Hi, kids. I'm Rex Roper, ace reporter for America's number one tabloid, the National Gasbag Chatterer. I'd like to tell you about some things that are so STRANGE, so BIZARRE, so downright SHOCKING that you may not believe they actually happened.

Are these things true? Well, you be the judge.

Ketchup Saves Boy's Life!

**Fargo, N.D.** Ten-year-old Tommy Farkus mistakenly ate liver for dinner last night.

Emergency teams rushed to the scene.

Tommy was treated for 6/7 hour in the ICFU (Incredibly Crummy Food Unit) at Fargo General Hospital. After another 3/4 hour of observation, he was released.

"I thought I was eating fish sticks," Tommy explained.

Doctors estimate that the boy swallowed three bites of liver weighing 3/5 ounce, 2/3 ounce, and 1/4 ounce.

"Ketchup saved him," said Dr. Janet Janetski. "He had so much ketchup on it, he couldn't even taste the liver."


1. How much time did Tommy spend in the hospital?

2. How much more time was spent treating Tommy than observing him?

3. How many ounces of liver did Tommy eat in all?

4. How many ounces of liver did Tommy eat in his first 2 bites?
5. How much liver did Tommy eat in his last 2 bites?

6. How much more liver did Tommy eat in his third bite than his second bite?

**ADDING FRACTIONS WITH LIKE DENOMINATORS**

To add \( \frac{8}{9} + \frac{5}{9} \):

1. Add the numerators
   \[ \frac{8}{9} + \frac{5}{9} = \frac{13}{9} \]

2. Change the sum to a mixed number, if necessary.
   \[ \frac{13}{9} = 1 \frac{4}{9} \]

7. What fraction of victims either panics or groans?

8. What fraction of victims yowls or covers the stuff with ketchup?

9. What fraction of victims panics, yowls, or covers with ketchup?

10. What fraction panics, groans, yowls, or covers with ketchup?

**Dog Eats Homework (Really?)**

Bobville, SC: Mike, a mutt owned by 12-year-old Brenda Sykes, became the first dog in history to actually eat a homework assignment.

Brenda had been making a scale model of Washington D.C. out of foot-long frankfurters when the hungry pup struck.

"He ate the whole thing," Brenda said.

Among the monuments destroyed were a \( \frac{1}{12} \)-pound frankfurter White House, a \( \frac{5}{12} \)-pound frankfurter Pentagon, a \( \frac{7}{12} \)-pound frankfurter Capitol, and an \( \frac{11}{12} \)-pound frankfurter Smithsonian.

(See questions on the next page)
**Oh No! 99!**

**Overview**

While older elementary students are typically engaged with larger whole numbers, many still need and benefit from practice with mental addition and subtraction of smaller numbers. In this two-person card game, players attempt to force their partner to be the one to push their jointly accumulating score above 99. The game provides practice with adding and subtracting while also giving students the chance to think strategically.

**Materials Needed**

A deck of playing cards (jokers removed) for each pair of students.

**Card Values and Operations**

- Aces: add 1
- Jacks: subtract 10
- Queens: wild cards that can represent any other card in the deck
- Kings: add zero
- All others (2–10): add their face value

**Directions for Playing the Game**

1. One player shuffles the cards and deals four cards to each player. The undealt cards remain in a stack, face down.
2. Players take turns playing one card at a time, adding or subtracting the value of their card to or from their jointly accumulating score.
3. Each time a player plays a card, he or she must replace it with the top card on the face-down stack.
4. Play continues until one player forces his or her partner to go over the score of 99.
IN THE CLASSROOM WITH CAREN

Introducing the Activity

To introduce Oh No! 99! to Kathleen Gallagher's fifth graders, I'd planned to go through a sample game with the whole class and then send them off to play with partners. I began by asking the class to join me in a circle on the rug, so everyone would have a good view of the deck of cards. After some adjusting of desks and bodies, we were ready to begin.

"I brought a lot of decks of cards here today because you're going to learn a card game," I announced. "Is it poker?" asked Chip, to a round of giggles.

"No, it's not poker and it doesn't involve gambling," I replied. "It's a game you play with a partner that helps you with math. I like this game a lot. It gives you a lot of practice adding numbers in your head. It makes you think."

I walked to the board and wrote Oh No! 99! "This game is called Oh No! 99! As I explain the game you might start to get some ideas about how it got its name. Now before I show you how to play, there are a few important things you need to know about the cards." I wrote A, J, Q, K on the board. "In Oh No! 99! an ace means add one point," I explained. I wrote add 1 next to the A. "A jack means subtract ten. A queen is wild. That means you can assign the queen the value of any other card in the deck."

"It can be any number?!" asked Jeannette, wide eyed.

"Can it be 1,000?" asked Kenneth.

"Well, it can't be 1,000, because there's no other card in the deck worth 1,000. It's wild, but it's not that wild," I told the class. I proceeded with my explanation. "A king means you don't add or subtract anything. For the rest of the cards in the deck, add whatever their number is: an eight means add eight, a three means add three. I'm going to leave this information up on the board, because you might need it when you play with your partner. The object of the game is to make your partner go over 99. So I want to force you to put down a card that makes the total 100 or more, and you want to try to get me to do the same. Can you guess why this game is called Oh No! 99?!" I asked.

"Because if there's 99 already, you're in trouble," offered Enrique.

"You got it," I agreed. "Before you go back to your tables, though, we're going to play one game together so you can see what the game is like and I can answer any questions. Since this is a game for partners, I'll play with the whole class as my partner. We're going to take turns adding cards to the discard pile."

I dealt four cards to myself and four cards to the class. I dealt all the cards face up, although I explained that when they played by themselves, they'd keep their cards hidden so their partner wouldn't know what they had.

"Okay, I'll go first, and I'll put down this seven. And since I put down a card, I need to pick another card from the top of the deck. I always want
to have four cards in my hand. Now it's your turn. Who would like to choose a card for the class?” Many hands shot up. I called on Greg.

“I'll use the nine,” Greg announced.

I put the nine on top of the seven.

“So now what's the total for the pile?” I asked the class.

“Sixteen!” they responded in unison.

“Good,” I said, “whenever you put a card down on the pile you have to tell what the new total is. Partners need to check each other and pay attention to make sure both of you know the total.”

We continued playing. I called on various students to make a choice for the class, and had the whole class tell me the new total after each card was added. When the total of the pile was 88, I asked Jenny to choose a card for the class to play. Many students tried to influence her.

“Don't use the ace yet,” advised Miguel.

“Put down the nine,” suggested Annabel.

Realizing the advice was being motivated by strategic thinking, I stopped the game to point this out: “I'm noticing that many of you have ideas about which card to play next. Would anyone like to explain your thinking?” I called on Kate.

“I think we should save the ace.”

“Why is that?” I asked.

“Because with an ace you only have to add one, and that's a low number. If the cards get up to a high number like 97 or 98, we can use the ace to make you go over 99.”

Many students nodded in agreement.

“Okay,” I continued, “Kate thinks you should hold onto your ace and save it for later. Does anyone have an idea about which card you might want to play next?”

“Use the nine,” said Ana, “because then the total will be 97 and that's close to 99. If you don't have a low card or a jack, queen, or king, we could win.”

The class played the nine, and my next play put the score over 99. I then sent the students off to their tables to play the game in pairs. “Remember,” I told them, “this game is important for two reasons. First, it gives you a lot of practice adding in your head. Second, when the total gets close to 99, you have to do a lot of thinking to plan a good strategy.”

Observing the Students

The students returned to their seats, and I circulated around the room. At first, I just wanted to make sure everyone understood the game and was playing with a partner. Then I spent some time observing individual games. Several students were quite animated and couldn't resist showing their cards to friends nearby. Miguel, for one, was proudly flashing his picture cards to anyone in his vicinity. I issued a few gentle reminders for students to stay in their seats and focus on the game.

I noticed that while many students were quickly and easily calculating the totals mentally, others
were more hesitant; some were even using their fingers. I was surprised to see Chip use his fingers to add 10 to 43. Adding ten should come automatically to most fifth graders, especially one like Chip, who came from a very traditional math program. However, it was clear that he had not made the base ten connection in this context. While Chip was certainly capable of adding 10 and 43 on paper using the standard algorithm, he did not see the significance of the relationship between the two numbers nor that there was a very predictable pattern when adding ten.

**A Writing Assignment**

After about fifteen minutes, even though everyone was still very involved in playing the game, I called everyone back together. I wanted to see what kinds of strategies they were using at this point, and I wanted them to have the opportunity to hear some of their classmates' thinking about the game so far. I illustrated a hypothetical situation on a projected transparency.

"Imagine," I said, "that you're playing Oh No! 99! and the total is up to 87. Your four cards are a six, a queen, an ace, and a king. Which card would you play next? As you think about this, pay attention to why you're choosing a particular card. I'm going to give you ten minutes of quiet writing time so you can tell me your ideas on this question. Make sure you put your name and date on the paper. Are there any questions?"

"You just want us to tell you which card we would use?" asked Jon.

"That's part of it," I answered, "but I also want to know why you would choose that card instead of any of the others. You might even want to tell me which card you definitely wouldn't want to use and why."

Most students chose either the six or the queen as their next card on the pile. Traci wrote, I would put the six down because you should get rid of your high cards and save your low cards as you get in the high 80s and 90s. A, Q, K, are not high cards because a Q is a wild card and you can use it as a 10. A K is a 1. Neal made a convincing argument for the queen (see figure 2.1): I would put down the queen as a ten so the total would be 97. 97 is a high number and if your partner has only numbers higher than four you win. If they have numbers less than four or a king, queen, jack, or ace and they lay down a queen as a two or a two, you can put down the king.

After reading through the class papers (additional examples are shown in figures 2.2 and 2.3), I realized the question didn't dig deeply enough into the methods the students used for adding the numbers. I got a general feel for their thinking about the cards, but the prompt I used focused more on strategy. I wanted to ask a question about the game that encouraged the children to tell me more explicitly how they were combining numbers. Did they use what they knew about place value to help? Were they merely counting on? Did they have more than one way to add numbers? I decided I would try to focus on these questions next time.
Teaching the Standard (Computation) With Children’s Literature

*Books dealing with this subject that could be incorporated into mathematics instruction.

2 x 2 = BOO!: A Set of Spooky Multiplication Stories by Loreen Leedy (Holiday House, 1995).

Amanda Bean’s Amazing Dream: A Mathematical Story by Cindy Neuschwander (Scholastic Press, 1998).

Anno’s Mysterious Multiplying Jar by Masaichiro and Mitsumasa Anno (Philomel, 1983).

Bunches and Bunches of Bunnies by Louise Mathews (Scholastic Press, 1991).


The Great Divide by Dayle Ann Dodds (Candlewick Press, 1999).


King Bidgood’s in the Bathtub by Audrey Wood (Harcourt Brace, 1985).

Mice Twice by Joseph Low (Macmillan, 1986).

Mission Addition by Loreen Leedy (Holiday House, 1997).


A Remainder of One by Elinor J. Pinczes (Houghton Mifflin, 1995).


Shark Swimathon by Stuart Murphy (HarperCollins, 2000).


What Comes in 2’s, 3’s, and 4’s? by Suzanne Aker (Simon & Schuster, 1990).
Standard 3 — Algebra and Functions

Algebra is a language of patterns, rules, and symbols. Students at this level represent relationships with numeric equations and use those equations to solve problems. They continue number patterns involving multiplication and use some of the rules for multiplication to check results. They begin to develop the concept of a function and the relationship between numbers and number lines.
Standard 3
Algebra and Functions

Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number and functional relationships.

3.3.1 Represent relationships of quantities in the form of a numeric expression or equation.
Example: Bill’s mother gave him money to buy three drinks that cost 45 cents each at the concession stand. When he returned to the bleachers, he gave 25 cents change to his mother. Write an equation to find the amount of money Bill’s mother originally gave him.

3.3.2 Solve problems involving numeric equations.
Example: Use your equation from the last example to find the amount of money that Bill’s mother gave him, and justify your answer.

3.3.3 Choose appropriate symbols for operations and relations to make a number sentence true.
Example: What symbol is needed to make the number sentence $4 \_ 3 = 12$ true?

3.3.4 Understand and use the commutative* and associative* rules of multiplication.
Example: Multiply the numbers 7, 2, and 5 in this order. Now multiply them in the order 2, 5, and 7. Which was easier? Why?

3.3.5 Create, describe, and extend number patterns using multiplication.
Example: What is the next number: 3, 6, 12, 24, ...? How did you find your answer?

3.3.6 Solve simple problems involving a functional relationship between two quantities.
Example: Ice cream sandwiches cost 20 cents each. Find the costs of 1, 2, 3, 4, ... ice cream sandwiches. What pattern do you notice? Continue the pattern to find the cost of enough ice cream sandwiches for the class.

3.3.7 Plot and label whole numbers on a number line up to 10.
Example: Mark the position of 7 on a number line up to 10.

* commutative rule: the order when multiplying numbers makes no difference (e.g., $5 \times 3 = 3 \times 5$), but note that this rule is not true for division
* associative rule: the grouping when multiplying numbers makes no difference (e.g., in $5 \times 3 \times 2$, multiplying 5 and 3 and then multiplying by 2 is the same as $5$ multiplied by $3 \times 2$), but note that this rule is not true for division
Example Algebra and Functions Lesson
Lesson Plan Content Page

Name: Abby Land
Lesson Topic: Algebraic Patterns

INTASC Principle: The professional educator understands content. (#1)

IN State Standard: Standard 3 Algebra and Functions- Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number and functional relationships.

IN State Indicator: 3.3.5- Create, describe, and extend number patterns using multiplication. Example: What is the next number: 3, 6, 12, 24, ...? How did you find your answer?

Annotated Bibliography:
Instructional:
This binder provides the teacher with many different worksheets having to do with creating and identifying patterns. These work sheets can be copied from the binder for use in the classroom. The binder is available in the Reading Room on the fourth floor of the Robert Bell Building.

This book contains an entire unit of lessons focused on algebraic patterns. These lessons focus on different types of patterns and involve the use of different manipulatives. Many of the ideas used in this lesson were pulled from this book. It can be found on the fourth floor of the Robert Bell Building in the Reading Room.

Informational:
At the start of the patterns unit this book contains a few pages that explain to the teacher what the book is teaching. This helps the teacher understand the basic concepts of the unit. This book is available in the Reading Room on the fourth floor of the Robert Bell Building.
Lesson Plan

IN State Standard: Standard 3 Algebra and Functions- Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number and functional relationships.

IN State Indicator: 3.3.5- Create, describe, and extend number patterns using multiplication. Example: What is the next number: 3, 6, 12, 24, ...? How did you find your answer?

Lesson Objective: The students will recognize patterns in mathematical contexts, and the rules that produce such patterns. The students will also be able to extend and make generalizations about the patterns.

Materials/Media: Hundreds boards (one for each exploration per student), colored pencils or crayons (one set for each student), overhead, overhead sheet, overhead marker, isosceles right triangle sheets (one for each student), overhead set of isosceles right triangles, scissors (one pair per student), blackboard, chalk, 30 tiles of one color and 30 of another (one set for each student), overhead copies of figures 1.6 and 1.7, and the pattern worksheets packet (one for each student).

New Information:
• Growing patterns involve a progression from one step to the next.
• These “growing patterns” are generally referred to as sequences.
• Growing patterns may have both increasing and decreasing sequences.
• Growing patterns can involve addition, subtraction, multiplication, division, or two or more of these methods.

Motivation: “You will be creating colorful designs by following the rules for selecting numbers from the hundred boards.” Discuss the multiples of numbers such as 2, 3, and 4 as a class to begin thinking about number combinations. Select two or three of the rules from the list below to have the students complete, or allow the students to choose their own. Explain to the students that each rule will need to be done on a separate hundred board (it does not matter if you use the 0-99, or 1-100 board) and tell them they should use a different color for each rule. The teacher should make sure to write the rules the students will be completing on an overhead.

Example Rules for Hundred-Board:
• Numbers with a 2 in them.
• Numbers with a 4 in them.
• Numbers that are multiples of 3.
• Numbers with a 7 in them.
• Numbers that are multiples of 5.
• Numbers with a 0 in them.
• Numbers having both digits the same. (Multiples of 11.)
• Numbers whose digits add to 9 (For example: in 63, 6+3=9). (Multiples of 9.)
Allow the students time to look over their completed designs. Encourage the students to look for distinct and similar patterns in their designs, and to describe their findings to the class. (Identifying Similarities/Differences) Tell the students to try and figure out why their design turned out the way it did. Ask the students questions like, “Do any boards have the same pattern? How are the boards alike and how are they different? Is there a difference between the boards that have multiples of even numbers and those that have multiples of odds? What do you notice about numbers that are multiples of both 3 and 5? What is the difference between these patterns?” The teacher will allow the class time to discuss each question after it is asked, before moving on to the next. (Questions, Cues, Advance Organizers) “By now you have discovered these interesting designs are actually mathematical patterns. Today we will be learning more about mathematical patterns and discovering how to continue these patterns.” Goal for Learner

Procedure:

1. New Information: The patterns we have seen so far are called growing patterns and they are the type of patterns we will continue to look at today. These patterns are called growing patterns because they may increase or decrease from step to step. In growing patterns these steps are referred to as sequences. Growing patterns can involve addition, subtraction, multiplication, division, or a combination of two or more of these methods. By finding the pattern that leads from one sequence to the next, you will be able to continue the pattern and learn the mathematics behind the pattern.

2. Modeling: The first pattern activity we are going to do involves finding out how many triangles we will need to make a worm who is getting bigger as he ages. Have the students cut the triangles out of their sheets. Now show the students on the overhead how big the worm is when it is one-day-old. Record the worms growth in a chart listing in one column, its age (in days) and in the other column, its growth (how many triangles it is). For example: Day 1 is 4 triangles. Now model the two-day old worm for the students on the overhead. Count the number of triangles and record this information in the chart.

3. Guided Practice: Have the students model a three-day-old worm on their desks while you model the same worm on the overhead. Tell the students to count the number of triangles making up the worms body and record this number in the chart. Explain to the students, “We are now going to find out how many triangles it would take to build a ten-day-old worm. However, in order to do this we must first find the mathematical pattern that is being used.” Guide the students through the process of finding this pattern. Encourage the students to look at the pattern and see how it grows (by two every time), then encourage the students to see how the worm began (as two center triangles and 2 on the sides). Now help the students by leading them to the equation by taking steps that seem appropriate for the level at which your class is working. (You may need to encourage students to work together to model more worms.) Answer: (age in days x 2) + 2 = number of triangles, or (age in days +1) x 2 = number of triangles.
4. **Check for Understanding:** Ask the students, "How many triangles would a six-day-old worm have? What about a twelve-day-old worm? Why do you know this? Could you figure out a new pattern if the worm had three heads meaning there were always three triangles on the ends of the worm instead of the original two?" Allow the students time to discuss the answers. *(Questions, Cues, Advance Organizers)*

5. **Practice/Application:** "Alfredo Gomez is designing square patios. Each patio has a square garden area in the center. Alfredo uses brown tiles (substitute color 1 that is being used here) to represent the soil of the garden. Around each garden, he designs a border of white tiles (substitute color 2 that is being used here). The pictures (put figure 1.6 on the overhead) show the three smallest square patios he can design with brown (color 1) tiles for the garden and white (color 2) tiles for the border." Tell the students, "You now need to model these three patios on your desk and record the data in a table similar to this one (place figure 1.7 on the overhead)." *(Non-linguistic Representations)* Allow the students time to build the first three patios and count the number of tiles involved. Encourage the students to figure out the pattern that is being used to build the patios. To do this, they may need to build more patios and do other experiments. Explain to the students once they have found the pattern, they should fill the data into their chart for Patio 6. One example of a possible pattern that works is: total brown tiles = patio number x patio number and total white tiles = (4 x patio number) + 4. Give the students time to come up with the pattern, and allow them to work with partners, or give them hints if they have too many problems.

6. **Closure:** The teacher will ask the students, "What did you learn during today’s lesson?" The teacher and the students will discuss the answers. Then the teacher will ask the students, "How could knowledge of mathematical patterns help you in the real world?" (An example of this was the tiling a patio activity). The teacher and students will discuss possible responses to this question. *(Questions, Cues, Advance Organizers)*

**Evaluation of Student Learning:** The teacher will pass out the pattern worksheets packet to each student (each packet contains pattern worksheets 1, 2, 3, 4, and 5). Depending on how much practice the students need the teacher will choose if they want to assign the whole packet, or only select worksheets. These worksheets will then be collected and graded to evaluate the students’ knowledge of patterns.

**Lesson Extension:** If there is time left over the teacher will encourage the students to draw a pattern of their own after showing an example. This activity will incorporate art into the classroom and show the students that patterns exist in both art and mathematics.
### Hundred Board (0–99)

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Hundred Board (1–100)

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<td>71</td>
<td>72</td>
<td>73</td>
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<td>94</td>
<td>95</td>
<td>96</td>
<td>97</td>
<td>98</td>
<td>99</td>
<td>100</td>
</tr>
</tbody>
</table>

Navigating through Algebra in Grades 3–5
Blocks

Pattern Blocks

Isosceles Right Triangles
Fig. 1.2.
Worms made with isosceles right triangles

Fig. 1.3.
A table showing the relationship between the age of a "worm" and the number of triangles in its "body"

<table>
<thead>
<tr>
<th>Age (Days)</th>
<th>Number of Triangles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>?</td>
</tr>
</tbody>
</table>

Fig. 1.6.
The three smallest square patios with gardens that Alfredo can design

Fig. 1.7.
A suggested table for organizing the data for the Tiling a Patio problem
Patterns 1

The building with 1 cube for its tower takes 6 cubes to build.

The building with 2 cubes for its tower takes 7 cubes to build.

The building with 3 cubes for its tower takes 8 cubes to build.

<table>
<thead>
<tr>
<th>Cubes in Tower</th>
<th>Cubes in All</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Patterns 2

A kite with 1 cube for its tail takes 10 cubes to build.

A kite with 2 cubes for its tail takes 11 cubes to build.

A kite with 3 cubes for its tail takes 12 cubes to build.

How many cubes to build a kite with 10 cubes for its tail?

How many cubes to build a kite with 100 cubes for its tail?

<table>
<thead>
<tr>
<th>Cubes in Tail</th>
<th>Cubes in All</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
### Patterns 3

A space house with 1 floor takes 3 cubes to build.

A space house with 2 floors takes 6 cubes to build.

A space house with 3 floors takes 9 cubes to build.

How many cubes to build a space house with 10 floors?

<table>
<thead>
<tr>
<th>Floors</th>
<th>Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

How many cubes to build a space house with 100 floors?
# Patterns 4

How many cubes for the carports when there are 10 cars?

How many cubes for the carports when there are 100 cars?

<table>
<thead>
<tr>
<th>Cars</th>
<th>Cubes in Carports</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Patterns 5

A well with 1 layer takes 8 bricks.

A well with 2 layers takes 16 bricks.

A well with 3 layers takes 24 bricks.

How many bricks to make a well with 10 layers?

How many bricks to make a well with 100 layers?

<table>
<thead>
<tr>
<th>Layers</th>
<th>Bricks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
More Teaching Ideas for Algebra and Functions
Teaching Ideas Bibliography (Algebra and Functions)

*Contains “I Spy Patterns” and “That’s Odd!” on pages 48, 49, 50, and 82, and 61, 62, and 63.

*Contains the “Building a Wall” and “Cover Up” ideas on pages 38, 39, 40, 41, 93, and 94, and 26, 17, 18, 29, and 90.

*Contains the “Writing Equations” activity on pages 86, 87, 88, 89, and 110.
Getting Ready

What You’ll Need

Cuisenaire Rods, 2 reds, 2 light green, 1 purple, and 1 yellow per pair

Strips of paper, 10 cm by 4 cm, at least 10 strips per pair, cut from 1-centimeter grid paper, page 110

Overhead Cuisenaire Rods and/or 2 centimeter grid paper transparency (optional)

Overview

Children work with a specified group of Cuisenaire Rods and explore the many different equations they can represent with them. In this activity, children have an opportunity to:

- write addition sentences
- use the properties of addition and equality

The Activity

Introducing

- Invite a volunteer to build and display a train using two or three rods.
- Invite another volunteer to build another train that is equal in length to the first one. At this point, the trains might look like these:

\[ \text{Red, } g \text{, and Purple, } p \text{, in the top train; } y \text{, yellow, in the bottom train.} \]

- Ask children to describe how the two trains are related. Have them use letter names for the rods, addition signs, and an equal sign. Children may suggest an equation like this:

\[ w + g + p = y + g \]

- Then rearrange the rods in each train and ask children to write equations that reflect the new arrangements.
On Their Own

**How many equations can you model using a certain group of Cuisenaire Rods?**

- Work with a partner. Use the following set of 6 Cuisenaire Rods: 2 red, 2 light green, 1 purple, and 1 yellow.
- Choose any of the rods to form 2 trains that are equal in length. Here is one example:
- Write as many equations as you can that describe how the two trains are related. Write each equation on its own strip of paper.
- Continue forming trains of equal length and writing equations.
- Find as many different pairs of equal-length trains as you can and record the matching equations.

The Bigger Picture

**Thinking and Sharing**

Make and post the following four labels: 3 rods, 4 rods, 5 rods, 6 rods. Ask volunteers, one pair at a time, to post an equation that uses only three rods. Once children agree that there are no duplicates, ask for four-rod equations. Continue in this manner until all the different equations that the class has found are posted.

Use prompts such as these to promote class discussion:
- What did you notice about the posted equations?
- How did you use the rods to help you find equations?
- Once you found an equation, could you change it to come up with another one? How?
- Do you think you have found all of the possible equations? Why?
- Did you organize your search in any special way? Explain.
- Which equations represent the same set of rods?

**Extending the Activity**

1. Have children repeat the activity with fewer than six rods.
2. Have children repeat the activity with different collections of six rods. Have them compare their results with the results of the original activity.
Where's the Mathematics?

As children complete this activity they can deepen their understanding of properties of addition by applying them to the search for new sentences. Some children may use the rods to build every sentence; others may not actually use the rods at all but may manipulate symbols to make new sentences. For example, having found one equation, \( r + g = y \), they can use their understanding of the commutative property of addition to find the sentence \( g + r = y \). If children think of the equal sign (\( = \)) as meaning “the same as,” they can justify rewriting \( r + g = y \) as \( y = r + g \)—an illustration of the symmetric property of equality. Again applying the symmetric property, a fourth equation can be written; that is, if \( g + r = y \), then \( y = g + r \).

There are 46 possible equations. It is unlikely that an individual pair of children—or even the whole class—will find all the sentences. Sorting the equations according to the number of rods used can help children to see what is missing.

There are 6 equations using three rods. These fall into two “families of equations,” one family involving \( r, g, y \) and the other involving \( r, r, p \):

| \( r + g = y \) | \( r + r = p \) |
| \( g + r = y \) | \( p = r + r \) |
| \( y = r + g \) |               |
| \( y = g + r \) |               |

There are 16 equations using four rods: eight involving \( r, g, p, y \); four involving \( r, r, g, g \); and four involving \( r, g, g, p \):

| \( r + y = g + p \) | \( r + g = a - r \) | \( r + s = g + g \) |
| \( r + y = p + g \) | \( r + g = r - g \) | \( p + r = g + g \) |
| \( y + r = g + p \) | \( g + r = r + g \) | \( g + g = r + p \) |
| \( y + r = p + g \) | \( g + r = g + r \) | \( g + g = p + r \) |
| \( g + p = r + y \) |               |               |
| \( g + p = y + r \) |               |               |
| \( p + g = r + y \) |               |               |
| \( p + g = y + r \) |               |               |
There are 24 equations using five rods: twelve involving r, r, g, g, p; and twelve involving r, r, g, p, y.

<table>
<thead>
<tr>
<th>p + g = g + r + r</th>
<th>y + g = r + r + p</th>
</tr>
</thead>
<tbody>
<tr>
<td>p + g = r + g + r</td>
<td>y + g = r + p + r</td>
</tr>
<tr>
<td>p + g = r + r + g</td>
<td>y + g = p + r + r</td>
</tr>
<tr>
<td>g + p = g + r + r</td>
<td>g + y = r + r + p</td>
</tr>
<tr>
<td>g + p = r + g + r</td>
<td>g + y = r + p + r</td>
</tr>
<tr>
<td>g + p = r + r + g</td>
<td>g + y = p + r + r</td>
</tr>
<tr>
<td>g + r + r = p + g</td>
<td>r + r + p = y + g</td>
</tr>
<tr>
<td>g + r + r = g + p</td>
<td>r + r + p = g + y</td>
</tr>
<tr>
<td>r + g + r = p + g</td>
<td>r + p + r = y + g</td>
</tr>
<tr>
<td>r + g + r = g + p</td>
<td>r + p + r = g + y</td>
</tr>
<tr>
<td>r + r + g = p + g</td>
<td>p + r + r = y + g</td>
</tr>
<tr>
<td>r + r + g = g + p</td>
<td>p + r + r = g + y</td>
</tr>
</tbody>
</table>

In the 46 equations above, the sum of the rods on each side of the equal sign is equivalent to an even number of white rods; this allows half to be on one side of the equal sign and half on the other side. There are no sentences that involve all six rods because the sum of these rods is equivalent to 19 white rods, and it is impossible to form two trains of the same length from 19 white rods.

The process of searching for all the equations involves making a systematic listing; this is the kind of thinking used to solve permutation problems such as, "How many different ways can three textbooks—a math book, a science book, and a history book—be arranged on a bookshelf?" A systematic approach can enhance children's ability to reason mathematically. Using letters to represent the rods provides an exposure to the kind of symbolic thinking children will use in algebra.
**Getting Ready**

**What You'll Need**

- Color Tiles, 100 of mixed colors per child
- 5 x 5 Color Tile grids,* 4 per child, page 93
- Cover-Up Product List, page 94
- Dice, 1 pair per group
- Tape
- Overhead Color Tiles and/or Color Tile grid paper transparency (optional)

**Overview**

In this game for two to four players, children build Color Tile arrays to cover squares on a hundreds board according to a roll of a pair of dice. In this activity, children have the opportunity to:

- create arrays to represent products
- use the commutative property of multiplication
- use the distributive property
- look for patterns in the rolls of a pair of dice

**The Activity**

Some children may break up their arrays in ways that fail to maintain the factors. In the example at the right, showing the product 12 by reorganizing the array into two rows of six would not be acceptable.

*Have children create their 10-by-10 game boards by cutting out four 5 x 5 grids and taping them together to form a square configuration.

**Introducing**

- Tell children that you rolled a pair of dice and that a 4 and 3 came up. Show them how to model the outcome 4 x 3 with a Color Tile array of four rows with three tiles in each row.
- Ask children to use their Color Tiles to model the outcome as 3 x 4 using three rows with four tiles in each row.
- Show children some ways to break up the first array into individual factors consisting of rows of tiles.
- Have children show ways to break up the second array into factors.

---

**Standard 3.3.4**
On Their Own

Play Cover Up!

Here are the rules.

1. This is a game for 2 to 4 players. The object is to be the first to completely cover a 10-by-10 game board with Color Tiles.

2. Each player gets a game board. All players share a list of products from 1 to 36. After each roll of the dice, players write the 2 numbers rolled next to the product of those numbers. For example, if a player rolls a 3 and a 4, that player would write $3 \times 4$ next to his or her product, 12.

3. Players take turns rolling the dice and making a Color Tile array to place on their game boards. Examples of arrays for 3 and 4 are shown to the right.

4. If a whole array does not fit, a player may break it up into parts. The parts must show the numbers rolled.

   - Okay for $3 \times 4$ or $4 \times 3$
   - Not okay for $3 \times 4$ or $4 \times 3$

5. Any player who rolls numbers for which no array will fit loses a turn.

6. The winner is the player who completely covers his or her board first.

   - Play several games of Cover Up.
   - Be ready to talk about your games and strategies.

The Bigger Picture

Thinking and Sharing

Invite children to talk about their games and describe some of the thinking they did.

Use prompts such as these to promote class discussion:

- Did the arrays for some numbers fit more easily than others? Why?
- When your board began to fill up, which number pairs did you want to roll? Explain.
- Which products from 1 through 36 could you never roll? Why?
- Were some products rolled more often than others? Which ones? Do you think this would happen if you played again? Explain.
- Which was better, leaving “holes” on the board between your arrays or keeping your arrays close together? Why?
**Writing**

Ask children to imagine that they are about to play *Cover Up* again. Have them explain what they think is the most important thing to do in order to fill their game board as quickly as possible.

**Where's the Mathematics?**

Children gain practice in applying the properties of multiplication as they decide how to make or break up their Color Tile arrays to fit on their game boards. They employ the commutative property of multiplication as they consider which way to represent each product. Most children will discover that it is usually better to use the smaller number rolled as the number of tiles per row. For example, a child who rolls 4 and 6 for a product of 24 will find that making six rows of four tiles will likely provide more fitting options than would making four rows of six. However, there are times—perhaps at the beginning of the game—when four rows of six would fit nicely. Here are some of the ways that six rows of four tiles can be broken up.

As children break up their arrays, positioning the parts wherever they fit on their boards, they are employing the distributive property of multiplication.
Extending the Activity

Have a pair of children play this version of Cover Up using a 10-by-10 game board and just two colors of tiles: The first player chooses a color and makes arrays in that color when he or she rolls products through 18. (Rolling a product greater that 18 loses a turn.) The second player uses the other color to show only the products from 20 through 36. (Rolling a product less than 20 loses a turn.) If both players, in succession, roll a product that cannot fit on the board, the game ends. The winner is the player who has covered the greater number of spaces on the board.

Children will come to realize that not all the products from 1 to 36 can be rolled and that some products will come up more often than others because there are more ways to roll them. This is shown in the chart on the right. Children will be surprised to note that 18, or half, of the numbers in their product list can never be rolled!

As a game progresses, the way in which the empty squares on a board are configured is also a significant factor. The two grids below each have 32 empty spaces.

<table>
<thead>
<tr>
<th>Product</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 x 1</td>
</tr>
<tr>
<td>2</td>
<td>1 x 2, 2 x 1</td>
</tr>
<tr>
<td>3</td>
<td>1 x 3, 3 x 1</td>
</tr>
<tr>
<td>4</td>
<td>1 x 4, 4 x 1, 2 x 2</td>
</tr>
<tr>
<td>5</td>
<td>1 x 5, 5 x 1</td>
</tr>
<tr>
<td>6</td>
<td>1 x 6, 6 x 1, 2 x 3, 3 x 2</td>
</tr>
<tr>
<td>7</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>2 x 4, 4 x 2</td>
</tr>
<tr>
<td>9</td>
<td>3 x 3</td>
</tr>
<tr>
<td>10</td>
<td>2 x 5, 5 x 2</td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>2 x 6, 6 x 2, 3 x 4, 4 x 3</td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3 x 5, 5 x 3</td>
</tr>
<tr>
<td>16</td>
<td>4 x 4</td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>3 x 6, 6 x 3</td>
</tr>
<tr>
<td>19</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>4 x 5, 5 x 4</td>
</tr>
<tr>
<td>21</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>4 x 6, 6 x 4</td>
</tr>
<tr>
<td>25</td>
<td>5 x 5</td>
</tr>
<tr>
<td>26</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>5 x 6, 6 x 5</td>
</tr>
<tr>
<td>31</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>6 x 6</td>
</tr>
</tbody>
</table>

This player loses a turn by rolling 6,6; 6,5; 6,4; or 5,5.

This player loses a turn only by rolling 6,6.

Through comparing game boards that look like these, children will note that it is better to fill the boards tightly rather than leaving "holes."
# COVER-UP PRODUCT LIST

<table>
<thead>
<tr>
<th>Product</th>
<th>Numbers rolled</th>
<th>Product</th>
<th>Numbers rolled</th>
<th>Product</th>
<th>Numbers rolled</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>13</td>
<td></td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>14</td>
<td></td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>15</td>
<td></td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>16</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>17</td>
<td></td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>18</td>
<td></td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>19</td>
<td></td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>20</td>
<td></td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>21</td>
<td></td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>22</td>
<td></td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>23</td>
<td></td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>24</td>
<td></td>
<td>35</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>36</td>
<td></td>
</tr>
</tbody>
</table>
I Spy Patterns
Grades 3–4

Goals
The students will—
- partition the given array into different parts;
- translate visual patterns into numerical expressions;
- explore how equivalent numerical expressions represent the commutative and associative properties of operations.

Prior Knowledge
The students should have had some experiences exploring visual patterns and translating such patterns into number sentences.

Materials and Equipment
You may want to give the students manipulatives to copy the array of objects. Have available one copy of the “I Spy Patterns” blackline master for each pair or group of students.

Classroom Environment
Students may work in pairs or groups of three. Allow sufficient time at the end of the activity for students to report and discuss their findings.

Activity
Engage
Distribute the copies of the blackline master and explain the following situation to the students:

Investigators from “CAP Operations” (the letters stand for commutative and associative properties) have discovered an unusual array of twenty-five diamonds in a secret room of an abandoned house. They suspect that the patterns in the array have some numerical codes. Your task is to find as many ways as you can to partition the array on the worksheet (see fig. 2.8).

Ask the following questions to guide the students’ thinking:

- What numerical pattern can you describe from the array of diamonds? (The students will likely find it easy to see that the numbers of diamonds in the columns produce the pattern $1 + 3 + 5 + 7 + 5 + 3 + 1$.)
- What other patterns do you see? (Point out that in the center of the array there is a $3 \times 3$ square [see fig. 2.9], and then ask the following question.)
- What numerical pattern can you see in this array now? (The numerical pattern can be expressed as $9 + 4 + 4 + 4$ [the nine diamonds in the center and the four triangles, each composed of four diamonds]).

This activity has been adapted from David Young, Cookie Combo: AMS Education Foundation, December 1995. pp. 48–50.
Elicit from the students that
\[ 9 + (4 + 4 + 4 + 4) = (9 + 4 + 4 + 4) + 4, \]
and explain that the equation demonstrates the associative property of addition (i.e., the result of addition is the same no matter which pair of adjacent elements is added first).

Use this example also to demonstrate the commutative property (i.e., the results of addition do not differ when the order in which the elements are used changes):
\[ 9 + (4 + 4 + 4 + 4) = (4 + 4 + 4 + 4) + 9 \]

Use your discretion when labeling these properties. Sometimes students memorize the labels without any understanding of the property. It is better to talk about the ideas the properties express: the commutative property involves the order of the addends or factors, and the associative property concerns grouping the terms of the operation.

Ask the students what the following equation demonstrates:
\[ (4 + 4 + 4 + 4) + 9 = (4 \times 4) + (3 \times 3) \]
(This equation demonstrates the definition of multiplication.)

**Explore**

Have the students partition the arrays on their copies of the “I Spy Patterns” blackline master. Listen to their discussions, and observe the ways in which they draw different patterns. Emphasize the translation of the visual patterns into numerical patterns. Guide the students’ thinking using questions such as the following:

- What numerical pattern does this partition give you?
- What is another way we can write the numerical pattern for that particular partition?
- Why are these two number expressions the same? (We changed only the order, or we put the numbers in a different group.)

**Extend**

Increase the total number of diamonds in the array to the following and ask the students to identify various possibilities for partitioning the arrays:

- Forty-one (see fig. 2.10)
- Sixty-one (see fig. 2.11)
If you examine the numerical patterns as the array is enlarged, you will find the following:

- **Top**: 9, 16, 25, ...
- **Center**: 7, 9, 11, ...
- **Bottom**: 9, 16, 25, ...

**Assessment Ideas**

Have the student explain the reasons for the equivalence of two numerical expressions that illustrate one of the properties discussed.

**Where to Go Next in Instruction?**

Examining patterns such as these and writing equivalent versions of the numerical expressions contribute to the development of mental-computation skills. An approach in mental computation focuses on different ways to express an operation.

The Building Houses activity, which follows, extends the ideas of dependent variables in equations to a system of three equations in three unknowns. In it, students are encouraged to use trial and error to determine how many "houses" go on each "island." The students are asked first to verbalize each relation in a given problem and then to express each problem as a set of three equations.
I Spy Patterns

Name ____________________________

Navigating through Algebra in Grades 3–5
BUILDING A WALL

Overview

Children create a two-color pattern for a Color Tile wall. They extend the pattern, then determine how many tiles would be needed to repeat the pattern a given number of times. In this activity, children have the opportunity to:

- connect visual patterns to numerical patterns
- organize and record data
- use patterns to make predictions

Introducing

- Show children this wall made by standing red and blue Color Tiles on end. Ask them to copy it with their Color Tiles.

- Establish that the basic pattern in the wall consists of two different colors of tiles that repeat.

- Ask how many tiles there are of each color and how many tiles there are in all.

- Then have children use their Color Tiles to copy this basic pattern for a wall.

- Ask volunteers to predict how many tiles of each color there would be if this pattern were repeated until there was a total of nine tiles.

- Have children extend their walls to check their predictions.

Getting Ready

What You'll Need

Color Tiles, about 40 each of red and blue per pair
Color Tile grid paper, several sheets per pair, page 90
Crayons
Tape
Overhead Color Tiles and/or Color Tile grid paper transparency (optional)

The Activity

If some children find it too difficult to stand the tiles up on end, suggest that they build a path instead of a wall, laying the tiles flat rather than standing them up.
On Their Own

Can you figure out the number of red and blue Color Tiles you will need to make a wall that is 10 patterns long?

• Work with a partner. Use red and blue Color Tiles to make a pattern for a wall. A pattern has 2 or more tiles of different colors in a particular order.
• When you have agreed on your pattern, stand up the tiles to look like a real wall.
• Use Color Tiles to make walls that are 1 pattern long, 2 patterns long, and so on up to 5 patterns long.
• Record each wall on a strip of grid paper. You may need to tape some of the strips together.
• For each wall, record the number of red tiles used, the number of blue tiles used, and the total number of tiles used.
• Look for patterns in your data. Predict how many tiles of each color you will need and how many tiles you will need altogether to make a wall that is 10 patterns long. Write down your prediction.
• Now check your prediction by completing your 10-pattern wall.
• Look for a way to make predictions for even longer walls.
• Be ready to discuss any number patterns you found in your data.

The Bigger Picture

Thinking and Sharing

Invite pairs of children to post their results.

Use prompts such as these to promote class discussion:
• How do these basic wall patterns differ from each other?
• What number patterns did you find in your data?
• If you know the number of red tiles in a wall, how could you find the total number of tiles in the wall?
• If you know the total number of tiles in a wall, how could you find the number of red tiles?
• If this wall had 100 tiles, how could you find the number of blue tiles?
• How could you find the number of red tiles, blue tiles, and tiles in all if this pattern were repeated 100 times?
**Where's the Mathematics?**

By creating their basic two-color pattern, then repeating it to build their wall, children become aware that patterns repeat in predictable ways. Some children may have difficulty identifying the basic patterns in the walls built by others. This may be due either to faulty repetition of the patterns themselves or to children's inability to visually isolate a part from a whole. If the latter is the case, it might be helpful to lead children in "reading" some wall patterns aloud, pausing between the repetitions of the patterns. For example, to help children identify the basic pattern in the wall shown here, you might lead children in chanting, “blue, blue, red, blue, red”; (pause) “blue, blue, red, blue, red”; (pause) “blue, blue, red, blue, red”; (pause) “blue, blue, red, blue, red”; (pause) “blue, blue, red, blue, red.”

This chart shows an example of the data that children might come up with. With data organized in this fashion, children may be able to predict the number of tiles of each color and the number of tiles in all, in the tenth pattern without actually extending the wall that far.

<table>
<thead>
<tr>
<th>Number of pattern</th>
<th>Number of red tiles</th>
<th>Number of blue tiles</th>
<th>Total number of tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>2nd</td>
<td>4</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>3rd</td>
<td>6</td>
<td>9</td>
<td>15</td>
</tr>
<tr>
<td>4th</td>
<td>8</td>
<td>12</td>
<td>20</td>
</tr>
<tr>
<td>5th</td>
<td>10</td>
<td>15</td>
<td>25</td>
</tr>
</tbody>
</table>
**Extending the Activity**

Have pairs create wall patterns with three colors of tiles. Have them extend their walls for several patterns. Then ask them to write a few sentences that describe their walls without identifying the actual pattern. Direct pairs to exchange descriptions of their walls, then work together to build each other's walls from the descriptions.

Children who recognize the sequence of patterns as shown in the second, third, and fourth columns—counting by twos, threes, and fives—may continue the numerical patterns to conclude that by the end of the tenth pattern, there will be 20 red tiles and 30 blue tiles for a total of 50 tiles. Some children will simply double all the numbers for the fifth patterns and wind up with the data for the tenth. From this data, children could also work backward to determine that if a wall made from this pattern were 100 tiles long (4 x 25), it would need to have 60 blue tiles (4 x 15).

Some children may need to extend their data tables to find the numbers of tiles of each color and tiles in all in 100 patterns. Others may realize that they can merely multiply the number of each color and the number of tiles in all contained in one pattern by 100 to find the solution.

Those children with a more highly developed understanding of patterning will sense that the ratio of the numbers of tiles within a pattern remains constant for any number of repetitions of the patterns. They may also be able to identify other number relationships, noting, for example, that the difference between the number of red and blue tiles (blue minus red) is equal to the pattern number. Noticing all this, a child may realize that, for the hundredth pattern, the difference between the numbers of red and blue tiles must be a number whose difference is 100 and that the total number of tiles must be 10 times the total number of the tenth pattern.

<table>
<thead>
<tr>
<th>Number of pattern</th>
<th>Number of red tiles</th>
<th>Number of blue tiles</th>
<th>Total number of tiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>10th</td>
<td>20</td>
<td>30</td>
<td>50</td>
</tr>
<tr>
<td>100th</td>
<td>200</td>
<td>300</td>
<td>500</td>
</tr>
</tbody>
</table>
That's Odd!
Grades 4–5

Goals
The students will—
• observe various patterns in an array;
• represent observed visual patterns as numerical patterns;
• represent a numerical pattern as a functional relationship.

Prior Knowledge
The students should recognize odd numbers. In grade 3, students should have had experiences examining patterns. In grade 4, students should be familiar with translating visual patterns into numerical patterns.

Materials and Equipment
A set of one hundred counters for each student or pair of students is needed for this activity.

Classroom Environment
This activity may be carried out by students in pairs or in small groups. Be sure to have available tables or other surfaces on which students can set up the counters to make the arrays for the activity.

Activity
Engage
Explain to the students that they will be exploring number patterns. Begin by having the students place their counters in the array in figure 3.6. Ask the students how many counters are used to make this array. It is obvious at first glance that the counters in the rows produce the following pattern: \(1 + 3 + 5 + 7 + 9 = 25\).

Explore
Calling the students’ attention to the triangular array, lead the class in investigating the pattern produced by the sums of consecutive odd numbers. For example, the sum of the first two consecutive odd numbers \((1 + 3)\) is 4, the sum of the first three consecutive odd numbers \((1 + 3 + 5)\) is 9, the sum of the first four consecutive odd numbers \((1 + 3 + 5 + 7)\) is 16, and the sum of the first five consecutive odd numbers \((1 + 3 + 5 + 7 + 9)\) is 25. Ask the following questions to guide the students’ investigation:

- How many circles are added to each new row? (Two circles are added to each new row.) Is this consistent? (yes)
- What is the difference between the numbers of circles in the first and second rows? (2) The second and third rows? (2)
- Is there a pattern in these differences? (Yes, “Add 2” is the pattern for the subsequent rows.)
Have the students extend the pattern, then ask these questions:

- How many counters will be needed to construct the next row? (eleven)
- What will be the total number of counters used in the new array? (thirty-six)
- Is there anything special about these numbers? (The total number of counters for each array is a perfect square.)
- Can you find a rule for determining the number of counters in any given row in the array? What is the rule? (The rule is \(2n - 1\), where \(n\) is the number of the row.)
- How many counters are in the tenth row? (nineteen)
- Could one of the arrays have a total of sixty counters? (no) Explain. (Sixty is not a perfect square.)

**Extend**

Ask the students what would happen if the counters were rearranged:

- Could the counters be arranged in another shape? (yes) What is it? (a square)
- Do the sums of these consecutive odd numbers form squares? (Yes. For example, with sixteen counters, we can make a \(4 \times 4\) square. With thirty-six counters \((1 + 3 + 5 + 7 + 9 + 11)\), we can make a \(6 \times 6\) square.)
- Does this pattern hold true for all sums of consecutive odd numbers?

Encourage the students to partition the array in figure 3.7 into groups of odd numbers of counters. Note that the counters can be arranged to form L shapes (see fig. 3.8). Ask the following questions:

- How many counters are in one leg of the L for the number 3? (two)
- What is the third odd number? (5)
- What is the relationship between the number of counters in one leg of an L and the position in the sequence of odd numbers of the number represented by the L? (The \(n\)th odd number has \(n\) counters on each leg.)
- What connection is there between \(n\) counters on each leg and the \(n\)th odd number being \(2n - 1\)? (Remember not to count the corner counter twice!)

Ask the students to visualize the sum of the consecutive odd numbers by nesting the Ls. That is, the L for the first odd number fits with the L for the second odd number to form a \(2 \times 2\) square that then fits with the L for the third odd number to form a \(3 \times 3\) square, and so on (see fig. 3.9). Ask the students to describe the relationship between the sums of consecutive odd numbers and the squares that are formed by combining the L arrangements. One possible method follows:
1 = 1 \times 1
1 + 3 = 2 \times 2
1 + 3 + 5 = 3 \times 3
1 + 3 + 5 + 7 = 4 \times 4
1 + 3 + 5 + 7 + 9 = 5 \times 5

Assessment Ideas

At all grade levels, you may want to use the tasks and questions posed in the tasks to obtain an informal assessment of the students' comprehension of the task and the concepts involved.

Where to Go Next in Instruction?

You may want to address the following questions in subsequent activities:

- What do arrays of consecutive even numbers look like? Here is one array of consecutive even numbers:

\[
\begin{array}{cccc}
\text{O } & \text{O} \\
\text{O } & \text{O } & \text{O} \\
\text{O } & \text{O } & \text{O } & \text{O} \\
\text{O } & \text{O } & \text{O } & \text{O } & \text{O} \\
\end{array}
\]

- What is a "rule," or function, for the \(n\)th consecutive even number? (The rule for the \(n\)th consecutive even number is \(2n\), where \(n\) is the number of the row in the array above.)

- What is a rule for the sum of \(n\) consecutive even numbers? (The rule for the sum of \(n\) consecutive even numbers is \(n(n+1)\), but it may be beyond the grasp of most students at this level.)

- Will the sum of consecutive even numbers result in square numbers as well? (no) Explain. (If we add \(2 + 4 = 6, 2 + 4 + 6 = 12, 2 + 4 + 6 + 8 = 20\), we notice that the sums are not perfect squares. Have the students determine if the pattern holds for the next few sums of consecutive even numbers.)

The sum of \(n\) consecutive odd numbers is \(n^2\).
Teaching the Standard (Algebra and Functions) With Children’s Literature

*Books dealing with this subject that could be incorporated into mathematics instruction.


A Grain of Rice by Helena Clare Pittman (Hastings House, 1986).

Insides, Outsides, Loops, and Lines by Herbert Kohl (Freeman, 1995).

The King’s Chessboard by David Birch (Penguin Puffin, 1988).

Number Patterns Make Sense (A Wise Owl Book) by Howard Fehr (Holt, Rinehart, & Winston, 1965).

The Rajah’s Rice: A Mathematical Folktale From India by David Barry (W. H. Freeman, 1994)


Spaghetti and Meatballs for All! by Marilyn Burns (Scholastic Press, 1994).

Two Ways to Count to Ten by Ruby Dee (Henry Holt and Co., 1988).
Standard 4 — Geometry
Students learn about geometric shapes and develop a sense of space. They identify quadrilaterals and learn about right angles as a basis for comparing other angles. They describe and classify three-dimensional shapes. They use the basic terms point, line, and line segment to describe shapes. They also develop the concept of mirror-image symmetry and draw lines of symmetry.
Standard 4
Geometry
Students describe and compare the attributes of plane and solid geometric shapes and use their understanding to show relationships and solve problems.

3.4.1 Identify quadrilaterals* as four-sided shapes.
   Example: Which of these are quadrilaterals: square, triangle, rectangle?

3.4.2 Identify right angles in shapes and objects and decide whether other angles are greater or less than a right angle.
   Example: Identify right angles in your classroom. Open the classroom door until it makes a right angle with one wall and explain what you are doing.

3.4.3 Identify, describe, and classify: cube, sphere*, prism*, pyramid, cone, cylinder.
   Example: Describe the faces of a pyramid and identify its characteristics.

3.4.4 Identify common solid objects that are the parts needed to make a more complex solid object.
   Example: Describe and draw a house made from a prism and a pyramid.

3.4.5 Draw a shape that is congruent* to another shape.
   Example: Draw a triangle that is congruent to a given triangle. You may use a ruler and pencil or the drawing program on a computer.

3.4.6 Use the terms point, line, and line segment in describing two-dimensional shapes.
   Example: Describe the way a triangle is made of points and line segments and how you know it is a triangle.

3.4.7 Draw line segments and lines.
   Example: Draw a line segment three inches long.

3.4.8 Identify and draw lines of symmetry in geometric shapes (by hand or using technology).
   Example: Use pencil and paper or a drawing program to draw lines of symmetry in a square. Discuss your findings.

3.4.9 Sketch the mirror image reflections of shapes.
   Example: Hold up a cardboard letter F to a mirror. Draw the letter and the shape you see in the mirror.

3.4.10 Recognize geometric shapes and their properties in the environment and specify their locations.
   Example: Write the letters of the alphabet and draw all the lines of symmetry that you see.
* quadrilateral: a two-dimensional figure with four sides
* sphere: round ball like a baseball
* prism: solid shape with fixed cross-section (a right prism is a solid shape with two parallel faces that are congruent polygons and other faces that are rectangles)
* congruent: two figures that are the same shape and size
Example Geometry Lesson
Lesson Plan Content Page

Subject: Math
Lesson Topic: Geometry (Tangram Fun!)

INTASC Principle: The professional educator understands content. (#1)

IN State Standard: Standard 4 Geometry- Students learn about geometric shapes and
develop a sense of space. They identify quadrilaterals and learn about right angles as a
basis for comparing other angles. They describe and classify three-dimensional shapes.
They use the basic terms point, line, and line segment to describe shapes. They also
develop the concept of mirror-image symmetry and draw lines of symmetry.

IN State Indicator: 3.4.4- Identify common solid objects that are the parts needed to
make a more complex solid object.

Annotated Bibliography:
Instructional:
Company Of America, Inc., 1996.
This book provides the idea for the shape shut-out game that is used in this lesson.
The book also contains the shape shut-out game board that can be photocopied and used
in the classroom. This book is available in the math reading room on the fourth floor of
the Robert Bell building.

Kitt, Nancy A. “Math 391 Class Handouts.” Ball State University, Muncie, IN:
This is a three-page handout containing a page on tangram directions, a page that
is a tangram ready to be cut, and a tangram shape sheet. All three of these pages will be
used in the beginning of the lesson before the students are given their own foam
tangrams. These notes were taken in my Math 391 class taught by Ms. Nancy Kitt and
they are now part of my own personal collection of teaching materials.

Informational:
Cathcart, George W., Yvonne M. Pothier, James H. Vance, Nadine S. Bezuk. Learning
Mathematics: In Elementary and Middle Schools. Upper Saddle River, NJ: Merrill
This book provides information on geometry and this is where basic information I
needed to know before I taught the lesson was found. This book was my textbook for my
Math 391 class, and is now part of my own personal collection.

Kitt, Nancy A. “Math 391 Class Notes.” Ball State University, Muncie, IN: 11/7/2002.
These notes provided many different pieces of information on tangrams and their
uses. The notes provided lots of helpful information on how to use the tangrams in the
classroom. The notes were taken in my math methods class with Ms. Nancy Kitt, and are
now part of my own personal collection of resources.
Lesson Plan

IN State Standard: Standard 4 Geometry- Students learn about geometric shapes and develop a sense of space. They identify quadrilaterals and learn about right angles as a basis for comparing other angles. They describe and classify three-dimensional shapes. They use the basic terms point, line, and line segment to describe shapes. They also develop the concept of mirror-image symmetry and draw lines of symmetry.

IN State Indicator: 3.4.4- Identify common solid objects that are the parts needed to make a more complex solid object.

Lesson Objective: The students will describe and compare spatial relationships using tangram pieces.

Materials/Media: Lesson plan, tangram cut-out sheets, scissors for each student, tangram shape sheets, sets of foam tangrams for each student, and one shape cut-out game board for every two students.

New Information:
- Tangrams can be used to show spatial relationships between shapes.
- Tangrams (shapes) can be put together to build new shapes.

Motivation: The teacher will give each student a tangram cut-out sheet and a pair of scissors. The teacher will direct the students to cut out the large square along the black lines. Then while demonstrating each step, the teacher will read the following directions and ask the students to compare and describe the relationships between the pieces. “Fold along diagonals AC and BD. Cut along the diagonal fold AC. What do you notice about these two shapes? Now fold and cut along DX. Label the two triangles 1 and 2. What do you notice about these two shapes? What is the relationship between these two triangles and the other triangle? Fold B to X and cut along YZ. Label this triangle 3. What do you notice about this triangle in relationship to triangles 1 and 2? Fold C to midpoint X and cut along the fold. Label this triangle 4. What do you notice about triangle 4? Cut off the square and label it 5. What do you notice about the square in relationship to the other pieces? Fold the remaining piece from X to Z. Cut along the fold in order to cut off the right triangle. Label this triangle 6. What is the relationship between number 6 and the other pieces? Label the remaining parallelogram 7. Could two shapes be placed together to make a parallelogram? What do you notice about this parallelogram?” (Questions, Cues, Advance Organizers and Identifying Similarities and Differences) “Today we will be using tangrams to explore the relationships between different shapes, and to build shapes using those we already have.” Goal for Learner

Procedure:
1. New Information: Many times an object or shape can be made by putting together two or more objects or shapes. Tangrams are shapes that can do just this; they can make different or larger shapes.
2. **Modeling:** Give each student the shape sheet and a set of foam (or plastic) tangrams. Model for the students the following questions on the shape sheet:
   - “Can you use one piece to make a square?” Show the students how one piece can be used to make a square since they already have a square piece.
   - “Can two pieces be used to make a triangle?” Show the students how you can use two of the triangles to make one larger triangle.
   - “Can you make a rectangle using three pieces?” Show the students how to use one square and two of the triangles to form a rectangle. *(Questions, Cues, Advance Organizers)*

3. **Guided Practice:** The students will work on completing the shape sheet with a partner; however, each student should fill in their answers on their own sheet. The teacher should encourage the students to use their tangram pieces to model the questions on the sheet. *(Non-linguistic Representations)*
   While the students are working on the shape sheet, the teacher will walk around the classroom to monitor the students' progress, and to assist the students if they have any problems.

4. **Check for Understanding:** Once the students are done with their shape sheets, or once the teacher has decided enough time has passed, the teacher will check over the sheet with the students. The teacher will ask each question on the shape sheet (or as many as they instructed the students to complete) to the entire class. The teacher should instruct the students to all answer with yes or no when the question is asked. If there is a lot of confusion, or a conflicting idea on a question, the teacher can have a student explain their answer to the class. However, if confusion is still present, the teacher should explain the question.

5. **Practice/Application:** After checking over the shape sheet as a class the teacher will give each pair of students a shape shut-out game board. The teacher will introduce the game by doing a thinking activity with the students. The teacher will tell the students to work together to try and fit both their sets of tangrams exactly within the grid lines on the top half of the game board. This will challenge the students to work together to fit both their tangram colors on the grid.
   The teacher will give the following rules for the game:
   - You will play the game will the partner you have been with the entire lesson.
   - Each player will have his/her own color of tangrams to use.
   - The players will take turns placing pieces on the grid (within the lines).
   - The object of the game is to be the last one who can place a tangram down with it still fitting in all the grid lines.
   - Therefore, if one player cannot place a piece down in the grid lines, the other player is the winner.
6. **Closure:** Discuss with the students which pieces were the most difficult to play with. "Was it better to be the first or second player to begin the game?" Then discuss with the students what they have learned about the tangrams by asking the students, "what do we know about the tangram pieces in relationship to each other?" Encourage the students to be specific in their answers; for example, pieces 1 and 2 are equal while 6 and 4 are \( \frac{1}{2} \) of pieces 5, 7, and 3. (Questions, Cues, Advance Organizers)

**Evaluation of Student Learning:** This lesson will not be formally evaluated; instead the teacher will listen carefully to the students’ answers during the "check for understanding" section of the lesson. This way the teacher will be able to see which groups could possibly use more help.

**Lesson Extension:** If the teacher has extra time after a few rounds of the shape shut-out game, they can turn the game into a tournament. They can do a single elimination, best of one game, or best of three game tournament to find the class shape shut-out champion. Then the champion can share their winning strategy with the class.
TANGRAM DIRECTIONS

1. Cut out the square.

2. Fold along both diagonals.

3. Cut along AC.

4. Fold and cut along DX. Label the two triangles 1 and 2.

5. Fold B to X. Cut along YZ. Label the triangle 3.

6. Fold C to the midpoint X. Cut along the fold. Label the triangle 4.

7. Cut off the square. Label it 5.

8. Fold the remaining piece from X to Z.

9. Cut along the fold to get a right triangle. Label it 6.

10. Label the parallelogram 7.
TANGRAMS
TANGRAM SHAPE SHEET

Which shapes can you make with your tangram pieces? Draw a sketch of your solution.

<table>
<thead>
<tr>
<th>Number of pieces used</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape Made</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Square</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Triangle</td>
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<tr>
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<td>Trapezoid</td>
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<tr>
<td>Parallelogram</td>
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<tr>
<td>Rhombus</td>
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</tr>
<tr>
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<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
More Teaching Ideas for Geometry
Teaching Ideas Bibliography (Geometry)

*Contains the “Building Solids” lesson on pages 26, 27, 28, 29, 30, 104, and 105.

*Contains the “Symmetry Search” activity on pages 74, 75, 76, 77, and 101.

*Contains the “Congruent Shapes” and “Recover the Symmetry” ideas on pages 30, 31, 32, 33, and 91, and 46, 47, 48, and 49.

*Contains the “Tan-Angles” lesson on pages 74, 75, 76, 77, and 110.
Getting Ready

What You'll Need
Tangrams, 1 set per child
Overhead Tangrams (optional)
Clock faces with movable hands (optional)

Overview

Children make a variety of Tangram shapes, then count and identify the angles, using a right angle as the benchmark. In this activity, children have the opportunity to:

- make and compare a variety of polygons
- notice how the sides and angles of a polygon are related
- determine if an angle is smaller, larger, or the same size as a right angle

Introducing

- Ask children to draw four clock faces, showing 2 o'clock, 3 o'clock, 4 o'clock, and 7 o'clock respectively, while you do the same at the board.
- Explain that the amount of space between the two hands of the clock is called an angle and that angles come in many sizes. Draw an arrow on each clock face to show the space (see below).
- Below the 3 o'clock drawing write “right angle.” Point out the square corner. Illustrate by placing the Tangram square between the hands. Explain that all angles with square corners are called right angles.
- Ask children to decide which drawings show angles smaller than a right angle and which show angles larger than a right angle.
- Verify each identification by testing it with the Tangram square.
- Label each of the drawings as shown.
On Their Own

How big are the angles of a Tangram shape?

• Work with a partner. Pick a Tangram piece and count how many angles it has.
• Decide if each angle in the piece is smaller, larger, or the same size as a right angle. Record your findings.
• Do this for each of the other Tangram pieces.
• Now, you and your partner each choose 2 or more Tangram pieces. Each of you use your pieces to make a shape you like. Be sure that a side of each piece is touching a side of another piece.
• Record your shapes and the following information:
  • the number of sides in your shape
  • the number of angles in your shape
  • which angles are right angles
  • which angles are smaller than a right angle
  • which angles are larger than a right angle.
• Compare your shapes and make a list of what you notice.

The Bigger Picture

Thinking and Sharing

Ask if anyone made a shape with fewer than three sides, then discuss why not. Ask if anyone made a shape with three sides. Have those children post their shapes. Next invite volunteers who made shapes with four sides to post their shapes. Continue with five sides, six sides, and so on until everyone has posted their shapes and all agree there are no duplicates.

Use prompts such as these to promote class discussion:

• What kind of angles does the small Tangram triangle have? the square? the parallelogram? the medium triangle? the large triangle?
• How did your shape compare to your partner’s?
• Which angles were the hardest to figure out? the easiest? Why?
• What do notice about the shapes that are posted?
• Is it possible to have shapes with no right angles? with one right angle? two? three? only right angles?
Extending the Activity

1. Have children sort their shapes in other ways, such as right angles/no right angles, symmetry/no symmetry, parallel sides/no parallel sides, perpendicular sides/no perpendicular sides.

Teacher Talk

Where’s the Mathematics?

This activity gives children the opportunity to build mental images of different-size angles and establish the right angle as a benchmark for estimating the size of any angle. Children should develop visual images of angles before they are taught the mathematical names associated with angles to measure angles in degrees. This assures that learning that mathematicians call an angle less that $90^\circ$ acute, an angle greater than $90^\circ$ obtuse, an angle equal to two right angles straight, and an angle bigger than two right angles reflex, is simply attaching new names to a concept children are already familiar with. Having a mental image of a right angle makes it easier to understand how to use a protractor and to recognize that the end product when drawing a $150^\circ$ angle cannot look like this:

\[
\begin{array}{c}
\text{(improper angle)}
\end{array}
\]

There are only three different angles in the Tangram set—a right angle, an angle half that size, and an angle one and half times that size.

The parallelogram has two pairs of the smaller angle and two pairs of the larger angle. The square has four right angles, and all of the triangles have one right angle and two smaller angles. Children are often surprised that the angles in the small Tangram triangle exactly match the angles in each of the other two triangles, since the triangles themselves are not the same size.

From the posted shapes, children can conclude that the number of sides in a polygon is always the same as the number of angles. They should also conclude that it is possible for a shape to have no right angles. Some may notice that all the three-sided shapes are always triangles and none have more than one right angle.
2. Explain that angles are measured in degrees and that a right angle has 90°. Then have children figure out the number of degrees in each angle of each of the Tangram pieces.

3. Have children find the total number of degrees in each Tangram piece and in the shapes they made.

It takes time for children to become skilled both at recognizing an angle in a shape and how it compares to a right angle. Often the orientation of an angle creates some difficulty. Have children who are having such difficulty, draw clock faces with times like these: 10 minutes after 9, 5 minutes before 9, and 30 minutes after 3. Then help them identify each one as less than, greater than, or equal to a right angle.

Children may describe reflex angles, angles greater than two right angles, as angles in corners (marked with arrows below) and may find them particularly hard to identify.
Building Solids
Grades 3-4

Goals

- Build a variety of solids from materials supplied
- Discover the relationship among the numbers of faces, edges, and vertices of solids
- Discover relationships between two- and three-dimensional figures

Prior Knowledge

Students should be familiar with properties (number of sides and angles) of many two-dimensional shapes, including triangles, squares, rectangles, pentagons, hexagons, and octagons. They also should be able to recognize congruent shapes.

Materials and Equipment

- A set of approximately sixteen “sticks”—for example, toothpicks or coffee stirrers—of each of four different lengths (two to eight inches)—for each pair of students
- A variety of “fasteners” (e.g., gumdrops, miniature marshmallows, clay)
- One copy for each pair of students of the blackline masters “Two- and Three-Dimensional Shapes” and “Counting Parts of Solids”
- For each pair of students, a set of three-dimensional wooden or plastic shapes—including one each of a cube, a square pyramid, a cylinder, a cone, and a sphere—and a variety of prisms such as a rectangular prism, a triangular prism, a pentagonal prism, a hexagonal prism, or an octagonal prism

Important Geometric Terms

Cube, pyramid, prism, cylinder, sphere, cone

Polyhedra (singular, polyhedron): Solids whose faces are polygons

Polygon: A closed two-dimensional figure that is made up of line segments that intersect only at their end points

Faces: Polygonal regions that make up the surface of a solid

Edges: The line segments created by the intersection of two faces of a solid

Vertices (singular, vertex): The points of intersection of two or more edges

Learning Environment

Students work in pairs to construct models of solids.

This activity has been adapted from Battista and Clements (1998).