.

Day II. The objective of the second day was for the students to be able to compare fractions and do addition and subtraction of fractions with like denominators. The experimenter asked the students to model and compare fractions with their Cuisenaire rods. The same strategy was used for the teaching of the addition and subtraction of
fractions with like denominators.

**Day III.** The objectives of this day were to review the addition and subtraction of fractions with like denominators and introduce the addition and subtraction of fractions with unlike denominators. Due to the convocation the week before, twenty minutes was eliminated from this lesson also. The students were not allowed time to practice the addition or subtraction of fractions with unlike denominators.

**Day IV.** The students were given the attitude survey again. The faction test was administered and no time limit was set. Each student received a pencil as a reward for participating in the experiment.

*Classroom III*

**Day I.** The experimenter explained the purpose of the experiment and that the students were being asked to complete the test even though they did not receive any lessons on the material. They were asked to do their best. They were given the test with no time limit set. A pencil was given to each student as a reward for participating in the study.

**Design and Analysis**

The design of the study was quasi-experimental because the subjects used were already assigned to the classroom by the principal. This elementary school was chosen because it had three fourth grade classrooms, two for the teaching treatments and one for the control group. Classrooms were randomly assigned by the experimenters to treatments. Fourth graders were chosen for this study because they had completed only introductory lessons on fractions. These lessons included discussion about the use of and
the naming of fractions. This information was obtained from consulting with the classroom teachers and examining the textbook. This elementary school was chosen because it had three fourth grade classrooms, two for the teaching treatments and one for the control group.

The Statistical Package for the Social Sciences (SPSS-X) was used to analyze the data. The fraction test was analyzed. The frequency of correct answers was computed for the total test score for each subject. The test was then broken down into traditional type questions and performance based questions, and the frequency of correct answers for each type was computed for each subject. Means and standard deviations were found and used to describe the total scores, the scores on the traditional questions, and the scores on the performance based questions for each classroom. An analysis of variance was performed on all three sets of questions to see if there was a significant difference between groups on the test scores.

Finally, means and standard deviation were used to describe the scores of the pre- and post-attitude surveys. An repeated measures analysis of variance was performed to find any significant differences and or interaction effects between the groups on both sets of scores.
RESULTS

Are there differences in achievement on fraction problems between children taught using the traditional method, and those taught using the hands-on method with manipulatives?

The means and standard deviations for the total scores of all three groups on the fraction test are reported in Table 1. The traditional group scored on the average two points more than the group that used manipulatives, and the group using manipulatives scored about three points more than the control group. Standard deviations were similar for all three classrooms.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Standard Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>10.68</td>
<td>2.83</td>
<td>19</td>
</tr>
<tr>
<td>Manipulatives</td>
<td>8.79</td>
<td>3.72</td>
<td>19</td>
</tr>
<tr>
<td>Control</td>
<td>6.00</td>
<td>3.07</td>
<td>19</td>
</tr>
</tbody>
</table>

A one-way ANOVA was computed to compare the mean scores of the three classrooms on the total fraction test. The difference was statistically significant, \( F(2,54) = 10.11, p < .0002 \). A summary of these results is reported in Table 2. A Tukey HSD test indicated that the mean for the traditional group (10.68) was not significantly different from the mean of the manipulative group (8.79). However, both of these means did differ significantly from the mean of the control group (6.00), \( p < .05 \).
TABLE 2

SUMMARY OF ANOVA FOR TOTAL TEST SCORES

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>210.98</td>
<td>2</td>
<td>105.49</td>
<td>10.11</td>
<td>.0002</td>
</tr>
<tr>
<td>Within groups</td>
<td>563.26</td>
<td>54</td>
<td>10.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>774.24</td>
<td>56</td>
<td>115.92</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Are there differences in how the two groups perform on the types of problems that only ask for an answer, and those requiring a description of the problem solving process?

The means and standard deviations for traditional questions are reported in Table 3. As this table indicates, difference between the groups ranged from approximately one and one third to one and one half points.

TABLE 3

MEANS AND STANDARD DEVIATIONS FOR TRADITIONAL QUESTIONS

<table>
<thead>
<tr>
<th>Method</th>
<th>Mean</th>
<th>Standard Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>6.84</td>
<td>1.64</td>
<td>19</td>
</tr>
<tr>
<td>Manipulatives</td>
<td>5.53</td>
<td>2.12</td>
<td>19</td>
</tr>
<tr>
<td>Control</td>
<td>4.00</td>
<td>1.83</td>
<td>19</td>
</tr>
</tbody>
</table>
A one-way ANOVA was used to compare the mean traditional scores for the three classrooms on the fraction test. These results were significant $F(2, 54) = 10.97, p < .0001$. A summary of these results is reported Table 4. A Tukey HSD test indicated that there was not a significant difference between the mean of the traditional group (6.84) and the mean of the manipulative group (5.83). Both of these means did, however, vary significantly from the mean of the control group (4.00), $p < .05$.

**TABLE 4**

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>between groups</td>
<td>76.88</td>
<td>2</td>
<td>38.44</td>
<td>10.97</td>
<td>.0001</td>
</tr>
<tr>
<td>within groups</td>
<td>189.26</td>
<td>54</td>
<td>3.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>266.14</td>
<td>56</td>
<td>41.94</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The means and standard deviations for the performance-based questions are reported in Table 5. Table 5 shows that the means and standard deviations for the performance-based questions of the fraction test of knowledge were also very similar for the traditional, manipulative, and control groups.

A one-way ANOVA was also used to compare the mean scores on the performance-based questions of the fraction test. These results were also significant $F(2, 54) = 5.83, p < .005$. A summary of these results is reported in Table 6.
A Tukey HSD test indicated that the mean for the traditional group (3.84) was not significantly different from the mean of the manipulative group (3.26). Both of these means was found to be significantly different from the mean of the control group (2.00), $p < .05$.

TABLE 6

SUMMARY OF ANOVA FOR PERFORMANCE BASED TEST QUESTIONS

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>$F$</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>33.72</td>
<td>2</td>
<td>16.86</td>
<td>5.83</td>
<td>.0051</td>
</tr>
<tr>
<td>within groups</td>
<td>156.21</td>
<td>54</td>
<td>2.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>189.93</td>
<td>56</td>
<td>19.75</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Are there differences in the groups' attitudes toward mathematics, specifically fractions, between the two groups that vary between pre and post testing?

Means and standard deviations for the pre- and post-attitude scale for Classroom I and Classroom II are reported in Table 7. Table 7 shows that there was very little difference in the pre-test attitude scores of the traditional and manipulative classrooms.

**TABLE 7**

<table>
<thead>
<tr>
<th>Treatment</th>
<th>PRE</th>
<th></th>
<th>POST</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
<td>Mean</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>Traditional</td>
<td>25.88</td>
<td>3.22</td>
<td>21.79</td>
<td>4.16</td>
</tr>
<tr>
<td>Manipulative</td>
<td>26.68</td>
<td>3.33</td>
<td>27.11</td>
<td>3.96</td>
</tr>
</tbody>
</table>

Table 7 also shows that after receiving the traditional lesson about fractions, Classroom I's mean attitude score dropped several points. However, Classroom II, that received the hands-on lesson using manipulatives, experienced a very slight drop in their mean attitude scores.

A two-way ANOVA showed a significant interaction. Attitude depended not only on the groups, but also on time. Table 8 gives a summary of the two-way ANOVA.

Summaries of Simple Effects Analyses are given in Tables 9 and 10. As can be seen in Table 9 there were no significant differences between the attitudes of the two
groups at the pretest. However, at the post-test time the attitudes between the two groups were significantly different.

Table 8

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between groups</td>
<td>189.27</td>
<td>1</td>
<td>189.27</td>
<td>8.85</td>
<td>.005</td>
</tr>
<tr>
<td>res. b.s.</td>
<td>727.34</td>
<td>34</td>
<td>21.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>total b.s.</td>
<td>916.61</td>
<td>35</td>
<td>2.76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time</td>
<td>73.57</td>
<td>1</td>
<td>73.57</td>
<td>15.79</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>Group X Time</td>
<td>107.34</td>
<td>1</td>
<td>107.34</td>
<td>29.04</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>res. w.s.</td>
<td>158.43</td>
<td>34</td>
<td>4.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total w.s.</td>
<td>339.34</td>
<td>36</td>
<td>9.43</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This interaction effect is portrayed schematically in Figure 1.

Fig. 1 Math Attitudes Pre and Post by Group
Table 10 shows that over time the attitudes of the manipulative group did not change, but the attitudes of the traditional group changed significantly.

**TABLE 9**

**SUMMARY OF SIMPLE EFFECTS ANALYSIS TRADITIONAL VS. MANIPULATIVE, HOLDING TIME OF TESTING CONSTANT**

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRAD. vs. MAN at pre-test</td>
<td>5.77</td>
<td>1</td>
<td>5.77</td>
<td>.27</td>
<td>&gt; .05</td>
</tr>
<tr>
<td>TRAD. vs. MAN at post-test</td>
<td>268.45</td>
<td>1</td>
<td>268.45</td>
<td>12.55</td>
<td>&lt; .01</td>
</tr>
<tr>
<td>res. b.s.</td>
<td>727.34</td>
<td>34</td>
<td>21.39</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 10**

**SUMMARY OF SIMPLE EFFECTS ANALYSIS, PRE VS. POST TESTING, HOLDING TREATMENT CONSTANT**

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre vs. Post TRAD.</td>
<td>169.88</td>
<td>1</td>
<td>169.88</td>
<td>36.02</td>
<td>&lt; .01</td>
</tr>
<tr>
<td>Pre vs. Post MAN</td>
<td>1.68</td>
<td>1</td>
<td>1.68</td>
<td>.36</td>
<td>&gt; .05</td>
</tr>
<tr>
<td>res. w.s.</td>
<td>158.43</td>
<td>34</td>
<td>4.66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
DISCUSSION AND CONCLUSIONS

To answer the first two questions of this study, no significant differences were found between the traditional and the manipulative groups' performance on the fractions test. This includes the comparisons between the two groups on the total test scores, traditional test scores, and the performance-based test scores.

The experimenters offer the following explanations for the obtained results. First, the unit of lessons is considered short compared to what a classroom teacher would normally spend in the teaching of the fraction concept. Because of an unexpected convocation, Classroom I and Classroom II were unable to spend time practicing the addition and subtraction of fractions with unlike denominators. These types of problems were presented on the fraction test of knowledge, but few students were able to correctly answer them, whether they appeared in the traditional or performance-based format.

Secondly, the experimenters observed that during the manipulative lessons some students spent much time building and playing with the Cuisenaire rods instead of following the lessons. Because of the difficulty of making sure that 19 students are on task at all times and not playing, the experimenters feel that the lessons would have benefited the students more if conducted in small groups.

Finally, when students are taught with manipulatives they must transfer the concrete knowledge (Cuisenaire rods) to abstract knowledge (pencil and paper). This is a very difficult transition for students to grasp and this may have been a factor in why the manipulative group did not score higher than they did. The students did use the
Cuisenaire rods on several of questions that required an explanation. However, they might not have been aware that the Cuisenaire rods could be used to solve traditional types of questions.

This information is important because it shows the that teaching of the transition from concrete to abstract is necessary. It is likely that students will eventually encounter both types of questions and will need the knowledge of how to solve both types of questions. Therefore it is essential that lessons for transition be incorporated into concrete teaching methods.

The third question of the current study asked whether difference in attitudes between the children taught by the traditional method and the children taught by the concrete, hands-on method could be determined. In fact, attitudes of the children taught by the traditional method declined significantly from pre- to post-testing. On the other hand, attitudes of the children taught by the concrete, hands-on method showed no such decrease and, while not significant, increased slightly. While experimenter attitudes may have influenced the results, they are consistent with previous findings by Harrison, Brindley, and Bye (1989).

The authors attribute this finding to several different factors. The Cuisenaire rods were a new experience for the children. At the elementary school level, math is typically taught in the traditional way. The children using manipulatives may have looked forward to a break in the daily routine of seat work with only pencil and paper. The manipulative method allowed the children to become actively involved in the lessons and work thought
problems and solve them right along with the teacher. Students also seemed to enjoy the time that was given to them each day to explore with the Cuisenaire rods on their own or in groups.

On the other hand, in the traditional classroom the chalkboard drills and work sheets closely resembled their daily routine. They seemed to become bored and frustrated when assigned to do work sheets individually at their seats. They seemed to express the most enjoyment when playing the fraction card games in groups.

These results are important for several reason. If an enjoyment of a subject is acquired at an early age then the children are less likely to have the "I can't" attitude that limits their performance and is so difficult to change. Also children are more likely to study mathematics problems on their own when work is required out of class. This positive attitude also affects the attitude of the teacher. The teacher might be more willing to try other hands-on activities in mathematics and in other subjects. More positive attitudes may also lead to continued study of mathematics in high school even after all required courses are taken. This is important because advanced mathematics skills are needed when competing for jobs and places in higher education.

Further research in the area needs to concentrate on well-designed and executed empirical comparisons of methods, with emphasis on providing for transfer of knowledge using one method to test questions which might reflect other levels of understanding.
References


Appendix A
Cuisenaire rods
Appendix B
Attitude Survey¹

¹ Adapted from McKenna, M. C., & Kear, D. J. (1990). Elementary reading survey. The Reading Teacher, 43(9), 630-634.
How do you feel when it is math time at school?

How do you feel about doing math homework?

How do you feel about your math book?

How do you feel about working on math with a friend?
How do you feel about working on math in your spare time?

How do you like working on fractions?

How do you feel about learning new things in math?

How do you feel about doing math instead of playing?
Appendix C
Fraction Test of Knowledge
1. Which part is shaded?

2. Name the fraction shown in this model:

3. \[ \frac{3}{5} + \frac{1}{5} = \]

4. Add the shaded parts and draw your answer in the blank circle.

5. Which of these models shows the fraction \( \frac{1}{3} \)?

6. Which of these models show equal fractions?
7. Circle the smallest fraction: \( \frac{1}{8}, \frac{1}{12}, \frac{1}{2}, \frac{1}{5} \)

8. Explain why \( \frac{1}{2} \) is larger than \( \frac{1}{3} \).

9. Draw two models that both represent the fraction \( \frac{2}{3} \).

10. \( \frac{3}{8} - \frac{1}{8} = \frac{2}{8} \)

11. Circle the larger fraction: \( \frac{1}{3} \) or \( \frac{1}{4} \)

12. \( \frac{4}{12} + \frac{3}{6} = \frac{7}{12} \)

13. Circle the fractions that are equal: \( \frac{2}{4}, \frac{2}{3}, \frac{1}{2}, \frac{1}{3} \)
14. A candy bar was shared equally by 6 people. If each person received one piece, what fraction of the candy bar did each get?

Explain how you got your answer.

15. \[ \frac{2}{5} - \frac{1}{5} = \]

Draw a model of this problem.

16. \[ \frac{4}{6} - \frac{2}{3} = \]

17. Add the shaded parts and draw your answer in the blank rectangle.
18. Pick the fraction that matches the model:

\[
\frac{1}{8} \quad \frac{1}{5} \quad \frac{1}{2} \quad \frac{1}{3}
\]

19. \( \frac{2}{3} - \frac{1}{6} = \) \\

20. \( \frac{1}{2} + \frac{2}{4} = \) \\

Draw a model of this problem.

21. \( \frac{1}{8} + \frac{2}{4} = \) \\

22. Pam and Jane helped their mothers bake brownies. Both girls use the same size pan. Pam cuts her brownies into 8 equal size pieces and Jane cuts her brownies into 9 equal size pieces. Will Pam or Jane get a larger piece of brownie? Explain your answer.
Appendix D
Lesson Plans
Day 1 - Introduction to Fractions

8:15 - 8:25

Introduce ourselves

Introduce why we are here -
   “We are here to introduce you to fractions. We will be here for the 
   next four days. Three days we will teach you about fractions and on the 
   fourth day there will be a test. This test will not effect your grade in 
   math. We are interested in how students learn math. Please give us your 
   full attention and do your best for us. Feel free to ask any questions 
   anytime.”

Attitude Survey
   “Please be honest about your feelings. Do not put your name on the 
   survey. The “smiley” Garfield means - you really enjoy/like. The next 
   Garfield mean - not my favorite. The third Garfield means - not my worst. 
   The last Garfield mean I do not enjoy/dislike.

8:25 - 8:40

Introduction to Fractions

Materials: Four large blue circles with magnetic tape on back and several 
   different pie pieces that will represent fractions of the blue circle (1/2, 
   1/4, 1/3, 1/6, 1/8). The fractions will be orange. Various fraction 
   games, overhead projector, and transparencies.

Objective: 
   At the conclusion of the introductory lesson on fractions, the students 
   will be able to recognize and name the fractions modeled on the board.

Introduction:
   “Where have you seen fractions used?”
   “When have you used fractions?”
   ** The teacher will make a list of 10 of the students responses on the 
   board for each question.
Teach:

**After each question that is asked by the teacher, time will be allowed for the students to answer.**

1. The teacher will start with the blue circles on the board (these represent the whole). "What do the blue circles represent?"
2. The teacher will then model several different fractions and the student will be asked to name the fractions correctly.
3. The teacher will ask if there are any question or if anything needs to be repeated.
4. (The games the students will be playing have been designed to help practice and reinforce the students skills of recognizing and naming fractions.) The teacher will explain each game and how they are to be played. (See attached sheets for games and rules.)
5. The class will be divided into groups and each group will have a game to play.
6. The teacher will walk around the room and help.
Day 2 - Comparing Fractions and Equivalent Fractions

8:15 - 8:20

Review of Mondays Lesson - have students name fractions modeled on the board.

Materials: Blue circles, orange pie pieces, overhead projector, games, transparencies, and handout

Objective: At the conclusion of the lesson on comparing fractions and equivalent fractions, the student will be able to complete the handouts with 80% accuracy or better.

8:20 - 9:15

Introduction: “It is not only important for people to name fractions but it is also important for people to compare fractions.”

“In what ways have you compared fractions?”

**The teacher will make a list of ten of the student responses on the board.

“Will you have more pizza if you have a 1/2 or a 1/3 of the pizza?”

“Will you have more pizza if you have a 1/2 or a 2/4 of the pizza?”

“These last two questions will be answered as we go through todays lesson.”

Teach:

1. “When we write a fraction there is a top and bottom number. The top number is called the ___________? (numerator). The bottom number is called the ___________? (denominator). Keep this in mind.

2. The teacher will model two fractions, using the blue circles and orange pie pieces, that are obviously not equivalent. The students will be asked various question referring to which fraction is more or less than
the other. This step will be repeated several times using different fractions.

3. The teacher will model two fractions, using the blue circles and orange pie pieces, that are not so obviously equivalent, and again ask the students to comment on which one is more or less than the other. This step will be repeated several times using different fractions.

4. The mathematical symbols for “greater than” and “less than” will be covered. The alligator story will be told to help the students remember what each symbol means.

5. The teacher will ask the students if they have formed any theories about comparing fractions. The teacher will record on the board the student responses.

** The teacher is looking for the relationship of the denominators. The teacher wants the students to realize and know that the larger the denominator, the smaller the size of each piece of pie. If the students do not mention this fact on their own the teacher will use some probing questions to see if the students can make the relationship. “Let’s look at the denominators of some of the fractions. Now look at the size of each piece of pie, what do you notice?”

6. The teacher will refer back to the question, “Will you have more pizza if you have 1/2 or 1/3 of the pizza?” The students should be able to state the correct answer (1/2).

7. The teacher will model equivalent fractions. The students will be asked to name the fractions. This step will be repeated several times. The teacher will ask the this question again and wait for the students to answer, “Will you have more pizza if you have 1/2 or 2/4 of the pizza?”

**Practice:**

Handouts (the students will be encouraged to ask and check answers with their neighbors)

Games (available for those who finish early and have completed the handouts with 80% accuracy)
Day 3 - Addition / Subtraction With Common Denominators

8:15 - 8:25

Review comparing fractions and equivalent fractions. Time will be allowed for the students to voice any opinions or questions they have about the lessons so far. The teacher should mention that equivalent fractions will be very important when adding and subtracting fractions.

Materials: Overhead projector, transparencies, blue circles, pie pieces, games, and handouts

Objective:
At the conclusion of the lesson on the addition and subtraction of fractions, the students will be able to complete the handouts with 70% accuracy or better.

8:25 - 8:40

Introduction:
"It is important for us to be able to add and subtract fractions."
"When have you added fractions?"
"When have you subtracted fractions?"
**The teacher will record the students' responses on the board.

Teach:
1. The teacher will explain the concepts of adding fractions together that have a common denominator. The teacher will model the addition of 1/4 + 1/4 with the blue circles and orange pie pieces. The teacher will show that when added together 1/4 + 1/4 = 1/2. Two more example will be done with the circles. The teacher will then do some examples that are written in the abstract form. The students will be asked to answer these examples. The teacher will explain that the concept of adding fractions is similar to that of adding whole numbers.

2. In a similar manner, the subtraction of fractions will be taught. As in whole numbers it is very important that the smaller value be "taken away" from the larger value.
3. The students will be presented with fractions, that do not have common denominators, to add. The students will be told that to add fractions that do not have a common denominator they must make common denominators. (Only if a student asks why will the teacher model pictorially a problem on a transparency for the students.) The teacher will review equivalent fractions. (Some students might find it easier to think about changing unlike denominators to common denominators by looking for equivalent fractions.) The teacher will also stress that multiplication can and will be used most often.

Steps in adding two fractions that do not have a common denominator.
   a. Look for a common factor.
   b. Which denominator(s) need to be changed?
   c. What needs to be multiplied to the denominator(s) so that the problem has common denominators?
   d. Whatever you multiply to the denominator you must multiply to the numerator of that same fraction!!!!
   e. You may now add the fractions.

Several examples will be done with the teacher leading the class.

4. The same steps will be taken when teaching the subtracting of fractions that do not have common denominators. Several examples will be done with the teacher leading the class.

Practice:
   Handouts (the students will be encouraged to ask and check answers with a neighbor)
   Games (available for those who finish early and have completed the handouts with 70% accuracy or better)
Day 1 - Introduction to Fractions

8:15 - 8:25

Introduce ourselves

Introduce why we are here -

"We are here to introduce you to fractions. We will be here for the next four days. Three days we will teach you about fractions and on the fourth day there will be a test. This test will not effect your grade in math. We are interested in how students learn math. Please give us your full attention and do your best for us. Feel free to ask any questions anytime."

Attitude Survey

"Please be honest about your feelings. Do not put your name on the survey. The "smiley" Garfield means - you really enjoy/like. The next Garfield mean - not my favorite. The third Garfield means - not my worst. The last Garfield mean I do not enjoy/dislike.

8:25 - 8:40

Introduction to Fractions

Materials: Four large blue circles with magnetic tape on back and several different pie pieces that will represent fractions of the blue circle (1/2, 1/4, 1/3, 1/6, 1/8). The fractions will be orange. Cuisenaire rods, overhead projector, overhead projector cuisenaire rods, and transparencies.

Objective:

At the conclusion of the introductory lesson on fractions, the students will be able to recognize and name the fractions modeled with cuisenaire rods.

Introduction:

"Where have you seen fractions used?"
"When have you used fractions?"
** The teacher will make a list of 10 of the students responses on the board for each question.

Teach:

**After each question that is asked by the teacher, time will be allowed for the students to answer.

1. The teacher will start with the blue circles on the board (these represent the whole). “What do the blue circles represent?”

2. The teacher will then model several different fractions and the student will be asked to name the fractions correctly.

3. The teacher will ask if there are any question or if anything needs to be repeated.

4. The students will be given the Cuisenaire rods to explore. The teacher will observe the students.

5. The students will share their discoveries with the class.

6. The teacher will model several fractions using the overhead projector cuisenaire rods and the students will be asked to model and name each fraction.
Day 2 - Comparing Fractions and Equivalent Fractions

8:15 - 8:20

Review of Mondays Lesson - have students name fractions modeled on the board.

Materials: Blue circles, orange pie pieces, cuisenaire rods, overhead cuisenaire rods, overhead projector, transparencies, and handouts.

Objective:
At the conclusion of the lesson on comparing fractions and equivalent fractions, the student will be able to complete the handouts with 80% accuracy or better.

8:20 - 9:15

Introduction:
“It is not only important for people to name fractions but it is also important for people to compare fractions.”
“In what ways have you compared fractions?”
**The teacher will make a list of ten of the student responses on the board.
“Will you have more pizza if you have a 1/2 or a 1/3 of the pizza?”
“Will you have more pizza if you have a 1/2 or a 2/4 of the pizza?”
“These last two questions will be answered as we go through todays lesson.”

Teach:
1. “When we write a fraction there is a top and bottom number. The top number is called the _________? (numerator). The bottom number is called the _________? (denominator). Keep this in mind.
2. The teacher will model two fractions, using the overhead cuisenaire rods, that are obviously not equivalent. The students will be asked various question referring to which fraction is more or less than the
other. The students will be given the chance to model the fractions and make new examples that will be shared with the teacher. The students will be asked to explain why their two fractions are not equivalent.

3. The teacher will model two fractions, using the overhead cuisenaire rods, that are not so obviously equivalent, and again ask the students to comment on which one is more or less than the other. The students will be given the chance to model the fractions and make new examples that will be shared with the teacher. The students will be asked to explain why their two fractions are not equivalent.

4. The mathematical symbols for “greater than” and “less than” will be covered. The alligator story will be told to help the students remember what each symbol means.

5. The teacher will ask the students if they have formed any theories about comparing fractions. The teacher will record on the board the student responses.

** The teacher is looking for the relationship of the denominators. The teacher wants the students to realize and know that the larger the denominator, the smaller the size of each piece of pie. If the students do not mention this fact on their own the teacher will use some probing questions to see if the students can make the relationship. “Let’s look at the denominators of some of the fractions. Now look at the size of each piece of pie, what do you notice?”

6. The teacher will refer back to the question, “Will you have more pizza if you have 1/2 or 1/3 of the pizza?” The students should be able to state the correct answer (1/2). The students will model and explain their answer.

** The teacher will record all the different ways that the students modeled 1/2. The teacher will compare the different ways and ask the students if they see any relationship.

7. The teacher will model equivalent fractions. The students will be asked to name the fractions. The students will be asked to model other equivalent fractions and share them with the class. The teacher will ask the this question again and wait for the students to answer, “Will you have more pizza if you have 1/2 or 2/4 of the pizza?” The students will be asked to model and explain their answer.

Practice:

Handouts (the students will be encouraged to model, draw, and explain the answers with their neighbors)
Day 3 - Addition / Subtraction With Common Denominators

8:15 - 8:25

Review comparing fractions and equivalent fractions. Time will be allowed for the students to voice any opinions or questions they have about the lessons so far. The teacher should mention that equivalent fractions will be very important when adding and subtracting fractions.

Materials: Cuisenaire rods, overhead cuisenaire rods, overhead projector, transparencies, blue circles, pie pieces, and handouts

Objective: At the conclusion of the lesson on the addition and subtraction of fractions, the students will be able to complete the handouts with 70% accuracy or better.

8:25 - 9:15

Introduction: “It is important for us to be able to add and subtract fractions.”
“When have you added fractions?”
“When have you subtracted fractions?”
**The teacher will record the students’ responses on the board.

Teach:
1. The teacher will explain the concepts of adding fractions together that have a common denominator. The teacher will model the addition of 1/4 + 1/4 with the overhead cuisenaire rods. The teacher will show that when added together 1/4 + 1/4 = 1/2. The students will be asked to model the examples also. Two more example will be done with the overhead cuisenaire rods. The teacher will then do some examples that are written in the abstract form. The students will be asked to model and explain the answers to these examples. The teacher will explain that the concept of adding fractions is similar to that of adding whole numbers.

2. In a similar manner, the subtraction of fractions will be taught. As in whole numbers it is very important that the smaller value be “taken
away" from the larger value. The students will be asked to model and explain why this fact is true about subtraction.

3. The students will be presented with fractions, that do not have common denominators, to add. The students will be told that to add fractions that do not have a common denominator they must make common denominators. The students will be asked to model why and explain. The teacher will help guide the students to discover this property. The teacher will review equivalent fractions. The students will be encouraged to find a way of changing unlike denominators to common denominators by looking for equivalent fractions. The teacher will ask the students if they find any relationship. If the students do not discover that one can multiply to get common denominators then the teacher will show the students.

Steps in adding two fractions that do not have a common denominator:

a. Look for a common factor.
b. Which denominator(s) need to be changed?
c. What needs to be multiplied to the denominator(s) so that the problem has common denominators?
d. Whatever you multiply to the denominator you must multiply to the numerator of that same fraction!!!(
e. You may now add the fractions.

Several examples will be done with the teacher leading the class and the students modeling (both will be using the cuisenaire rods).

4. The same steps will be taken when teaching the subtracting of fractions that do not have common denominators. Several examples will be done with the teacher leading the class and the students modeling.

Practice:
Handouts (the students will be encouraged to model, draw, an explain the answers with a neighbor)

Review:
Appendix E
Pie-Shaped Fractional Pieces
Appendix F
Worksheet, Traditional Group, Day I

Fill in the blanks.

G shows \(\frac{1}{4}\). ___ shows \(\frac{3}{4}\). ___ shows \(\frac{2}{3}\). ___ shows \(\frac{11}{25}\).

K shows ___. B shows ___. L shows ___. C shows ___.

___ shows \(\frac{0}{8}\). ___ shows \(\frac{5}{10}\). ___ shows \(\frac{8}{8}\). ___ shows \(\frac{3}{8}\).

___, ___ show less than one half shaded.

___, ___ show one half shaded.

___, ___ show more than one half shaded.

___, ___ show one whole unit shaded.
Appendix G
Worksheets, Traditional Group, Day II 3

Shade the squares. Then use $>,$ $=$, or $<$ to make each statement true.

$>$ means "is greater than"

$=$ means "is equal to"

$<$ means "is less than"
\( \frac{1}{2} \) shaded. Fractions equal to \( \frac{1}{2} \).

We can say that: \( \frac{1}{2} = \) \( \) \( \) \( \) \( \) \( \).

\( \frac{3}{4} \) shaded. Shade fractions equal to \( \frac{3}{4} \).

We can say that: \( \frac{3}{4} = \) \( \) \( \) \( \) \( \).

Shade \( \frac{1}{3} \). Shade fractions equal to \( \frac{1}{3} \).

We can say that: \( \frac{1}{3} = \) \( \) \( \) \( \) \( \).
Appendix H
Fraction Card Games
Appendix I
Worksheet, Hands-On Group, Day I*
FROM ... Everything’s Coming Up Fractions with Cuisenaire Rods

RIDDLE: How Many Hamburgers Can You Eat On An Empty Stomach?

The answer to this riddle is written in code at the bottom of the page. To break this code, use rods to work the problems below. If the statement is true, circle the letter in the column labeled true. If the statement is false, circle the letter in the column labeled false. Match the circled letters with the problem numbers to answer the riddle at the bottom of the page.

<table>
<thead>
<tr>
<th>Prob.</th>
<th>True</th>
<th>False</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Y</td>
<td>R</td>
<td>Red is (\frac{1}{3}) of dark green.</td>
</tr>
<tr>
<td>2</td>
<td>D</td>
<td>F</td>
<td>Green is (\frac{1}{3}) of brown.</td>
</tr>
<tr>
<td>3</td>
<td>H</td>
<td>C</td>
<td>Yellow is (\frac{1}{3}) of (orange &amp; purple).</td>
</tr>
<tr>
<td>4</td>
<td>O</td>
<td>I</td>
<td>Purple is (\frac{1}{3}) of (orange &amp; red).</td>
</tr>
<tr>
<td>5</td>
<td>T</td>
<td>H</td>
<td>Dark green is (\frac{1}{3}) of (orange &amp; blue).</td>
</tr>
<tr>
<td>6</td>
<td>N</td>
<td>L</td>
<td>Black is (\frac{1}{3}) of (orange &amp; orange).</td>
</tr>
<tr>
<td>7</td>
<td>T</td>
<td>S</td>
<td>Yellow is (\frac{1}{3}) of (orange &amp; yellow).</td>
</tr>
<tr>
<td>8</td>
<td>M</td>
<td>X</td>
<td>Dark green is (\frac{1}{3}) of (orange &amp; brown).</td>
</tr>
<tr>
<td>9</td>
<td>P</td>
<td>B</td>
<td>White is (\frac{1}{4}) of purple.</td>
</tr>
<tr>
<td>10</td>
<td>R</td>
<td>C</td>
<td>Purple is (\frac{1}{4}) of (orange &amp; dark green).</td>
</tr>
<tr>
<td>11</td>
<td>I</td>
<td>E</td>
<td>Yellow is (\frac{1}{4}) of (orange &amp; orange).</td>
</tr>
<tr>
<td>12</td>
<td>G</td>
<td>S</td>
<td>Black is (\frac{1}{4}) of (orange &amp; orange &amp; blue).</td>
</tr>
<tr>
<td>13</td>
<td>U</td>
<td>J</td>
<td>Dark green is (\frac{1}{4}) of (orange &amp; orange &amp; purple).</td>
</tr>
<tr>
<td>14</td>
<td>R</td>
<td>A</td>
<td>Brown is (\frac{1}{4}) of (orange &amp; orange &amp; orange).</td>
</tr>
<tr>
<td>15</td>
<td>E</td>
<td>I</td>
<td>Black is (\frac{1}{4}) of (orange &amp; orange &amp; brown).</td>
</tr>
<tr>
<td>16</td>
<td>N</td>
<td>Y</td>
<td>Brown is (\frac{1}{4}) of (orange &amp; orange &amp; orange &amp; red).</td>
</tr>
</tbody>
</table>

Riddle Answer

```
\[
\begin{array}{cccc}
\underline{4} & \underline{16} & \underline{6} & \underline{1} \\
\underline{1} & \underline{4} & \underline{13} & \underline{10} \\
\end{array}
\begin{array}{cccc}
\underline{12} & \underline{7} & \underline{4} & \underline{8} \\
\underline{11} & \underline{12} & \underline{16} & \underline{4} \\
\end{array}
\begin{array}{cccc}
\underline{14} & \underline{2} & \underline{7} & \underline{15} \\
\underline{15} & \underline{8} & \underline{9} & \underline{7} \\
\end{array}
\begin{array}{cccc}
\underline{10} & \underline{15} & \underline{14} & \underline{7} \\
\underline{16} & \underline{4} & \underline{7} & \underline{1} \\
\end{array}
\]```
IRB MATERIALS AND PERMISSION FROM COWAN
PROJECT TITLE: Manipulative Versus Traditional Teaching for Math Concepts: Instruction-Testing Match

PROJECT DATES: Begin 11/9/92 End 11/19/92
(Allow 3 weeks for IRB review. Project dates should include ONLY the period of time involving human subjects and begin date shall be AT LEAST 3 weeks after receipt of protocol by the IRB.)

FUNDING SOURCES: Ball State University Honors College Undergraduate Fellowship Program

PRINCIPAL INVESTIGATOR (PI): Maile Keagle DEPARTMENT: Elementary Ed.

CO-PRINCIPAL INVESTIGATOR: Angela Brummett DEPARTMENT: Psych. Science

PI RANK (circle one): Undergraduate Masters Doctoral Faculty Other:

PI ADDRESS INFORMATION: (please include street, city, state & zip)
CAMPUS 3716 N. Tillotson Muncie, IN 47304 Telephone 317-284-5041
HOME 15755 Noonan Rd. Hickory Corners, MI 49060 Telephone 616-671-5614

If PI is a student: (See Back) Faculty Supervisor (please print) Dep. Educational Psychology Faculty Supervisor Signature Date 10/6/92

PI Signature Date

Co-PI Signature Date

REVIEW PROCEDURE REQUESTED:
☑ Exempt Review
☐ Expedited Review (check one)

IRB USE ONLY—BELOW THIS LINE

Received:

Initial Review: Initial Action: Approved Pending
☐ Deferred

DISCUSSION – SPECIFICATIONS:
IRB EXEMPT STATUS CHECKLIST

Principal Investigator: Angela Brummett/Maile Keagle

If you believe your proposed research is in ONE or MORE of the six categories of research which are exempt from the Code of Federal Regulations for the protection of human subjects, indicate the most appropriate category(s) that apply to the proposed project.

EXEMPT CATEGORIES

1. Research conducted in educational settings, involving normal educational practices, such as:
   (a) Research on regular and special education instruction strategies, (or)
   (b) Research on the effectiveness of, or the comparison among, instructional techniques, curricula, or management methods.

2. Research using standardized educational tests (cognitive, diagnostic, aptitude, achievement) and the information gathered will be recorded in such a way that subjects CANNOT be identified either directly or indirectly.

3. Research involving survey or interview procedures, EXCEPT where ALL of the following conditions exist:
   (a) Responses are recorded in such a manner that the subjects can be identified directly or indirectly, and
   (b) The responses, if they become known outside the research, could reasonably place the subject at risk of criminal or civil liability, or be damaging to the subject's financial standing or employability, and
   (c) If the research deals with sensitive aspects of the subject's own behavior, such as illegal conduct, drug or alcohol use, or sexual behavior

* Category 3 does not apply to research where children are subjects.

** 4. Research involving the observation (including observation by participants) of public behavior, EXCEPT where ALL of the conditions listed above in #3 (i.e., 3a,b,c) also exist.

** Category 4 does not apply to research involving children as subjects if the investigator(s) participates in the behavior being observed.

5. Research involving the collection or study of existing data, documents, records, pathological specimens, or diagnostic specimens, and these sources are publicly available; or if the information is recorded, it is recorded by the investigator in such a manner that subjects CANNOT be identified directly or indirectly.

6. Research involving a category specifically added to this list by the Department of Health and Human Services and published in the Federal Register.

In signing this exemption form, the principal investigator agrees that the category(s) checked above do strictly apply to the proposed research.

Maile L. Keagle
Angela J. Brummett

P.I. Signature

Date
This study is interested in investigating different teaching and assessment methods for the concept of fractions. The subjects for this study will be three fourth grade classes at Cowan Elementary School in Muncie, Indiana.

Angela Brummett and Maile Keagle will be teaching a lesson in two of the classrooms for approximately one hour on each of three days. The lessons will center around the concept of fractions. The first group will be taught using "traditional" methods such as rote memorization and practice drills using worksheets. The second group will be taught using a "hands-on" method. These children will be taught and encouraged to use Cuisenaire Rods, a teaching aid that allows children to manipulate and discover relationships between fractions.

On a fourth day both classrooms will be given the same test. This test will include items that simply ask for an answer to fraction problems, and also items that require application of the knowledge that the children have learned during the lesson. The second type of items will also require the children to explain how they arrived at each answer. It is hypothesized that both groups of children will do well on the first type of item, but children taught using the hands-on method will score higher on the second type of item. A third classroom that has not received any type of lesson will also be given the test for comparison purposes.

The children that participate in this study will not be required to place their names on the tests, and all data will remain anonymous.
Angela Brummett and Maile Keagle, under the supervision of Dr. Betty Gridley, have the permission to carry out the study described on the attached page at Cowan Elementary School. Plans for this study have been discussed with all staff members involved in the project. All lesson plans and testing materials will be seen and approved by the appropriate staff members before any portion of the project is started.

Signed
Position

Signed
Position

Signed
Position

Signed
Position
TO: Maile Keagle and Angela Brummett  
3716 N. Tillotson  
Muncie, IN 47304

FROM: Elizabeth Glenn, Chair  
Institutional Review Board

DATE: October 8, 1992

RE: Human Subjects Protocol I.D. - #IRB 93-60

Your protocol entitled "Manipulative Versus Traditional Teaching for Math Concepts..." has recently been approved as an exempt study by the Institutional Review Board. Such approval is in force during the project dates 11/9/92 to 11/19/92.

It is the responsibility of the P.I. and/or faculty supervisor to inform the IRB:

- when the project is completed, or
- if the project is to be extended beyond the approved end date,
- if the project is modified,
- if the project encounters problems,
- if the project is discontinued.

Any of the above notifications should be addressed in writing to the Institutional Review Board, c/o the Office of Academic Research & Sponsored Programs (2100 Riverside Avenue). Please reference the above identification number in any communication to the IRB regarding this project. Be sure to allow sufficient time for extended approvals.

slm

pc: Betty Gridley