THE INSTITUTE OF ACTUARIES

AN HONORS THESIS
SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
for the degree
HONORS BACHELOR OF SCIENCE
by
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BALL STATE UNIVERSITY
MUNCIE, INDIANA
FEBRUARY, 1976
The arms of the Institute of Actuaries
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I. INTRODUCTION

As an actuarial science student at Ball State University diverting my attentions to a more culturally oriented endeavor at that University's London Centre, I was confronted with the unique opportunity to study a kindred society of my chosen profession. The spirit of cooperation at the Institute was exceptional and left me with an appreciation of the venerable ways of the English that I will retain long after my stay in London. I was also impressed by the fraternal atmosphere that seemed to exist in stately old Staple Inn.

Observing the quiet dignity and proud traditions of the Institute, it becomes obvious that it provides an excellent atmosphere for an aspiring student. It is this tradition that facilitates the purpose of the Institute of Actuaries, that being: to elevate the status of the profession and provide for the instruction of its students.
II. PURPOSE

The Institute of Actuaries is an interesting organization with a long proud history. The purpose of this paper is to provide a basic understanding of its history and functions. In attempting to do this, several specific areas of interest have been isolated. These areas include the history of the Institute, its place of residence, its structure, its members and finally some of the ties that exist between it and the Society of Actuaries in the United States.
III. A BRIEF HISTORY OF THE INSTITUTE

In the early days of the actuarial profession, it became increasingly apparent that the formation of an organization in which actuaries could meet and confer would be extremely beneficial to the advancement of the profession. An association of managers of life assurance companies had existed in Edinburgh for some time prior to these initial efforts to form an actuarial society in 1848. These initial efforts were immediately confronted by two conflicting schools of thought. A relatively small, but influential, group of actuaries thought their purposes would best be served by an association, the "Actuaries Club," of managers to discuss professional problems. The larger group envisioned a more comprehensive association. This association would not only provide an opportunity for discussion, but also serve to elevate the status of the profession and facilitate the instruction of students entering it. The majority prevailed and thus, since its inception, July 8, 1848, the Institute has placed great emphasis on its character as a learned society. (1,11)

The first president of the Institute, Mr. John Finlaison, was elected on November 11, 1848. (2,3) Finlaison was in the twilight of an illustrious career. He was the keeper of the Admiralty Records from 1809 until 1822 and Actuary of the National Debt from 1822 until 1851. He compiled the first Navy list in 1814. His report in 1929 resulted in the Northampton Table being superseded by the Finlaison Table for
the Sale of Annuities by the Government. One morning in 1831 he was having breakfast with Augustus de Morgan, who happened to have a copy of the experience of the Royal Assurance Corporation. Before breakfast, Finlaison compiled the first mortality table. (3,6)

From its inception the Institute, which originally numbered around one hundred and thirty members, began to carry out the activities which are familiar to present members. To ensure the high academic standards of the Institute, a system of examinations for prospective members was devised and a library was established. Meetings were also begun for the discussion of business problems. In 1850 the Assurance Magazine was officially adopted as the Journal of the Institute of Actuaries. It was also during this period that a large number of the Scottish members of the Institute resigned and apparently formed the Faculty of Actuaries in Scotland.

The first home of the Institute was at No. 12 Chatam Place Blackfriars. In 1852, it was decided to seek refuge with the Statistical Society of London. This alliance continued at three addresses until the Institute moved to Staple Hall Inn in 1887. This seems to have been a fortunate decision as it has remained at that same address for eighty-nine years. (1,12)

In 1881, in order to enhance the dignity and authority of the profession, the Institute merged with the renegade Actuaries' Club and was granted a Royal Charter of Incorporation. The Actuaries Club has developed into a social club within the Institute that still meets after every council meeting. (1,13)
The years between the late 1880's and World War I were good ones for the Institute as it prospered and enjoyed the installation of such new wonders as electric lights and telephones in Staple Inn. Then World War I interrupted the prosperity as eighty-two members of the Institute were lost in the tragic war. (2,135-170)

The Institute of Actuaries Student Society was formed in 1910 and the first woman was admitted to the Institute in 1920.

In 1937 the Actuarial Tuition Service was inaugurated. This is a joint organization of the Institute and the Faculty of Actuaries in Scotland which provides tuition for the main examinations of both bodies.

The history of the Institute was again interrupted by World War II. Forty-two members were killed and in August of 1944, Staple Hall was destroyed by a German fire bomb. (4,22)

The examinations were revised in 1949, 1959 and 1966 in order to broaden the scope and enhance the quality of training. The Institute has also established a unified set of texts and continues to encourage a professional and academic attitude. (1,13)
IV. STAPLE INN

A visitor to the Institute inevitably must fall prey to the mystique of its home, Staple Hall Inn. It has changed little since a nineteenth century visitor described it in the following manner:

A wanderer in Holborn, who strayed through the arch into the courts behind, would find there a pleasant quiet quadrangle where the noise of the traffic was hushed. Turning into it out of the clashing streets "imparts to the relieved pedestrian the sensation of having put cotton in his ears and velvet soles on his boots. It is one of those nooks where a few smokey sparrows twitter in smokey trees, as though they called to another, let us play in the country."

So Charles Dickens described the scene in *Edwin Drood*. After setting this scene at Staple Inn, he goes on to describe it on a December afternoon.

Neither wind nor sun, however, favored Staple Inn one December afternoon towards six o'clock, when it was filled with fog and candles shed murky and blurred rays through the windows of all its then occupied sets of chambers; notably from a set of chambers in a corner house in the inner quadrangle, presenting in black and white over its ugly portal the mysterious inscription:

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P J T
I747
```
in which set of chambers, never worrying about the inscription, unless to bethink himself at odd times on glancing up at it, that haply it might mean perhaps John Thomas or perhaps Joe Tyler, sat Mr. Grewgious writing by his fire.

Another author, Nathaniel Hawthorne, remembered the Inn on a bright summer day.

In Holborn still, I went through another arched entrance . . . and here likewise seemed to be offices; but in a court opening inward from this, there was a surrounding seclusion of quiet dwelling-houses, with beautiful green shrubbery and grass plots in the court, and a great many summer flowers in full bloom. The windows were open; it was a lovely summer afternoon, and I have a sense that bees were humming in the court, though this may have been suggested by my fancy, because the sound would have been so well suited to the scene . . . In all the hundreds of years since London was built, it has not been able to sweep its rolling tide over that little island of quiet. (4,20)

The peaceful court and stately buildings of the Inn are the product of a long and curious history dating back to the first activity at the site of the Inn in the late thirteenth century. At this time the first building, probably a wool staple or market, hence the name, was erected. Some of the original stain glass from this building is now displayed in the Inn. (4,8)

Some time around the year 1400, Staple Inn began to be used by lawyers rather than merchants. Within a span of fifty years, the Inn had become well established as an Inn of Chancery. Sir John Fortescue, Lord Chief Justice and afterwards Lord Chancellor to Henry VI, refers to the Inns of Chancery in De Laudibus Legum Angliae (1468-71) "... there are . . . ten lesser inns, and sometimes more, which are called Inns of Chancery." The Inns of Chancery were used to train young lawyers before they entered the Inns of Court. An entire society of well to do merchants and lawyers lived within the confines of the hall and
in 1580 the Fellows of the Society of Staple Inn financed the construction of a new hall. It was during this period that the Inn enjoyed its greatest prosperity and much of the stained glass in the Inn today dates from these times. (4,9-16)

Due to a restructuring of the legal education system in the early nineteenth century, the association between that profession and the Inn was dissolved. One by one the Inns of Chancery disappeared and their sites were put to more profitable uses. Eventually the Society of Staple Inn sold its inheritance to a building firm. This caused great controversy as many feared that one of the few remaining survivals of old London would disappear, but it was purchased by the Prudential Assurance Co. with the public spirited intention of preserving this unique relic.

Staple Inn was then restored and leased to the Institute of Actuaries. "As the President of the Institute said at the time, the public have been no losers by the change of proprietorship. The hall has become the centre of a branch of scientific education and the Inn as a whole has had the benefit of 'active and intelligent government.'" (4,20)

There the Institute remained, the dignity of the profession and the proud history of the Inn complementing each other perfectly, for fifty-seven years. Then, on a warm August night in 1944, a flying bomb destroyed large portions of Staple Hall.

"For eleven years the Institute was without a home, though with kind friends who lent accommodation. Then in 1955, the Institute returned to the rebuilt Staple Inn, with premises which matched the beauty of the earlier Inn."
Using the same design with old timbers and panelling the hall was reproduced. Fortunately, as in the great cathedrals of London, the stained glass windows had been removed for safe keeping and are now displayed in the new hall. The Institute also received many furnishings from kindred societies. (4,23)
V. THE STRUCTURE OF THE INSTITUTE

The management of the affairs of the Institute is carried out by a council of thirty Fellows which is subject to the control of the General Meetings of the Institute. Five new members are elected to this committee each year at the General Meeting. A President, not more than four Vice-Presidents, a Treasurer and at least two Honorary Secretaries are chosen annually by the Council. All of these honorary officers must be chosen from the council except the President who may also be a former council member.

Because there is much detailed work to do, extensive use of the committee system is employed. These committees are chaired by a Vice-President or some other senior member or former member of the council. The seventeen committees are: Dinners, Education, Finance, Financial Standards of Long Term Insurance Business, General Purposes, International Relations, Legislation, Library, Manpower, Professional Conduct Investigation, Professional Guidance, Public Relations, Recruitment, Research, Review, Scope of the Profession and Sessional Meetings. There are also four joint committees with the Faculty of Actuaries, these are: Actuarial Tuition Service, Consulting Practice, Research and Text Books. There are six basic classifications for members of the Institute. To be classified as a Fellow of the Institute (F.I.A.) an applicant must be at least twenty-three and must pass the Intermediate Examination and one set of subjects of the Final Examination. To be an Associate
of the Institute (A.I.A.) an applicant must be twenty-one years of age and have passed all subjects of the Intermediate Examination. The status of Associate is automatically awarded any Fellow in the Society of Actuaries or the Faculty of Actuaries. A Student must have a university degree or a grade of B in the English equivalent to an advanced high school mathematics course. The other three titles, Honorary Member, Honorary Overseas Member and Affiliate Member, are awarded by the Council when they deem it appropriate.

Ordinary General Meetings are held on the fourth Monday of each month from October to April except December. The main purpose of these meetings is to discuss business problems. The Annual General Meeting is held in June. At this meeting committee reports are read and officers are elected. (1,15-22,69-72)
VI. AN ACTUARY IN ENGLAND

Though the application of mathematics entails a very wide spectrum, life assurance is still the principal sphere of occupation of actuaries in Great Briton. Some sixty percent of actuaries in the United Kingdom are employed by life assurance companies. Fellows of the Institute are involved in all phases of these companies, but, of course, most are engaged in the actuarial work which is the basis for the business. Actuaries are also playing an increasingly important role in fire, accident and auto insurance.

Approximately ten percent of all actuaries are in consulting practice. In this capacity actuaries are often required to investigate a wide variety of business problems.

A further five percent of the Fellows of the Institute are employed in government service, the majority in the Government Actuary's Department. This department is responsible for providing actuarial advice to any government department requiring it and is particularly concerned with the finances of the various social insurance schemes and of pension schemes in the public service. Other actuaries occupy directing posts in the statistical branches of government departments.

Another five percent are on the London Stock Exchange or are employed by Stock Exchange firms. The work involves investment analysis and the control and organization of research departments whose activities range from visiting companies for firsthand information, to
the application of computer techniques, to investment statistics and forecasts.

About six percent of the Fellows are engaged directly in industry and commerce. Some manage pension funds or investments for large corporations, some are in merchant banks and others are concerned with computer development. A growing number are engaged in operational research. (5,8-9)

The final category of actuarial employment is that of education. Six actuaries are university professors and one is a Professor of Actuarial Science at the City University in London. This is quite a part from the teaching that is conducted on a part time basis within the Institute or Faculty. A considerable part of this teaching is likely to pass to the university in the future and it is probable that more actuaries will be attracted to academic careers. (1,10)
VII. SOME TIES WITH THE SOCIETY

Because this paper is a study of the Institute as it compares to its American counterpart, the Society of Actuaries, this section is devoted to the interactions between the two.

According to Dr. John Beekman, a professor of Mathematical Sciences at Ball State University and an Associate of the Society, until the early 1950's, English texts were being used to prepare for some of the American actuarial examinations. New Institute textbooks are used as aids in lecture preparation by some American professors.

In 1939, during the Jubilee of the Actuarial Society of America, the Institute donated a complete bound set of the Journal of the Institute of Actuaries.

During the last ten years, the Society has sponsored research conferences at many Universities including Michigan, Yale, Harvard, Brown and others and English actuaries have attended some of these. The Journal of the Institute of Actuaries is sometimes studied by Americans of that profession, and the Transactions of the Society of Actuaries is studied by some of their English counterparts.

The gesture which perhaps is most indicative of the friendly relations between the two organizations dates back to the German bombing of World War II. The members of the Society offered to house the children of the Institute's members until the danger passed. In return the Institute's members took in the relatives of American actuaries that were stationed in England.
## A SYLLABUS OF EXAMINATIONS FOR THE SOCIETY

### Associate Examinations

<table>
<thead>
<tr>
<th>Part</th>
<th>Time Allowed</th>
<th>Subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Three hours</td>
<td>General mathematics</td>
</tr>
<tr>
<td>2</td>
<td>Three hours</td>
<td>Probability and statistics</td>
</tr>
</tbody>
</table>
| 3    | Three hours  | a) Numerical analysis  
b) Theory of interest |
| 4    | Five hours   | Life contingencies |
| 5    | Five hours   | a) Demography  
b) Principles underlying the construction of mortality and other tables  
c) Elements of graduation of mortality tables and other series  
d) The sources and characteristics of the principal mortality and disability tables (including the methods used in their construction and graduation) and of the principal mortality and disability investigations  
e) Risk theory |

### Fellowship Examinations

<table>
<thead>
<tr>
<th>Part</th>
<th>Time Allowed</th>
<th>Subjects</th>
</tr>
</thead>
</table>
| 6    | Six hours    | a) Life, Health, and Pension Coverages (Basic)  
b) Life, Health, and Pension Coverages (Advanced)  
c) Marketing of Insurance Products  
d) Selection of Risks  
e) Life, Health, and Pension Coverages (Advanced) |
| 7    | Six hours    | a) Investment of Life Insurance and Pension Funds and Valuation of Assets (Basic)  
b) Valuation of Liabilities (Basic)  
c) Investment of Life Insurance and Pension Funds and Valuation of Assets (Advanced)  
d) Valuation of Liabilities (Advanced) |
<table>
<thead>
<tr>
<th>Part</th>
<th>Time Allowed</th>
<th>Subjects</th>
</tr>
</thead>
</table>
| 8    | Six hours    | a) Gross Premiums, Distribution of Surplus, and Pension Funding (Basic)  
b) Contract Values and Changes (Basic)  
c) Expense Analysis  
d) Gross Premiums, Distribution of Surplus, and Pension Funding (Advanced)  
e) Contract Values and Changes (Advanced) |
| 9    | Six hours    | a) Social Insurance (Basic)  
b) Life and Health Insurance Accounting  
c) Life Insurance Law  
d) Life Insurance Taxation  
e) Social Insurance (Advanced) |
A SYLLABUS OF THE EXAMINATIONS FOR THE INSTITUTE

Intermediate Examination

Subject (1)—Mathematics
One Paper (3 hours)

Subject (2)—Probability and Elementary Statistics
One Paper (3 hours)


Subject (3)—Compound Interest
One Paper (3 hours)

Subject (4)—Life and Other Contingencies
Two Papers (3 hours each)
Construction of mortality, sickness, multiple-decrement and other similar tables from graduated data; determination and use of the functions based thereon. Values of and premiums for annuities and assurances on one or more lives. Values of and contributions for sickness benefits, pension benefits, disability and widows' and orphans' benefits.

Subject (5)—Statistics
One Paper (3 hours)
Subject (6)—The Analysis of Mortality and Other Actuarial and Demographic Statistics
One Paper (3 hours)

Subject (7)—Economic Background to Finance and Investment
One Paper (3 hours)

Final Examination

Subject (la)—Finance and Investment (Less Advanced Level)
Two Papers (3 hours each)
Investment policy for insurance companies, pension funds and other investors. Types of investment including gilt-edged stocks, debentures, preference shares, ordinary shares, mortgages, property, life interests and reversions. Analysis and selection of investments. The broad principles of the supervision and administration of investments. Economic and other factors affecting rates of interest.

Subject (lb)—Finance and Investment (More Advanced Level)
Two Papers (3 hours each)
(a) The making of investment policy to meet the needs of the investor, whether individual or corporate. A general knowledge of United Kingdom legislation and taxation, as affecting various types of investor, will be required.
(b) The appraisal and comparison of fixed interest securities.
(c) Investment analysis, including the scrutiny of the trend of relevant economic and other statistics in order to estimate the probable future progress of a particular ordinary share.
(d) The consideration of the investment overseas of United Kingdom funds. Candidates should be able to illustrate general principles by particular reference to investment in the United States and Australia.
(e) Investment in and on the security of property in the United Kingdom.
(f) Investment in and on the security of life interests and reversions.
(g) The valuation of investments.
(h) Economic background and sources of information in those aspects which bear directly upon investment.

(i) Ancillary problems of investment, including the underwriting of new issues and the organization of an investment department.

Subject (2a)—Life Offices—Ordinary and Industrial (Less Advanced Level)

Two Papers (3 hours each)

(a) Office rates of premium for assurances and annuities depending on life and other contingencies, surrender values of life policies and other contracts.

(b) General principles of valuation of the liabilities and assets of life assurance companies and the analysis and distribution of surplus. Elementary methods of recording and grouping annuities and insurances for valuation, and of calculating the liabilities therefor.

(c) Investment of life assurance and annuity funds; tax as affecting such funds.

(d) General provisions of British legislation relating to life assurance companies.

Subject (2b)—Life Offices—Ordinary and Industrial (More Advanced Level)

Two Papers (3 hours each)

A more advanced and extended treatment of the subjects comprising the Syllabus for Subject (2a).

Subject (3a)—Pension Funds and Friendly Societies (Less Advanced Level)

Two Papers (3 hours each)

(a) Rates of contribution for retirement benefits, widows' and orphans' benefits, sickness benefits and death benefits.

(b) General principles of valuation and the analysis and treatment of surplus or deficiency.

(c) Investments of pension and widows' and orphans' funds; tax as affecting such funds; investments of friendly societies.

(d) General provisions of British legislation relating to pension and widows' and orphans' funds, and to friendly societies.

(e) Actuarial and demographic aspects of national insurance schemes with special reference to the British scheme; the integration of national insurance benefits with those of private funds.

Subject (3b)—Pension Funds and Friendly Societies (More Advanced Level)

Two Papers (3 hours each)

A more advanced and extended treatment of the subjects comprising the Syllabus for Subject (3a).

Subject (4)—More Advanced Statistics and Operational Research

Two Papers (3 hours each)

(a) Direct probability distributions. Moments and cumulants; characteristic functions. The theory of sampling and sampling distributions. Estimation, significance and hypothesis testing, including curve-fitting and graduation tests. Correlation, regression and analysis of variance. Elements of decision theory. Practical applications of statistical processes.
IX. INTERMEDIATE SUBJECT I

Time allowed—3 hours

The following statements may be used in writing ALGOL programs:

identifier ; = read;
print (expression);

The number of marks allocated is shown at the end of each question.

1. $f(x)$ is a function of the fourth degree in $x$, and a student has calculated and tabulated the following values of $f(x)$:

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1.5738</td>
<td>1.5828</td>
<td>1.5991</td>
<td>1.6200</td>
<td>1.6490</td>
<td>1.6858</td>
<td>1.7313</td>
<td>1.7866</td>
</tr>
</tbody>
</table>

He has formed a difference table and found that the fifth differences are not zero. Given that he made only one error in his calculations,

(i) find and correct the error,
(ii) write an ALGOL program to compute and print the correct values of $f(x)$ for $x = 1(0.01)5$ and $x = 6(0.02)8$.

2. Derive the three-point Gauss integration formula

$$
\int_{-1}^{+1} f(x) dx = \frac{1}{3} \left[ 5f(-\sqrt{3}) + 8f(0) + 5f(\sqrt{3}) \right]
$$

3. The radiation intensity $I$ from a radioactive body at time $t$ is given by the formula $I = I_0 e^{-at}$, where $I_0$ and $a$ are constants. $n$ values of $I$ are observed at times $t_1, t_2, \ldots, t_n$. Use the method of least squares to obtain formulae for $I_0$ and $a$.

4. One method of sorting a set of $n$ numbers, $a_1, a_2, \ldots, a_n$, into descending order of size is as follows:

(a) the first number in the set, $a_1$, is compared in turn with succeeding numbers until a larger number, $a_t$, is found; $a_t$ is then interchanged with $a_1$, resulting in a new first number $a'_1$. The new first number, $a'_1$, is then compared in turn with the remaining numbers in the set starting with $a_{t+1}$ and, if a larger number, $a_u$, is found, $a_u$ and $a'_1$ are interchanged. This process continues similarly until all numbers up to and
including \( a_n \) have been brought into comparison with the first number as amended from time to time, and any necessary interchanges of numbers have been made, so that the resulting first number is the largest in the set.

(b) When (a) has been completed, the same method of comparison is used between the second number and the third and succeeding numbers, and any necessary interchanges of numbers are made, so that the resulting second number is the second largest in the set.

(c) When (b) has been completed, the process is repeated in turn with the third and subsequent numbers, until the whole set is in descending order of size.

Write an ALGOL program to sort a set of \( n \) different numbers into descending order of size. Use the method of sorting described above.

(Using FORTRAN)

```fortran
DO 30 NCOUNT = 1, N
   ABIG = A (1)
   IPT = 1
   DO 20 I = 2, N
      IF (ABIG .GT. A(I)) GO TO 20
         ABIG = A(I)
         IPT = I
   20 CONTINUE
WRITE (6, 103) NCOUNT, ABIG
103 FORMAT (' ', 13, F10.2)
   A (IPT) = 0.0
30 CONTINUE
```

5 If \( \frac{dy}{dx} = y + x^2 \) and \( y = 1 \) when \( x = 0 \), use a Runge-Kutta 4th Order method to find \( y \) to three decimal places when \( x = .06 \).
X. INTERMEDIATE SUBJECT II

Time allowed—3 hours

The number of marks allocated is shown at the end of each question.

1. The game of "Spoof" is played as follows:
Each player holds \( x \) coins in his clenched fist, where \( 0 \leq x \leq 4 \). The probability that a player holds \( x \) coins is \( p_x \) and is known by all the players. Each player in turn has one guess as to the total number of coins held by all the players and states his guess to the other players. A player is not allowed to guess a number which has already been guessed by a previous player. When every player has had his turn, each player opens his hand so that the winning player, if any, can be determined.

   (i) When the number of players is three and \( p_x \) is one-fifth for each value of \( x \), calculate

   (a) the expected total number of coins held by all the players,

   and

   (b) the most likely total number of coins held by all the players.

   (ii) In the general case, where \( p_x \) is different for each value of \( x \) and where there are \( n \) players, state, with reasons, whether the first player's guess should be based on the expected or the most likely total number of coins held by all the remaining players, if he wishes to maximize his chance of guessing correctly. State briefly why it may not be true that the first player's chance of guessing correctly is greater than that of any other player.

\[
\begin{align*}
\text{f}(x) &= \text{probability that there are } x \text{ coins} \\
(1) \text{a.) } E[x] &= \sum_{x=1}^{12} x \cdot f(x) = \\
&= \frac{1(3) + 2(6) + 3(10) + 4(15) + 5(18) + 6(19) + 7(18) + 8(15) + 9(10) + 10(6) + 11(3) + 12(1)}{125} \\
&= \frac{750}{125} = 6
\end{align*}
\]
b.) The most likely total number of coins is the one with the largest probability \( f(x) \).

\[
f(6) = \frac{19}{125}
\]

The expected value and most likely total are the same only because the function is symmetric about the mean.

(ii) To maximize the chances of guessing correctly, a player must consider the number of coins in his hand and add the expected value of the other players' total to it. In doing this he arrives at a more accurate estimate but he is also lending the other players information as to the number of coins in his hand. Thus he is exchanging information for the right to guess first.

2. Two men, A and B, meet every day at the local inn for lunch between 12.00 noon and 12.10 p.m. Each man is equally likely to arrive at any moment in the ten minute interval. Given that on a particular day A arrived first and B arrived within five minutes of A, what is the probability that A arrived before 12.03 p.m.?\[\text{\( \frac{1}{5} \)}\]

3. (i) Define, and discuss the relative advantages of, each of the following measures of location:

(a) arithmetic mean,
(b) median,
(c) mode.

(ii) A precision engineering firm occasionally purchases quantities of ball bearings in two sizes. Two companies A and B each manufacture both sizes, but in the past those of diameter 1 centimetre have been purchased from company A and those of diameter 3 centimetres have been purchased from company B. The actual diameters of a sample of 10 from each company are as follows:

<table>
<thead>
<tr>
<th>Diameter in centimetres</th>
</tr>
</thead>
<tbody>
<tr>
<td>A 1.05 1.02 1.00 0.96 0.96 0.99 1.01 1.04 0.97 0.97</td>
</tr>
<tr>
<td>B 3.10 3.00 3.00 2.92 3.07 2.95 2.98 3.02 3.03 2.90</td>
</tr>
</tbody>
</table>

Calculate, for each sample, the mean and an appropriate measure of dispersion, and explain to what extent they could be used to help the firm decide whether there is a case for purchasing all ball bearings from one company in future.
A large company operates regular training courses for its graduate employees. At the end of each course it holds a series of five tests which are taken in ascending order of difficulty. If a candidate fails a test he cannot go on to take the others. Experience has shown that the probability that a candidate who takes a test passes it is one-half.

The company awards a cash bonus for each test passed, starting at 10 for the first test, 20 for the second test, and so on up to 50 for the fifth test.

(i) What is the probability that at the end of a course on which there are 10 candidates the total company payout in bonuses will be less than 50?

(ii) In the case of n candidates, describe briefly how you would calculate the probabilities of all the various total payouts, using simulation techniques with the aid of a computer. You are not required to write a computer program.

5 (i) Explain the reasoning underlying the derivation of the constants b and c in the regression equation

\[ y = bx + c \]

(ii) What is the main limitation in the technique of estimating values of related variables from a regression equation of this type, and how can the extent of the limitation be determined in a particular case? (16)

(1,90-92)
XI. SUMMARY

Though the tests of the Institute ask several detailed questions rather than the many multiple choice questions in the Society's first examinations, their subject matter and purpose are quite similar.

Through their various programs, the Institute and Society have been vital to the development of the profession in their respective countries. The Institute of Actuaries seems to be an institution that transcends time in its ability to serve the profession and its illustrious history is overshadowed only by the promise of its future.
BIBLIOGRAPHY


