A Unit Plan for Exponential and Logarithmic Functions

An Honors Thesis (HONRS 499)

by

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May 4, 2002
Abstract

This project includes two sets of unit plans. The first set, a unit plan and accompanying lesson plans, are original plans made for a traditional high school schedule for a precalculus class. The original unit is a twelve-day unit. In addition to the unit plan and lesson plans are supplementary materials for the unit. These materials include warm-up handouts, a project description, project assessment rubrics, a computer activity handout, a quiz with a key, and a test with a key. To complete the original unit, a list of websites for students to use in the unit is also provided. The second set, another unit plan and accompanying lesson plans, are modified plans made for a block schedule that I implemented during my student teaching at Anderson High School. These plans are modified from the original plans, and form an eleven-day unit. Included with the modified plans are daily reflections and other materials used. These materials are the same as in the original unit, except that there are some additional worksheets and activities included. To complete the modified unit, assessment data from the unit is provided. Finally, a textbook analysis is included, along with a bibliography of the resources used to create the unit, important websites, and a NCTM Standards note and justification. The two sets of plans show how plans can be modified to form a successful unit that can be implemented in an actual classroom setting, regardless of the type of schedule. The success of the modified lesson can be judged based on the assessment data included.
Acknowledgements

I would like to thank the people who helped me at the various stages of this project to prepare the unit plan, lesson plans, and all of the supplemental materials. Their encouragement, criticisms, and suggestions have been invaluable to me. First I would specifically like to thank Dr. David Thomas, my thesis advisor, for his helpful advice. His ideas and hints helped to make this unit much better, especially in the area of technology. Thank you Dr. Thomas for your valuable and kind support of this project, and also for your insight and wisdom to prepare me to teach mathematics. A second person I would like to specifically thank is Mr. Peter Gast, my cooperating teacher at Anderson High School, for not only helping me develop into a better teacher, but also develop into a better planner. His timely advice and support for my planning and gathering of materials greatly assisted me with the completion and implementation of this project in an educational setting. Thank you Mr. Gast for your support to help my unit become a success in the classroom at Anderson High School.
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2. Supplemental Materials for the Unit

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5. Unit Data from the Implementation of the Unit at Anderson High School

6. Textbook Analysis

7. Bibliography

8. Important Websites

9. NCTM Standards Note and Justification
### Aaron Gogel

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<th>Textbook: Merrill Advanced Mathematical Concepts</th>
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<tbody>
<tr>
<td><strong>Unit Goals:</strong> By the end of the unit, should be able to: use the properties of exponents and logarithms, solve equations by using the properties of exponents and logarithms, and graph exponential and logarithmic functions.</td>
</tr>
<tr>
<td><strong>NCTM &amp; Indiana Standards:</strong> Problem solving, communication, reasoning, connections, algebra, representation, functions, data analysis, mathematical structure</td>
</tr>
</tbody>
</table>

### Instructional Materials & Technologies:
- Classroom set of graphing calculators, graph paper, overhead projector

### Information Resources:
- Computer lab for Internet and use of programs

### Instructional Overview:

<table>
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<th>Lesson Title/Activity</th>
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<td>Population Growth/Compound Interest</td>
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<td>5</td>
<td>11.4 Logarithmic Functions</td>
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<td>Pg. 626-628, 19-39 odd, 40-52 even, 56; Writing for journal: #2</td>
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<td>6</td>
<td>Quiz</td>
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<td>Outline of Chapter Project</td>
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<td>7</td>
<td>11.5 Common Logarithms</td>
<td>629-633</td>
<td>Answers to computer activity</td>
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<tr>
<td>8</td>
<td>11.6 Exponential and Logarithmic Equations</td>
<td>636-640</td>
<td>Pg. 639-640, 16-28, 29-39 odd, 42, 44; Writing for journal: Population growth model and question</td>
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<tr>
<td>10</td>
<td>11.7 Natural Logarithms</td>
<td>641-645</td>
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<tr>
<td></td>
<td>Finish chapter project</td>
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<td></td>
<td>Pg. 643-645</td>
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<td></td>
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<td>13-24 odd, 25-30, 31-39 odd</td>
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<td>1-44</td>
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<td>12</td>
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<td></td>
<td>Read 12-1 Pg. 656 - 660</td>
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**Assessment Overview:**

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<th>Instruments &amp; Procedures</th>
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<tr>
<td><strong>Formative</strong></td>
<td>Students will be informed on a daily basis how they are doing in the class. Students will be given the opportunity to write in journals on particular problems as well as others to gather feedback on a weekly basis.</td>
<td>Homework assignments are graded in class to show students their performance with the exercises. Journals with comments about problems as well as concerned questions about understanding will be returned in the middle and end of the unit.</td>
</tr>
<tr>
<td><strong>Summative</strong></td>
<td>Students’ achievement will be summarized through one mid-unit quiz, a project, and one exam. This achievement summary will be based on their performance with material discussed and covered.</td>
<td>The quiz will be held approximately at the middle of the unit and will cover material discussed in 11.1 – 11.4, with the unit project following. The unit test will take place at the end of the unit. The test will be assessed fairly to meet unit goals. Group members with a rubric will score group participation.</td>
</tr>
<tr>
<td><strong>Grading</strong></td>
<td>Grading will consist of points awarded for homework, the quiz, the test, the project, and especially participation in class and on the project. Each of these will be a percentage of unit grade.</td>
<td>Grading breakdown for unit: Homework/journals: 20% Quiz: 15% Project: 15% Test: 40% Class and Group Participation: 10%</td>
</tr>
</tbody>
</table>
## Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Rational Exponents</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Use properties of exponents.</td>
</tr>
<tr>
<td></td>
<td>- Evaluate and simplify expressions containing rational exponents</td>
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<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
</tr>
<tr>
<td></td>
<td>Communication</td>
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<td>Reasoning</td>
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<td>Connections</td>
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<td>Algebra</td>
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<td>Mathematical Structure</td>
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<td><strong>Instructional Materials &amp; Information Resources</strong></td>
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<td></td>
<td>Transparencies</td>
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<td></td>
<td>Chalk and chalkboard</td>
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<tr>
<td><strong>Introduction</strong></td>
<td>Do warm-up problem</td>
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<td></td>
<td>Attendance</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Introduce chapter project: populations of countries using PowerPoint and Excel</td>
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<tr>
<td></td>
<td>Discuss portfolios and journals</td>
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<td></td>
<td>Discuss scientific notation</td>
</tr>
<tr>
<td></td>
<td>List properties of exponents and examples of each</td>
</tr>
<tr>
<td></td>
<td>Put students in groups, assign a property, have them give examples of how it is used, and have them show how the property can be derived from the definitions of exponents. Put on overhead.</td>
</tr>
<tr>
<td></td>
<td><strong>Example #1:</strong></td>
</tr>
<tr>
<td></td>
<td>The diameter of the planet Jupiter is $1.427 \times 10^5$ km. What is its volume?</td>
</tr>
<tr>
<td></td>
<td>Introduce rational exponents</td>
</tr>
</tbody>
</table>
**Example #2:** Evaluate the following:
1. \(625^{1/4}\)
2. \(3^{1/2} * 21^{1/2}\)

Define rational exponents

**Example #3:** Evaluate \(81^{5/4}\)
Note: it is easier to first calculate the fourth root of 81 and then raise it to the fifth power of 81.

Have select students put the following examples on the board:

**Example #4:** Express: fourth root of \(27x^4 y^3\) using rational exponents

**Example #5:** Express \((5a)^{2/3} b^{5/3}\) using radicals

**Example #6:** Simplify third root of \(a^4 b^8\)

**Closure**
Have each student recite one of the properties of exponents

**Assignment**
15-33 odd, 36-52 even, 53-56
Writing for journal: #1
# Precalculus Lesson Plan

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</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Exponential Functions</td>
</tr>
</tbody>
</table>
| **Lesson Goals** | By the end of the lesson, students should be able to:  
- Graph functions on a graphing calculator.  
- Evaluate expressions with irrational exponents.  
- Graph exponential functions.  
- Graph exponential inequalities |
| **NCTM Standards Addressed** | Problem Solving  
Communication  
Reasoning  
Connections  
Algebra  
Functions |
| **Instructional Materials & Information Resources** | Graphing calculators (classroom set)  
Overhead projector  
Graphing Calculator view screen for the projector  
Chalk and chalkboard |
| **Introduction** | Do warm-up problem  
Attendance  
Ask for questions from homework  
Collect homework |
| **Instructional Sequence & Questions** | Sketch the graph of the following situation:  
In the first week of this month, I put one dollar in the savings. Each week there after, I put away double the previous week's amount.  
Discuss graphs.  
Graphing calculator activity  
Graphing exponential functions with the class  
Graph \( y = 3^x \) and \( y = (1/3)^x \) on the same set of axes on calculator  
**Q:** How do these graphs compare? Is there some sort of reflection?  
\( y = 9^{2+x} \)  
**Q:** What happened to the graph? Was it shifted?  
Introduce irrational exponents:  
**Q:** What is an irrational number? |
Using calculators: evaluate:
\[5\sqrt{2}\]
\[0.4^\pi\]

Introduce exponential functions

After graphing them on the calculator:
Discuss the behavior of the graphs of \(y = 2^x\) and \(y = 2^{-x}\)
Compare the values of \(y = 2^x\) and \(y = 2^{-x}\) on the intervals \(-10 < x < 0\) and \(0 < x < 10\).

Note that exponential functions are transformed the same way other functions are.

Using graphing calculators:
Graph the functions \(y = (1/2)^x\), \(y = (1/2)^x + 1\), \(y = (1/2)^x - 2\), and \(y = 3(1/2)^x\) on the same set of axes. Then describe the transformations of the parent graph \(y = (1/2)^x\) that have taken place to form each of the other graphs.

Describe annuities: give an example on page 609.

Inequalities: similar to other inequalities:
Graph \(y \geq 3^x + 1\)

<table>
<thead>
<tr>
<th><strong>Closure</strong></th>
<th>How do the graphs (y = (1/2)^x) and (y = 2^{-x}) compare?</th>
</tr>
</thead>
</table>
| **Assignment** | Pg. 612-613  
14-22, 29-34, 39-43  
Writing for journal  
pg. 611  
#1 |
**Precalculus Lesson Plan**

<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>The Number e</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Use the exponential function ( y = e^x )</td>
</tr>
<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
</tr>
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<td></td>
<td>Communication</td>
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<td></td>
<td>Reasoning</td>
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<td></td>
<td>Connections</td>
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<td></td>
<td>Functions</td>
</tr>
<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set)</td>
</tr>
<tr>
<td></td>
<td>Overhead projector</td>
</tr>
<tr>
<td></td>
<td>Graphing Calculator view screen for the projector</td>
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<tr>
<td></td>
<td>Chalk and chalkboard</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>Do warm-up problem</td>
</tr>
<tr>
<td></td>
<td>Attendance</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Take questions over the homework and collect</td>
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<tr>
<td></td>
<td>Introduce the number ( e ) and its applications</td>
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<tr>
<td></td>
<td>Group work:</td>
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<tr>
<td></td>
<td>Present the formula for compound interest</td>
</tr>
<tr>
<td></td>
<td>Have groups calculate the value of an account with a principal of $1000 after three years at 6% interest compounded annually, quarterly, monthly, daily, and hourly.</td>
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<tr>
<td></td>
<td>Note how the sequences of values appear to converge to a limit.</td>
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<tr>
<td></td>
<td>This limiting value ( Pe^n ).</td>
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<tr>
<td></td>
<td>Calculate the limiting value.</td>
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<td></td>
<td>Discuss values for ( e ).</td>
</tr>
<tr>
<td><strong>EX:</strong> Find the value of ( e^{-1.27} )</td>
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<tr>
<td>**Discuss graphs involving ( e ).</td>
<td></td>
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<tr>
<td><strong>EX:</strong> Using graphing calculator: graph: ( y = 2e^x + 1 )</td>
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<tr>
<td><strong>Q:</strong> On what interval is this function increasing?</td>
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</tr>
<tr>
<td><strong>Describe applications:</strong> Newton's law of cooling: ( y = ac^{-kt} + c )</td>
<td></td>
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</tbody>
</table>
**EX:** An object is moved from a house heated to 70 degrees to the outdoors where the temperature is 15 degrees. What will the temperature of the object after 30 minutes? Assume $a = 55$ and $k = 0.55$.

<table>
<thead>
<tr>
<th>Closure</th>
<th>Describe one of the applications that use the exponential function $y = e^x$</th>
</tr>
</thead>
</table>

| Assignment       | Pg. 617-619  
13-23 odd, 26-34 even, 35  
Writing for journal: pg. 616  
#1                  |
# Precalculus Lesson Plan

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<thead>
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<tbody>
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<td><strong>Lesson Title</strong></td>
<td>Population Growth/Compound Interest Demonstration and Activity Powerpoint review</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
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<tr>
<td></td>
<td>- Understand various types of population growth.</td>
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<td></td>
<td>- Use a spreadsheet to calculate the value of an investment after a specified number of years.</td>
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<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
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<td>Communication</td>
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<td></td>
<td>Reasoning</td>
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<td>Connections</td>
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<td></td>
<td>Functions</td>
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<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set)</td>
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<td>Computer Lab</td>
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<td><strong>Introduction</strong></td>
<td>Do warm-up problem</td>
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<tr>
<td></td>
<td>Attendance</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Ask questions from the homework.</td>
</tr>
<tr>
<td></td>
<td>Focus on #35: say that we will look at this type of curve today</td>
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<tr>
<td></td>
<td>Logistic Equation: from <em>Mathematics Teacher</em> article</td>
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<td></td>
<td>Go to computer lab:</td>
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<td></td>
<td>Reintroduce chapter project</td>
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<td></td>
<td>Discuss populations and countries to look at.</td>
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<tr>
<td></td>
<td>Have students open PowerPoint and briefly introduce its use for the project demonstration.</td>
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<td></td>
<td>Demonstrate population growth demonstration and program</td>
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<td></td>
<td>On each computer, have population growth demonstration from Microsoft Excel loaded onto every computer for the students to experiment with.</td>
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<td></td>
<td>Have the students write the characteristics of each type of graph and what happens as values of each equation are changed.</td>
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<tr>
<td></td>
<td>After students have finished exploring with population growth models, have them open a new sheet in Excel.</td>
</tr>
<tr>
<td><strong>Closure</strong></td>
<td>Discuss the different types of growth models.</td>
</tr>
<tr>
<td>---------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td><strong>Assignment</strong></td>
<td>Compound Interest Spreadsheet</td>
</tr>
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</table>

Introduce the formula for compound interest: \( A = P(1+r/n)^{nt} \)

Distribute handout for compound interest activity

Have the students turn to pg. 620 in their textbooks. Explain the processes of Excel, and how to type things in.

Using this as their guide, have them print out a sheet for 50 years of account balances.

Have the students modify the program to print comparisons of the amount accumulated in accounts with two different interest rates.
## Precalculus Lesson Plan

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</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Logarithmic Functions</td>
</tr>
</tbody>
</table>
| **Lesson Goals** | By the end of the lesson, students should be able to:  
- Graph logarithmic functions on a graphing calculator.  
- Evaluate expressions involving logarithms.  
- Solve equations involving logarithms.  
- Graph logarithmic functions and inequalities. |
| **NCTM Standards Addressed** | Problem Solving  
Communication  
Reasoning  
Connections  
Functions |
| **Instructional Materials & Information Resources** | Graphing calculators (classroom set)  
Overhead graphing calculator  
Chalk and chalkboard  
Overhead projector  
Transparencies |
| **Introduction** | Do warm-up problem sheet  
(simplification)  
Attendance |
| **Instructional Sequence & Questions** | Ask for questions from each student's spreadsheet results.  
Using graphing calculators, graph logarithmic functions:  
**Example #1:** Graph \( y = \log_4 x \).  
**Q:** Can I type this function into my calculator?  
I must apply change of base formula!  
Remind students that logarithms with a base other than 10 must be rewritten as a quotient.  
**Q:** What is the range and domain of this function?  
**Example #2:** Graph \( y = \log (x + 6) \).  
**Q:** Do I have to apply the change of base here? What is the range and domain of this function?  
Discuss what changes can happen if things are added to the equations.  
Have students graph the following and state the range and domain of |
each: \( y = \log_3 (x+4) \) and \( \log_6 (3-x) \).

Introduce the definition of logarithmic function: relate it to the exponential function.

Do several examples of converting between the two.

Example #3: Write \( \log_{10} 0.01 = -2 \) in exponential form.
Q: What is my \( x \)? a? y? -> use these to write exponential equation.

Example #4: Evaluate the expression \( \log_8 64 \).

Have the students write the properties of exponents.
Use these properties to show the properties of logarithms:
Product property
Quotient property
Power property
Property of equality

On overhead: have students do the following with assistance from classmates:
Solve each equation.
\[
\log_b 5 = -1/3. \quad \rightarrow \quad 1/125 \\
\log_{10} (2x+5) = \log_{10} (5x-4) \quad \rightarrow \quad 3 \\
\log_3 (4x+5) - \log_3 (3-2x) = 2 \quad \rightarrow \quad 1
\]

<table>
<thead>
<tr>
<th>Closure</th>
<th>How are logarithms related to exponents?</th>
</tr>
</thead>
<tbody>
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<td>Assignment</td>
<td>Pg. 626-627</td>
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<td>19-39 odd, 40-52 even, 56</td>
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<td></td>
<td>#1</td>
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<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Mid-Chapter Quiz and Chapter Project</td>
</tr>
</tbody>
</table>
| **Lesson Goals**         | By the end of the lesson, students should be able to:  
                          - Demonstrate their knowledge of exponential and logarithmic functions through a quiz.  
                          - Find population data in the library or on the Internet.  |
| **NCTM Standards Addressed** | Problem Solving  
                          Communication  
                          Reasoning  
                          Connections  
                          Functions  |
| **Instructional Materials & Information Resources** | Graphing calculators (classroom set)  
                          Chalk and chalkboard  
                          Library for books and computer use  |
| **Introduction**         | Do warm-up problem: Twizzler's Activity  
                          Attendance  |
| **Instructional Sequence & Questions** | Ask for questions from homework or anything from chapter.  
                          Do review examples of logarithms:  
                          **Example #1**  
                          a. Evaluate \( \log_{48} 0 \)  
                          b. Evaluate \( \log_{7} -1 \)  
                          **Example #2**  
                          Solve \( \log_{x} 36 = 2 \)  
                          **Example #3**  
                          Simplify:  
                          \( \log_{5} x = (1/3) \log_{5} 64 + 2 \log_{5} 3 \)  
                          Distribute mid-unit quiz (15 minutes)  
                          After everyone is finished with quiz, announce project outlines for each group are due at the end of class.  
                          Go to the library for use of computers and books for project research and for use of powerpoint for each pair.  |
<table>
<thead>
<tr>
<th>Closure</th>
<th>Ask for questions about chapter projects, and give approval. Announce that computer lab will be used next time.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assignment</td>
<td>Outline of Chapter Project</td>
</tr>
</tbody>
</table>
### Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Exponential and Logarithmic Functions in the Computer Lab</td>
</tr>
</tbody>
</table>
| **Lesson Goals**             | By the end of the lesson, students should be able to:  
- Demonstrate their reasoning skills with the shifting of exponential and logarithmic functions by changing values of their relative equations. |
| **NCTM Standards Addressed** | Problem Solving  
Communication  
Reasoning  
Connections  
Functions |
| **Instructional Materials & Information Resources** | Graphing calculators (classroom set)  
Chalk and chalkboard  
Computer Lab for computers |
| **Introduction**             | Do warm-up problem in the room  
Attendance |
| **Instructional Sequence & Questions** | Go over quiz.  
Discuss progress, results with projects  
May have to review PowerPoint  
Announce computer lab day: use of internet worksheet on exponential and logarithmic functions  
Take class to computer lab to work on computer activity.  
Have them go to the site: [http://homepage.mac.com/apgogel](http://homepage.mac.com/apgogel)  
Click on explogact to download.  
Also open Function Probe: explain its use  
***If Function Probe is inaccessible, take graphing calculator case along for the students to use instead***  
Answer the questions to the activity on their own sheet of paper.  
When finished, groups can work on their PowerPoint presentations if they are not finished |
<table>
<thead>
<tr>
<th>If not finished with activity, students can print out remaining sheets for completion at home.</th>
</tr>
</thead>
</table>
| **Closure** | Ask for questions on their computer lab activities  
Briefly discuss next class's topic and shifting of graphs |
| **Assignment** | Answers to computer activity |
# Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Common Logarithms</td>
</tr>
</tbody>
</table>
| **Lesson Goals**             | By the end of the lesson, students should be able to:  
- Find common logarithms and antilogarithms of numbers.  
- Use common logarithms to compute powers and roots |
| **NCTM Standards Addressed** | Problem Solving  
Communication  
Reasoning  
Connections  
Functions |
| **Instructional Materials & Information Resources** | Graphing calculators (classroom set)  
Chalk and chalkboard |
| **Introduction**             | Do warm-up problem (5-Minute Check)  
Attendance |
| **Instructional Sequence & Questions** | Have students look at calculators and determine the bases for the logarithmic functions listed on them.  
Q: Why are these included on a calculator rather than some of the other logarithms? -> common logarithms!  
Define common logarithms: Logarithms with base 10  
Evaluate some common logarithms together:  
log 1000  
log 100  
log 10  
log 1  
log 0.1  
log 0.01  
Use definitions to evaluate logarithms:  
**Example 1:**  
Given that log 3 = 0.4771, evaluate:  
a. log 0.0003  
b. log 3000 |
Describe what the characteristic and mantissa are for a common logarithm.

**Example 2:** Psychology learning curve
Using \( u_n = kn^b \), find the number of hours to build 45th engine at a truck company assuming that the learning rate 82% and 510 hours were used to build the first one. And calculate the company will save on labor if the average worker earns $11.85 per hour.

Describe antilogarithms: \( \log x = a \), \( x = \text{antilog } a \)

**Example 3:** Chemistry-pH of solutions
Find the concentration of hydrogen ions of tomato juice if the juice has a pH of 4.1
Use the fact that \( \text{pH} = \log (1/H^+) \)

Discuss the solutions of solving problems with powers and roots: use logarithms and properties of logarithms!

**Example 4:** Use logarithms to evaluate: \( 16^{(4.9)^3} \)
\[ \frac{596^{(1/3)}}{596^{(1/3)}} \]

**Closure**
Q: What are the characteristic and mantissa for a common logarithm?
Do 2-3, 4-16 evens together

**Assignment**
Pg. 632-633
17-27 odd, 30-38 even, 40
Writing for journal:
Pg. 632 #1
## Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
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</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Exponential and Logarithmic Equations</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Use a graphing calculator to solve exponential and logarithmic equations and inequalities.</td>
</tr>
<tr>
<td></td>
<td>- Solve exponential and logarithmic equations.</td>
</tr>
<tr>
<td></td>
<td>- Solve exponential and logarithmic inequalities.</td>
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<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
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<tr>
<td></td>
<td>Communication</td>
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<td></td>
<td>Reasoning</td>
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<td>Connections</td>
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<tr>
<td></td>
<td>Functions</td>
</tr>
<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set)</td>
</tr>
<tr>
<td></td>
<td>Chalk and chalkboard</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>Do warm-up problem</td>
</tr>
<tr>
<td></td>
<td>(Change of base formula problem)</td>
</tr>
<tr>
<td></td>
<td>Attendance</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Ask and answer questions from homework assignment.</td>
</tr>
<tr>
<td></td>
<td>Collect homework</td>
</tr>
<tr>
<td><strong>Q:</strong> How could I solve the following equation: $3x^2 - 4 = x + 2$? Are there more than one method that I could use?</td>
<td></td>
</tr>
<tr>
<td><strong>Do graphing calculator activity: solving exponential and logarithmic equations and inequalities.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Solve the following by graphing together:</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Example #1:</strong> Solve $3^{4x^2} = (0.2)^{3x-1}$ to the nearest hundredth. What is the solution?</td>
<td></td>
</tr>
<tr>
<td><strong>Example #2:</strong> Solve $\log_4 (x + 2) \leq \log_5 (3x - 1)$ to the nearest hundredth. What is the solution?</td>
<td></td>
</tr>
<tr>
<td><strong>Have the students solve:</strong> $\log_4 (x - 3) &gt; \log_5 (2x - 7)$</td>
<td></td>
</tr>
<tr>
<td><strong>Start with solutions to exponential and logarithmic equations.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Describe Example:</strong> radioactive decay and half-life: $y = y_0 e^{\frac{\ln(2)}{T} t}$</td>
<td></td>
</tr>
</tbody>
</table>
Discuss keys to solving exponential equations:
1.) take the log of both sides of the equation.
2.) simplify using properties of logarithms.
3.) evaluate with a calculator.

**Example #3:** Solve the equation \(3.9^{x+1} = 28\)
How can we solve this? Use logarithms!!

**Example #4:** Half-life- P-32 is a radioactive substance with half-life of 14.3 days. How long would it take to reduce a 100 gram sample of P-32 to 15 grams?
Which equation do I apply here?
Note how the power property of logarithms allows us to transform the original equation into a linear equation in \(x\).

Have the students do the following population growth example:
**Example #5:** Population growth – A bacterial colony doubles every 45 minutes. How much time will it take for the population to increase 5 fold?
Which equation do I apply here?

**Example #6:** Solve \(5^{x^3} > 10^{x^6}\)
What do I do to both sides first?
Let the students complete the rest of the example.

Remind students of change of base formula: \(\log_a n = \log n / \log a\)

<table>
<thead>
<tr>
<th><strong>Closure</strong></th>
<th>Why can we use logarithms to solve equations involving variables that are exponents?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assignment</strong></td>
<td>Pg. 639-640</td>
</tr>
<tr>
<td></td>
<td>16-28, 29-39 odd, 42, 44</td>
</tr>
<tr>
<td></td>
<td>Writing for journal: Give an example of an exponential function that models population growth, and write why it is and give some values.</td>
</tr>
</tbody>
</table>
# Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Natural Logarithms</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Find natural logarithms of numbers.</td>
</tr>
<tr>
<td></td>
<td>- Solve equations using natural logarithms</td>
</tr>
<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
</tr>
<tr>
<td></td>
<td>Communication</td>
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<td></td>
<td>Reasoning</td>
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<td></td>
<td>Connections</td>
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<tr>
<td></td>
<td>Functions</td>
</tr>
<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set)</td>
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<tr>
<td></td>
<td>Chalk and chalkboard</td>
</tr>
</tbody>
</table>

**Introduction**
- Do warm-up problem
- Attendance

**Instructional Sequence & Questions**
- Ask and answer questions from homework assignment.
- Collect homework assignment
- Describe exponential growth and decay of populations: give general formula: \( y = ne^{kt} \) where:
  - \( y \) is the final amount, \( n \) is the initial amount, \( k \) is a constant, and \( t \) is time.
  - \( k > 0 \): growth, \( k < 0 \): decay

  **Q:** Where have we seen this type of equation before? -&gt; compound interest? Continuous?
  When would my bank account double in value given the equation: \( A = Pe^{rt} \)

- Define natural logarithms: used to solve problems involving \( e \).

  **Q:** What is the \( \ln \ e \)? -&gt; 1!

**Example #1:**
A major highway was constructed 5 years ago to accommodate a population of up to 40,000 commuters. It was estimated that the commuter population at that time was about 25,000. Today, there are
about 31,000 cars commuting on the highway each day.

a. If the commuter population continues to grow at this rate, when will the highway need to be upgraded again?

b. How many commuters will the upgraded highway have to accommodate to meet the demand for an additional 10 years?

Change the values in the previous example for the students to do:

a. 10 years ago: 50,000; 15,000 at that time; 40,000 today
b. for an additional 5 years

Describe antilogarithms for natural logarithms.

**Example #2**: Suppose two languages split off from a common ancestral language 1000 years ago. What portion of the words from the ancestral language would you expect to find in each of them today?

Use the following equation: \( n(r) = -5000 \ln r \), where \( r \) is the percent of the words from the ancestral language that are common to both languages now.

**Example #3**: When would my bank account double in value given the equation: \( A = Pe^n \) and with my rate being 5%?

Ask the students when it would triple.

Allow time at the end of class for students to finish chapter project.

If time, allow for some presentations

<table>
<thead>
<tr>
<th><strong>Closure</strong></th>
<th>Compare and contrast the common and natural logarithmic functions, and give examples of applications of each.</th>
</tr>
</thead>
</table>
| **Assignment** | Finish chapter project  
Pg. 643-645  
13-23 odd, 25-30, 31-39 odd  
Writing for journal:  
Pg. 643 #2 |
# Precalculus Lesson Plan

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Unit Review and Presentation of Projects</td>
</tr>
</tbody>
</table>
| **Lesson Goals**             | By the end of the lesson, students should be able to:  
- Recall main concepts from the unit. |
| **NCTM Standards Addressed** | Problem Solving  
Communication  
Reasoning  
Connections  
Functions |
| **Instructional Materials & Information Resources** | Graphing calculators (classroom set)  
Chalk and chalkboard  
Projector and computer for powerpoint presentations |
| **Introduction**             | Do warm-up problem (ln and e sheet)  
Attendance |
| **Instructional Sequence & Questions** | Pass back papers  
Ask and answer questions from homework assignment and collect it.  
PowerPoint presentation of chapter projects: Allow 5 minutes per group to present their project.  
Distribute rubric for grading for class to score each group.  
Grade each group based on their outline, presentation, and data.  
Also give extra credit!  
Ask for questions on any type of problem in unit.  
Use page 647 for in-class review:  
Use #1-9 from teacher's edition  
Do ln word problems to do in class as an application review for the test. |
| **Closure**                  | Ask for any questions for the test |
| **Assignment**               | Study for unit test  
Pg. 646-648  
1-44 |
<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Unit Test</td>
</tr>
</tbody>
</table>
| **Lesson Goals**                 | By the end of the lesson, students should be able to:  
- Demonstrate knowledge of concepts from unit on a test. |
| **NCTM Standards Addressed**     | Problem Solving  
  Communication  
  Reasoning  
  Connections  
  Functions |
| **Instructional Materials & Information Resources** | Graphing calculators (classroom set)  
  Chalk and chalkboard  
  Projector and Computer Cart |
| **Introduction**                 | Attendance  
  Discuss grades on projects  
  Do warm-up problem (exponential and logarithmic problems)  
  Show class population growth demonstration |
| **Instructional Sequence & Questions** | Ask and answer questions from review assignment.  
  Ask for any general questions regarding test .  
  Distribute Unit Test |
| **Closure**                      | Announce the start of series and sequences: use example of \(e\) |
| **Assignment**                   | Read 12-1 Pg. 656 – 660 |

**Precalculus Lesson Plan**
#35 From Textbook: Growth of a population of Organisms

Limited Growth: Using the Logistic Equation

\[ n = \frac{M}{1+b^c t} \]

\[ M = 200 \]
\[ b = 20 \]
\[ c = 0.35 \]

<table>
<thead>
<tr>
<th>Time</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.52381</td>
</tr>
<tr>
<td>1</td>
<td>13.25051</td>
</tr>
<tr>
<td>2</td>
<td>18.29541</td>
</tr>
<tr>
<td>3</td>
<td>25.00389</td>
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<td>4</td>
<td>33.71579</td>
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<td>5</td>
<td>44.68796</td>
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<tr>
<td>6</td>
<td>57.98566</td>
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<td>7</td>
<td>73.37102</td>
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<tr>
<td>8</td>
<td>90.24451</td>
</tr>
<tr>
<td>9</td>
<td>107.6981</td>
</tr>
<tr>
<td>10</td>
<td>124.6923</td>
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<tr>
<td>11</td>
<td>140.2923</td>
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<tr>
<td>12</td>
<td>153.8566</td>
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<tr>
<td>13</td>
<td>165.1059</td>
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<tr>
<td>14</td>
<td>174.0748</td>
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<td>15</td>
<td>181.0036</td>
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<td>16</td>
<td>186.2271</td>
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<td>18</td>
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<td>198.7317</td>
</tr>
<tr>
<td>24</td>
<td>199.1046</td>
</tr>
<tr>
<td>25</td>
<td>199.3682</td>
</tr>
</tbody>
</table>

What do you notice about the significance of \( M \)?

What would happen if you changed \( M \)?
Teaching the Logistic Function in High School

Make no bones about it: this article is highly biased in favor of teaching the logistic curve in high school. Both the exponential and logarithmic curves are staples of the precalculus curriculum. These functions are usually introduced in a third-year mathematics course. They reappear in the calculus curriculum for both Advanced Placement calculus and first-semester college calculus. The logistic function, a natural capstone to teaching exponential curves, is a logical extension of mathematics instruction to a variety of applications. It should be part of the repertoire of functions that we teach to eleventh- and twelfth-grade students.

The classic application of the logistic function is to model exponential growth with limiting factors. For example, the spread of disease and the profitability of a new business idea appear exponential in their early stages of growth. In later stages, the curve passes through a point of inflection, and growth slows as the curve rises to an upper limit. When the problem is the spread of disease, the adaptation of the population to the disease and the size of the population are limiting factors. In business, highly profitable ideas attract competition, which, in turn, inhibits untrammeled growth.

The function takes the form

\[ f(x) = \frac{L}{1 + a \cdot e^{-bx}} \]

The numerator, \( L \), is the upper limit to growth. The constants \( a \) and \( b \) affect the rapidity with which the function approaches \( L \). The detail with which the class analyzes this function depends on the ability of the class. An AP calculus student can be expected to analyze this model in terms of both the function and derivative. At that level, the constant \( b \) can be developed as the product of some constant, \( k \), and \( L \), the upper limit. The students in my lower-level calculus class can use the chain rule to find the derivative and can use their calculators to find the point of inflection. My precalculus students can successfully solve the function for values of \( x \) when they are given \( f(x) \). They have a better intuitive grasp of the meaning of the numbers that they derive than they often do when they solve exponential equations, even though the logistic function is more complex. My emphasis in this article is on precalculus students, although I will try to point out extensions that can apply to calculus students.

After a unit on exponential and logarithmic functions, I introduce the logistic function by example. The introduction is simple: I tell my class that I heard a great joke, but I am only going to tell it to two people. They each can tell it to two more people, and so on, while we keep track of the number of iterations that are required before everybody has heard the joke. We do not keep track of who tells the joke to whom, as we do not limit telling the joke only to those who have not heard it. As I point out, the activity does not work in that manner. Students will talk between periods, pass notes, and corner their friends at lunch. Sooner or later, someone that they tell will have already heard the joke. In fact, as time passes, fewer and fewer people have not already heard the joke. If we restrict the population to mathematics students, or simply to my class, we have a clear upper limit on the total number of people in the population.

A plot showing the spread of the joke might show the time periods along the x-axis. The y-axis can represent the number of people who have heard the joke. Figure 1a shows the spread of the joke during the first six time periods. I used a program that...
I wrote for the TI-83 graphing calculator (see program 1) to generate data and set up the calculator before class. With the calculator connected to a viewscreen, I ask students how they would model the data at all stages. Opposed to exponential curves, I use the viewscreen gram I wrote for the TI-83 graphing calculator (see Vol. 95, No.4. April works well before class. With the calculator connected to a

The class spends fifteen to twenty minutes experimenting and organizing information. I try to make this process efficient by asking students to post their results as soon as most groups have them. I can then reserve time to close the class with a set of questions, such as those in figure 3.

Although the problems on sheet 1 are designed for precalculus students, a calculus teacher can readily use them to help students move into more complicated analyses. For instance, the logistic function makes an excellent introduction to limits.

I ask students to investigate some fundamental questions

Effects of changing $L$
- Negative $L$ reflects the curve in the $x$-axis.
- Larger $L$ raises the upper limit.
- Smaller $L$ lowers the upper limit.

Effects of changing $a$
- Larger $a$ shifts the curve to the right.
- $0 < a < 1$ shifts the curve to the left.

Effects of changing $b$
- Larger $b$ makes the curve rise steeply.
- $0 < b < 1$ makes the curve shallower.

Fig. 2
The logistic function

\[ f(x) = \frac{30}{1 + 10e^{-0.5x}}. \]

describes the spread of good news through my family. The function \( f(x) \) gives the numbers of family members who have heard the news after \( x \) days. Many family members are close by or in constant contact. Others will learn by using snail mail. The fathers are typically the last to learn anything. The teacher can ask the following questions about this function:

- How would you interpret a fractional day? A fractional person? How should you round these quantities—up, down, or algebraically?
- How many family members could get the news?
- How many people have heard the news after five days?
- Graph this function on a graphing calculator, and estimate the number of days that must pass before at least twenty-five people have heard the news. How would you answer this question algebraically?
- What is the lowest value of \( x \) for which your calculator reports that all thirty members of my family have heard the news? Is the calculator correct, or did it give up?
- Solve \( f(x) \) directly, that is, algebraically, for \( f(x) = 27 \). Report \( x \) to five places, and then round to the nearest day.
- Create a new graph showing how many people heard the news (for the first time) on any given day. Graph these data on the same screen as our logistic function. How would you estimate these data? This graph is the rate-of-change graph, or the graph of the first derivative.

Calculus students should be able to differentiate the logistic function and use the derivative and a calculator to find the point of inflection. Part of the appeal of the logistic function is the number of ways in which it can be used to illustrate important concepts in advanced mathematics.

I typically tell students to discuss the problems in their groups and write up their responses individually for homework. The next day, I ask students to share their discoveries. If this activity is too independent for the class, I give them a few simple problems for practice and use the next day to work on the remaining problems. An advanced group can discuss the change in slope over the point of inflection, limits, and relative rates of change. This discussion—at whatever level—is an important way to verify the progress of the class. It is also a great lead-in to a more independent project.

On the third and fourth days, I typically conclude this activity with the logistic function and assign a project. I have found that students better retain and understand material if they participate in mathematics from a variety of approaches and activities. Writing and exploring are crucial to understanding; communication skills are crucial to success.

Communication skills are crucial to success

The project that I designed for this activity is given on sheet 2. It uses the program LOGIST, which I wrote for the TI-83 (see program 1). The project mimics the spread of an infection through a given population and asks students to use the power of the graphing calculator to create a model for this miniepidemic.

The program asks students to choose a size for the population and then select the number of time periods to examine. A typical population size is between fifteen and thirty, and the number of time periods is between seven and sixteen. The program then starts the simulation by randomly selecting a "carrier" to bring the infection to the population. At the end of each time period, the calculator reports the total number of people infected and the number of people who have been newly infected during that time period. Students appreciate their ability to experiment with different parameters: we typically try a few in class before they leave for their next class.

The most successful students experiment several times before transcribing the data for the "good" experiment. They make a chart on a separate piece of paper, on which they show both the total number of infections and new infections. They perform the regression on the total infection data to obtain the logistic function and include a graph with Plot 1 displaying total infections against time, Plot 2 with new infections, and the regression line. I think that clearly modeling these steps in the classroom is important so that students can see the kind of detail needed in making a sophisticated analysis of experimental data.

I ask students to try regressing different functions against the time and infection data to determine the function that best describes the behavior of the data. They write a brief paragraph describing the spread of the infection through the target group. When this project is successful, my only role is to move from group to group, answering questions and indicating strengths and weaknesses in their arguments. This project has sometimes been unsuccessful because students failed to participate actively in student-originated discussion. In those groups, I have given strict guidelines on the parameters of the investigation and checked back frequently to ensure progress. This less-successful outcome has been the exception rather than the rule.

As with any curriculum topic, teaching the logistic function must advance the students' understanding of mathematics. The explicit goal in teaching is furthering mathematics; my implicit goal is to help my students acquire and assess knowledge. Students spend the third and fourth days of this activity discussing the project and writing. This activity closes the entire unit on exponential functions and maintains connections with the course-long investigation of functions.
Using Microsoft Excel
Population Growth and Compound Interest

First go to:
http://homepage.mac.com/apgogel

Download:
Maths331#1.xls

Look at the different types of population growth.
The last one is from your homework assignment #35.

Compound Interest

\[ A = P \left(1+\frac{r}{n}\right)^{nt}\]

Example:

<table>
<thead>
<tr>
<th>Principal</th>
<th>Interest rate of 9% compounded monthly for 2 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1000)</td>
<td>(B19)</td>
</tr>
<tr>
<td>(12)</td>
<td>(B20)</td>
</tr>
<tr>
<td>(0.09)</td>
<td>(B21)</td>
</tr>
<tr>
<td>Year</td>
<td>Balance</td>
</tr>
<tr>
<td>1 (at A23)</td>
<td>(B19\left(1+B21/B20\right)^{(B20*A23)})</td>
</tr>
<tr>
<td>2 (at A24)</td>
<td>(B19\left(1+B21/B20\right)^{(B20*A24)})</td>
</tr>
</tbody>
</table>

Type in Letter and number location of your values to go with the equation. Put an = sign first!!
You only have to do it once! Click and Drag to complete a larger table!

Assignment (use page 620 from textbook for reference)

Now complete and print out a list of balances for 50 years!!
Principal: 5000
Interest Rate of 6.5%
Compounded Quarterly

Start with First Balance Equation!
Then click on the lower right corner of the cell and drag down to 50!
If you have questions, ask Mr. Gogel.

Extra Credit
Modify the program so you can print comparisons of balances for two different interest rates for the same time, rate, principal, and number of times compounded
Precalculus warm-up

Simplify:
8. \[ \sqrt[3]{x(x - 1)} \]
9. \[ \frac{u + 1}{\sqrt{u}} \]
10. \[ x^2(\sqrt{x} - \sqrt[3]{x} + 3) \]

11. Give a decimal approximation of \( e \) to 3 places.

12. \( e^0 = \) __________

13. \[ \frac{e}{\sqrt{e}} \]

14. In calculus, what is the derivative of \( y = e^x \)?
   Do you know why?

---

Precalculus warm-up - put on board!

Simplify:
8. \[ \sqrt[3]{x(x - 1)} \]
9. \[ \frac{u + 1}{\sqrt{u}} \]
10. \[ x^2(\sqrt{x} - \sqrt[3]{x} + 3) \]

11. Give a decimal approximation of \( e \) to 3 places. \( \approx 2.718 \)

12. \( e^0 = \) __________

13. \[ \frac{e}{\sqrt{e}} = e^{\frac{1}{2}} \]

14. In calculus, what is the derivative of \( y = e^x \)?
   Do you know why?

Because the slope of any tangent line to \( y = e^x \) is the function value at that point!

\[ e^x \text{ at } x = 0 \quad \lim_{x \to 0} y = x + 1 \]
Twizzlers Warm-up

Complete the following:

1.) Take one Twizzler. Measure and record its length.

2.) Cut or bite the Twizzler exactly in half. Eat on half, measure and record the other half.

3.) Continue this process until the Twizzler is gone or you cannot cut or measure anymore.

   Discuss: Mathematically speaking, is the Twizzler ever supposed to be completely gone?

4.) Using the data that you have created, plot your data based on the length versus the number of cuts.

   Discuss: Based on your plot, what kind of function does it look like?

5.) Write a function that would model your plot.

Twizzlers Warm-up – answers in inches

Complete the following: (Answers may vary based on choice of measurement)

1.) Take one Twizzler. Measure and record its length in inches or cm. (8 inches)

2.) Cut or bite the Twizzler exactly in half. Eat on half, measure and record the other half. (4 inches)

3.) Continue this process until the Twizzler is gone or you cannot cut or measure anymore. (2 in., 1 in. 

   $\frac{1}{2}$ in., $\frac{1}{4}$ in.,...)

   Discuss: Mathematically speaking, is the Twizzler ever supposed to be completely gone? (no)

4.) Using the data that you have created, plot your data based on the length versus the number of cuts.

   Discuss: Based on your plot, what kind of function does it look like? (An exponential function)

5.) Write a function that would model your plot. ($y = \left(\frac{1}{2}\right)^x$ It would exactly model the data.)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
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<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>0.125</td>
</tr>
</tbody>
</table>
1. Evaluate $81^{3/4}$

2. Evaluate $\frac{25^{3/4}}{25^{1/4}}$

3. Graph the equation $y = 0.1^x$

Evaluate each expression to the nearest ten thousandth.

4. $e^{3.8}$

5. $5\sqrt{3}e$

6. $e^{2/5}$

Solve each equation.

7. $\log_x 512 = 3$

8. $\log_5 5 = x$

9. $\log_5 (x^2 - 30) = \log_5 6$
1. Evaluate $81^{3/4}$

$$\left(\frac{4\sqrt[3]{81}}{3}\right)^3 = 27$$

3. Graph the equation $y = 0.1^x$

Evaluate each expression to the nearest ten thousandth.

4. $e^{3.8}$

44.7011

5. $5\sqrt[3]{e}$

14.2783

6. $e^{2.5}$

1.4918

Solve each equation.

7. $\log_5 512 = 3$

$x^3 = 512$

$x^3 = 8^3$

$x = 8$

8. $\log_5 5 = x$

$(5^2)^x = 5^1$

$x = 2$

9. $\log_5 (x^2 - 30) = \log_5 6$

$x^2 - 30 = 6$

$x^2 = 36$

$x = \pm 6$
Precalculus warm-up and review

**ln and e**

Evaluate each:

1. \( \ln 1 = \)
2. \( e^0 = \)

3. \( \ln e = \)
4. \( \ln e^2 = \)

5. Write \( e^{-1} \) as a fraction:
6. Write \( e^{-5} \) as a fraction:

7. \( \ln (1/e) = \)
8. \( \ln (1/e^3) = \)

9. \( \ln \sqrt{e} = \)
10. \( \ln \sqrt[3]{e^2} = \)

11. \( \ln 0 = \)
12. \( \ln -1 = \)

13. \( \ln (-e) = \)
14. \( e^{\ln x} = \)

15. \( e^x \cdot e^x = \)
Precalculus warm-up and review

**ln and e**

Evaluate each:

1. \( \ln 1 = 0 \)  
2. \( e^0 = 1 \)

3. \( \ln e = 1 \)  
4. \( \ln e^2 = 2 \)

5. Write \( e^1 \) as a fraction:
   \[
   \frac{1}{e}
   \]

6. Write \( e^5 \) as a fraction:
   \[
   \frac{1}{e^5}
   \]

7. \( \ln (1/e) = -1 \)  
8. \( \ln (1/e^7) = -7 \)

9. \( \ln \sqrt{e} = \frac{1}{2} \)  
10. \( \ln \sqrt[3]{e^2} = \frac{2}{3} \)

11. \( \ln 0 = \phi \)  
12. \( \ln -1 = \phi \)

13. \( \ln (-e) = \phi \)  
14. \( e^{\ln x} = x \)

15. \( e^x \cdot e^x = e^{2x} \)
The following are some sites that may help:

http://www.census.gov/ipc/www/idbrank.html

http://www.census.gov/cgi-bin/popclock

http://www.censusindia.net/ (I suggest that you use this site and country!!!)

http://www.popnet.org/

http://www.ibiblio.org/lunarbin/worldpop

http://www.un.org/popin/data.html

www.yahoo.com -> population

www.google.com -> population

**Grading:**

Your group will be graded based on the following:

Completion of required slides: (10 points)
- title
- table
- graph
- explanation
- sources

Presentation (5 points)

Accuracy of data (5 points)

Individual contribution to group (5 points)

Opportunity for extra credit is also available.
<table>
<thead>
<tr>
<th>Name</th>
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<tbody>
<tr>
<td>Precalculus Population Growth PowerPoint Presentations</td>
</tr>
<tr>
<td>Grading:</td>
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<tr>
<td>Completion of required slides: 2 pts each</td>
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<tr>
<td>• Title</td>
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<tr>
<td>• Table</td>
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<td>• Graph</td>
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<tr>
<td>• explanation</td>
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<tr>
<td>• sources</td>
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<tr>
<td>Presentation</td>
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<tr>
<td>Accuracy of data</td>
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<tr>
<td>Individual contribution to group</td>
</tr>
<tr>
<td>Extra Credit Using Excel:</td>
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<tr>
<td>Total</td>
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Name ____________________

Precalculus Population Growth PowerPoint Presentations | Group: ____________________

Grading:

Completion of required slides: 2 pts each
- Title
- Table
- Graph
- explanation
- sources

Presentation
Accuracy of data
Individual contribution to group
Extra Credit Using Excel:
Total / 25 points
Rate your team members based on their role in this project: (1 - 5) with 5 being best.

Member #1: ___________________ Score: _____

Member #2: ___________________ Score: _____ (if applicable)

Rate your team members based on their role in this project: (1 - 5) with 5 being best.

Member #1: ___________________ Score: _____

Member #2: ___________________ Score: _____ (if applicable)

Rate your team members based on their role in this project: (1 - 5) with 5 being best.

Member #1: ___________________ Score: _____

Member #2: ___________________ Score: _____ (if applicable)
Precalculus Test: Chapter 11
Exponential and Logarithmic Functions

Name __________________________ Date ____________ Period ______

Non-calculator Section

1. Evaluate \( \sqrt{\frac{32}{32^\frac{1}{8}}} \).

2. Express \( \sqrt[4]{64a^4} \) using rational exponents.

3. Express \( \sqrt[4]{2^4 \cdot 4} \) using radicals.

4. Evaluate \( \frac{8^{\frac{1}{5}}}{8^{\frac{2}{5}}} \).

5. Express \( \sqrt[5]{5ab^{20}} \) using rational exponents.

6. Express \( a^{\frac{4}{5}} \cdot b^{\frac{1}{3}} \) using radicals.

7. Write \( 0.2^3 = 0.008 \) in logarithmic form.

8. Write \( \log_7 (\sqrt{7})^6 = 3 \) in exponential form.

9. Write \( 4^{-2} = \frac{1}{16} \) in logarithmic form.

10. Solve \( \log_x \frac{1}{216} = -3 \).
Name ________________________

Graph each equation or inequality. (2 points each)

11. \( y = 0.5^x + 2 \)

12. \( y \leq 4^x \)

Solve each equation. (2 points each)

13. \( \log_4 (2x - 1) = \log_4 16 \)

14. \( \log_5 3 + \frac{1}{2} \log_5 36 = \log_5 x \)

15. \( \log x 64 = 3 \)

16. Evaluate \( \log_{27} 81 \).

17. Solve \( \log_2 0.125 = x \)
Precalculus Test: Chapter 11
Exponential and Logarithmic Functions

Calculator Section

Solve each equation using logarithms. Round your answer to the nearest ten-thousandth.

18. \( 5^x = 45 \)  
   \( x = \) ___________ (+2)

19. \( 6^{x+2} = 10.3 \)  
   \( x = \) ___________ (+2)

20. Ms. Cubbatz invested a sum of money in a certificate of deposit that pays 8% interest compounded continuously. Recall that the formula for the amount in an account earning interest compounded continuously is \( A = Pe^{rt} \). If Ms. Cubbatz made the investment on January 1, 1989 and the account is worth $12,000 on January 1, 1993, what was the original amount in the account?

   \( P = \) ___________ (+4)

21. To the nearest dollar, find the future value of $500 invested at 9% for 4 years in an account that is compounded continuously.

   \( A = \) ___________ (+4)

22. Mide Kallenberg deposited some money in a bank that earns 5.6% interest compounded continuously. How long would it take to double the amount of money in Mr. Kallenberg’s account?

   \( t = \) ___________ (+4)
Bonus Problems

1.) After 13 years, 2.1 pounds of radioactive material remain from a 7-pound sample. Using the equation, \( y = y_0 \cdot 0.5^{t/T} \), find the half-life of the material.

2.) Explain why \( \ln 0 = x \) has no real solutions.
1. Evaluate $\frac{\sqrt{32}}{32^{\frac{1}{2}}}$. 
\[ \frac{2}{4} = \frac{1}{2} \]

2. Express $\sqrt[6]{64b^4}$ using rational exponents.
\[ 4^{\frac{1}{6}} b^{\frac{4}{6}} = 2b^{\frac{3}{3}} \]

3. Express $2^{\frac{1}{2}} x^\frac{4}{5}$ using radicals.
\[ \sqrt[5]{2x^4} = \sqrt[5]{2} \cdot \sqrt[5]{x^4} \]

4. Evaluate $\frac{8^{\frac{1}{3}}}{8^{\frac{1}{8}}}$. 
\[ 8^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2} \]

5. Express $\sqrt[4]{35abc^{10}}$ using rational exponents.
\[ 35^{\frac{1}{4}} a^{\frac{1}{4}} b^{\frac{2}{4}} c^{\frac{5}{4}} = 35^{\frac{1}{4}} a^{\frac{1}{4}} b^{\frac{5}{4}} \]

6. Express $a^{\frac{2}{3}} b^{\frac{1}{3}}$ using radicals.
\[ \sqrt[3]{a^2 b} \]

7. Write $0.2^3 = 0.008$ in logarithmic form.

8. Write $\log_7 (\sqrt{7})^6 = 3$ in exponential form.
\[ \sqrt{7}^3 = 3 \rightarrow 7^\frac{3}{2} = (\sqrt{7})^3 \]

9. Write $4^{-2} = \frac{1}{16}$ in logarithmic form.
\[ \log_4 \frac{1}{16} = -2 \]

10. Solve $\log_2 \frac{1}{216} = -3$. 
\[ 2^{-3} = \frac{1}{216} \]
\[ x^3 = 216 \]
Precalculus Test: Chapter 11
Exponential and Logarithmic Functions

Name

**Calculator Section**

Graph each equation or inequality. (2 points each)

11. \( y = 0.5^x + 2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>4.4</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>2.25</td>
</tr>
<tr>
<td>-2</td>
<td>6</td>
</tr>
</tbody>
</table>

12. \( y \leq 4^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>( \frac{1}{16} )</td>
</tr>
<tr>
<td>-1</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Solve each equation. (2 points each)

13. \( \log_4 (2x - 1) = \log_4 16 \)

\[ 2x - 1 = 16 \]
\[ 2x = 17 \]
\[ x = \frac{17}{2} \]

13. \( x = \frac{17}{2} \) (+2)

14. \( \log_5 3 + \frac{1}{2} \log_5 36 = \log_5 x \)

\[ \log_5 3 + \log_5 36 = \log_5 x \]
\[ \log_5 18 = \log_5 x \]
\[ x = 18 \]

14. \( x = 18 \) (+2)

15. \( \log_x 64 = 3 \)

\[ x^3 = 64 \]
\[ x = 4 \]

15. \( x = 4 \) (+2)

16. Evaluate \( \log_7 81 \).

\[ 2^7 x = 81 \]
\[ 3^4 = 81 \]
\[ x \cdot \sqrt[4]{3} = 81 \Rightarrow 27 \cdot 81 = 81 \]

16. \( x = \sqrt[3]{81} \) (+2)

17. Solve \( \log_2 0.125 = x \)

\[ 2^x = 0.125 \]
\[ 2^x = \frac{1}{8} \]
\[ 2^x = 2^{-3} \]

17. \( x = -3 \) (+2)
Precalculus Test: Chapter 11
Exponential and Logarithmic Functions

*Calculator Section*

Solve each equation using logarithms. Round your answer to the nearest ten-thousandth.

18. \(5^x = 45\)
   \[\log_5 45 = \log_5 5^x\]
   \[x \log_5 5 = \log_5 45\]
   \[x = \frac{\log_5 45}{\log_5 5}\]
   \[x \approx 2.3652\]

19. \(6^{x+2} = 10.3\)
   \[\log_6 10.3 = \log_6 6^{x+2}\]
   \[(x+2) \log_6 6 = \log_6 10.3\]
   \[x+2 = \frac{\log_6 10.3}{\log_6 6}\]
   \[x = \frac{\log_6 10.3}{\log_6 6} - 2\]

20. Ms. Cubbatz invested a sum of money in a certificate of deposit that pays 8% interest compounded continuously. Recall that the formula for the amount in an account earning interest compounded continuously is \(A = Pe^{rt}\). If Ms. Cubbatz made the investment on January 1, 1989 and the account is worth \(12,000\) on January 1, 1993, what was the original amount in the account?
   \[12,000 = Pe^{(0.08)(4)}\]
   \[12,000 = P(1.372)\]
   \[P = 8,713.79\]

21. To the nearest dollar, find the future value of \(500\) invested at 9% for 4 years in an account that is compounded continuously.
   \[A = 500e^{0.09 \cdot 4}\]
   \[A = 716.66\]
   \[A = 717\]

22. Mide Kallenberg deposited some money in a bank that earns 5.6% interest compounded continuously. How long would it take to double the amount of money in Mr. Kallenberg's account?
   \[A = Pe^{rt}\]
   \[2P = Pe^{0.056t}\]
   \[2 = e^{0.056t}\]
   \[\ln(2) = \ln(e^{0.056t})\]
   \[\ln(2) = 0.056t \ln(e)\]
   \[\ln(2) = 0.056t\]
   \[t = \frac{\ln(2)}{0.056} = 12.38\text{ yrs}\]
Bonus Problems

1.) After 13 years, 2.1 pounds of radioactive material remain from a 7-pound sample. Using the equation, \( y = y_0 \cdot 0.5^{t/T} \), find the half-life of the material.

\[
2.1 = 7 \cdot 0.5^{13/T} \\
3 = 0.5^{13/T} \\
\log 3 = \log 0.5^{13/T} \\
\log 3 = \frac{13}{T} \log 0.5 \\
\frac{\log 3}{\log 0.5} = \frac{13}{T} \\
T = 7.48 \text{ years}
\]

2.) Explain why \( \ln 0 = x \) has no real solutions.

Answers may vary


\[ e^x \neq 0 \] - there is no number that could be an exponent on \( e \) that equals zero. \( e \) to any power is never equal to zero.
Other Student Materials/Websites

- For Compound Interest Computer Activity Lesson:
  A look at the Logistic Equation
  
  http://homepage.mac.com/apgogel
  Download: Maths331#1.xls

- For Exponential and Logarithmic Functions Computer Activity:
  
  http://homepage.mac.com/apgogel
  Download: explogact
## Aaron Gogel

### Textbook: Merrill Advanced Mathematical Concepts

### Unit Goals: By the end of the unit, should be able to: use the properties of exponents and logarithms, solve equations by using the properties of exponents and logarithms, and graph exponential and logarithmic functions.

### Instructional Materials & Technologies: Classroom set of graphing calculators, graph paper, overhead projector

### Information Resources: Computer lab for Internet and use of programs

### Instructional Overview:

<table>
<thead>
<tr>
<th>Day</th>
<th>Lesson Title/Activity</th>
<th>Pages</th>
<th>Assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.1: Rational Exponents</td>
<td>598-604</td>
<td>Pg. 602-604 6: 15-33 odd, 36-52 even, 54</td>
</tr>
<tr>
<td>Tues 3/19</td>
<td>Introduce chapter project</td>
<td></td>
<td>8: 14-34 even, 35-51 odd, 56</td>
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<tr>
<td></td>
<td>Graphing calculator activity with exponential functions</td>
<td></td>
<td></td>
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<tr>
<td>2</td>
<td>PT3 DAY</td>
<td>606-613</td>
<td>Read 11.2</td>
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<tr>
<td>Thur. 3/21</td>
<td>Movie: Exponential Functions</td>
<td></td>
<td>Practice 11.2</td>
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<tr>
<td>3</td>
<td>11.2 Exponential Functions</td>
<td>606-619</td>
<td>Pg. 611-613 6: 39 - 43</td>
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<td>Mon. 4/1</td>
<td>11.3 The Number e</td>
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<td>8: 39 - 43</td>
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<td>Pg. 617-619 6: 13-17 odd 26, 28, 35</td>
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<td>8: 12-16 even 25, 27, 35</td>
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<td>4</td>
<td>Computer Lab</td>
<td>620</td>
<td>Compound Interest Spreadsheet</td>
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<td>Wed. 4/3</td>
<td>Population</td>
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<td>Growth/Compound Interest Demonstration and Activity</td>
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<td>Powerpoint review</td>
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<td>5</td>
<td>11.4 Logarithmic Functions</td>
<td>622-628</td>
<td>Practice 11-4</td>
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<tr>
<td>Fri. 4/5</td>
<td>Graphing calculator activity with logarithmic functions</td>
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<tr>
<td>6</td>
<td>Quiz</td>
<td></td>
<td>Outline of Chapter Project for powerpoint</td>
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<tr>
<td>Tues. 4/9</td>
<td>Library for chapter project research and work on powerpoint</td>
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<tr>
<td>7</td>
<td>Exponential and Logarithmic Functions Computer Activity</td>
<td></td>
<td>Answers to activity</td>
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<tr>
<td>Thur. 4/11</td>
<td>with Function Probe</td>
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<tr>
<td>Date</td>
<td>Schedule</td>
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</tbody>
</table>
| 8 Mon. 4/15 | **11.6 Exponential and Logarithmic Equations**  
Graphing calculator activity: equations and inequalities  
**11.7 Natural Logarithms**  
Finish chapter project  
Practice 11-6 and 11-7 |
| 9 Wed. 4/17 | **Unit Review**  
Chapter Project Presentations  
Page 646-648  
1-44 |
| 11 Fri. 4/19 | **Unit Test**  
Read 12-1 Pg. 656 - 660 |

**Assessment Overview:**

<table>
<thead>
<tr>
<th>Formative</th>
<th>Goals</th>
<th>Instruments &amp; Procedures</th>
</tr>
</thead>
</table>
| Students will be informed on a daily basis how they are doing in the class.  
Students will be given the opportunity to write in journals on particular problems as well as others to gather feedback on a weekly basis. | Homework assignments are graded in class to show students their performance with the exercises. |

<table>
<thead>
<tr>
<th>Summative</th>
<th>Goals</th>
<th>Instruments &amp; Procedures</th>
</tr>
</thead>
</table>
| Students’ achievement will be summarized through one mid-unit quiz, a project, and one exam.  
This achievement summary will be based on their performance with material discussed and covered. | The quiz will be held approximately at the middle of the unit and will cover material discussed in 11.1 – 11.4, with the unit project following. The unit exam will take place at the end of the unit.  
The exam will be assessed fairly to meet unit goals.  
Group members with a rubric will score group participation. |

<table>
<thead>
<tr>
<th>Grading</th>
<th>Goals</th>
<th>Instruments &amp; Procedures</th>
</tr>
</thead>
</table>
| Grading will consist of points awarded for homework, the quiz, the exam, the project, and especially participation in class and on the project. Each of these will be a percentage of unit grade. | Grading breakdown for unit:  
Homework: 15%  
Quiz: 15%  
Project: 15%  
Exam: 40%  
Class and Group Participation: 15% |

**Other:**

Students will also be graded by a rubric for class participation.
<table>
<thead>
<tr>
<th>Aaron Gogel</th>
<th>Precalculus</th>
<th>Lesson Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Textbook</strong></td>
<td>Merrill Advanced Mathematical Concepts</td>
<td></td>
</tr>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Chapter 9 test and Chapter 11 introduction</td>
<td></td>
</tr>
</tbody>
</table>
| **Lesson Goals** | By the end of the lesson, students should be able to:  
- Show their level competency with the concepts from the chapter on a test |  |
| **NCTM and Indiana Standards Addressed** | Problem solving, communication, reasoning, connections, representation, functions, geometry |  |
| **Instructional Materials & Information Resources** | Graphing calculators (classroom set)  
Dry erase markers and board |  |
| **Introduction** | Do warm-up problem |  |
| **Instructional Sequence & Questions** | Ask for questions for the test  
Do example problems based on each question |  |
| **Questioning:**  
- follow up  
- confirm  
- redirect | When everyone is finished, discuss next chapter project with regards to population: have everyone think of a country whose population they would like to model |  |
| Questioning:  
- follow up  
- confirm  
- redirect | If time, discuss and review properties of exponents: have students give examples for each:  
Product property \( a^m a^n = a^{m+n} \)  
Power of a power \( (a^n)^m = a^{mn} \)  
Power of a quotient \( \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \)  
Power of a product \( (ab)^n = a^n b^n \)  
Quotient property \( \frac{a^m}{b^n} = a^{m-n} \)  
**Examples** |  |
| **Closure** | Announce covering of exponential and logarithmic functions next. |  |
| **Assignment** | Read first part of chapter 11 |  |

\[
b^{\frac{m}{n}} = \sqrt[n]{b^m}
\]

\[
b^{\frac{m}{n}} = \sqrt[n]{b^m}
\]
# Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Rational Exponents and Exponential Functions with Graphing Calculators</td>
</tr>
</tbody>
</table>
| Lesson Goals           | By the end of the lesson, students should be able to:  
- Use properties of exponents.  
- Evaluate and simplify expressions containing rational exponents  
- Graph exponential functions on a graphing calculator and note changes in graphs. |
| NCTM Standards Addressed | Problem Solving  
Communication  
Reasoning  
Connections  
Algebra  
Mathematical Structure |
| Instructional Materials & Information Resources | Graphing calculators (classroom set)  
Overhead projector  
Graph transparency (pre-made)  
Graphing Calculator view screen for the projector  
Chalk and chalkboard |
| Introduction           | Do warm-up problem |
| Instructional Sequence & Questions | Introduce chapter project: populations of countries  
Discuss scientific notation  
List properties of exponents and examples of each  
Put students in groups, assign a property, have them give examples of how it is used, and have them show how the property can be derived from the definitions of exponents.  
Example #1: The diameter of the planet Jupiter is $1.427 \times 10^5$ km. What is its volume?  
\[ V = \frac{4}{3} \pi r^3 \rightarrow 1.521 \times 10^5 \text{ km}^3 \]
Introduce rational exponents  
\[ a^{\frac{m}{n}} = \sqrt[n]{a^m} \]
Example #2: Evaluate the following:
1. \(625^{1/4} = \sqrt[4]{625} = 5\)
2. \(3^{1/2} \times 21^{1/2} = \sqrt{3} \times \sqrt{21} = \sqrt{63} = 3\sqrt{7}\)

Define rational exponents
\(b^{1/n} = \sqrt[n]{b}\)

Example #3: Evaluate \(81^{5/4}\)
\(\left(\sqrt[4]{81}\right)^5 = (3)^5 = 243\)
Note: it is easier to first calculate the fourth root of 81 and then raise it to the fifth power of 81.

Example #4: Express: fourth root of \(27x^4 y^3\) using rational exponents
\(\sqrt[4]{27x^4 y^3} = x\sqrt[4]{27} y\sqrt[4]{3}\)

Example #5: Express \((5a)^{2/3} b^{5/2}\) using radicals
\(\frac{5\sqrt[3]{a^2}}{\sqrt[2]{b}} = \frac{5\sqrt[6]{a^2} \sqrt[3]{b}}{\sqrt[2]{b}}\)

Example #6: Simplify third root of \(a^4 b^3\)
\(\sqrt[3]{a^4 b^3} = ab\sqrt[3]{a b}\)

Sketch the graph of the following situation:
In the first week of this month, I put one dollar in the savings. Each week there after, I put away double the previous week's amount.

Discuss graphs
- Get out overhead graph.
Graphing calculator activity
Graphing exponential functions with the class
Graph \(y = 3^x\) and \(y = (1/3)^x\) on the same set of axes on calculator
Q: How do these graphs compare? Is there some sort of reflection?
Reflection about \(y = x\).
Graph \(y = 9^{2+x}\)
Q: What happened to the graph? Was it shifted?
Shifted left 2
\(y = 9^{2+x} - 1 \rightarrow \text{shifted down 1}\)

Closure
Have each student recite one of the properties of exponents

Assignment
Pg. 602-604
6: 15-33 odd, 36 – 52 even , 54
8: 14 – 34 even, 35 – 51 odd, 56
11.1: Rational Exponents

Today's lesson in precalculus served primarily as an introduction to exponential and logarithmic functions. Its purpose was to be a review of the basic concepts behind exponential functions. After discussing the test given the previous class meeting, the rules of exponents were readdressed and reviewed, and the simplification of expressions containing both rational exponents and radicals were also reviewed. Since these topics were review, the students had no trouble with it, and the topics were covered fairly quickly.

I think the best parts of the lesson were the group work on the rules of exponents and the graphing calculator activity with exponential functions. First, I put students in groups to give examples involving one of the five rules of exponents and to prove them. I think that it is important for students to not only give their own examples, but to be able to justify or prove them as well. This not only facilitates cooperative groups, but discussion as well. The rules of exponents were review for the students, and they are important for the study of this chapter.

Besides the group work, I think that the graphing calculator activity was a great introduction to exponential functions. Using the overhead graphing calculator, I showed four examples of exponential functions, each showing an introduction to the basic shape of the functions, and what values cause shifts to the graphs. This activity set me up to teach the next section on exponential functions. I think that it is important to use my resources regarding technology to give a visual representation of the types of graphs of exponential functions.
**Precalculus Lesson Plan**

<table>
<thead>
<tr>
<th>Textbook</th>
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<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>PT3 Day: Substitute Teacher Plan</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Evaluate expressions with irrational exponents.</td>
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<tr>
<td></td>
<td>- Graph exponential functions.</td>
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<td></td>
<td>- Graph exponential inequalities</td>
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<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
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<td></td>
<td>Communication</td>
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<td>Reasoning</td>
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<td>Connections</td>
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<td></td>
<td>Algebra</td>
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<tr>
<td></td>
<td>Mathematical Structure</td>
</tr>
<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>TV and VCR</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>SUBSTITUTE:</td>
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<tr>
<td></td>
<td>Take attendance</td>
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<tr>
<td></td>
<td>Leave list of absentees</td>
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<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Show movie over exponential functions</td>
</tr>
<tr>
<td></td>
<td>After movie, give students assignment</td>
</tr>
<tr>
<td><strong>Closure</strong></td>
<td>None</td>
</tr>
<tr>
<td><strong>Assignment</strong></td>
<td>Read 11.2</td>
</tr>
<tr>
<td></td>
<td>Practice 11.2</td>
</tr>
</tbody>
</table>
Exponential Functions

Use a calculator to evaluate each expression to the nearest ten thousandth.

1. $3\sqrt{2}$
2. $4\sqrt{2}$
3. $5\sqrt{6}$

Graph each equation.

4. $y = 2^x - 1$
5. $y = 3^x - 2$
6. $y = -2^x + 1$
7. $y = 2^{-x} - 1$

Graph each inequality.

8. $y > 2^x$
9. $y \geq (0.5)^x$
\textbf{Key}

1. 41.73
2. 7.10
3. 51.54

\textbf{11:2}
Precalculus Lesson Plan

<table>
<thead>
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<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
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<tbody>
<tr>
<td>Lesson Title</td>
<td>Exponential Functions and the Number e</td>
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<tr>
<td>Lesson Goals</td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Graph functions on a graphing calculator.</td>
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<td>- Evaluate expressions with irrational exponents.</td>
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<tr>
<td></td>
<td>- Graph exponential functions.</td>
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<tr>
<td></td>
<td>- Graph exponential inequalities</td>
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<tr>
<td></td>
<td>- Use the exponential function $y = e^x$</td>
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<tr>
<td>NCTM Standards Addressed</td>
<td>Problem Solving</td>
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<tr>
<td></td>
<td>Communication</td>
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<td>Connections</td>
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<td></td>
<td>Algebra</td>
</tr>
<tr>
<td></td>
<td>Functions</td>
</tr>
<tr>
<td>Instructional Materials &amp; Information Resources</td>
<td>Graphing calculators (classroom set)</td>
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<tr>
<td></td>
<td>Overhead projector</td>
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<td></td>
<td>Graphing Calculator view screen for the projector</td>
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<tr>
<td></td>
<td>Chalk and chalkboard</td>
</tr>
<tr>
<td>Introduction</td>
<td>Do warm-up problem - 5 min. Check</td>
</tr>
<tr>
<td></td>
<td>Questions from homework</td>
</tr>
<tr>
<td></td>
<td>Collect homework assignments: Pg 602 and Practice 11.2</td>
</tr>
<tr>
<td>Instructional Sequence &amp; Questions</td>
<td>Review from before spring break: - 5 min.</td>
</tr>
<tr>
<td></td>
<td>Introduce irrational exponents:</td>
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<tr>
<td></td>
<td>Using calculators: evaluate:</td>
</tr>
<tr>
<td></td>
<td>$\sqrt{2} \approx 1.41$</td>
</tr>
<tr>
<td></td>
<td>$\pi \approx 3.14$</td>
</tr>
<tr>
<td></td>
<td>$0.4\pi \approx 1.26$</td>
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<tr>
<td></td>
<td>Introduce exponential functions $y = a^x$</td>
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<td></td>
<td>After graphing them on the calculator:</td>
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<tr>
<td></td>
<td>Discuss the behavior of the graphs of $y = 2^x$ and $y = 2^{-x}$</td>
</tr>
<tr>
<td></td>
<td>Compare the values of $y = 2^x$ and $y = 2^{-x}$ on the intervals $-10 &lt; x &lt; 0$ and $0 &lt; x &lt; 10$. Reflect across $y$ - axis</td>
</tr>
<tr>
<td></td>
<td>Intersect? $(1, 2)$ - $y$ - int</td>
</tr>
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<td></td>
<td>Values greater!</td>
</tr>
<tr>
<td></td>
<td>Note that exponential functions are transformed the same way other functions are.</td>
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</tbody>
</table>
|                           | Using graphing calculators:
Graph the functions \( y = (1/2)^x \), \( y = (1/2)^x - 2 \), \( y = (1/2)^x + 1 \), and \( y = 3(1/2)^x \) on the same set of axes. Then describe the transformations of the parent graph \( y = (1/2)^x \) that have taken place to form each of the other graphs. Have students describe movements.

Describe any similarities or differences between the graphs.

Inequalities: similar to other inequalities:
Graph \( y \geq 3^x \) - solid or dashed

Introduce the number \( e \) and its applications

Group work:
Present the formula for compound interest
Have groups calculate the value of an account with a principal of $1000 after three years at 6% interest compounded annually, quarterly, monthly, daily, and hourly.
Note how the sequences of values appear to converge to a limit.
This limiting value is \( e \).

Using technology:
Calculate the limiting value.

Discuss values for \( e \).
EX: Find the value of \( e^{1.27} \)

Discuss graphs involving \( e \).

EX: Using graphing calculator: graph:
\( y = 2e^x + 1 \)
Q: On what interval is this function increasing?

Describe applications: Newton's law of cooling:

EX: An object is moved from a house heated to 70 degrees to the outdoors where the temperature is 15 degrees. What will the temperature of the object after 30 minutes? Assume \( a = 55 \) and \( k = 0.55 \).

Closure:
How do the graphs \( y = (1/2)^x \) and \( y = 2^x \) compare?
Describe one of the applications that use the exponential function \( y = e^x \)

Assignment
Pg. 612-613
39-43
Pg. 617-619
6: 13-17 odd, 26, 28, 35
8: 12-16 even, 25, 27, 35
#35 From Textbook: Growth of a population of Organisms

Limited Growth: Using the Logistic Equation

\[ \frac{n}{M} = \frac{1}{1 + be^{at}} \]

- \( M = 200 \)
- \( b = 20 \)
- \( c = 0.35 \)

<table>
<thead>
<tr>
<th>Time</th>
<th>Population</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>9.52381</td>
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<tr>
<td>1</td>
<td>13.25051</td>
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<td>2</td>
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<td>199.1046</td>
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<tr>
<td>25</td>
<td>199.3682</td>
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</tbody>
</table>

What do you notice about the significance of \( M \)?

What would happen if you changed \( M \)?
Make no bones about it: this article is highly biased in favor of teaching the logistic curve in high school. Both the exponential and logarithmic curves are staples of the precalculus curriculum. These functions are usually introduced in a third-year mathematics course. They reappear in the calculus curriculum for both Advanced Placement calculus and first-semester college calculus. The logistic function, a natural capstone to teaching exponential curves, is a logical extension of mathematics instruction to a variety of applications. It should be part of the repertoire of functions that we teach to eleventh- and twelfth-grade students.

The logistic function, 

$$f(x) = \frac{L}{1 + a \cdot e^{-bx}}$$

is algebraically difficult but visually elegant. It describes the behavior of data in business and in the sciences. The ready availability of graphing technology and computer algebra systems (CASs) has removed the barriers to manipulating the logistic function. It is accessible to us in a manner that was unimaginable ten years ago.

The classic application of the logistic function is to model exponential growth with limiting factors. For example, the spread of disease and the profitability of a new business idea appear exponential in their early stages of growth. In later stages, the curve passes through a point of inflection, and growth slows as the curve rises to an upper limit. When the problem is the spread of disease, the adaptation of the population to the disease and the size of the population are limiting factors. In business, highly profitable ideas attract competition, which, in turn, inhibits untrammelled growth.

The function takes the form

$$f(x) = \frac{L}{1 + a \cdot e^{-bx}}$$

The numerator, \(L\), is the upper limit to growth. The constants \(a\) and \(b\) affect the rapidity with which the function approaches \(L\). The detail with which the class analyzes this function depends on the ability of the class. An AP calculus student can be expected to analyze this model in terms of both the function and derivative. At that level, the constant \(b\) can be developed as the product of some constant, \(k\), and \(L\), the upper limit. The students in my lower-level calculus class can use the chain rule to find the derivative and can use their calculators to find the point of inflection. My precalculus students can successfully solve the function for values of \(x\) when they are given \(f(x)\). They have a better intuitive grasp of the meaning of the numbers that they derive than they often do when they solve exponential equations, even though the logistic function is more complex. My emphasis in this article is on precalculus students, although I will try to point out extensions that can apply to calculus students.

After a unit on exponential and logarithmic functions, I introduce the logistic function by example. The introduction is simple: I tell my class that I heard a great joke, but I am only going to tell it to two people. They each can tell it to two more people, and so on, while we keep track of the number of iterations that are required before everybody has heard the joke. We do not keep track of who tells the joke to whom, as we do not limit telling the joke only to those who have not heard it. As I point out, the activity does not work in that manner. Students will talk between periods, pass notes, and corner their friends at lunch. Sooner or later, someone that they tell will have already heard the joke. In fact, as time passes, fewer and fewer people have not already heard the joke. If we restrict the population to mathematics students, or simply to my class, we have a clear upper limit on the total number of people in the population.

A plot showing the spread of the joke might show the time periods along the x-axis. The y-axis can represent the number of people who have heard the joke. Figure 1a shows the spread of the joke during the first six time periods. I used a program that

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I wrote for the TI-83 graphing calculator (see program 1) to generate data and set up the calculator before class. With the calculator connected to a viewscreen, I ask students how they would model the data thus far. After discussing parabolic curves as opposed to exponential curves, I use the viewscreen to display an exponential curve fitted to the data, as shown in figure 1b. The joke is spreading quickly, because not too many people have yet heard it, it is new for most of the people to whom it is told. So far, the exponential curve is a good fit.

The next four time periods tell a different story. The people who have heard the joke keep trying to tell it to other people, but they catch only a few more people who have not yet heard it. After ten time periods, the pattern shown in figure 1c is clear. The joke spreads quickly at first and then slows down. The pattern calls for a "logistic" model—an exponential function with limits to growth. Figure 1d shows a logistic curve fitted to the data. The original regressed exponential curve is clearly inappropriate in the latter stages. The logistic regression, however, is an excellent match for the data at all stages.

The students can easily model this situation. The teacher puts all the students' names on cards in a hat and brings a silly joke to class. The teacher tells the joke to two students and lets them randomly pick two name cards. They tell the joke only to the people whose names they have drawn. The students then put those cards back in the hat. On each successive round, everyone who has heard the joke draws two cards, tells the joke to the students whose names are on the cards, and then puts the cards back into the hat. This method is a fairly robust one for generating a good logistic curve. It works well if this random card-pairing method is used each time that students tell the joke. It does not work at all if they specifically locate the people who have not yet heard the joke. This experiment is probably best done with a random-number generator, but it is much more fun if the names are on cards drawn from a hat. The teacher should monitor the progress of the experiment and build a graph as the experiment progresses. The final step is to use the calculator's regression function to estimate the logistic function suggested by the data.

As with all the functions that my classes study, I ask the students to investigate certain fundamental questions. I group the class into threes and ask them to determine how the logistic curve is affected by changes to the various constants and their signs. I give each group an index card and a basic equation in which \( L > 0 \) and \( a > 0 \) are 1:

\[
f(x) = \frac{10}{1 + e^{10}}
\]

I ask the students to set a standard viewing window and change one constant at a time to isolate its effect on the function. This method makes good pedagogical sense: it allows students to construct knowledge and emphasizes the control that students have over the characteristics of the function. In ten or fifteen minutes, the teacher can reasonably expect students to isolate such effects as those shown in figure 2. In a typical class session, with students who are familiar with working in groups—and who are familiar with my weird examples and questions, I give students five to ten minutes of introduction and review on the logistic function. The class spends fifteen to twenty minutes experimenting and organizing information. I try to make this process efficient by asking students to post their results as soon as most groups have them. I can then reserve time to close the class with a set of questions, such as those in figure 3.

Although the problems on sheet 1 are designed for precalculus students, a calculus teacher can readily use them to help students move into more complicated analyses. For instance, the logistic function makes an excellent introduction to limits.

### Effects of changing \( L \)
- Negative \( L \) reflects in the x-axis.
- Larger \( L \) raises the upper limit.
- Smaller \( L \) lowers the upper limit.

### Effects of changing \( a \)
- Larger \( a \) shifts the curve to the right.
- \( 0 < a < 1 \) shifts the curve to the left.

### Effects of changing \( b \)
- Larger \( b \) makes the curve rise steeply.
- \( 0 < b < 1 \) makes the curve shallower.
The logistic function

\[ f(x) = \frac{30}{1 + 10e^{-0.2x}}. \]

describes the spread of good news through my family. The function \( f(x) \) gives the numbers of family members who have heard the news after \( x \) days. Many family members are close by or in constant contact. Others will learn by using snail mail. The fathers are typically the last to learn anything. The teacher can ask the following questions about this function:

- How would you interpret a fractional day? A fractional person? How should you round these quantities—up, down, or algebraically?
- How many family members could get the news?
- How many people have heard the news after five days?
- Graph this function on a graphing calculator, and estimate the number of days that must pass before at least twenty-five people have heard the news. How would you answer this question algebraically?
- What is the lowest value of \( x \) at which \( f(x) \) is less than 0.5? Is there a fractional value of \( x \) at which \( f(x) \) is less than 0.5?
- Solve \( f(x) = 27 \) for \( x \). Report to five places, and then round to the nearest day.
- Create a new graph showing how many people have heard the news (for the first time) on any given day. Graph these data on the same screen as our logistic function. How would you estimate these data? This graph is the rate-of-change graph, or the graph of the first derivative.

**Fig. 3**
A set of questions

---

**Communication skills are crucial to success**

Calculus students should be able to differentiate the logistic function and use the derivative and a calculator to find the point of infection. Part of the appeal of the logistic function is the number of ways in which it can be used to illustrate important concepts in advanced mathematics.

I typically tell students to discuss the problems in their groups and write up their responses individually for homework. The next day, I ask students to share their discoveries. If this activity is too independent for the class, I give them a few sample problems for practice and use the next day to work on the remaining problems. An advanced group can discuss the change in slope over the point of inflection, limits, and relative rates of change. This discussion—at whatever level—is an important way to verify the progress of the class. It is also a great lead-in to a more independent project.

On the third and fourth days, I typically conclude this activity with the logistic function and assign a project. I have found that students better retain and understand material if they participate in mathematics from a variety of approaches and activities. Writing and exploring are crucial to understanding; communication skills are crucial to success.

The project that I designed for this activity is given on sheet 2. It uses the program LOGIST, which I wrote for the TI-83 (see program 1). The project mimics the spread of an infection through a given population and asks students to use the power of the graphing calculator to create a model for this miniepidemic.

The program asks students to choose a size for the population and then select the number of time periods to examine. A typical population size is between fifteen and thirty, and the number of time periods is between seven and sixteen. The program then starts the simulation by randomly selecting a "carrier" to bring the infection to the population. At the end of each time period, the calculator reports the total number of people infected and the number of people who have been newly infected during that time period. Students appreciate their ability to experiment with different parameters: we typically try a few in class before they leave for their next class.

The most successful students experiment several times before transcribing the data for the "good" experiment. They make a chart on a separate piece of paper, on which they show both the total number of infections and new infections. They perform the regression on the total infection data to obtain the logistic function and include a graph with Plot1 displaying total infections against time, Plot2 with new infections, and the regression line. I think that clearly modeling these steps in the classroom is important so that students can see the kind of detail needed in making a sophisticated analysis of experimental data.

I ask students to try regressing different functions against the time and infection data to determine the function that best describes the behavior of the data. They write a brief paragraph describing the spread of the infection through the target group. When this project is successful, my only role is to move from group to group, answering questions and indicating strengths and weaknesses in their arguments. This project has sometimes been unsuccessful because students failed to participate actively in student-originated discussion. In those groups, I have given strict guidelines on the parameters of the investigation and checked back frequently to ensure progress. This less-successful outcome has been the exception rather than the rule.

As with any curriculum topic, teaching the logistic function must advance the students' understanding of mathematics. The explicit goal in teaching is furthering mathematics; my implicit goal is to help my students acquire and assess knowledge. Students spend the third and fourth days of this activity discussing the project and writing. This activity closes the entire unit on exponential functions and maintains connections with the course-long investigation of functions.
# Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Population Growth/Compound Interest Demonstration and Activity Powerpoint review</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Understand various types of population growth.</td>
</tr>
<tr>
<td></td>
<td>- Use a spreadsheet to calculate the value of an investment after a specified number of years.</td>
</tr>
<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving Communication Reasoning Connections Functions</td>
</tr>
<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set) Chalk and chalkboard Computer Lab</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>Do warm-up problem</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Ask questions from the homework.</td>
</tr>
<tr>
<td></td>
<td>Focus on #35: say that we will look at this type of curve today</td>
</tr>
<tr>
<td></td>
<td>Go to computer lab:</td>
</tr>
<tr>
<td></td>
<td>Reintroduce chapter project</td>
</tr>
<tr>
<td></td>
<td>Discuss populations and countries to look at.</td>
</tr>
<tr>
<td></td>
<td>Have students open powerpoint and briefly introduce its use for the project demonstration.</td>
</tr>
<tr>
<td></td>
<td>Demonstrate population growth demonstration and program</td>
</tr>
<tr>
<td></td>
<td>On each computer, have population growth demonstration from Microsoft Excel loaded onto every computer for the students to experiment with.</td>
</tr>
<tr>
<td></td>
<td>Have the students write the characteristics of each type of graph and what happens as values of each equation are changed.</td>
</tr>
<tr>
<td></td>
<td>After students have finished exploring with population growth models, have them open a new sheet in Excel.</td>
</tr>
</tbody>
</table>
Using Microsoft Excel
Population Growth and Compound Interest

First go to:
http://homepage.mac.com/apgogel

Download:
Maths331#1.xls

Look at the different types of population growth.
The last one is from your homework assignment #35.

Compound Interest

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

Example:

<table>
<thead>
<tr>
<th>Principal</th>
<th>Interest rate of 9% compounded monthly for 2 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 :B19</td>
<td></td>
</tr>
</tbody>
</table>

| Times compounded | 12 :B20 |
| Rate             | 0.09 :B21 |

<table>
<thead>
<tr>
<th>Year</th>
<th>Balance</th>
<th>1 (at A23)</th>
<th>B19 * (1 + B21 / B20) * (B20 * A23)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 (at A24)</td>
<td>B19 * (1 + B21 / B20) * (B20 * A24)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Type in Letter and number location of your values to go with the equation. Put an = sign first!! You only have to do it once! Click and Drag to complete a larger table!

Assignment (use page 620 from textbook for reference)

Now complete and print out a list of balances for 50 years!!
Principal: 5000
Interest Rate of 6.5%
Compounded Quarterly

Start with First Balance Equation!
Then click on the lower right corner of the cell and drag down to 50!
If you have questions, ask Mr. Gogel.

Extra Credit
Modify the program so you can print comparisons of balances for two different interest rates for the same time, rate, principal, and number of times compounded
## Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Logarithmic Functions</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Graph logarithmic functions on a graphing calculator.</td>
</tr>
<tr>
<td></td>
<td>- Evaluate expressions involving logarithms.</td>
</tr>
<tr>
<td></td>
<td>- Solve equations involving logarithms.</td>
</tr>
<tr>
<td></td>
<td>- Graph logarithmic functions and inequalities.</td>
</tr>
<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
</tr>
<tr>
<td></td>
<td>Communication</td>
</tr>
<tr>
<td></td>
<td>Reasoning</td>
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<tr>
<td></td>
<td>Connections</td>
</tr>
<tr>
<td></td>
<td>Functions</td>
</tr>
<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set)</td>
</tr>
<tr>
<td></td>
<td>Overhead graphing calculator</td>
</tr>
<tr>
<td></td>
<td>Chalk and chalkboard</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>Do warm-up problem</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Ask for questions from each students spreadsheet results.</td>
</tr>
<tr>
<td></td>
<td>Using graphing calculators, graph logarithmic functions:</td>
</tr>
<tr>
<td></td>
<td>Example #1: Graph $y = \log_4 x$.</td>
</tr>
<tr>
<td></td>
<td>Q: Can I type this function into my calculator?</td>
</tr>
<tr>
<td></td>
<td>I must apply change of base formula!</td>
</tr>
<tr>
<td></td>
<td>Remind students that logarithms with a base other than 10 must be rewritten as a quotient.</td>
</tr>
<tr>
<td></td>
<td>Q: What is the range and domain of this function?</td>
</tr>
<tr>
<td></td>
<td>Example #2: Graph $y = \log (x + 6)$.</td>
</tr>
<tr>
<td></td>
<td>Q: Do I have to apply the change of base here? What is the range and domain of this function?</td>
</tr>
<tr>
<td></td>
<td>Discuss what changes can happen if things are added to the equations.</td>
</tr>
<tr>
<td></td>
<td>Have students graph the following and state the range and domain of</td>
</tr>
<tr>
<td></td>
<td><strong>Overhead</strong></td>
</tr>
<tr>
<td></td>
<td>$\log_a x = \frac{\log x}{\log a}$</td>
</tr>
<tr>
<td></td>
<td>$\log_4 x = \frac{\log x}{\log 4}$</td>
</tr>
</tbody>
</table>
Introduce the definition of logarithmic function: relate it to the exponential function

\[ x = a^y \rightarrow \log_a x = y \]

Do several examples of converting between the two.

**Example #3:** Write \( \log_{10} 0.01 = -2 \) in exponential form.

**Q:** What is my \( x \)?

**Example #4:** Evaluate the expression \( \log_8 64 \).

Have the students write the properties of exponents.

Use these properties to show the properties of logarithms:
- **Product property**
  \[ \log_a (xy) = \log_a x + \log_a y \]
- **Quotient property**
  \[ \log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y \]
- **Power property**
  \[ \log_a (x^r) = r \log_a x \]
- **Property of equality**
  \[ \log_a x = \log_a y \rightarrow x = y \]

On overhead: Have students do the following with assistance from classmates:

**Solve each equation.**

1. \( \log_{10} 5 = -1/3 \) \( \rightarrow 1/125 \)
2. \( \log_{10} (2x+5) = \log_{10} (5x-4) \) \( \rightarrow 3 \)
3. \( \log_3 (4x+5) - \log_3 (3-2x) = 2 \) \( \rightarrow 1 \)

**Closure**

How are logarithms related to exponents?

**Assignment**

Practice 11-4
Precalculus warm-up

Simplify:
8. \( \sqrt[3]{x(x-1)} \)

9. \( \frac{u+1}{\sqrt{u}} \)

10. \( x^2(\sqrt{x} - \sqrt[3]{x} + 3) \)

11. Give a decimal approximation of \( e \) to 3 places. 

12. \( e^0 = \) 

13. \( \frac{e}{\sqrt{e}} = \frac{e}{e^{1/2}} = e^{1/2} \)

14. In calculus, what is the derivative of \( y = e^x \)?

Do you know why?

Because the slope of any tangent line to \( y = e^x \) is the function value at that point!

\[ e^x \at x = 0 \lim y = x + 1 \]
Logarithmic Functions

Write each equation in logarithmic form.
1. \(2^5 = 32\)  
2. \(5^{-3} = \frac{1}{125}\)  
3. \(6^{-3} = \frac{1}{216}\)

Write each equation in exponential form.
4. \(\log_3 27 = 3\)  
5. \(\log_4 16 = 2\)  
6. \(\log_{10} \frac{1}{100} = -2\)

Evaluate each expression.
7. \(\log_7 7^3\)  
8. \(\log_{10} 0.001\)  
9. \(3^{\log_3 6}\)
10. \(\log_b b^{-4}\)  
11. \(\log_a a\)  
12. \(3^{4 \log_3 4}\)

Solve each equation.
13. \(\log_x 64 = 3\)  
14. \(\log_4 0.25 = x\)  
15. \(\log_4 (2x - 1) = \log_4 16\)  
16. \(\log_{10} \sqrt{10} = x\)

Graph each equation or inequality.
17. \(y = \log_2 x\)  
18. \(y < \log_{10} (x - 1)\)
Logarithmic Functions

Write each equation in logarithmic form.
1. \(2^5 = 32\)
   \(\log_2 32 = 5\)
2. \(5^{-3} = \frac{1}{125}\)
   \(\log_5 \frac{1}{125} = -3\)
3. \(6^{-3} = \frac{1}{216}\)
   \(\log_6 \frac{1}{216} = -3\)

Write each equation in exponential form.
4. \(\log_3 27 = 3\)
   \(3^3 = 27\)
5. \(\log_4 16 = 2\)
   \(4^2 = 16\)
6. \(\log_{10} \frac{1}{100} = -2\)
   \(10^{-2} = \frac{1}{100}\)

Evaluate each expression.
7. \(\log_7 7^3 = x\)
   \(7^x = 7^3\)
   \(x = 3\)
8. \(\log_{10} 0.001 = x\)
   \(10^x = 0.001\)
   \(x = -3\)
9. \(3^{\log_3 6}\)
   \(3^x = 6\)
10. \(\log_b b^{-4}\)
   \(-4\)
11. \(\log_a a\)
   \(1\)

Solve each equation.
13. \(\log_x 64 = 3\)
   \(x^3 = 64\)
   \(x = 4\)
14. \(\log_4 0.25 = x\)
   \(4^x = \frac{1}{4}\)
   \(x = -1\)
15. \(\log_4 (2x - 1) = \log_4 16\)
   \(2x - 1 = 4\)
   \(2x = 5\)
   \(x = \frac{5}{2}\)
16. \(\log_{10} \sqrt{10} = x\)
   \(10^{\frac{x}{2}} = \sqrt{10}\)
   \(10^x = 10^{\frac{1}{2}}\)
   \(x = \frac{1}{2}\)

Graph each equation or inequality.
17. \(y = \log_2 x\)
18. \(y < \log_{10} (x - 1)\)
As I reflect on today's lesson, I can point out several strong points. First, I feel that my warm-up problem sheet that I created was very good for preparing the class for calculus. The sheet involved simplification and evaluation of expressions involving radicals and \( e \). Also, the warm-up problems served as a review over the first three sections of the chapter. Another strong point to the lesson was the exploration of logarithmic graphs on the overhead graphing calculator. As the students and I graphed logarithmic functions, I asked them various things about range, domain, and shifts due to the changes in the function. This generated some discussion on how changes in the function cause changes in the domain and range, which is what I had hoped for. Along with this, the idea of inverse functions arose during our comparison of logarithmic and exponential functions. This discussion helped the students realize where logarithms come from and their relationship with exponential functions. The final strong point to the lesson was the fact that the students were engaged the entire lesson. Every student was interested in the subject mainly because it is a new topic that they had never seen before.

As a result of these strong points, I feel that the objectives of this lesson were met. The students were able to graph the logarithmic functions on their graphing calculators and see the basic shape that a logarithmic function takes. Also, through the examples that I had them do in class, the students were able to simplify and solve basic logarithmic expressions and equations. Most saw the relationship between the properties of exponents, the definition of logarithms, and the properties of logarithms, which are used to evaluate and simplify equations and expressions. I did see, however, that since logarithms are new to most of the students, I needed to spend a little more time working on more advanced expressions and equations to further their development with working with logarithms. Next class, we will do some more work with simplifying logarithmic expressions and evaluating logarithmic equations.

I will assess student learning based upon how well they do on the assignment given to them. The questions asked next time in class will determine how much more
time we need to spend on logarithms. I am planning on playing logarithmic bingo, which is a game that gives students more practice with converting equations from logarithmic form to exponential form and simplifying logarithms. If I taught this lesson again, I would have spent more time giving the students more examples to work on in class. Then, the students could do all of the problems themselves and ask questions about them. Due to my review of PowerPoint at the end of the period, I ran out of time to do this.
# Precalculus Lesson Plan

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<th><strong>Textbook</strong></th>
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<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Mid-Chapter Quiz and Chapter Project</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Demonstrate their knowledge of exponential and logarithmic functions through a quiz.</td>
</tr>
<tr>
<td></td>
<td>- Find population data in the library.</td>
</tr>
<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
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<td>Communication</td>
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<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set)</td>
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<td></td>
<td>Chalk and chalkboard</td>
</tr>
<tr>
<td></td>
<td>Library for books and computer use</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>Do warm-up problem (Application of Logarithms)</td>
</tr>
<tr>
<td></td>
<td>Given the equation $b = 100(2^t)$ (explain each part),</td>
</tr>
<tr>
<td></td>
<td>If a bacteria colony doubles every 20 minutes and there are 500 bacteria at noon, when will the population reach 16,000?</td>
</tr>
<tr>
<td></td>
<td>$\Rightarrow 1:40$ p.m.</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Ask for questions from homework or anything from chapter.</td>
</tr>
<tr>
<td></td>
<td>$\text{TWIZZLERS}$</td>
</tr>
<tr>
<td></td>
<td>Do review examples of logarithms:</td>
</tr>
<tr>
<td></td>
<td><strong>Example #1</strong></td>
</tr>
<tr>
<td></td>
<td>a. Evaluate $\log_{10}0$</td>
</tr>
<tr>
<td></td>
<td>b. Evaluate $\log_{7}-1$</td>
</tr>
<tr>
<td></td>
<td><strong>Example #2</strong></td>
</tr>
<tr>
<td></td>
<td>Solve $\log_{3}36 =2$</td>
</tr>
<tr>
<td></td>
<td><strong>Example #3</strong></td>
</tr>
<tr>
<td></td>
<td>Simplify: $\log_{5}x = (1/3) \log_{5}64 + 2 \log_{5}3$</td>
</tr>
<tr>
<td></td>
<td>Instead of quiz, play Logarithmic Bingo for logarithm review.</td>
</tr>
<tr>
<td></td>
<td>After everyone is finished with quiz, announce project outlines for each group are due at the end of class.</td>
</tr>
</tbody>
</table>
Go to the library for use of computers and books for project research and for use of PowerPoint for each pair.

<table>
<thead>
<tr>
<th>Closure</th>
<th>Ask for questions about chapter projects, and give approval. Announce that computer lab will be used next time.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assignment</strong></td>
<td>Outline of Chapter Project</td>
</tr>
</tbody>
</table>
Twizzlers Warm-up

Complete the following:

1.) Take one Twizzler. Measure and record its length.

2.) Cut or bite the Twizzler exactly in half. Eat on half, measure and record the other half.

3.) Continue this process until the Twizzler is gone or you cannot cut or measure anymore.
   Discuss: Mathematically speaking, is the Twizzler ever supposed to be completely gone?

4.) Using the data that you have created, plot your data based on the length versus the number of cuts.
   Discuss: Based on your plot, what kind of function does it look like?

5.) Write a function that would model your plot.

Twizzlers Warm-up – answers in inches

Complete the following: (Answers may vary based on choice of measurement)

1.) Take one Twizzler. Measure and record its length in inches or cm. (8 inches)

2.) Cut or bite the Twizzler exactly in half. Eat on half, measure and record the other half. (4 inches)

3.) Continue this process until the Twizzler is gone or you cannot cut or measure anymore. (2 in., 1 in. 
   \( \frac{1}{2} \) in., \( \frac{1}{4} \) in., ...)
   Discuss: Mathematically speaking, is the Twizzler ever supposed to be completely gone? (no)

4.) Using the data that you have created, plot your data based on the length versus the number of cuts.
   Discuss: Based on your plot, what kind of function does it look like? (An exponential function)

5.) Write a function that would model your plot. (\( y = \left(\frac{1}{2}\right)^x \) It would exactly model the data.)

---

**Twizzlers Measurements: Eating Half at a Time**

<table>
<thead>
<tr>
<th>Cuts</th>
<th>Length (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>0.5</td>
</tr>
<tr>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>0.125</td>
</tr>
</tbody>
</table>
# Logarithmic Bingo

Evaluate: Write in logarithmic form.  

<table>
<thead>
<tr>
<th>Evaluate:</th>
<th>Write in logarithmic form.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \log_2 4 ) = 2 \quad \quad 2^3 = 8 \quad \quad \log_2 8 = 3</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_3 81 ) = 4 \quad \quad 3^2 = 9 \quad \quad \log_3 9 = 2</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_5 125 ) = 3 \quad \quad 6^2 = (1/36) \quad \quad \log_6 (1/36) = -2</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_{25} 256 ) = 8 \quad \quad 25^{(1/2)} = 5 \quad \quad \log_{25} 5 = (1/2)</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_{10} 100,000 ) = 5 \quad \quad 9^{(3/2)} = (1/27) \quad \quad \log_9 (1/27) = (-3/2)</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_4 2 ) = (1/2) \quad \quad 32^{(4/5)} = 16 \quad \quad \log_{32} 16 = (4/5)</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_{16} 64 ) = (3/2) \quad \quad 5^3 = (1/125) \quad \quad \log_5 (1/125) = -3</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_8 32 ) = (5/3) \quad \quad 2^6 = 64 \quad \quad \log_2 64 = 6</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_2 (1/8) ) = -3 \quad \quad 10^{(1/10)} \quad \quad \log_{10} (1/10) = -1</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_{10} (1/100) ) = -2 \quad \quad 27^{(4/3)} = 81 \quad \quad \log_{27} 81 = (4/3)</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_{32} 2 ) = (1/5) \quad \quad 100^2 = 10000 \quad \quad \log_{100} 100000 = 2</td>
<td>\quad \quad \mid</td>
</tr>
<tr>
<td>( \log_4 128 ) = (7/2)</td>
<td></td>
</tr>
</tbody>
</table>
Logarithmic Bingo

The original plan for today's lesson was to take a quiz over the first half of the chapter and then proceed to the library to work on projects. These plans changed, however, when I noticed that the students needed extra time with logarithms, especially after discussing the homework assignment. Logarithms are a new concept for them, so I thought that it was necessary to spend more time on them. As a result of today's lesson in precalculus, I noticed that there were strong points that made the lesson successful. First, I chose to do a warm-up problem on an example of exponentials using twizzlers candy. The students gained a new insight on exponential functions and decay by measuring a twizzler after continually taking half away. Also, the class played logarithmic bingo, where students evaluated logarithmic expressions and wrote exponential equations as equations in logarithmic form. This activity involved repetitious use of logarithms, which was what I thought the students needed in order to sharpen their skills with logarithms. The game allowed them to further their understanding with not only converting from exponential form to logarithmic form, but also to evaluate logarithmic expressions.

Since I modified my plans based on how the students were doing with the concept of logarithms, my objectives changed as well. As a result of how well the logarithmic bingo went and its effectiveness with giving the students more practice with logarithms, I think that my objectives were met for this lesson. Because of this, we can now move on to the next topic during the next class period. Also, I assessed student learning by observing students place markers on their bingo cards. I also will assess further understanding by grading a homework assignment that was given last time.

If I taught this lesson again, I would not do it any differently. The only thing that I would consider changing is to give a quiz at the beginning of class, but one not containing logarithms. I need to assess the level of understanding that the students are at through some sort of summative evaluation, so I may give a short quiz next time to accomplish this.
1. Evaluate $81^{3/4}$

2. Evaluate $\frac{25^{3/4}}{25^{1/4}}$

3. Graph the equation $y = 0.1^x$

Evaluate each expression to the nearest ten thousandth.

4. $e^{3.8}$

5. $5\sqrt{3}e$

6. $e^{2/5}$

Solve each equation.

7. $\log_x 512 = 3$

8. $\log_{\sqrt{5}} 5 = x$

9. $\log_5 (x^2 - 30) = \log_5 6$
1. Evaluate \( 81^{\frac{3}{4}} \)
   \[
   
   \left(\sqrt[4]{81}\right)^{3}
   \]
   \[
   = 27
   \]

3. Graph the equation \( y = 0.1^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Evaluate each expression to the nearest ten thousandth.

4. \( e^{3.8} \)  
   \[
   44.7011
   \]

5. \( 5\sqrt{3}e \)  
   \[
   14.2783
   \]

6. \( e^{2.5} \)  
   \[
   1.4918
   \]

Solve each equation.

7. \( \log x \ 512 = 3 \)
   \[
   x^3 = 512
   \]
   \[
   x = 8
   \]

8. \( \log_{5} 5 = x \)
   \[
   (5^{\frac{1}{2}})^x = 5
   \]
   \[
   x = 2
   \]

9. \( \log_{5} (x^2 - 30) = \log_{5} 6 \)
   \[
   x^2 - 30 = 6
   \]
   \[
   x^2 = 36
   \]
   \[
   x = \pm 6
   \]
Precalculus Population Growth Project
With PowerPoint Presentation

Purpose:

The purpose of this project is to track the population growth of a particular country over a certain time period, and then see if the growth resembles an exponential function. You are then to present your findings to the class via a PowerPoint presentation.

Instructions:

1. You will need to find a partner to work on the project. You can decide the workload of each person in your group, as long as each person has a task.

2. Use the resources available to you to find the population history of countries. If you want you can track the population growth of the world as well. You may use books, Internet, etc. A list of Internet sites is provided for you, and I suggest that you use them.

3. After researching population statistics for countries, pick a country whose population growth statistics you want to use.

4. It is your choice to pick how far back in time you want to track population data. My suggestion is that the farther back you go in time, the better results you will have.

5. It is your choice to pick the number of time intervals that you want to track and find data points. My suggestion is to use population data on five or ten-year intervals.

6. After collecting your population data, create a PowerPoint presentation that includes the following:
   - A title slide including title with name of country, name of group members, class, and period.
   - A slide including a table of population data for your country.
   - A slide including a graph showing the growth of population
   - A slide containing your thoughts whether or not the growth of population is exponential or not.
   - A slide containing the sources from which you found your information.
   - (OPTIONAL/BONUS) Slides containing a data table from Excel and an Excel graph of an exponential function that closely matches your population growth data for your country.

If you have questions, feel free to ask for assistance with anything, and good luck!

(See reverse side)
The following are some sites that may help:

http://www.census.gov/ipc/www/idbrank.html
http://www.census.gov/cgi-bin/popclock
http://www.censusindia.net/ (I suggest that you use this site and country!!!)
http://www.popnet.org/
http://www.ibiblio.org/lunarbin/worldpop
http://www.un.org/popin/data.html
www.yahoo.com -> population
www.google.com -> population

Grading:

Your group will be graded based on the following:

Completion of required slides: (10 points)
- title
- table
- graph
- explanation
- sources
Presentation (5 points)
Accuracy of data (5 points)
Individual contribution to group (5 points)

Opportunity for extra credit is also available.
4/11/02 Indian Day
PowerPoint Research Projects

Doing population data research and creating a PowerPoint presentation was the objective for today's precalculus class. As a result of this lesson, I can say that it was quite successful. If I taught this lesson again, I would do it the same way. All of the students were productive during the entire period, mainly because I advised one person in each group to gather data while the other started creating the presentation.

Not only did the students gain experience researching the data on the Internet, but they also gained experience working with PowerPoint and Excel. As I was walking around the lab and assisting with questions and problems, I noticed that each group was getting better with Excel and PowerPoint as they gained experience with the programs. Also, the work with compound interest in the computer lab last week really helped with giving the students more experience with Excel.

Before class today, I distributed a handout including the purpose of the project, the exact instructions to follow, several useful websites, and my rubric for grading the project. Students were given the entire class time in the computer lab to research data, produce PowerPoint presentation, and add an Excel spreadsheet and graph for extra credit. My objective for the lesson today was accomplished, for most students completed the project during class. This is good because I was not going to allow any more class time to complete the project. The presentation of projects will be next week on Wednesday.
### Precalculus Lesson Plan

<table>
<thead>
<tr>
<th>Textbook</th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Title</td>
<td>Exponential and Logarithmic Functions in the Computer Lab</td>
</tr>
</tbody>
</table>
| Lesson Goals | By the end of the lesson, students should be able to:  
| | - Demonstrate their reasoning skills with the shifting of exponential and logarithmic functions by changing values of their relative equations. |
| NCTM Standards Addressed | Problem Solving  
| | Communication  
| | Reasoning  
| | Connections  
| | Functions |
| Instructional Materials & Information Resources | Graphing calculators (classroom set)  
| | Chalk and chalkboard  
| | Computer Lab for computers |
| Introduction | Do warm-up problem in the room  
| | Attendance |
| Instructional Sequence & Questions | Go over quiz.  
| | Discuss progress, results with projects  
| | May have to review powerpoint  
| | Announce computer lab day: use of internet worksheet on exponential and logarithmic functions  
| | Take class to computer lab to work on computer activity.  
| | Have them go to the site: http://homepage.mac.com/apgogel  
| | Click on explogact to download.  
| | Also open Function Probe: explain its use  
| | ***If Function Probe is inaccessible, take graphing calculator case along for the students to use instead***  
| | Answer the questions to the activity on their own sheet of paper.  
| | When finished, groups can work on their powerpoint presentations if they are not finished  
| | If not finished with activity, students can print out remaining sheets for
Precalculus Lesson Plan

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<td>Lesson Title</td>
<td>Exponential and Logarithmic Equations</td>
</tr>
<tr>
<td></td>
<td>Natural Logarithms</td>
</tr>
<tr>
<td>Lesson Goals</td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Use a graphing calculator to solve exponential and logarithmic equations and inequalities.</td>
</tr>
<tr>
<td></td>
<td>- Solve exponential and logarithmic equations.</td>
</tr>
<tr>
<td></td>
<td>- Solve exponential and logarithmic inequalities.</td>
</tr>
<tr>
<td></td>
<td>- Find natural logarithms of numbers.</td>
</tr>
<tr>
<td></td>
<td>- Solve equations using natural logarithms</td>
</tr>
<tr>
<td>NCTM Standards Addressed</td>
<td>Problem Solving</td>
</tr>
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<td>Functions</td>
</tr>
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<td>Instructional Materials &amp; Information Resources</td>
<td>Graphing calculators (classroom set)</td>
</tr>
<tr>
<td></td>
<td>Chalk and chalkboard</td>
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<tr>
<td></td>
<td>Overhead graphing calculator</td>
</tr>
<tr>
<td>Introduction</td>
<td>Attendance</td>
</tr>
<tr>
<td></td>
<td>Do warm-up problem (Change of base problem)</td>
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<tr>
<td></td>
<td>Remind students of change of base formula: ( \log_b n = \frac{\log_a n}{\log_a b} )</td>
</tr>
<tr>
<td>Instructional Sequence &amp; Questions</td>
<td>Q: How could I solve the following equation: ( 3x^2 - 4 = x + 2 )? Are there more than one method that I could use?</td>
</tr>
<tr>
<td></td>
<td>Do graphing calculator activity: solving exponential and logarithmic equations and inequalities.</td>
</tr>
<tr>
<td></td>
<td>Solve the following by graphing together:</td>
</tr>
<tr>
<td></td>
<td><strong>Example #1:</strong> Solve ( 3^{4x-2} = (0.2)^{3x-1} ) to the nearest hundredth. What is the solution?</td>
</tr>
<tr>
<td></td>
<td><strong>Example #2:</strong> Solve the equation ( 3.9^{x+1} = 28 ). How can we solve this? <strong>Use logarithms!!</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Application:</strong> Start with solutions to exponential and logarithmic equations.</td>
</tr>
<tr>
<td></td>
<td>Describe Example: growth, radioactive decay, and half-life: ( y = y_0 e^{rt} )</td>
</tr>
</tbody>
</table>
Example #3: Half-life - P-32 is a radioactive substance with half-life of 14.3 days. How long would it take to reduce a 100 gram sample of P-32 to 15 grams?

Which equation do I apply here?

Note how the power property of logarithms allows us to transform the original equation into a linear equation in x.

Have the students do the following population growth example:

Example #4: Population growth - A bacterial colony doubles every 45 minutes. How much time will it take for the population to increase 5 fold?

Which equation do I apply here?

Example #5: Inequalities

Have the students solve \(2^{x+5} > 3\). Works the same as an equation!

Using e:

Describe exponential growth and decay of populations: give general formula \(y = ne^{kt}\) where:

- \(y\) is the final amount, \(n\) is the initial amount, \(k\) is a constant, and \(t\) is time.

\(k > 0\): growth, \(k < 0\): decay

Q: Where have we seen this type of equation before? -> compound interest? Continuous?

Define natural logarithms: used to solve problems involving \(e\).

What is the \(\ln e\)? -> 1!

Do practice 9-11 pg 643: solve by taking the \(\ln\) of both sides!

Example #6: When would my bank account double in value given the equation: \(A = Pe^{rt}\) and with my rate being 5%?

Ask the students when it would triple.
Exponential and Logarithmic Equations

Solve each equation or inequality by using logarithms.

1. $8^x = 10$

2. $12^x = 18$

3. $2.4^x \leq 20$

4. $1.8^{x-5} = 19.8$

5. $4^{2x} + 1 = 15.2$

6. $3^{5x} = 85$

7. $x < \log_2 15$

8. $x \geq \log_3 12.3$

9. $x^{\frac{2}{3}} > 25.3$

Solve each equation.

10. $1500 = 6e^{0.043t}$

11. $1249 = 175e^{-0.04t}$

12. $\ln 6.7 = \ln (e^{0.21t})$

13. $\ln 724.6 = \ln (e^{6.3t})$
1. A highway system can support 500,000 cars. A city currently has 375,000 cars, up from 350,000 5 years ago. If the population continues to grow at this rate, when will the highway system need updated?

2. Bob invests money in a CD that pays 8% compounded continuously. \((A = Pe^{rt})\). If Bob invested on May 1, 1990 and the account is worth $20,000 on May 1, 2001, what was the original investment?

3. Kim is studying bacteria in Biology class. She knows that \(k = 0.658\) for a certain bacteria when \(t\) is measured in hours. How long would it take for 15 of these bacteria to multiply into 250 bacteria?

4. Heritage Hills H.S. has 1500 students and plays class 4A basketball. In 1998, Heritage Hills had 1580 students. High school teams less than 1200 enrollment play class 3A basketball. If the drop in enrollment continues at a constant rate, what year will Heritage Hills High School drop to 3A basketball?
Solve each equation or inequality by using logarithms.

1. \(8^x = 10\)
   \[
   \log_8 8^x = \log_8 10 \\
   x \cdot \log_8 8 = \log_8 10 \\
   x = \frac{\log_8 10}{\log_8 8} = 1.1073
   \]

2. \(12^x = 18\)
   \[
   \log_9 12^x = \log_9 18 \\
   x \cdot \log_9 12 = \log_9 18 \\
   x = \frac{\log_9 18}{\log_9 12} = 1.1632
   \]

3. \(2.4^x \leq 20\)
   \[
   \log_{2.4} 2.4^x \leq \log_{2.4} 20 \\
   x \cdot \log_{2.4} 2.4 \leq \log_{2.4} 20 \\
   x \leq \frac{\log_{2.4} 20}{\log_{2.4} 2.4} \approx \frac{3.4219}{1} \\
   x \leq 3.4219
   \]

4. \(1.8^{-5} = 19.8\)
   \[
   \log_9 1.8^{-5} = \log_9 19.8 \\
   (x-5) \log_9 1.8 = \log_9 19.8 \\
   x-5 = \frac{\log_9 19.8}{\log_9 1.8} \\
   x = 10.0795
   \]

5. \(4^{2x+1} = 15.2\)
   \[
   \log_4 4^{2x+1} = \log_4 15.2 \\
   2x+1 \log_4 4 = \log_4 15.2 \\
   2x+1 = \log_4 15.2 \\
   2x = 1.963 \\
   x = 0.9815
   \]

6. \(3^{5x} = 85\)
   \[
   \log_3 3^{5x} = \log_3 85 \\
   5x \log_3 3 = \log_3 85 \\
   5x = \frac{\log_3 85}{\log_3 3} \\
   x = 1.068
   \]

7. \(x < \log_2 15\)
   \[
   x < \frac{\log_2 15}{\log_2 2} \\
   x < 3.907
   \]

8. \(x \geq \log_3 12.3\)
   \[
   x \geq \frac{\log_3 12.3}{\log_3 3} \\
   x \geq 2.284
   \]

9. \(x^{\frac{2}{3}} > 25.3\)
   \[
   \log_9 x^{\frac{2}{3}} > \log_9 25.3 \\
   \frac{2}{3} \log_9 x > \log_9 25.3 \\
   10 \log_9 x > 2.105 \\
   x > 127.2567
   \]

10. \(1500 = 6e^{0.043t}\)
   \[
   \ln 1500 = \ln 6 + 0.043t \\
   \ln 250 - \ln e = 0.043t \\
   \ln 250 = 0.043t \ln e \\
   t = 128.41
   \]

11. \(1249 = 175e^{0.04t}\)
   \[
   \ln 1249 = \ln 175 + 0.04t \\
   \ln 7.137 = \ln e - 0.04t \\
   \ln 7.137 = 0.04t \ln e \\
   t = -49.13
   \]

12. \(\ln 6.7 = \ln (e^{0.21t})\)
   \[
   \ln 6.7 = 0.21t \ln e \\
   t = 9.06
   \]

13. \(\ln 724.6 = \ln (e^{6.3t})\)
   \[
   \ln 724.6 = 6.3t \ln e \\
   t = 1.05
   \]
Precalculus

In word problems

1. A highway system can support 500,000 cars. A city currently has 375,000 cars, up from 350,000 5 years ago. If the population continues to grow at this rate, when will the highway system need updated?

\[
y = n e^{kt}
\]

\[
375,000 = 350,000 e^{r(5)}
\]

\[
1.0714 = e^{5r}
\]

\[
ln 1.0714 = 5r \ln e
\]

\[
5 \ln 1.0714 = R
\]

\[
ln 1.3 = ln e^{0.137986}
\]

\[
R = \frac{0.137986}{5} = 0.0275972
\]

\[
500,000 = 375,000 e^{0.0275972t}
\]

\[
1.3 = \frac{e^{0.0275972t}}{e}
\]

\[
t = \frac{ln 1.3}{0.0275972} = 20.85
\]

2. Bob invests money in a CD that pays 8% compounded continuously. \((A = Pe^t)\). If Bob invested on May 1, 1990 and the account is worth $20,000 on May 1, 2001, what was the original investment?

\[
A = Pe^t
\]

\[
20,000 = Pe^{0.08(11)}
\]

\[
P = \frac{20,000}{e^{0.08(11)}}
\]

\[
P = \#8295.66
\]

3. Kim is studying bacteria in Biology class. She knows that \(k = 0.658\) for a certain bacteria when \(t\) is measured in hours. How long would it take for 15 of these bacteria to multiply into 250 bacteria?

\[
250 = 15 e^{0.658 t}
\]

\[
16.67 = e^{0.658 t}
\]

\[
ln 16.67 = ln e^{0.658 t}
\]

\[
ln 16.67 = 0.658 t \ln e
\]

\[
t = \frac{ln 16.67}{0.658}
\]

\[
t = 4.28 \text{ hours}
\]

4. Heritage Hills H.S. has 1500 students and plays class 4A basketball. In 1998, Heritage Hills had 1580 students. High school teams less than 1200 enrollment play class 3A basketball. If the drop in enrollment continues at a constant rate, what year will Heritage Hills High School drop to 3A basketball?

\[
1500 = 1580 e^{k(4)}
\]

\[
1994 = e^{k(4)}
\]

\[
ln 1994 = ln e^{4k}
\]

\[
ln 1994 = 4k \ln e
\]

\[
ln 1994 = k
\]

\[
k = \frac{ln 1994}{4}
\]

\[
1200 = 1500 e^{-0.013 t}
\]

\[
8 = e^{-0.013 t}
\]

\[
ln 8 = ln e^{-0.013 t}
\]

\[
ln 8 = -0.013 t \ln e
\]

\[
ln 8 = -0.013 t
\]

\[
t = \frac{ln 8}{-0.013}
\]

\[
t = 17.18 \text{ years}
\]
Today was the last day of new material, and it was a very busy and productive day. Not only was a lot of new material covered, but also based on my observations, the students understood the concepts discussed. There were several strong points to the lesson. First of all, the warm-up at the beginning of the period really got the students' minds back on logarithms. Also, the problems gave an introduction to what we were covering in class based on their prior knowledge of logarithms. Based on their knowledge of the properties of logarithms and the change of base formula, the students were able to complete the problems and gain some insight to what the topic was for the day. Another strong point to the lesson was the connections made to the real world. This lesson included many applications, mainly with finance, biology, and chemistry. Logarithms have many applications, and so I decided that it would be important for the students to see how they are used in the real world. Finally, a good strong point to the lesson was that I gave many examples for the students to try. I think that it is very important to do this with new material, mainly so that I can walk around the room to see if each student is completing the example and understanding the process of how to do the examples. Also, this gives me the opportunity to assist the students that may be struggling.

Based on how well the lesson went, I think that I met all of my objectives and goals for the lesson. By checking for understanding and scanning the room while the students were working on examples, I gathered that the students were able meet each of my objectives for the lesson. I came to this conclusion based on how well the students completed the examples and answered my questions. I used this information to assess student learning as well. I will also use the practice worksheet given to assess how well each of the students did with the concepts and objectives of the lesson, and if each objective was fully met.

If I was to teach this lesson again, I would try to make it a two-day lesson instead of doing it all in one day. I would teach the concepts behind the lesson on one day and
then spend the second day on applications, since there are so many of them. Also, while doing this, I would try to have more students work on the board or in groups during each lesson to get all of the students more involved.
**Precalculus Lesson Plan**

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<thead>
<tr>
<th>Textbook</th>
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<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Unit Review and Presentation of Projects</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to:</td>
</tr>
<tr>
<td></td>
<td>- Recall main concepts from the unit.</td>
</tr>
<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving</td>
</tr>
<tr>
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<td>Communication</td>
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<td></td>
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<td>Functions</td>
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<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set)</td>
</tr>
<tr>
<td></td>
<td>Chalk and chalkboard</td>
</tr>
<tr>
<td></td>
<td>Projector and computer for PowerPoint presentations</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>Do warm-up problem (ln and e sheet)</td>
</tr>
<tr>
<td></td>
<td>Attendance</td>
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<tr>
<td></td>
<td>- Take up disks - Copy to group</td>
</tr>
<tr>
<td></td>
<td>Go over warm-up</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Take questions from the homework and collect</td>
</tr>
<tr>
<td></td>
<td>PowerPoint presentation of chapter projects: Allow 5 minutes per group to present their project.</td>
</tr>
<tr>
<td></td>
<td>Distribute rubric for grading for class to score each group.</td>
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<tr>
<td></td>
<td>Grade each group based on their outline, presentation, and data.</td>
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<tr>
<td></td>
<td>Also give extra credit!</td>
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<tr>
<td></td>
<td>Ask for questions on any type of problem in unit.</td>
</tr>
<tr>
<td></td>
<td>Use page 647 for in-class review:</td>
</tr>
<tr>
<td></td>
<td>Use #1-9 from teacher's edition</td>
</tr>
<tr>
<td></td>
<td>Do ln word problems to do in class as an application review for the test.</td>
</tr>
<tr>
<td><strong>Closure</strong></td>
<td>Ask for any questions for the test</td>
</tr>
<tr>
<td><strong>Assignment</strong></td>
<td>Study for unit test</td>
</tr>
<tr>
<td></td>
<td>Pg. 646-648</td>
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<td>1-44</td>
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**ON TEST**

Rules of exponents
Simplifying rational exponents

$\text{Exp} \rightarrow \text{Log}$

$\text{Log} \rightarrow \text{Exp}$

Solving log eqns
Use prop of log
Explain exp

Log log to solve
Exp eqns

In word problems
1) \((\frac{1}{2})^{-3} \rightarrow 2^3 \rightarrow 8\)

2) \((\sqrt{81})^{\frac{3}{2}} \rightarrow (81^{\frac{1}{2}})^{\frac{3}{2}} \rightarrow 81^{\frac{3}{4}} \rightarrow \left(\frac{4\sqrt{81}}{3}\right)^3 \rightarrow (3)^3 \rightarrow 27\)

3) How are \(y = 2^x\) and \(y = 3^x\) related? Graph both!

4) What eqn used for int compounded continuously? \(A = Pe^{rt}\)

5) IF \(\log_b 3 = m\), what is \(\log_b 27\)?

\[b^m = 3\]
\[b^? = 27\]
\[b^? = 3^3\]
\[b^? = 3m\]

6) Domain, range of \(y = \log_b x\)

\(b – \) all + reals

\(R – \) all reals

7) \(\frac{3}{5}^x = 14\)

8) \(\ln 34.9 = ?\)

\(\ln 34.9 \approx 3.546 - 1.44\)
Precalculus warm-up and review

ln and $e$

Evaluate each:

1. $\ln 1 =$
2. $e^0 =$

3. $\ln e =$
4. $\ln e^2 =$

5. Write $e^1$ as a fraction:
6. Write $e^5$ as a fraction:

7. $\ln (1/e) =$
8. $\ln (1/e^7) =$

9. $\ln \sqrt{e} =$
10. $\ln \sqrt[3]{e^2} =$

11. $\ln 0 =$
12. $\ln -1 =$

13. $\ln (-e) =$
14. $e^{\ln x} =$

15. $e^x \cdot e^x =$
Precalculus warm-up and review

**In and e**

Evaluate each:

1. $\ln 1 = 0$
2. $e^0 = 1$
3. $\ln e = 1$
4. $\ln e^2 = 2$
5. Write $e^{-1}$ as a fraction: $\frac{1}{e}$
6. Write $e^{-5}$ as a fraction: $\frac{1}{e^5}$
7. $\ln \left(\frac{1}{e}\right) = -1$
8. $\ln \left(\frac{1}{e^2}\right) = -2$
9. $\ln \sqrt{e} = \frac{1}{2}$
10. $\ln \sqrt[3]{e^2} = \frac{2}{3}$
11. $\ln 0 = \phi$
12. $\ln -1 = \phi$
13. $\ln (-e) = \phi$
14. $e^{\ln x} = x$
15. $e^x \cdot e^x = e^{2x}$
Name ________________________________

Precalculus Population Growth PowerPoint Presentations  Group: ________________

Grading:

Completion of required slides: 2 pts each
  • Title ________
  • Table ________
  • Graph ________
  • explanation ________
  • sources ________ = ________ / 10 points
Presentation ________ / 5 points
Accuracy of data ________ / 5 points
Individual contribution to group ________ / 5 points

Extra Credit Using Excel: ________ / 5 points

Total ________ / 25 points

Name ________________________________

Precalculus Population Growth PowerPoint Presentations  Group: ________________

Grading:

Completion of required slides: 2 pts each
  • Title ________
  • Table ________
  • Graph ________
  • explanation ________
  • sources ________ = ________ / 10 points
Presentation ________ / 5 points
Accuracy of data ________ / 5 points
Individual contribution to group ________ / 5 points

Extra Credit Using Excel: ________ / 5 points

Total ________ / 25 points
Group: __________________

_______________________

_______________________

Rate your team members based on their role in this project: (1 – 5) with 5 being best.

Member #1: ________________  Score: _____

Member #2: ________________  Score: _____ (if applicable)

Group: __________________

_______________________

_______________________

Rate your team members based on their role in this project: (1 – 5) with 5 being best.

Member #1: ________________  Score: _____

Member #2: ________________  Score: _____ (if applicable)

Group: __________________

_______________________

_______________________

Rate your team members based on their role in this project: (1 – 5) with 5 being best.

Member #1: ________________  Score: _____

Member #2: ________________  Score: _____ (if applicable)
Reflective Analysis of Classroom Experiences  
at Anderson High School

4/17/02 Indian Day  
PowerPoint Presentations and Review

Today's precalculus lesson consisted of PowerPoint presentations of the students' projects involving population growth of countries and a review for the test on Friday. As I reflect back on today's lesson, I can note several strong points to my lesson today. First of all, my warm-up and review sheet was a strong point to the lesson. This quick review involving natural logarithms and $e$ not only gave the students practice with the rules of exponents, properties of logarithms, and using $\ln$ and $e$, but it also gave me time to set up the PowerPoint presentations and to do attendance. I really think that the warm-up sheet was really helpful for the students to start the review as well. Another strong point to my lesson was the order of which I conducted the lesson. The way I conducted the lesson was so that I had minimal downtime while still incorporating the technology with the presentations and the review at the end of class. A final strong point to my lesson was my review at the end of class. The class and I discussed what was going to be on the test, and everyone was involved in describing all that we have covered in this unit and solving problems for examples that I presented to them.

Because of the strong points of my lesson, my goals or objectives were met for the lesson. The students presented great projects to the class and each student in the groups participated with the presentation. Also, the students gained more experience working with technology to present their ideas and research, which I think is very important. Also, the class participated in a review discussion over the chapter. Since no new material was covered in this lesson, I did not have to assess student learning, but I will see how much the students know with regards to exponential and logarithmic functions on Friday when we review once more and take a test. Finally, if I taught this lesson again, I would not do it any differently. I really thought that today's lesson was a very successful one.
<table>
<thead>
<tr>
<th><strong>Textbook</strong></th>
<th>Merrill Advanced Mathematical Concepts</th>
</tr>
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<tbody>
<tr>
<td><strong>Lesson Title</strong></td>
<td>Chapter 11 Test</td>
</tr>
<tr>
<td><strong>Lesson Goals</strong></td>
<td>By the end of the lesson, students should be able to: - Demonstrate knowledge of concepts from unit on a test.</td>
</tr>
<tr>
<td><strong>NCTM Standards Addressed</strong></td>
<td>Problem Solving, Communication, Reasoning, Connections, Functions</td>
</tr>
<tr>
<td><strong>Instructional Materials &amp; Information Resources</strong></td>
<td>Graphing calculators (classroom set), Chalk and chalkboard</td>
</tr>
<tr>
<td><strong>Introduction</strong></td>
<td>Attendance, Discuss grades on projects, Do warm-up problem (exponential and logarithmic problems)</td>
</tr>
<tr>
<td><strong>Instructional Sequence &amp; Questions</strong></td>
<td>Ask and answer questions from review assignment. Ask for any general questions regarding test. Distribute Unit Test</td>
</tr>
<tr>
<td><strong>Closure</strong></td>
<td>Announce the start of series and sequences: use example of e</td>
</tr>
<tr>
<td><strong>Assignment</strong></td>
<td>Read 12-1 Pg. 656 – 660</td>
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</table>
Precalculus Test: Chapter 11
Exponential and Logarithmic Functions

Name ____________________

Date _______ Period ____

Non-calculator Section

1. Evaluate \( \frac{\sqrt{32}}{32^{\frac{3}{4}}} \).

2. Express \( \sqrt[3]{64b^4} \) using rational exponents.

3. Express \( 2^{\frac{1}{2}} \cdot x^{\frac{5}{4}} \) using radicals.

4. Evaluate \( \frac{g^{\frac{1}{3}}}{g^{\frac{4}{3}}} \).

5. Express \( \sqrt[3]{35ab^{30}} \) using rational exponents.

6. Express \( a^{\frac{3}{4}} \cdot b^{\frac{1}{4}} \) using radicals.

7. Write \( 0.2^3 = 0.008 \) in logarithmic form.

8. Write \( \log_7 (\sqrt{7})^6 = 3 \) in exponential form.

9. Write \( 4^{-2} = \frac{1}{16} \) in logarithmic form.

10. Solve \( \log_x \frac{1}{216} = -3 \).
Precalculus Test: Chapter 11
Exponential and Logarithmic Functions

Calculator Section

Solve each equation using logarithms. Round your answer to the nearest ten-thousandth.

18. \( 5^x = 45 \)  

19. \( 6^{x+2} = 10.3 \)

20. Ms. Cubbatz invested a sum of money in a certificate of deposit that pays 8% interest compounded continuously. Recall that the formula for the amount in an account earning interest compounded continuously is \( A = Pe^{rt} \). If Ms. Cubbatz made the investment on January 1, 1989 and the account is worth $12,000 on January 1, 1993, what was the original amount in the account?

21. To the nearest dollar, find the future value of $500 invested at 9% for 4 years in an account that is compounded continuously.

22. Mide Kallenberg deposited some money in a bank that earns 5.6% interest compounded continuously. How long would it take to double the amount of money in Mr. Kallenberg's account?
Bonus Problems

1.) After 13 years, 2.1 pounds of radioactive material remain from a 7-pound sample. Using the equation, \( y = y_0 \cdot 0.5^{t/T} \), find the half-life of the material.

2.) Explain why \( \ln 0 = x \) has no real solutions.
Exponential and Logarithmic Functions

Non-calculator Section

1. Evaluate \( \frac{\sqrt[3]{32}}{32^{\frac{1}{3}}}. \)
   \[ \frac{2}{4} = \frac{1}{2} \]

2. Express \( \sqrt[6]{64b^4} \) using rational exponents.
   \[ x^{\frac{4}{6}} b^{\frac{2}{6}} = \sqrt[3]{2b} \]

3. Express \( 2^\frac{1}{2} x^\frac{1}{5} \) using radicals.
   \[ \sqrt[5]{2x} \]

4. Evaluate \( \frac{\frac{1}{8}}{\frac{1}{3}} \).
   \[ \frac{1}{8} \cdot \frac{3}{1} = \frac{1}{2} \]

5. Express \( \sqrt[5]{35a^3b^5} \) using rational exponents.
   \[ 35^\frac{3}{5} a^{\frac{3}{5}} b \]

6. Express \( a^{\frac{2}{3}} b^{\frac{1}{2}} \) using radicals.
   \[ \sqrt[3]{a} \cdot \sqrt{b} \]

7. Write \( 0.2^2 = 0.008 \) in logarithmic form.
   \[ \log_{10} (0.008) = 3 \]

8. Write \( \log_7 (\sqrt{7})^3 = 3 \) in exponential form.
   \[ \log_7 7^3 = 3 \rightarrow 7^3 = (\sqrt{7})^6 \]

9. Write \( 4^{-3} = \frac{1}{16} \) in logarithmic form.
   \[ \log_4 \frac{1}{16} = -2 \]

10. Solve \( \log_2 \frac{1}{216} = -3. \)
    \[ x^{-3} = \frac{1}{216} \]
    \[ x^3 = 216 \]

Date 4/19 Period B+7
Graph each equation or inequality. (2 points each)

11. \( y = 0.5^x + 2 \)

<table>
<thead>
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<th>( x )</th>
<th>( y )</th>
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<tr>
<td>-1</td>
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12. \( y \leq 4^x \)

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<td>1</td>
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</tbody>
</table>

Solve each equation. (2 points each)

13. \( \log_4(2x - 1) = \log_4 16 \)

\[
2x - 1 = 16
\]
\[
x = 9
\]

14. \( \log_5 3 + \frac{1}{2} \log_5 36 = \log_5 x \)

\[
\log_5 3 + \log_5 6 = \log_5 x
\]
\[
\log_5 18 = \log_5 x
\]
\[
x = 18
\]

15. \( \log_x 64 = 3 \)

\[
x = 64
\]
\[
x = 4
\]

16. Evaluate \( \log_{27} 81 \).

\[
27^x = 81
\]
\[
x = \frac{4}{3}
\]

17. Solve \( \log_2 0.125 = x \)

\[
2^x = 0.125
\]
\[
x = \frac{1}{3}
\]
\[
2^x = 2^{-3}
\]
\[
x = -3
\]
Calculator Section

Solve each equation using logarithms. Round your answer to the nearest ten-thousandth.

18. \(5^x = 45\)
   \[
   \log_5 45 = x \\
   x = \frac{\log 45}{\log 5}
   \]
   \(x \approx 2.3652\)  

19. \(6^{x+2} = 10.3\)
   \[
   \log 6^{x+2} = \log 10.3 \\
   (x+2) \log 6 = \log 10.3 \\
   x + 2 = \frac{\log 10.3}{\log 6} \\
   x = \frac{\log 10.3}{\log 6} - 2
   \]
   \(x \approx -6.984\)  

20. Ms. Cubbatz invested a sum of money in a certificate of deposit that pays 8% interest compounded continuously. Recall that the formula for the amount in an account earning interest compounded continuously is \(A = Pe^{rt}\). If Ms. Cubbatz made the investment on January 1, 1989 and the account is worth $12,000 on January 1, 1993, what was the original amount in the account?
   \[
   12,000 = Pe^{.08 \cdot 4} \\
   12,000 = P(1.377) \\
   P = \$8,713.79
   \]

21. To the nearest dollar, find the future value of $500 invested at 9% for 4 years in an account that is compounded continuously.
   \[
   A = 500e^{.09 \cdot 4} \\
   A = 716.66 \\
   A = \$717
   \]

22. Mide Kallenberg deposited some money in a bank that earns 5.6% interest compounded continuously. How long would it take to double the amount of money in Mr. Kallenberg's account?
   \[
   A = Pe^{rt} \\
   2P = Pe^{.056t} \\
   2 = e^{.056t} \\
   \ln 2 = \ln e^{.056t} \\
   \ln 2 = .056t \ln e \\
   \frac{\ln 2}{.056} = t \\
   t = 12.38 \text{ years}
   \]
Bonus Problems

1.) After 13 years, 2.1 pounds of radioactive material remain from a 7-pound sample. Using the equation, \( y = y_0 \cdot 0.5^{\frac{t}{T}} \), find the half-life of the material.

\[
2.1 = 7 \cdot 0.5^{\frac{13}{T}}
\]

\[
3 = 0.5^{\frac{13}{T}}
\]

\[
\log_{0.5} 3 = \frac{13}{T} \log 0.5
\]

\[
\frac{\log 3}{\log 0.5} = \frac{13}{T}
\]

\( T = 7.48 \) years

2.) Explain why \( \ln 0 = x \) has no real solutions.

Answers may vary

\( e^x \neq 0 \) -- there is no number that could be an exponent on \( e \) that equals zero. \( e \) to any power is never equal to zero.
4/19/02 Indian Day  
Chapter 11 Review and Test

Since today was a test day, my only objective for the students was to show me how much they know and recall from our unit on exponential and logarithmic functions. There were, however, a couple strong points to my lesson. First, I gave warm-up problems that not only served to direct the students' minds to math, but to give me time to take attendance and distribute papers. Also, the warm-up was a set of problems that featured the key ideas from the unit and were chosen based on the areas that I thought that the students might have problems with. By doing an assessment of what I thought the class might have troubles with, I selected the correct problems to help correct this. Also, by having selected students write and explain the solutions to the problems, the students saw a different perspective of doing the problems. Hopefully this helped them with understanding the problems and their solutions.

My goal for the lesson was to provide an adequate but quick review and to administer a test to assess each student's knowledge with exponential and logarithmic functions. I really think that I met my goal for the lesson. The students benefited greatly from the warm-up problems for review. Also, I will assess student learning based on the results of the test that the students took. Finally, if I taught this lesson again, I would not do it any differently. The class was given plenty of time to take the test while receiving a short review as well.
### 6th Period Precalculus Class

#### Unit Grades

<table>
<thead>
<tr>
<th>Assign.</th>
<th>Pg. 602</th>
<th>WS 11-2</th>
<th>Pg. 617</th>
<th>Comp. Int</th>
<th>WS 11-4</th>
<th>Project</th>
<th>WS 11-6</th>
<th>Unit Test</th>
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#### Grade Distribution - Test

- A: 8
- B: 4
- C: 4
- D: 1
- F: 3

#### Grade Distribution - Overall Unit

- A: 8
- B: 8
- C: 2
- D: 2

#### Grading Scale:

- A: 95-100
- B: 85-94
- C: 70-84
- D: 60-69
- F: 0-59

Average: 42.23

%: 84.46%

%: 91%
Distribution of Test Scores
Sixth Period

Distribution of Unit Scores
Sixth Period
# 8th Period Precalculus Class

## Unit Grades

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Distribution of Test Scores
Eighth Period

Distribution of Unit Scores
Eighth Period
Textbook Analysis of Merrill Advanced Mathematical Concepts: Precalculus with Applications

Merrill Advanced Mathematical Concepts is a high school precalculus textbook copyrighted in 1997. The textbook is currently used at Anderson High School for a precalculus course. The publisher is Glencoe/McGraw-Hill, and the authors are Berchie W. Gordon, Lee E. Yunker, F. Joe Crosswhite, and Glen D. Vannatta.

This textbook covers mathematical topics included in seventeen chapters. It includes many reference and supplemental materials. In the back of the text, there is a glossary, answers to selected exercises, an index, a symbols page, and a formulas page. Applications and information about careers are spread throughout the book and serve as important elements in the textbook. Many computer and graphing calculator applications are also given in special sections. Graphing calculators are even requested to be used for particular problems and applications throughout the text.

Each chapter begins with a list of objectives and information about a career that relates to the content in the chapter. As far as the way the mathematical content is presented, basically each section of each chapter begins with a brief introduction and an application to lay the foundation for the section. This introduction to a concept and application is followed by an example. Many other brief introductions and definitions paired with examples subsequently follow.

In this textbook, the exercises are listed at the end of each section. A check for understanding is included first, which features guided practice problems and questions for communicating mathematics. Following the check, there are many practice exercises. At the end of every practice exercises set, there is a critical thinking problem along with applications and problem solving. There are also writing and graphing calculator questions in some sections. There are additional sections and segments that involve explorations with other subjects, applications, technology, and graphing calculator activities.

There are numerous examples throughout the book where connections between the subject matter, other disciplines, and real life are made. At the beginning of each chapter, a career goal section is featured. This section discusses a particular career where math from the chapter presented is used. Also, every chapter includes applications that
involve other subjects such as physics, chemistry, finance, banking, biology, business, geology, and even psychology. At the beginning of every chapter, a chapter project is suggested and explained. Teachers can use this to do a project involving the mathematics presented in the chapter. Other mathematical ideas, such as geometry, trigonometry, statistics, probability, and discrete mathematics are featured. In other words, this text includes a variety of ideas, applications, and disciplines for a well-rounded approach to precalculus.

In my opinion, I think that this is a great textbook. I was surprised that it includes everything that I would expect from a precalculus textbook. I even used the textbook while student teaching at Anderson High School. It covers all areas to successfully prepare for calculus and college. Also, I really liked the sections regarding writing, projects, and the use of technology with computers and especially graphing calculators for problems in each section. The separate graphing calculator activity sections are a great addition as well. I think that writing is very important to mathematics and the understanding of the subject. Also, I believe that projects are also important. Technology, of course, is essential today in the teaching of mathematics, especially with precalculus, and helpful in the understanding of mathematical concepts. All of these important ideas are presented in this textbook, along with a great array of applications of mathematics in many different subjects.

The NCTM standards are also well represented. Each standard for a precalculus course is featured in this textbook. I do not see any aspect of this book as problematic. I do, however, wish that I had a more updated version of this book, and one that features a Ti-83 calculator instead of a Ti-81, since a Ti-83 is more commonly used. The book starts where I want, with a unit on relations, functions, and graphs, for I think that precalculus students need to understand functions and their graphs before continuing with any other topic in precalculus. I think that this textbook is missing very little, and the use of examples in each section is emphasized, which is great. If this textbook were to be used in my school district, I would be very pleased. As stated before, it includes everything that I would want in a precalculus textbook. Also, I have used in during my student teaching experience, and the textbook was a great resource for me.
Bibliography of Resources Used for Unit Plan


Important Websites

- **A Unit Plan for Exponential and Logarithmic Functions**
  Honors College Project Online

  **Go to:**
  http://homepage.mac.com/apgogel

  Click on Honors Thesis folder

  Select the folder of your choice:
  - Lesson Plans
  - Other
  - Reflective Journals
  - Unit Plan

  Contains the entire project

- **Electronic Portfolio**
  **Under Construction**

  **Go to:**

  Features artifacts and other information with regards to the INTASC standards. Also includes other personal information with regards to education.
NCTM Standards Note and Justification

Most of the NCTM Standards are highlighted upon or satisfied in this unit. As noted in the Unit Plan, the NCTM Standards included in this unit are: problem solving, communication, reasoning, connections, algebra, representation, functions, data analysis, and mathematical structure. The following is the justification of my choices for the Exponential and Logarithmic Computer Activity with regards to the NCTM Standards.

Justification of Choices:

I believe that it is important for students to identify functions, especially exponential functions, and see how graph of each function changes as different values are placed on each of the functions. The previous activity took a basic concept, and placed different values at different locations of the function, and then analyzed how each value changed the graph of each function. This idea is prevalent throughout graphing of functions, and it is important for students to identify changes in graphs as the functions change.

According to the NCTM standards, “students should encounter new classes of functions,” which include exponential functions, and “They should begin to understand aspects of mathematical form and structure, such as that all quadratic functions share certain properties, as do all functions of other classes—linear, periodic, or exponential.” (p. 287) In this activity, students are to see that exponential functions share certain properties, and that slight differences in functions result in slight differences in their graphs. Finally, to reiterate my point for justification, the NCTM standards state that, “High school students should have substantial experience in exploring the properties of different classes of functions,”(p.298) which is what I included in this activity, but with exponential functions. Finally, the NCTM standards state that, “In addition, students should learn to recognize how the values of parameters shape the graphs of functions in a class. With access to computer algebra systems (CAS)—software on either a computer or calculator that carries out manipulations of symbolic expressions or equations, can compute or approximate values of functions or solutions to equations, and can graph functions and relations—students can easily explore the effects of changes in parameter
as a means of better understanding classes of functions.” (p.299) In the activity, Function Probe is used to discover changes in exponential function parameters.

Finally, from the July 2000 Journal for Research for Mathematics Education, “the topic of multiple representations of functions is important in secondary school mathematics curricula.” Eric J. Knuth, University of Wisconsin—Madison, presented this in his article, “Student Understanding of the Cartesian Connection: An Exploratory Study”, which finally justifies that it is important to look at different representations of functions, which also includes exponential functions.