Basic Bond Investment

An Honors Thesis (HONRS 499)

by

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Abstract

This paper will address and compare different types of bonds. The discussion will look at the prices of bonds, face values, and yield rates. How inflation affects investments will also be explored. Practical examples of saving through bonds will be given. This discussion is not meant to give advice, but is meant as background information.
Before discussion begins, some common symbols used throughout this paper should be described. These are described below:

\[ P = \text{price of the bond} \]

\[ F = \text{par value or face amount of the bond} \]

**Note:** For the purposes of this paper, the assumption has been made that the redemption value at maturity is the same as the face value. The redemption value is the amount received when the bond is cashed in. The redemption value does not have to be the same as the face value.

\[ r = \text{coupon rate of the bond} \]

\[ Fr = \text{amount of the coupon (face amount multiplied by the coupon rate)} \]

\[ i = \text{yield rate of a bond (interest rate earned on the bond, could also be the interest rate the bond holder wants to yield)} \]

\[ n = \text{number of payment periods to the redemption date} \]

The following three assumptions must be made for the purposes of this paper:

1. All payments will be made on specified dates.
2. The bonds discussed will have a fixed maturity date.
3. The price of the bond is calculated immediately following a coupon payment date.
What is a bond? A bond represents a debt owed by organizations like businesses and the government to the bond owner. Basically, a bond is a loan to the organization by the bond owner. The bond is issued by the organization to borrow money for a variety of reasons. These reasons could include expansion of the business, funding for a university project, and even road repair.

Bonds can be and are often sold before maturity. Obviously, bonds can also be sold anytime after issue. The face value, and coupon rates are fixed, as well as the maturity date (in the cases studied for the purpose of this paper). The only variable is the amount the purchaser wants to yield. The price depends on the yield desired. This is how the purchaser decides what will be the maximum amount he/she will invest in a bond. If the bond owner sells the bond before maturity, he/she probably will not receive the face value. The only way the face value is guaranteed is if the owner holds the bond until maturity.

The following equations will be used to determine the price of the bonds studied in this paper.

zero coupon bonds:

\[ P = \frac{F}{(1+i)^n} \]

coupon bond:

\[ P = Fr\bar{A}_{\frac{n}{i}} + F(1+i)^{-n}; \]

\[ \bar{A}_{\frac{n}{i}} = \frac{(1-(1+i)^{-n})}{i} \]

**Coupon Bonds**

A coupon bond is a bond with coupons that can be redeemed periodically as stated in the bond terms. These coupons can be quarterly, semiannual, monthly, etc... . The coupons are paid in addition to the
redemption amount. In addition to being a coupon bond, such a bond could also be registered or unregistered. In the case of a registered bond, the investor is stated in the records of the borrower. If the investor wishes to sell the bond, the new owner must be reported as the new owner/investor to the borrower. This is because the coupons will be paid to the person listed in the borrower’s records. In the case of the unregistered bond, the investor is not stated in the borrower’s records, and therefore, the coupons are paid to whoever has possession of the bond. The unregistered bond is sometimes called a bearer bond because the payments are made to whoever bears the bond (Kellison, 205). The main advantage of a coupon bond is the periodical payments. This can be considered a guaranteed source of income. This extra income is interest that is paid out, rather than being accumulated.

Example:

Sally is 25 and wants to save up for a new car. She has some extra money put back. She would like to invest this money in a coupon bond. This way by the age of 35 she can buy a new car and get a few extra dollars a year. How much should she invest if she wants $20,000 in 10 years.

The annual coupon rate is 3%, with a rate of return of 5%.

\[ P = 20,000(.03)A_{10.05} + 20,000(1.05)^{-10} \]
\[ P = 600A_{10.05} + 20,000(1.05)^{-10} \]
\[ P = 4,633.04 + 12,278.27 \]
\[ P = $16,911.31 \]

If Sally has this much to invest, she could buy her a new car now. but she would not have the extra $600 a year, for the next 10 years. Depending on Sally’s circumstances, this could be a good investment.
Example:

Ralph and Sue just had a baby. Ralph’s parents gave them $10,000 for the baby’s college fund. Sue’s parents gave them $5,000 for the baby’s college fund. This gives them a $15,000 total. They look into investing this money in a coupon bond so they can use the coupon money for baby expenses.

They invest in a bond with coupons that have an 8% converted quarterly interest rate. The bond also has an 8% yield rate converted quarterly. They want this investment for 18 years. What is the face value?

\[
15,000 = F(.02)A_{72}^{.02} + F(1.02)^{-72}
\]
\[
15,000 = F(.02)(1-(1.02)^{-72})/.02 + F(1.02)^{-72}
\]
\[
15,000 = F(.02)(37.984063) + F(.2403187)
\]
\[
15,000 = F(.7596813 + .2403187)
\]
\[
F = $15,000
\]

They will have a face value, and redemption value, equal to what they invested, as long as the bond matures in 18 years. If they sell in 18 years, and this is before maturity, they could either lose or gain capital. Remember, one is guaranteed a redemption value equal to the face value only if the bond is held until maturity. They will receive quarterly coupon payments of $300 (coupon payment amount = Fr = 15,000(.02) = 300). The couple may want more money saved for the baby’s college fund after 18 years. After they become more financially stable, they can reinvest the coupon payments and/or save more with the coupon payments.
Zero Coupon Bonds

A zero coupon bond is just what it says. It is a bond without periodic coupon payments. A zero coupon bond can be considered a bond with coupons at a rate of 0%. The interest earned will be the difference between the face value and the amount paid for the bond, rather than in the form of coupons. This reflects the fact that zero coupon bonds are sold at a discount. Because of this, one can also assume that the longer the maturity length, the greater the discount. This is due to the fact that there should be more interest earned with a longer duration until maturity. Even though nothing is paid until maturity, or the bond is resold, the interest accrued is taxed annually just as if it were paid out because this is the amount of the increased worth of the bond. This interest is known as “imputed interest,” or “phantom interest,” reflecting the idea that the interest is not actually paid out (Thau, 85). U.S. savings bonds are zero coupon bonds, but interest can be treated differently. The interest can be deferred until such a bond is cashed in, or it could be treated as previously discussed.

As with any bond the face amount is only guaranteed if one holds the bond until maturity. Therefore, if the bond is held until maturity there is virtually no risk. This is especially true in the case of the zero coupon bond because the amount earned is known and guaranteed. If one resells the bond before maturity, there could be a great gain or loss, therefore, becoming volatile. The simplest explanation of what is meant by volatility here is the possibility of price or value changes in either an upward or downward direction. The more volatile bond has a greater chance of price fluctuations. In this case the longer the maturity length, the more volatile. This is due to the fact that the longer a bond is held, there is a greater possibility of price or value changes because there is a greater amount of time for other factors to influence the price of the bond. The zero coupon bond is the most volatile of all bonds, for reasons discussed later under volatility.
Example:

George desires $150,000 at age 65 for his retirement. George is now 23. He is interested in a 5% rate of return converted annually. He is thinking of investing in a zero coupon bond. How much should he invest in this type of bond?

Using the equation stated in the beginning of this paper,

\[ P(1.05)^{42} = 150,000 \]
\[ P = 150,000/(1.05)^{42} \]
\[ P = 19,325.94 \]

we find that George should invest $19,325.94 in this zero coupon bond to earn $150,000 by the age of 65. This would be too much for the average 23 year old to invest at once. Therefore, George must consider investing a little each year, or even another investment instead of (or as well as) the zero coupon bond.

George first decides to consider investing $1000 a year for 15 years. George is still interested in a 5% interest rate. His approximate investment is followed below.

<table>
<thead>
<tr>
<th>yr#</th>
<th># of yrs invested</th>
<th>amount earned on each investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>42</td>
<td>(1000(1.05)^{42} = 7,761.59)</td>
</tr>
<tr>
<td>2</td>
<td>41</td>
<td>(1000(1.05)^{41} = 7,391.99)</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>(1000(1.05)^{40} = 7,039.99)</td>
</tr>
<tr>
<td>4</td>
<td>39</td>
<td>(1000(1.05)^{39} = 6,704.75)</td>
</tr>
<tr>
<td>5</td>
<td>38</td>
<td>(1000(1.05)^{38} = 6,385.48)</td>
</tr>
<tr>
<td>yr#</td>
<td># of yrs invested</td>
<td>amount earned on each investment</td>
</tr>
<tr>
<td>-----</td>
<td>------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>6</td>
<td>37</td>
<td>(1000(1.05)^{37} = 6,081.41)</td>
</tr>
<tr>
<td>7</td>
<td>36</td>
<td>(1000(1.05)^{36} = 5,791.82)</td>
</tr>
<tr>
<td>8</td>
<td>35</td>
<td>(1000(1.05)^{35} = 5,516.02)</td>
</tr>
<tr>
<td>9</td>
<td>34</td>
<td>(1000(1.05)^{34} = 5,253.35)</td>
</tr>
<tr>
<td>10</td>
<td>33</td>
<td>(1000(1.05)^{33} = 5,003.19)</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>(1000(1.05)^{32} = 4,764.94)</td>
</tr>
<tr>
<td>12</td>
<td>31</td>
<td>(1000(1.05)^{31} = 4,538.04)</td>
</tr>
<tr>
<td>13</td>
<td>30</td>
<td>(1000(1.05)^{30} = 4,321.94)</td>
</tr>
<tr>
<td>14</td>
<td>29</td>
<td>(1000(1.05)^{29} = 4,116.14)</td>
</tr>
<tr>
<td>15</td>
<td>28</td>
<td>(1000(1.05)^{28} = 3,920.13)</td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>TOTAL = $89,843.80</strong></td>
</tr>
</tbody>
</table>

This is over half of what George desired to have at retirement. In this example, George stopped investing at the age of 37. This is still a young age. This total could be made greater in two ways. 1) As he grows older and more stable financially, he can invest more each year. 2) He can also invest for a longer period of time. Also, the interest rate will probably fluctuate over such a fifteen year period so these numbers are not going to be exact, but because they will probably fluctuate in both directions these numbers may be a fairly close approximation.

Let's extend this example so that George invests until the age of 62.

<table>
<thead>
<tr>
<th>yr#</th>
<th># of yrs invested</th>
<th>amount earned on each investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>27</td>
<td>(1000(1.05)^{27} = 3733.46)</td>
</tr>
<tr>
<td>17</td>
<td>26</td>
<td>(1000(1.05)^{26} = 3555.67)</td>
</tr>
<tr>
<td>18</td>
<td>25</td>
<td>(1000(1.05)^{25} = 3386.35)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>24</td>
<td>1000(1.05)^{24} = 3225.10</td>
</tr>
<tr>
<td>20</td>
<td>23</td>
<td>1000(1.05)^{23} = 3071.25</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>1000(1.05)^{22} = 2925.26</td>
</tr>
<tr>
<td>22</td>
<td>21</td>
<td>1000(1.05)^{21} = 2785.96</td>
</tr>
<tr>
<td>23</td>
<td>20</td>
<td>1000(1.05)^{20} = 2653.30</td>
</tr>
<tr>
<td>24</td>
<td>19</td>
<td>1000(1.05)^{19} = 2526.95</td>
</tr>
<tr>
<td>25</td>
<td>18</td>
<td>1000(1.05)^{18} = 2406.62</td>
</tr>
<tr>
<td>26</td>
<td>17</td>
<td>1000(1.05)^{17} = 2292.02</td>
</tr>
<tr>
<td>27</td>
<td>16</td>
<td>1000(1.05)^{16} = 2182.87</td>
</tr>
<tr>
<td>28</td>
<td>15</td>
<td>1000(1.05)^{15} = 2078.93</td>
</tr>
<tr>
<td>29</td>
<td>14</td>
<td>1000(1.05)^{14} = 1979.93</td>
</tr>
<tr>
<td>30</td>
<td>13</td>
<td>1000(1.05)^{13} = 1885.65</td>
</tr>
<tr>
<td>31</td>
<td>12</td>
<td>1000(1.05)^{12} = 1795.86</td>
</tr>
<tr>
<td>32</td>
<td>11</td>
<td>1000(1.05)^{11} = 1710.34</td>
</tr>
<tr>
<td>33</td>
<td>10</td>
<td>1000(1.05)^{10} = 1628.89</td>
</tr>
<tr>
<td>34</td>
<td>9</td>
<td>1000(1.05)^{9} = 1551.33</td>
</tr>
<tr>
<td>35</td>
<td>8</td>
<td>1000(1.05)^{8} = 1477.45</td>
</tr>
<tr>
<td>36</td>
<td>7</td>
<td>1000(1.05)^{7} = 1407.10</td>
</tr>
<tr>
<td>37</td>
<td>6</td>
<td>1000(1.05)^{6} = 1340.10</td>
</tr>
<tr>
<td>38</td>
<td>5</td>
<td>1000(1.05)^{5} = 1276.28</td>
</tr>
<tr>
<td>39</td>
<td>4</td>
<td>1000(1.05)^{4} = 1215.51</td>
</tr>
<tr>
<td>40</td>
<td>3</td>
<td>1000(1.05)^{3} = 1157.63</td>
</tr>
</tbody>
</table>

TOTAL = $55,250.08  
+ PREVIOUS TOTAL $89,843.80  
$145,093.88
This is only a $4,906.12 difference of what George will have at age 65 and what George wants at age 65. This is if George always contributes $1000 a year to his investments. If he does what is suggested earlier, contributing more as he becomes more financially stable, he could have more than he wants at retirement.

**Treasury Notes**

Treasury notes usually mature in two to ten years. Smaller notes mature closer to two years, while larger notes mature closer to ten years. Prices can fluctuate due to interest rate changes that occur during the maturity period. The longer the maturity the more prices can fluctuate. Price changes are directly related to maturity length (Thau, 79). As with other types of bonds, if the note is resold before maturity, there may be either a capital loss or a capital gain (Thau, 79). There is a tax advantage to Treasury notes that one should look into. The interest earned on notes are exempt from state and local taxes, but federal taxes are collected on notes.

**Municipals**

Municipal bonds are issued with a public purpose, therefore, they are mainly issued by city, county, and state governments. Businesses with a public purpose can also issue municipal bonds, such as utility companies, hospitals, and universities (Thau, 99). Municipals are issued in multiples of $5000 denominations. Maturity lengths range from a few months to over thirty years. Most municipals are issued at face value with semi-annual coupons. Some of these will pay interest only after the bond has been outstanding for a specific number of years (Scott, 52). There are zero coupon municipals that are issued at large discounts, just as any other zero coupon bond. The main reason many investors choose municipals is that they are federally tax exempt. Sometimes municipals are also exempt from state and local taxes if you live in the state issuing
the bonds. This also depends on the laws of the state one lives in. The Tax Reform Act of 1986 made the municipal the last tax shelter available to the individual, therefore, also making the individual the primary investor to municipals (Thau, 100). There are a couple of things one must take into consideration when determining if they want to invest in a municipal. The first question to answer is, will investing in a municipal help reach one's investment goals. The second question to answer is, will one's tax bracket make it worth the tax exempt investment. If one falls within a low federal tax bracket, one should usually avoid municipal bonds and try to find an investment with a higher return. Of course, the opposite is also true, the higher one's tax bracket, the better chance a municipal will be a good investment. One can calculate the taxable equivalent, also called the tax equivalent yield, which is the percent one would have to earn in a taxable security to pass up the municipal and its tax exemption. This can be calculated as followed:

\[
\text{taxable equivalent} = \frac{\text{tax exempt yield}}{1 - \text{tax bracket}}.
\]

**Taxable equivalent:** (Tax equivalent yield) is the interest rate one needs to earn to make a taxable investment more worthwhile than the tax exempt investment.

**Tax exempt yield:** is the yield rate (interest rate) one will earn on the tax exempt investment.

**Tax bracket:** is the percentage of taxes one pays yearly.
Example:

Charles is looking into a municipal yielding 8%. This means Charles has the possibility of an 8% tax exempt yield. Charles is in a 17% tax bracket. Charles is wanting to know what kind of yield he would have to have to make a taxable investment more worthwhile than the municipal.

\[
\text{taxable equivalent} = \frac{0.08}{(1-0.17)} \\
= \frac{0.08}{0.83} \\
= 0.0964 \\
= 9.64\%
\]

For Charles to pass up the tax exempt municipal for another type of non tax exempt investment, he would have to earn 9.64%.

Example:

Paul has a choice of investments. One investment is a municipal earning 5%. The other is a zero coupon bond earning 6%. Paul is in a 15% tax bracket. Paul wants to know if the tax exemption will make up for a lower interest rate. In other words which is the better investment for Paul?

\[
\text{taxable equivalent} = \frac{0.05}{(1-0.15)} \\
= \frac{0.05}{0.85} \\
= 0.0588 \\
= 5.88\%
\]

Because Paul can earn 6% with the zero coupon bond, and he only needs to earn 5.88% to get a better investment than the 5% municipal, the zero coupon bond is the better investment for Paul at this time.
Example:

Now, Tim has a choice of investments. One investment is a municipal earning 8%. The other is a zero coupon bond earning 10%. Tim is in a 35% tax bracket. Which is the better investment for Tim?

\[
\text{taxable equivalent} = \frac{.08}{1-.35} = \frac{.08}{.65} = .1231 = 12.31\%
\]

Because Tim needs to earn 12.31% to make a taxable investment better than the 8% municipal, and the zero coupon bond is only offering 10%, the municipal is a better investment.

This process can also be used to determine whether Treasury notes are a better investment than another locally taxable investment.

Debenture Bonds

Debenture bonds are long term corporate bonds that are not backed with any particular assets. All assets not committed to any other debts are considered collateral for this bond. (Scott, 33)

Junk Bonds

Junk bonds are also called high-yield bonds. The borrowers of junk bonds may not be able to meet the terms stated. Investors are attracted to junk bonds for this high yield, but they are also willing to accept the high risk of nonpayment. Because of the high risk of nonpayment, junk bonds are not usually a good investment for individual investors. (Scott, 34)
Mortgage Bonds

In the case of a mortgage bond investors are given a first mortgage on part, or all, of the borrowers property as collateral. Because this type of collateral is secure, mortgage bonds are considered one of the safest investments. (Scott, 34)

Volatility and Interest Rate Response

All bonds have the potential for fluctuation due to interest rate changes, and therefore, are subject to a risk. When interest rates rise, prices drop in response. The opposite is also true, when interest rates fall, prices rise in response.

ILLUSTRATION:

Take a bond with the following conditions:
F = 1000  r = 3%  n = 10 years

\[ i = 4\% \quad i = 6\% \quad i = 8\% \]

\[ P = \$918.89 \quad \$779.20 \quad \$664.50 \]

This should illustrate the price differences and how one can gain or lose in response to interest rate changes.

The volatility of a bond corresponds to the coupon rate, as well as, the interest rate. The lower the coupon rate, the more volatile. As mentioned before, zero coupon bonds are the most volatile because there is no coupon, or a rate of 0%.
ILLUSTRATION:

Take a bond with the following conditions:
F = 1000  i = 6%  n = 10 years.

<table>
<thead>
<tr>
<th>r</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$558.39</td>
</tr>
<tr>
<td>2%</td>
<td>$705.60</td>
</tr>
<tr>
<td>3%</td>
<td>$779.20</td>
</tr>
<tr>
<td>4%</td>
<td>$852.80</td>
</tr>
</tbody>
</table>

If i goes up to 8 % then the following changes occur:

<table>
<thead>
<tr>
<th>r</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>$463.19</td>
</tr>
<tr>
<td>2%</td>
<td>$597.40</td>
</tr>
<tr>
<td>3%</td>
<td>$664.50</td>
</tr>
<tr>
<td>4%</td>
<td>$731.60</td>
</tr>
</tbody>
</table>

(These numbers are calculated with the formulas stated in the introduction)

The price of the zero coupon bond went down approximately 17%.
The price of the bond with a coupon of 2% went down approximately 15.3%.
The price of the bond with a coupon of 3% went down approximately 14.7%.
The price of the bond with a coupon of 4% went down approximately 14.2%.
This shows that the zero coupon bond has a larger potential for price fluctuations, therefore, being more volatile.

In the case of interest rates rising and prices dropping, a short maturity would be wise because more would be lost the longer the bond was held. In the opposite case, as interest rates fall and prices rise, one would benefit from a longer maturity because more would be gained. One way of protection from these fluctuations is to buy short term or intermediate term bonds. This would be to buy bonds from two to seven years or so. Keeping the length of maturity down decreases the time, and therefore, the potential for interest rate changes and price fluctuations. This could decrease the amount earned, but it could also be protection from a great loss.
Inflation

Inflation is a measure of the amount by which a good or service or a package of goods and services increase in price (Scott, 125). The consumer price index, or cpi, is the way of measurement of inflation through these goods and services. The cpi is shown with a basket of goods. This basket of goods has a price on it for a particular year, and inflation is calculated by comparing the price to the price of the same basket of goods in the year of comparison.

Example:

Our basket of goods contains a dozen eggs, pair of jeans, oak desk, and dining room table.

In year X this basket costs $350.
In year X+Y this basket costs $450.
The same goods have increased in cost by approximately 28.57%.
Inflation has increased from year X to year X+Y by 28.57%.
The cpi is also 28.57%.

The producer price index, or ppi, measure these products at the wholesale level. The ppi is calculated in the same manner as the cpi, only on the wholesale level. Inflation has been defined by The Webster Dictionary as being an increase in the amount of money in circulation, resulting in a relatively sharp and sudden fall in its value and rise in prices. Inflation is something that is very difficult to predict for a great amount of time into the future. It is for this reason that bond owners may have unexpected losses. One will know the amount and when one will receive payments, as stated on the bond, but one will not necessarily know...
what the amount will actually be worth (Scott, 127). The higher the rate of inflation during ownership of the bond, the fewer goods and services each interest and principal payment will buy. Of course, the opposite is also true. If inflation declines, the more goods and services each interest and principal payment will buy. If one wants to protect oneself from major damage due to inflation, the same solution is offered as was to the risk of interest rate changes, short to intermediate maturities will keep the possibilities of many changes down. One must remember inflation is a very difficult thing to predict.

Example:

Let's go back to our example with George. George has accumulated $145,093.88 at age 65. George accumulated this through a zero coupon bond. George wants to use this to supply himself with an annual income. He purchases an annuity with this amount.

The amount of the annuity payments depends on the length of the annuity and the interest rate.

\[ X = \text{the amount of the annuity payments} \]

\[ 145,093.88 = X A_{10|0.05} \]

is the formula to be used to figure the amount of \( X \).

Let's assume it is a 10 year annuity, with an interest rate of 5%. If he uses all of the $145,093.88 to purchase the annuity the amount of the annuity payments is found in the following way.

\[ 145,093.88 = X A_{10|0.05} \]

\[ X = 145,093.88 / A_{10|0.05} \]

Note: \( A_{10|0.05} \) is found as previously stated in the paper.
\[ A_{10}^{0.05} = 7.7217349 \]

\[ X = \frac{145,093.88}{7.7217349} \]

\[ X = 18,790.32 \]

The annuity payments are \$18,790.32

If inflation increases by 6% in the first year, his purchasing power declines by 6%.

What cost George \$18,790.32 the first year will cost \$19,917.74 \((18,790.32 \times 1.06)\) in the next year. Because of the increased cost, George can not purchase as much. His purchasing power has declined by 6%. His purchasing power will continue to fluctuate along with the inflation throughout the ten years of the annuity.

A background has now been presented in bond investment. There is a lot to consider when investing. Remember to consider interest rates, possibility of rate changes, tax advantages, and even the affects of inflation. Now, one should be able to know what to look for and be better prepared to research an investment.
Bibliography


