Computers and Mathematics Education

An Honors Thesis (Honors 499)

by

Melissa Renee Crawford

Dr. Ramon L. Avila

Ball State University

December, 1991

Expected date of graduation: May, 1992
Purpose of Thesis:

This thesis is devoted to the topic of computer use in education, and specifically in mathematics education. Among the major considerations are teacher computer literacy and features of software to be evaluated when purchasing software. Also mentioned are the results of studies that show the usefulness of computers in the classroom, specific software that could be used in mathematics classes, and the author's experience with various software pieces.
Computers can cut thirty percent of the time it takes for students to learn something. A topic that might take ten days to grasp could, on the average, be mastered in just seven days with the proper help from the computer. However, what is "proper help" from a computer? How should a teacher use the computer in the classroom? What are the results of research done with students and computers? What type of software should be used? Since mathematics and computers seem to be so closely related, how could computers be used to help teach mathematics? These and many other related questions will be answered in this paper. The specific way in which a computer should be integrated into the classroom will be considered. The results of studies done with computers in education will be mentioned. The problem of teachers sometimes feeling inadequate to teach computers will be spoken about, and solutions will be suggested. Different uses of the computer in the classroom will be mentioned, as well as some deficiencies in the computer's capabilities. Software features that an educator should consider will be spoken about and specific and general programs will be listed that can be used in most any mathematics classroom. Finally, the author will speak of some personal experiences with computers in the classroom.

The use of the computer that seems to be most beneficial to students is called instructional computer use, which is defined as "the appropriate integration of microcomputer-based learning activities with teacher's instructional goals and with the ongoing curriculum, which changes and improves on the basis of
feedback. It seems computer use that is consistent and that is linked with the students' text is most helpful to students. A teacher that has a computer in the classroom, and allows students to play games on the computer as a reward for doing something good is not using the computer in this manner. Benefits from computer use may not be as evident if the computer is just used sporadically and use is unrelated to the text.

Instructional computer use can take on many forms, depending on what types of learning are stressed when the computer is used, and which uses of the computer are most often implemented. The most beneficial type of instructional computer use is termed "orchestration cluster". In orchestration cluster, thinking ability and basic skills are stressed. The students are encouraged to use the computer to learn, and not just to have fun. When used in an orchestration cluster manner, the computer is used primarily for practicing, tutoring, and simulating.

Research shows that the proper use of the computer in the classroom not only increases the students' knowledge, but can also increase the students' confidence in his or her mathematical ability.

In one study done with students using the computer, the students played a game called "Algebra Arcade" for fifteen to twenty minutes each day at the end of class. The game scores students on how well they understand graphing. This four week experiment involved 848 students, with some using the computer and some not using the computer. The results showed that those
students using the computer had higher achievement and greater graphing ability. However, using the computer did not change the students' attitudes toward math.\(^1\)

Another study done on integrating the computer into the classroom was done with forty-eight students and covered a four-week period. About half of the students worked on the computer and half served as a control group. Pre-tests and post-tests were given to assess the students' progress. The results showed that the students who used the computer had a better abstract understanding of geometry. The Logo that the students had been working with made the relationships between geometric shapes and their definitions more clear than simply using verbal definitions. Although in this study the computer did not seem to increase student knowledge of geometric facts, it did increase students' confidence in their geometric abilities\(^1\). Therefore, as research shows, proper computer use with various programs can increase students' knowledge, understanding and confidence.

In addition, another study showed that the computer can be successful in identifying why students make the errors they do. The computer can often interpret if the error was caused by a simple addition error, or if the error was caused by the student's simply not knowing what to do to solve the problem. This could be very beneficial in helping students correct misconceptions that the teacher might not realize exist.\(^10\)

Another benefit of using computers in the classroom is the assistance computers can offer to handicapped students. For
visually impaired students, the computer can put large print on the screen until the student has had time to read it. For severely visually impaired students, speech synthesizers pronounce the things on the screen for them. Students who cannot use their hands to write because of muscle problems may be able to use the computer to type. Although some equipment for handicapped students is costly, with community support, the money could undoubtedly be found.

The responsibility of using the computer in the classroom falls on the individual teacher, whether the district supplies the money or not. Therefore, the teacher is also responsible to make sure he knows enough about the computer himself to be able to use the computer with the students. According to the authors of Teaching Mathematics and Science, there are eight things a computer literate teacher should know. The first is how to discuss the importance of the computer and its uses in society. A teacher who plans to use a computer with his students should be able to explain to these students why computers are important in our society today. The second is the laws dealing with hardware and software. A teacher must understand what software he can legally copy and what software he cannot legally copy. Next, a teacher should know the operational scheme of a computer. He should understand about the different types of memory, the different drives on the computer, and basic troubleshooting procedures. Also, he should understand the uses and limits of the computer in the classroom. He should not expect the computer
to take over his teaching for him. Furthermore, the teacher should know, in general, how to operate computers, including how to work with the operating system he will be using. A computer literate teacher should know how to select and use software. He should know how to make educated choices about hardware. Finally, although debated by some, the authors of this book believe a computer literate teacher should be able to read and write simple programs. If a teacher can at least partially do these eight things, he has the tools to successfully use a computer in his classroom.

There are three primary uses for a computer in the classroom: programming, teacher utilities, and computer-based instruction. Students can learn to program in many different languages, including BASIC, Pascal, Fortran, and C. Teacher utilities include a database for storing grades and attendance, a program that figures grades, and any other types of programs that help teachers. The computer can be especially helpful to mathematics teachers because of its ability to generate random numbers for math tests and quizzes. Computer-based instruction comes in eight forms. The first two of these are communicating through the use of a word processor and informing through the use of a bulletin board or a similar device. The third form of computer-based instruction is teacher-led demonstrating. The teacher can use the computer in classroom discussions to illustrate concepts such as limits and graphing principles. When demonstrating, the teacher can also use the computer to perform
tedious calculations, allowing the students to concentrate more on the results of the computations and less on the computations themselves. The fourth use of computer-based instruction is for evaluating students. This evaluation can be in the form of a computer test, game or drill. A computer can quickly determine a student's score on a test, game, or drill, and tell the teacher how much the student really understands. The last four forms of computer-based instruction are similar to each other. They are student practicing, computer-based tutoring, simulating, and gaming. The computer can be especially helpful for teachers in these four areas for two specific reasons. First, computers are excellent educational tools because active learning is much better than passive learning. Students will generally learn more, retain what they learn better, and enjoy learning more if they learn actively. The second reason the computer is so helpful to teachers when used in one of these four ways is because it can give students the individualized instruction that teachers often do not have the time to give. The computer can be the private tutor that many students need. With this personal tutor at their disposal, the students can quickly get through the material they understand and spend more time on the concepts with which they have more difficulty.

However, one must remember that there are many tasks the computer cannot do. It cannot answer all of our educational problems. It does not help very much in the classroom if it is just an add-on. To maximally benefit students and teachers, the
computer should be an integral part of the curriculum. The final thing that a computer is absolutely unable to do is replace the teacher. The computer is only able to free the teacher from non-teaching duties, such as grading and reviewing for the slower students.

Once a teacher has a computer in his or her classroom and has decided to use it as an integral part of the teaching, he or she must decide what type of software will most benefit the students. There are two main types of software. One is designed primarily to aid the teacher in classroom discussions. The second type is to be used primarily by individual or small groups of students.

Programs made to be used by the teacher for class discussions should have some specific features. These programs should stop often, giving the teacher the opportunity to talk to the class, and giving the students the opportunity to ask questions. The program should allow the teacher to determine the pace at which the material is presented, with opportunities for the teacher to repeat, skip, replace, or add steps in the program. A program that is going to be used in front of the class should have figures and characters large enough to be seen from the back of the room. Preview of future material is also an added benefit for which to watch.

Computer software that is purchased specifically for individual or small group use should also have some specific features. These programs should interact with the students.
This interaction should be frequent and high quality; that is, the questions posed to the students should be meaningful and more than simple "yes or no" questions. The programs used by students individually or in small groups should be capable of having a conversation with the students. These programs should involve problem-solving and thinking skills. This software should have content that is closely tied with the students' textbook, and should explain procedures in a similar manner as the textbook. Different levels of difficulty should be permitted because of the differing levels of student understanding. The program should be flexible enough to allow the students to investigate with different ways of doing a problem. These characteristics are the ones to look for when considering buying software for individual or small group use.

Independent of the setting in which a program is going to be used, all programs intended for classroom use should have some specific characteristics. The content should always be accurate and always be something that needs to be learned. The program should be long enough that the students can really learn something meaningful, but not too long to be finished in a small number of class periods. (From the author's experience, most any program, excluding games, that lasts more than three class periods becomes quite boring.) Charts and pictures that are related to the material are useful for students, and are especially helpful for the slower students who may need these pictorial illustrations to help them understand. All
educational programs should present the material sequentially in the steps the students need to understand, and be sure to not leave any steps out. Color graphics and animation are good additions to any program, and specifically to a program that is going to be used to teach children. Sound effects with the option of turning them off would be another flexible feature to look for. A program that is user-friendly and requires no programming knowledge is most useful for the majority of teachers and students. Added benefits to look for in a program are a review of the major topics at the end, repetition of key concepts throughout the program, and smooth transitions from one topic to the next. Real-life applications always make the material seem more practical to students, and thus are important to include in educational programs. All of these features need to be considered when buying educational software.

Some types of programs lend themselves especially well to whole-class instruction. One of these is graphing equations. Graphing equations can be used to illustrate the concept of the derivative and of limits. Graphing equations can also be used to experiment in problems such as adding numbers to functions to see how this affects the graph. Students can graph y=x, and compare this with graphs of equations such as y=x+3 and y=-5x. Another computer use that lends itself well to whole-class instruction is the illustration of numerical concepts. The teacher can use the computer to do many more examples much more quickly, and therefore the students can gain a better understanding of
concepts such as the quadratic formula and other concepts that require a lot of arithmetic\(^\text{10}\).

There are some obstacles to bear in mind when deciding to use a computer in the classroom. First of all, drastic curriculum changes would result if students were allowed to use the computer in the classroom. The texts would have to allow for computer time\(^\text{10}\). Another caution is that students could become dependent upon computers. Teachers must make sure students can do mathematics on their own without the computer. If the students become dependent upon the computer to help them through all their math, then no one will be around in the next generation to program these computers to do the math. Another problem to keep in mind involves the software used. Some programs try to make math appear fun by adding graphics and music and such. These things are great but should not be the focus of the program. The program should concentrate on using math to manipulate the program. The program should concentrate on the interesting aspects of math, and allow the student to use these aspects to make the program go\(^\text{10}\). Another problem to consider when using the computer is that often there are not enough computers for every student. It should be considered if there are enough for every two or three students, and if the activities will be beneficial with the students working in groups. One must also be sure the teacher knows enough about the computer to be using it in the classroom. Another problem that is quickly fading away is the shortage of courseware. In the past, very
little good software was available at prices that school systems could afford. However, each day more and more affordable software is being developed. A final problem to consider involves training the teachers who do not know how to use computers. Should this training be implemented after school, in the summer, or on weekends? How should the teachers be rewarded for attending this training? When asked these questions, teachers themselves suggested that the training be after school. The incentives they suggested were money, release time, or credits that would go toward buying a computer for the classroom. However, some of these teachers opposed incentives altogether saying that incentives would make some teachers come to the training simply for the incentives, and not because they really wanted to learn about computers.

The computer can be used in any high school math class. Programs are available to help teachers at almost any level of math.

In basic math classes, for example, there is a game called Darts that teachers can use. It helps students learn fractions. In the game, a balloon is place between two integers and students can use fractions, decimals, mixed numbers, and expressions using operations to determine the point on the number line to which the balloon corresponds. This makes learning fractions, which seems to be a least favorite topic of math students, a little more exciting.

Algebra I students can use the computer to find squares,
square roots, cubes, and cube roots\(^2\). The computer can be used to graph equalities and inequalities so that the students can see the solutions, even if they cannot factor the equalities or inequalities they are given\(^5\). As a fun activity, the computer can be used to generate perfect numbers. A perfect number is a number whose factors (excluding itself) sum to itself. As a break, some students will enjoy watching the computer generate these perfect numbers\(^4\). Students working with graphing parabolas can use the computer to discover how the changing values of A, B, and C affect the graph of \(y = A(X + B)^2 + C\). There are many graphing programs available for algebra students to experiment with, including the Equation Plotter by Sunburst. By experimenting with these graphing programs, students can discover many of the generalizations that exist in the graphing of parabolas, circles, ellipses, and hyperbolas. Many algebra students would also enjoy playing a game called Green Globs. In this game, the computer plots some green globs on a coordinate axis. The students have to develop equations whose graphs will pass through these green globs. The more green globs the students hit with each graph, the more points they earn. This game can be very good in helping students understand the graphs of different equations.

Geometry students could almost live at the computer and still learn all the material they would from a textbook. There are all kind of programs dealing with isometries, geometric shapes, and proofs.
First of all, geometry students can learn about isometries at the computer. The computer can be used to illustrate translations, rotations, reflections, and glide reflections. A typical problem a computer might give students to help them understand reflections would be:

Given that two electric plants are located on the same side of the bank of a river, and a pump located in the water will pump water to both plants, where in the water should the pump be placed so that the sum of the distances from the pump to each of the plants is a minimum?

The computer can quickly generate problems such as this one with a drawing to represent what the words say. The computer can also help students understand terms pictorially by drawing pictures of things such as the angle of incidence and the angle of reflection, and showing that the two angles are equal. The computer can demonstrate this fact with practical applications, such as golf, basketball, and pool.

Secondly, geometry students can learn all about geometric shapes using the computer. Logo is especially useful for helping geometry students see the pictorial representations that go with the verbal definitions of shapes. Using Logo, students can quickly catch on to the angle measures associated with various geometric shapes. More will be mentioned about Logo later. Programs also exist that can help students see the interrelationships between different shapes, such as polygons, quadrilaterals, parallelograms, and rectangles.

Finally, geometry students can learn how to do proofs on the
computer using the Geometry Proof Tutor program. This program generates proofs for the students to do. It allows the student to continue, even if the student is getting nowhere, until the student makes a logical error. The computer then tells the student what his error is. The student can then either give a different answer or ask for help. The tutor offers two types of help for the student. It will either list any rule the student has learned that the student asks to see, or it will hint as to what the next step is in the proof. The computer continues to give more and more hints as the student keeps making mistakes, and it will eventually give the student the answer after he has made enough errors. The tutor seems to be user-friendly. It catches mistakes such as calling two angles equal instead of congruent or naming an angle by two letters. The Geometry Proof Tutor was tested in geometry classrooms in the 1985-86 and 1986-87 school years. The slower students benefited from using the tutor, and the gifted students were challenged. However, it was found that students who shared computers while using the tutor did not improve as much as those who had the computer to themselves to work on. It seems this activity is most beneficial when used by individuals, and not groups. The students who worked with the tutor on the average improved their geometry scores a letter grade. The tutor seems to especially be beneficial to bright students who are bored or unmotivated. The tutor gives them some motivation. The tutor also helps with average students who simply lack confidence in their own
mathematical ability. The tutor offers the students practice and individualized attention with little or no peer pressure\(^\text{18}\). It seems to be a great way for students to learn to do proofs.

Statistics classes can also benefit from using the computer. The computer can endlessly generate random numbers for probability problems\(^\text{8}\). The computer can also figure the mean, standard deviation, and other measures of central tendency. It can demonstrate different types of graphs for the students, like pie graphs and box and whisker plots\(^\text{14}\).

Computer spreadsheets can be used by math students doing story problems. Spreadsheets will quickly do the calculations that are often tedious, allowing the student to focus more on the result of the problem than on the calculations\(^\text{3}\). This author once wrote a spreadsheet that explored Newton's Law of Cooling. This law deals with how long it will take an object to cool when it is taken from a hotter environment to a cooler one, such as taking a cake from the oven into a seventy degree Fahrenheit room. The spreadsheet can quickly calculate any variable in the formula when given the values for the other variables in the formula. Using the spreadsheet gives the students the chance to examine the results of Newton's Law of Cooling. The students can see more clearly how a change in the surrounding temperature will affect the final temperature, how a change in the original temperature will affect the final temperature and so on.

Students can gain an appreciation for the power of Newton's Law of Cooling. Other uses for spreadsheets include problems
figuring interest and many types of simulations.

A program available that can be used in most any math class is called muMath. This program can do anything from simple addition to calculus. It can even do limits and integrals. This program would be good for a school that did not have a lot of money to spend on a lot of different software. This one piece of software could be used throughout the entire high school math curriculum.

Junior high or even elementary math classes can benefit from using a computer in their classrooms. As mentioned earlier, muMath could be used for simple math, even at a junior high level. However, the computer use that will be emphasized here is the use of Logo. Seymour Papert, one of the primary writers of the Logo language, speaks of his experience with pre-teens using Logo. He says the students learn angle measures almost unconsciously. After having the computer generate all types of geometric shapes with varying angle sizes, the students begin to unconsciously recognize the approximate measures of angles. The use of variables becomes more than just a mathematical idea to students—it becomes a source of power. Variable use is introduced as a symbol that represents many different numbers. The students use variables to tell the computer to do something such as draw a spiral with circles of varying radius. The idea of a variable becomes familiar to students. Probably the most general benefit of using Logo comes from helping children realize that they can learn from their mistakes. When a mistake is made,
students come to realize that they do not have to start over. They can debug until the desired result is obtained. In math, this fact also holds true. A child with the wrong answer does not need to start over. This student should realize that going back over the problem and finding the error and correcting the misconception is more important than just getting the right answer.

When one looks at many college classes, it becomes obvious that high schools should at least acquaint students with classroom computer use. Stanford University, for example, has logic courses that are fully computer dependent. The logic program has two parts. The first part uses the computer to lecture to the students. The computer screen is used to summarize the main points of the computer's lecture. The rate of the lecture can be varied and students can repeat sections if they wish. The second part of the logic program has the student prove a particular result using logic. The computer then checks the student's work and offers assistance if necessary. This is just one example of why high schools should be using computers in their classrooms.

This author has worked with computers in some college classes. In one class, Green Globs and an equation plotter were used. The equation plotter allowed the user to enter equations, and the program would plot the equation that had been entered. From doing this, with some guidance, a user should easily see how adding, subtracting, and multiplying numbers by different
equations affected the graphs of these equations. For example, multiplying the $x$ in $y=x$ by a $-1$ makes the slope of the line opposite of $y=x$. In working with Green Globs, a student will use this knowledge of graphing to try to get a high score. This author worked with the Geometric Supposers in another class. The Geometric Supposers will draw almost any two-dimensional shape that high school students need to learn about. It will also draw bisectors, altitudes, and other lines and points associated with these geometric shapes. The Geometric Supposers can measure the degree of angles and the length of line segments, helping students come to conclusions about angle bisectors, line bisectors, congruent and similar shapes, and other geometric concepts. From this author's experience using the computer in classes, it is believed that high school students would not only enjoy using these programs, but would also be able to learn math by coming to conclusions on their own.

Using some of the ideas mentioned in this paper (especially the software considerations), this author reviewed a demonstration disk entitled "Utilizing Computers to Teach Secondary Mathematics". This disk was put out by the Asbury Park Board of Education as a demonstration of a series of programs they have developed to assist teachers of secondary mathematics from subjects such as general math all the way up to calculus. The demonstration disk had six parts: transforming the standard form for the equation of a line into the slope-intercept form, finding the vertex and axis of symmetry of parabolas, determining
if two triangles are congruent using the SSS, SAS, ASA, and AAS triangle congruence theorems, determining the period and amplitude of trigonometric functions, finding the values of definite integrals, and figuring the unit price. Overall, the program was quite impressive, giving the student positive reinforcement for correct answers and showing the way to get the correct answer when an incorrect answer is given. The examples done in the program were explained adequately so that students at most any level of understanding could comprehend them. However, the one major problem with the program is that it did not tell the user how to stop the program. It seems once a student starts in a certain area, they are stuck there until they turn the computer off. This major downfall of the program made it difficult to completely appreciate the program's good features.

When used alongside the text, the computer can help students increase their mathematical knowledge and understanding and their own confidence in their mathematical ability. However, before attempting to use the computer in the classroom, a teacher should be sure he or she is computer literate. Once a teacher meets the above-mentioned qualifications for computer literacy, he must decide if he wants to use the computer to teach computer programming or for computer-based learning or simply for his own use, such as for storing grades or test generation. No matter how a teacher decides to use a computer in his classroom, he must choose appropriate software. As mentioned, a teacher should know what characteristics to look for in software. The
characteristics vary according to the use intended for the software. There are many problems one must face when using a computer in the classroom, such as shortage of time, computers, and software. Another problem to consider is teacher training and whether the teachers will be rewarded for attending training. Although there has been a shortage of courseware in the past, each day new courseware is being developed. At present, there are already enough educational programs to help students at any level of mathematics from basic math to calculus. Implementing the computer into a classroom in an organized manner requires a lot of planning on the part of the teacher. However, from my research and own personal experiences with the computer, it seems the results—the increased student interest, increased student understanding, and increased student self-confidence—are well worth the trouble it is to the teacher.
WORKS CITED


