Where Learning and Teaching Meet: Heuristics and the Information Processing Model

An Honors Thesis (HONRS 499)

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Muncie, Indiana
May 1991

May 5, 1991 Graduate
Introduction

Learning and instruction are quite distinct activities for most of us: we easily recognize when we are functioning as a student, or "learner;" likewise, we recognize when a person is functioning as a teacher, or "instructor." These two activities are closely interrelated. However, most of us have had the experience of being able to learn easily from the instruction we were given, as well as not being able to learn from the instruction. If we have had the responsibility for instructing, we realize even more that the boundary between learning and instructing is somewhat blurred. Still, the two activities seem to be quite distinguishable. This has raised an interesting question in my mind: are there better ways to teach? Rather, is there research on ways to teach that suggests insights into how learning and instructing are related?

The Information Processing Model

Theories of learning describe the way that individuals learn, including how they gain knowledge, comprehension or skill. Learning also involves mastery of a subject or topic by fixing it in the mind or memory through experience or study. The research that I will discuss relates primarily to the development of learning in students of elementary or secondary school levels. Theories of instruction, on the other hand, deal with the methods used to produce learning. Defined as imparting knowledge, instruction involves techniques to transfer the material in a way that an individual can attempt to gain mastery of it. The transfer of material through instruction is usually performed by teachers. These two different theories, though appearing to be reciprocal in nature, are presented in research as two separate entities. John Woodward reinforces this point in "Procedural Knowledge in Mathematics: The Role of the Curriculum,"
saying, "traditionally, theories of learning and instruction have been distinct and at times, estranged disciplines."¹ Woodward and others have developed one model of cognition that has worked toward bridging the gap between these two areas of education. Information processing, a theory of learning, provides a basis for the use of certain instructional methods and reinforces certain teaching techniques in order to connect learning by students with instruction by teachers. I will first give a description of information processing, and later present examples of mathematical topics to which I will apply the theory. This may suggest some methods and techniques that teachers can use to further develop learning as represented by the information processing model.

Information processing takes the view that knowledge is the basis of action. A goal of information processing, therefore, is to identify what has to be learned in order to produce a specified performance. Information processing evaluates how individuals have learned by evaluating how they perform. "The goal of an information processing analysis of any task or knowledge domain is to specify the knowledge and processing activities that underlie performance," states James W. Pellegrino and Susan R. Goldman in "Information Processing and Elementary Mathematics."²

One area of learning where information processing can be applied is mathematics. The information processing model says that performance in mathematics is based on two basic types of mathematical knowledge, declarative and procedural. Declarative knowledge is an "interrelated network" containing the basic facts, and is considered to be the knowledge of things that are true or

¹Woodward, p. 242.
²Pellegrino and Goldman, p. 24.
false. In other words, it is basic factual knowledge that can be verbalized. However it can be "compiled" or abbreviated into larger units. For example, we know that two plus two is four and two plus five is seven from memory, which is declarative knowledge. By compiling, or combining, or organizing these units or facts, — these elements of declarative knowledge — we could derive that twelve plus nine is twenty-one. But this knowledge, for a student in the primary grades, would not be declarative knowledge. Most of the mathematics taught in the primary grades, such as addition, subtraction, multiplication, and division facts of single-digit numbers, is declarative knowledge because it comes strictly from memorization. This declarative knowledge could be thought of as rote knowledge. The teacher's goal at this level is for students to be able to give the answer directly from memory without applying any procedure to get the answer.

Algorithms and Rote Knowledge

Procedural knowledge, or knowledge of how to do something, on the other hand, includes the methods that can be used to derive answers for problems lacking prestored answers. This type of knowledge involves applying some process — some aspect of procedural knowledge — to a set of conditions, including facts or declarative knowledge, to arrive at the desired answer. Thus, obtaining twenty-one as the answer to twelve plus nine in the example above is procedural knowledge. A student who had not yet achieved this level would have to apply his declarative knowledge and make generalizations about

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3 Pellegrino and Goldman, p. 24.
5 Pellegrino and Goldman, p. 24.
single-digit addition to work this problem. Some facts, such as multiplication of double-digit numbers, can be answered by most elementary level students even though the answers might not be stored in direct memory. Obtaining these answers may require the application of some algorithm, indicating procedural knowledge. In fact, very often procedural knowledge is like an algorithm. In an algorithm, each step is very specific, the next step to be performed is very clear, there is an outcome, and there is an ending or stopping point. The four criteria of an algorithm are frequently present in procedural knowledge in mathematics.

The initial objective and goal of most educators is to teach students how to apply their declarative knowledge — usually what they have memorized — to their procedural knowledge — algorithms they are learning. Students, though, with practice and experience should be able to transfer procedural knowledge back into declarative knowledge. For example, when learning to add two columns of numbers, a student is acquiring procedural knowledge. However, with practice and as his skill develops, this procedure becomes "automatic" — almost like a reflex — and would become more like declarative knowledge. Thus, declarative knowledge is incorporated into procedural knowledge at first, and then this procedural knowledge might later become declarative knowledge with practice or repeated use.

The Executive Function

There are several different names for the stage or level that develops declarative into procedural knowledge in information processing: the "executive function" or "strategic thinking"⁶, or "knowledge compilation."⁷

Knowledge compilation is the process by which the skill transits from the declarative stage to the procedural stage. It consists of the subprocesses of composition, which collapses sequences of productions into single productions, and proceduralization, which embeds factual knowledge into productions. Once proceduralized, further learning processes operate on the skill to make the productions more selective in their range of applications. These processes include generalization, discrimination, and strengthening of productions.8

Acquisition of knowledge has stages of development — producing more and more complex "productions." Declarative knowledge takes place first, in most cases. The executive function then functions when students apply this first knowledge by making generalizations and specifications about it to develop procedural knowledge. Twenty plus thirty is fifty, and five plus seven is twelve, are examples of declarative knowledge for some students. Being able to combine these facts to add twenty-five to thirty-seven and get sixty-two indicates a procedure — that is, procedural knowledge or an algorithm — involving "carrying." The executive function, in this case, is making generalizations from single-digit addition to two-column addition.

It is established and accepted by many educators that these areas of knowledge in mathematics exist and are very important cycles of learning, especially the facilitating area of executive function. John Woodward emphasizes how "one is able to retrieve information faster, and what were separate steps increasingly turn into more orchestrated units" as declarative knowledge becomes more procedural through practice.9 What he is saying, by my interpretation, is that — in most if not all of the mathematics that primary and secondary students learn — declarative knowledge is rote memorization

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8Anderson, p. 369.
and procedural knowledge is algorithmic applications. According to his quote, he feels that procedural knowledge can be called upon faster and more concisely than declarative knowledge. On the contrary, I feel that algorithms (i.e. procedural knowledge) becomes rote, or declarative, through practice. Once achieving this transfer from procedural back to declarative knowledge - able to be recited from memory - this knowledge is more organized and easily obtainable. Thus, I think declarative knowledge, as developed from practicing procedural knowledge, is retrieved faster and exists in more orchestrated units. The goal, then, it seems to me, in developing higher levels of facility with mathematics, is twofold: first, to expand that area of mathematical knowledge that can be called upon instantly — that is, teaching should be directed to transferring more and more of procedural knowledge to declarative; second, to enhance the learner's ability to bring to bear his executive function on developing new procedural knowledge.

George Polya's Work On Heuristics

Actual instructional methods or materials on how to implement the executive area of knowledge, the area between the declarative and the procedural, into teaching mathematics is very hard to obtain and, when obtainable, is very broad-based. The only sound, applicable advice given on this concern is by Pellegrino and Goldman, who say that "a primary one [mechanism] is strengthening of both declarative and procedural knowledge units through experience and practice."¹⁰ Some mathematics teachers interpret this statement to mean that students need to be "drilled" with problem after problem until memorization occurs. However, there are other more constructive

methods that can be used in mathematics teaching to encourage the
development of both new declarative and new procedural knowledge.

One method I propose that could provide this experience and practice to
achieve procedural level of knowledge is the use of heuristics in teaching.
Heuristic, meaning "serving to discover," aims to study the methods and rules
of discovery and invention. Heuristic patterns and methods that can be
used in teaching have been proposed by several authorities but most contemp-
orary material really dates back to George Polya and his book, How To Solve
It (1957). The foundation of information pertaining to the evolution of heuristic
can be credited to older writers, such as Pappus, Descartes, Leibnitz, Balzano,
and Hadamard, who to various degrees provide a logical and psychological
background to the concept. Polya himself developed what is known as modern
heuristic, which he characterized as "endeavors to understand the process of
solving problems, especially the mental operations typically useful in this
process." Heuristics for Polya is, above all else, intended to aid a person in
developing the operations needed for solving problems. Another point that
Polya makes, which reinforces the valuable use of heuristics to develop
procedural knowledge, is "experience in solving problems and experience in
watching other people solving problems must be the basis on which heuristic
is built." By watching other people solving problems, Polya meant for a
student to watch a teacher solve a problem from start to finish, observing the
teacher's thought and mechanical processes. From this observation, the stud-

11 Polya, p. 112.
12 Polya, p. 130.
13 Polya, p. 130.
ent may begin to understand and attempt to imitate the teacher. Thus, heuristic mechanisms involve repeated practice and experience which in turn help students incorporate the facts of declarative knowledge into the methods of procedural knowledge. Krulik and Rudnick also emphasize this link to procedural knowledge when they point out that heuristics provide the direction needed by all people to approach, understand and obtain answers to problems that confront them, which, in turn, is developing procedural knowledge.\(^{14}\)

Heuristics or heuristic patterns can further be described as a set of suggestions and questions that a person must follow and ask himself in order to resolve a dilemma or solve a problem.\(^{15}\) This statement may sound obvious and simple, but heuristics should not be confused with algorithms, which are specific step-by-step directions that lead to a particular desired answer. Each algorithm has to be applied to a specific set of objects, whereas heuristics are more general. Most heuristic patterns can be applied to all classes of problems. This distinction between heuristics and algorithms reinforces the categories of knowledge already discussed: that algorithms are types of procedural knowledge, but heuristics are types of executive knowledge.

Polya presented a four-step plan that included several detailed heuristic techniques. These techniques deal with the mental operations used in problem solving which he referred to in his definition of modern heuristic. He revealed each of the four steps and its importance in the following quote:

Each of these phases has its importance. It may happen that a student hits upon an exceptionally bright idea and jumping all


preparations blurts out with the solution. Such lucky ideas, of course, are most desirable, but something very undesirable and unfortunate may result if the student leaves out any of the four phases without having a good idea. The worst may happen if the student embarks upon computations or constructions without having understood the problem. It is generally useless to carry out details without having seen the main connection, or having made a sort of plan. Many mistakes can be avoided if, carrying out his plan, the student checks each step. Some of the best effects may be lost if the student fails to reexamine and to reconsider the solution.\(^16\)

These four steps: understand the problem, make a plan, carry out the plan, and examine the solution, have been included in many algebra textbooks to help students solve word problems. However, the elaboration on each point is very brief and is specific only to that section of the book. There are also few guidelines for the teacher to follow in successfully implementing these steps. This plan is not generally presented in any other type of mathematics textbook other than first year algebra books, which indicates a need for further research in this area. Although heuristics are acknowledged as a tool for teaching problem solving, proper attention and use appear not to be given to them.

Included in the first phase, "understanding the problem," are questions that students might ask themselves that would aid in their understanding, such as "What is the unknown?, What are the data?, and What is the condition?"\(^17\) Polya points out that these questions are of the greatest importance to the problem-solver because he focuses his attention on the principal part of the problem that links the unknown to the data which helps form the solution. Some additional suggestions that are made to further clarify

\(^{16}\)Polya, p. 6.

\(^{17}\)Polya, p. xvi.
understanding are: describe the problem setting and visualize the action; re-
state the problem in your own words; organize the information; draw a figure
or chart; use suitable notation; and separate the various parts of the condi-
tion.18

In considering the second phase of the heuristic patterns — devising a plan — the key is to select some kind of a strategy that will help solve the problem. This step is considered by most students to be the most difficult heuristic of all. A strategy is "that part of the problem-solving process which provides the direction the problem solver should take in finding the answer."19 Strategies are not as problem-specific as are algorithms, and are often used in combination. One important approach that is usually very suc-
cessful in clarifying all four steps of the plan is to have the problem-solver find a similar, simpler problem having the same or similar unknown.20 It would be even more helpful if this related problem was previously solved by the student because he would then have some direction for solving the problem at hand. Other suggestions for devising a plan are various strategies that can be used to help solve the problem: pattern recognition, simplification and reduction, experimentation and simulation, guess and test, logical deduction, organized listing, and working backwards. Although this list is not exhaustive, use of these strategies will help enhance the problem-solving abilities of students. The difficult part for most students is selecting the right strategy or combination of strategies to use. Krulik and Rudnick comment on


19Krulik and Rudnick, Sourcebook. p. 5.

20Polya, p. 9.
this difficulty, saying "proper strategy selection is the result of repeated
exposure to lots and lots of problems."\(^{21}\) This exposure is an element of the
development of procedural knowledge, as mentioned before.

When carrying out the plan or strategy, obtaining the solution is the
key. One problem-solving subskill that might be useful in this step is recog-
nizing when there is insufficient data to reach an answer. When working
toward the solution, the student should check each step. A student may con-
vince himself of the correctness of a step in his reasoning, either "intuitively"
or "formally."\(^{22}\) Although advanced reasoning may not take place early in a
person's exposure to solving problems, it will come more easily and naturally
with practice. Polya points out that what is needed most in this step is
patience.\(^{23}\)

The final step, examine the solution, includes asking oneself, "Can I
check the result?, Can I check the argument?, Can I derive the result differ-
ently?, Can I use the result, or the method, for some other problem?"\(^{24}\) If a
student does not participate in this fourth step, he is missing an important
and instructive phase of the work. Polya pointed out that, "By looking back at
the completed solution, by reconsidering and reexamining the result and the
path that led to it, they [students] could consolidate their knowledge and
develop their ability to solve problems."\(^{25}\) Although this step does include


\(^{22}\)Polya, p. 13.

\(^{23}\)Polya, p. 12.

\(^{24}\)Polya, p. xvii.

\(^{25}\)Polya, p. 15.
checking for mechanical errors, the important part is examining the methods used and evaluating their use in other situations. Krulik and Rudnick feel one way students can evaluate the problem in this manner is to ask "what if" questions, involving changing the circumstances of the original problem.\textsuperscript{26}

I have provided one heuristic plan for problem-solving, developed by Polya. However, there are several sets of heuristics that can be used with varying number of steps to follow. The set of heuristics that students or teachers choose to use does not really matter, as long as some set is implemented, which is what Krulik and Rudnick point out: "What is important is that students learn a heuristic model, develop an organized set of 'questions' to ask themselves, and then constantly refer to them when confronted by a problem situation."\textsuperscript{27} To learn the use of heuristics, attention should be paid to the instructional methods used by the teacher, as pointed out by Krulik and Rudnick when saying, "... we must do more than merely hand the heuristics to the students - rather, instruction must focus on the thinking that the problem solver goes through as he or she considers a problem. It is the process - not the answer - that is problem solving."\textsuperscript{28} The pedagogy of problem solving used by teachers can aid students in successfully applying heuristics to problems. The goal is to develop "methods that the teacher can employ in the classroom to assist students in the utilization of heuristics in their development of problem-solving skills."\textsuperscript{29}

\textsuperscript{26}Krulik and Rudnick, \textit{Handbook}, p. 29.

\textsuperscript{27}Krulik and Rudnick, \textit{Handbook}, p. 39.

\textsuperscript{28}Krulik and Rudnick, \textit{Handbook}, p. 21.

\textsuperscript{29}Krulik and Rudnick, \textit{Handbook}, p. 39.
Pedagogical Methods For A Heuristic Model

There are sixteen pedagogical methods suggested by Krulik and Rudnick in *Problem Solving: A Handbook for Teachers* to use in teaching heuristics. One suggestion for teachers is to "create an atmosphere of success."30 This point includes choosing problems carefully, beginning with relatively simple problems to ensure success and progressing to more advanced, multi-step problems. A teacher should also encourage his students to solve problems, which can be done by finding problems that are of interest to the students. It is very important to teach students how to read the problems, critically and carefully, for meaning. A technique that might be helpful in developing reading for understanding is to have the students underline or circle the key words in the problem and discuss these words in class. Further suggestions for teachers are to include the students in the problem by using their names in the problem or letting them act out or experiment with the problem, if possible. A teacher can also have his students create their own problems, which requires them to know the anatomy of a problem. Some directions or facts might need to be provided in order to get the students started in writing problems, such as a set of answers for which the students are to provide problems.

Having students work together in small groups or pairs is a suggestion made that is not normally used in a traditional classroom. Group brainstorming and problem solving are techniques of the business world that can be of beneficial use in a mathematics classroom. In having students share ideas, the teacher should provide guidance and practice in the particular skills involved. While brainstorming, there should be no evaluation of any kind and everyone

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should be encouraged to provide as many ideas as possible by using their imaginations.

Some further pedagogical suggestions made are: encourage the use of freehand drawings; suggest alternatives when the present approach has yielded all possible information; raise creative, constructive questions; emphasize creativity of thought and imagination; encourage students to use a calculator; take advantage of computer programming; have students flow-chart their own problem-solving progress; use strategy games in class; include problems that have more than one step; and do not teach new mathematics while teaching problem solving.31

Although I did not elaborate on these methods in any detail, they are successful teaching techniques that can be used to implement heuristics into the material being taught. To carry this point further, they are instructional methods that can be used to try and develop the executive function in students. I wanted to point out these methods, even though the focus of my paper is on information processing, heuristics, and problem solving.

Conclusion

Heuristics, as George Polya presented them, are meant to be used for problem solving. Problem solving is a process. "It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation."32 Thus, solving word problems requires procedural knowledge. A student must be able to use algorithms to arrive at

31Krulik and Rudnick, *Handbook*.

an answer to a problem of this nature. Heuristics, as related to problem-solving, is the executive function between the declarative knowledge of the facts and the procedural knowledge of solving. Students should be exposed to problem-solving throughout their mathematics curriculum to be given the opportunity to develop higher levels of knowledge. The National Council of Teachers of Mathematics Professional Standards reinforces this point by saying, "Teachers should engage students in mathematical discourse about problem solving. This includes discussing different solutions and solution strategies for a given problem, how solutions can be extended and generalized, and different kinds of problems that can be created from a given situation." As well as engaging students in problem solving, teachers should also evaluate their students' levels of knowledge - declarative, executive, procedural - and attempt to present instruction for the problem at the appropriate level.

The next section will provide two examples of problem-solving, each based on the same problem. In these examples, I will describe how to apply heuristics as a teaching technique. I will also make comments on the type of knowledge being demonstrated, declarative or procedural. However, the teacher will have to observe and evaluate each student in order to determine his level.

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\[^{33}\text{NCTM, p. 95.}\]
PROBLEM

What is the sum of the first 100 positive integers?

DISCUSSION I

In understanding the problem, the teacher would make sure the students know what "the sum of the first 100 positive integers" means. The question means:

1 + 2 + 3 + ... + 98 + 99 + 100

We assume the students are at a level where this knowledge should be declarative knowledge.

There are several different approaches that might be taken to arrive at the answer to this problem. One approach would be simply to add the first number to the second number and add this sum to the third number and add this sum to the fourth number, and so on. This method would involve a student demonstrating declarative knowledge — just adding from memory.

A teacher would want the students to go beyond this declarative approach and try to find a procedure that could be used to obtain this answer, as well as other similar sums. Heuristics can be used to help find this procedure. One heuristic would be to look at a similar but simpler problem, such as the sum of two, three, or four positive integers. The students should examine these cases and try to find a pattern that could be generalized.
Looking at the simpler problems, they find:

\[
\begin{align*}
1 & = 1 \\
1 + 2 & = 3 \\
1 + 2 + 3 & = 6 \\
1 + 2 + 3 + 4 & = 10
\end{align*}
\]

As they look at the last case where there are four elements, the teacher directs the students to look for a pattern — a use of heuristic methods. Here is what the teacher helps them to discover. If they add the first and the last integer, and if they add the second and the second to last integer, the same sum is produced — five. This sum is equal to the number of elements, four, plus one. To get the answer for the problem, however, they need to carry it one step further. The answer is ten, which is two times the five that they have. How do they know to take two times the number? Because they are pairing up the numbers, dividing the number of elements they have by two will give them the number of pairs they have. Two is equal to the number of elements (four) divided by two. They have now discovered a pattern.

The question that the students must be led to ask is, "Will this always work?" Try it on the previous sum: here the students might see \(1 + 2 + 3\) as one-and-a-half pairs; that is, the sum is \(1 + 3\) (the only "pairing") times \(3/2\), 6. Try again: how about \(1 + 2 + 3 + 4 + 5\)? This is two-and-a-half pairs of 6 each, or 6 times \(5/2\). It works! The technique that must be clearly recognized — and this may require the teacher stating it, explicitly — is that problems with more easily recognizable patterns are presented first as an introduction to more complex problems.
Once the students have recognized the pattern, they should make a generalization concerning \( n \) elements. The students are using their "executive function" when using heuristics to discover the pattern. When making a generalization from this pattern, when procedures are being recognized, procedural knowledge is being developed. What should the students conclude? They discovered from the above example that they need to take the number of elements and add one: \( n + 1 \); they then need to multiply this sum by the number of pairs they have, which is equal to the number of elements divided by two: \( \frac{n}{2} \). Their generalized pattern is:

\[
1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}
\]

A teacher would have the students check the pattern for various cases, such as \( n=1 \), \( n=2 \), and \( n=3 \). To answer the problem with which they started, the students would know that \( n=100 \). They can carry out the process — apply their newly created procedural knowledge — obtaining 100 times \((100 + 1)\) divided by 2, which is 5050. Thus, the sum of the first 100 positive integers is 5050.

The students derived the "formula" through the use of their executive functions — with the aid of a simpler problem and pattern recognition — and proceduralization or their procedural knowledge — generalizing and acquiring an algorithm.

**DISCUSSION II**

A second approach that might be taken for this problem is to use
figurate numbers to represent the sum of integers. The teacher would need to present the definition and perhaps some historical background for figurate numbers. For this particular problem, they need to use "triangular" numbers to represent the positive integers referred to in the problem and "oblong" numbers to arrive at the formula. To illustrate:

\[\begin{array}{c|c}
1 & 1 + 2 = 3 \\
1 + 2 + 3 = 6 & 1 + 2 + 3 + 4 = 10 \\
\end{array}\]

Notice that as the numbers increase, they form similar triangles. These four triangular numbers show the sum of the first four positive integers. Oblong numbers show the sum of positive even integers, as illustrated below:

\[\begin{array}{c|c}
2 & 2 + 4 = 6 \\
2 + 4 + 6 = 12 & 2 + 4 + 6 + 8 = 20 \\
\end{array}\]

Here, the first four oblong numbers show the sum of the first four even numbers.

While they look at an oblong number, such as \(2 + 4 + 6 = 12\), they again want to try to find a pattern utilizing by using their executive functions. They have three integers in this example, making \(n=3\). The answer they are looking for is twelve, which is four times the number of elements they have. But four is equal to the number of elements we have plus one: \(n+1\). The geometry of figurate numbers makes this discovery relatively easy to lead students to see: the "height" of the oblong number times its "width." Therefore, if they generalize their example, they find that the sum of \(n\) positive even integers is:
Thus far, the students should have used their executive function to look for a pattern and their procedural knowledge to develop an algorithm to use. At this point, since this result is so intuitively simple, and depending on the amount of practice a student has using the formula, he may transfer it from procedural knowledge to declarative knowledge.

However, he must further use his executive functions to answer the question at hand — the sum of the first 100 positive integers. We know that the triangular numbers represent the positive integers. We also know that the formula for the oblong numbers, or the positive even integers, is \( n(n+1) \). If we look at the numbers pictorially again, we notice that each oblong number is made up of two triangular numbers.

\[
\begin{align*}
2 & \quad 2 + 4 = 6 \\
2 + 4 + 6 & \quad 2 + 4 + 6 + 8 = 20
\end{align*}
\]

To then obtain the formula for a triangular number, we need to divide the formula for oblong numbers by two. Thus, the formula for triangular numbers is:

\[
1+2+3+\ldots+n = \frac{n(n+1)}{2}
\]
We can then use this formula to answer the problem as we did in Discussion I. In working the problem using this geometric approach of oblong and triangular numbers, some students may be able to "see" the patterns more easily. In this second solution, a student has to first use his executive function to arrive at the "oblong number formula." This knowledge may then remain at the procedural level or become declarative, depending on the student's level and how much he practices using the formula. The student's executive function and procedural knowledge must be used a second time to obtain the "triangular number" formula.
Bibliography


