MOTIVATIONAL IDEAS FOR THE TEACHING OF MATHEMATICS

by

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As a prospective teacher of mathematics, I have found myself searching for a way to make students (at the junior high level, particularly) realize that mathematics can be fun and that one does not have to be a genius to be successful in a mathematical endeavor. The key to the answer seemed to be motivation. Not motivation in the sense of creating little mathematicians; rather, motivation to instill in the students an appreciation for mathematics and an understanding that math is not "a big, bad grizzly bear." This may seem like an impossible dream, but for the teacher who is willing to accept the challenge, the opportunities are unlimited.

One discussion of motivation included the following paragraph:

Motivated is a description we apply to behavior which is directed towards the satisfaction of some need. If we say that a certain piece of behavior seems motiveless to us, we mean that we do not know, and cannot even guess, what need is satisfied by means of it. So questions about motives are usually, in disguise, questions about needs.¹

This point of view raises another question. That is, how does a teacher convince junior high students that they "need" mathematics?

fields of study, the business world, industry, and other professions.\textsuperscript{2} Many times they see mathematics as having little or no value, or fail to associate it with experience (that is, reality) and common sense. As a result there develops a feeling of mystification and befuddlement.\textsuperscript{3}

Taking these problems into consideration forces one to conclude that the only way to motivate students in a mathematics classroom is to make their learning experiences fun. Richard R. Skemp, a mathematician-psychologist, sums it up very well: "That we experience pleasure from any activities... is the most powerful incentive to learning; mathematics or any other subject."\textsuperscript{4} Now the question is how to make a mathematics class fun.

Perhaps one of the most important aspects is the classroom routine. To be a good teacher one needs more than just a single aid or one special technique. Following is a short list of basic ideas which could add some spice to a dull classroom.

1. Vary the order of basic projects which are to be included during each period from day to day.

2. Share some of the more routine basic responsibilities with appropriate students; i.e., involve the students in necessary jobs in the classroom.

3. Try to prepare activities that will permit a student-learning-centered classroom rather than a teacher-dominated classroom. In


\textsuperscript{4}Skemp, p. 135.
addition to providing a change of pace, learning by doing is a most effective method.

4. One should avoid long periods of time on any one type of activity or class organization.

5. Since junior high school students usually like to be active and find it difficult to be confined to a chair all day, occasional opportunities for them to move around the room during the period could work wonders.

6. Don't make a habit of announcing exactly what activities will take place that day—surprise them!

As a general rule, students enjoy socializing or doing things with other people, especially at the junior high level where they are beginning to identify more and more with their peer group. Mathematical games can often be fun and provide opportunities for drill work and/or introducing a new idea. Certainly, too many games could become just as tiring and boring as regular classwork, so one must be careful not to abuse an effective technique. Mathematical games and puzzles can definitely be justified as an instructional technique, for there is abundant evidence that moderate and careful use of such devices does add a great deal of interest to a classroom situation.

The following is a mathematical game played with two different colored dice. One color (say, black) signifies a move to the right on a number line. The other color (red) implies a move to the left. After constructing a number line

6Butler, p. 143.
randomly select a "winner's spot" and mark it with an X or some other indicator. (See illustration below)

Students could work in small groups with the following rules:

Determine (by any acceptable method) which player will go first.

First player rolls the two dice. Assume he comes up with a 6 on the black die and a 3 on the red one. This tells the player to move 6 spaces to the right and 3 spaces to the left - in effect, a move of 3 spaces to the right, starting at the point zero.

The players continue taking turns until someone lands on the "winner's spot." The idea of numbers being to the left of zero can be viewed as a concrete example of negative numbers.

If your class is more "sophisticated" or needs more of a challenge, apply the same idea to a two-dimensional grid. In this case the two different colors, black and red, correspond to \((x,y)\) coordinates. Activities such as these can be related to ordered pairs, graphing, and even provide some drill work in adding and subtracting.

Another possibility is very much like the game Battleship. This game is played on a standard two-dimensional grid and is based on an understanding of the ordered pair \((x,y)\). It would probably be most appropriate after completing a unit on integers and prior to one on graphing equations of straight

lines. Due to the nature of the game, couples or very small groups divided into two teams are the most reasonable ways to organize the classroom. For the average 40-50 minute class the size of the grid should probably be limited to six units from zero in the positive and negative directions.

Provide each player with a single sheet of paper having two grids on it. Each player places his own battleships (3) on one grid and records his shots at the opponents ships on the other. A ship consists of three adjacent points (horizontally, vertically, diagonally). Each player (or team) takes turns at firing three shots. The game ends when a player (or team) has lost all his ships.8

A question every teacher probably asks himself is "How can I introduce this unit, or topic, and capture the students' attention?" It seems logical that in order for effective learning to take place, a person's curiosity should be aroused. That is, the surrounding stimuli should be such that the students are attracted to what is going on. The elements of novelty and sheer intellectual curiosity can be very effective stimuli for creating interest.

Consider the following statement as an attention-getter for a unit on proportion: "Did you know that the ant's brain is the heaviest in proportion to its weight of all animals?"9

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8 William R. Bell, "Cartesian Coordinates and Battleship," The Arithmetic Teacher, 21 (May 1974), 421.

This could be followed by a short discussion of what is meant by proportion. Then, present Table I to the students and ask them if they can discover a way to calculate proportion.

<table>
<thead>
<tr>
<th>ANIMAL</th>
<th>BRAIN WEIGHT</th>
<th>BODY WEIGHT</th>
<th>%</th>
<th>PROPORTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sparrow</td>
<td>.0052 lb.</td>
<td>.125 lb.</td>
<td>4.2</td>
<td>1:24</td>
</tr>
<tr>
<td>Horse</td>
<td>3.306 lb.</td>
<td>1322.76 lb.</td>
<td>.25</td>
<td>1:400</td>
</tr>
<tr>
<td>Ant</td>
<td>.00005 lb.</td>
<td>.001 lb.</td>
<td>5.0</td>
<td>1:20</td>
</tr>
<tr>
<td>Elephant</td>
<td>12.26 lb.</td>
<td>6134.1 lb.</td>
<td>.20</td>
<td>1:500</td>
</tr>
</tbody>
</table>

Once students have found a method of calculating proportion, present questions such as the following:

The proportion of brain weight to body weight of an average whale is 1:40,000.

His body weight is 20,125 lb.

His brain weight is ________.

The percentage of brain weight is ________%.

Suppose your brain is 2.5% of your total weight. See if you can figure out how much YOUR brain weighs!10

A unit on probability could be introduced by having the students participate in the following activity. Begin with five containers which have in them a mixture of black and white beads.

Container A has 25 white beads, 5 black beads
Container B has 20 white beads, 10 black beads
Container C has 15 white beads, 15 black beads
Container D has 10 white beads, 20 black beads
Container E has 5 white beads, 25 black beads

Tell the students each container has 30 beads, some black, some white. They are to sample 20 beads (replacing the bead each time),

10Crouse, p. 109.
record their samples, then guess which container they have.\(^{11}\)

Of course, one has to realize that mathematics cannot be all games, so at times the students will have to settle down in their seats and do some work on their own. The challenge here is to find ways of making this enjoyable, also. For example, consider a class which is working on solving equations. They can perhaps gain a little more satisfaction by doing something similar to the worksheet that follows.\(^{12}\) (See worksheet on next page) If the students are not far enough along to know the meaning of \(i\), make up some other equations for them to solve. This is also an excellent way to incorporate some drill work. For those of you who are curious, the solution is:

AN EXCESS OF INDIVIDUALS SKILLED IN THE PREPARATION OF EDIBLES IMPAIRS THE QUALITY OF A THIN DERIVATIVE OF HEAT. (In other words, "Too many cooks spoil the broth.")\(^{13}\)

Another possibility would be cross-number puzzles. At times the students may benefit from some problems involving percent, decimals, fractions, etc. Take up some problems and put them into the form of a crossword puzzle. This adds a touch of variety and makes drill work a bit easier to take (provided the cross-number puzzles are not too long and involved).

Occasionally, students might enjoy making a discovery simply

\(^{11}\)Bruce C. Burt, "Drawing Conclusions From Samples," The Arithmetic Teacher, 16 (Nov. 1969), 539.


\(^{13}\)Boyle, p. 165.
An Algebra Adage

Evaluate the following equations for clues to the sentence above.

1. $(3 + i)(3-i) = A$
2. $(5 + 4i)^2 = B + 40i$
3. $3i(2-i) - 2i(3 + 2i) = C$
4. $(1 + i)^3(1-i)i = D$
5. $(1-i)^2(1+i)^2 = E$
6. $2i(1-3i) = 2(i + 2) + H$
7. $(1 + 2i)(2-3i) - (2-5i)(1 + 3i) = I$
8. $8i(1-i) = 2(4 + 3i) = J$
9. $i^{63} = L$
10. $(3 + 2i)(-5 + 7i) - (7i - 29) = K$
11. $M = 3 - 4i$
12. $(3 + 2i)(5i - 2) = 0 + (5i - 16) = N$
13. $(4i)(-2i) = P$
14. $i(2 + i)(3 - 4i) = Q + i(7 - 5i) = R$
15. $\frac{2i}{2-i} - \frac{4i}{1-2i} = \frac{P}{R}$
16. $\frac{1}{i} + \frac{1}{2i} = S$
17. $x^2 - 6x + 73 = 0$
18. $z^2 + 6z + 34 = 0$

Now that you have done the easy part, see if you can shorten the phrase into a well-known proverb!
to satisfy their intellectual curiosity. Present them with a challenge: How many blocks are there in any given triangular pile? Distribute to each student a copy of each of the three following worksheets. Here the students are led through a programmed discovery of a number pattern by studying a triangular pile of blocks. It might be a good idea to have on hand 20-30 cubes so the students could manipulate the cubes and see a concrete model if they so desired.

Surprise is usually an effective technique in the classroom. How would your students react if you told them that \( \frac{\sqrt{2}}{2} \) was actually equal to 1? The average rate (for the total trip) of a car going into the country at 45 mph and returning at a rate of 55 mph may yield some surprising results:

Once students reach the level of understanding how to set up equations, the following card trick could prove to be an interesting challenge. This came to me through a fellow student, who encountered the problem in a mathematics class.

One begins by taking a regular deck of playing cards. Place the first card face up on a table. Then (starting from the number represented by the card: ace=1, jack=11, queen=12, king=13) start counting until you reach 13, placing a card (face up) on top of the first one for each number. For example, assume the first card you place face up is a 9. You would place four additional cards on top of the 9 (9+4=13) and turn the pile over. Continue this process until you no

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15 Johnson, p. 13.
1. How many blocks are in this simple pile? ____________

2. Including any hidden blocks, how many are in this two-layer pile? ____________

3. How many layers are in this triangular pile?
   How many blocks are in this pile? ____________

The pile in question 3 has three layers and a total of ten blocks. If your answers were different, try to find your mistakes.

4. Draw a picture of a similar triangular pile but with four layers.
   How many blocks are in the pile? ____________

5. Can you imagine a triangular pile with forty-five layers, or one million layers, or n layers? ____________
   Is there any limit to the number of layers such a triangular pile might have? ____________

If you follow the directions on the next two sheets, you will discover a method for finding the total number of blocks in a triangular pile of any number of layers. For example, you will be able to find that there are exactly 1,353,400 blocks in a 200-layer triangular pile without counting them all.
Complete the sketches and numbers in this table.

<table>
<thead>
<tr>
<th>Sketch of the layer as viewed from above</th>
<th>Number of blocks in this layer</th>
<th>Total number of blocks in the triangular pile through this layer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Layer 2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Layer 3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>Layer 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Layer 8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As a check, the total through layer 8 should be 120. If your answer is different, find your mistake.
1. Notice that you can group layers 8 and 7 together to obtain a perfect square.

\[
36 + 28 = 64 \\
64 = 8 \times 8 = 8^2 \\
64 \text{ is a perfect square.}
\]

2. What perfect square is obtained if you group together in the same way the next two layers, layers 6 and 5? layers 4 and 3? layers 2 and 1?

3. List the four perfect squares obtained in order starting with the smallest.

Are they squares of even or odd numbers?

4. Find the sum of the perfect squares listed.

Does it agree with your total through layer 8 as given on sheet 2?

5. Now group the layers again, only start with the odd-numbered layer, layer 7. What perfect square do you get when you group layers 7 and 6? layers 5 and 4? layers 3 and 2?

6. List the four perfect squares obtained, including the one block left over. Start with the smallest square, one.

Are they squares of even or odd numbers?

7. Find the sum of the perfect squares listed.

Does it agree with your total through layer 7 as given on sheet 2?

8. By now you should have discovered how to find the total number of blocks through any given layer in a triangular pile.

What numbers would you add to find the total number of blocks through layer 12? Note that 12 is an even number.

\[
\_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = \_\_\_
\]

9. What numbers would you add to find the total number of blocks through layer 17? Note that 17 is an odd number.

\[
\_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ = \_\_\_
\]

10. Describe in words the method used to find the number of blocks in a triangular pile through any given layer \(n\).
longer have enough cards to complete a pile.

When the process is completed, have someone choose three stacks of cards and you pick up those remaining. Turn up the top card on two of the piles. By adding 10 to the combined face value of these two cards, and subtracting that number from the number of cards in your hand, you can predict what the numerical value of the top card of the third pile will be. After doing this a few times, challenge the students to discover WHY this will work.

One thing to remember about motivation is that students will rapidly lose interest in something they do not (or cannot) understand. Success is necessary to some degree is one is to expect cooperation and effective learning to take place. As a general rule, therefore, most lessons should be completed in one class period. That is, students should not be left "in the dark" for very long. Try to have each student experience some success in each class period. If they cannot solve a problem or come to a desired conclusion, they should be helped before they lose interest! Allowing students to work on their own and discover something is fine as long as they are not left alone TOO long.

From this point of view it is easy to see why some students prefer to make paper airplanes. They can construct the airplane and fly it almost immediately, knowing if their attempt was a success or a failure. Students need positive reinforcement, especially at the junior high level. In addition to positive reinforcement, they need to have material
presented which is somewhat challenging. Many times 7th and 8th grade mathematics is nothing more than a review of elementary level mathematics.

A fairly new concept in junior high mathematics concerns motion geometry and transformations. Although it has not been integrated into all schools it appears to be a very promising experience. The fantastic thing about this innovation is the idea of a geoboard. A geoboard is nothing more than a flat board into which nails have been driven in a regular pattern, but its implications are tremendous.

There are three types of geoboards: 1) the square grid geoboard, 2) the circular geoboard, and 3) the isometric grid geoboard (see illustration below), each structured for a particular type of activity, or study.16

Various geometric figures can be formed by stretching rubber bands around appropriate nails. A variety of topics can be taught using the geoboard, such as the idea of an image, area, circles and angle measure, equilateral triangles, and other closed figures. In an activity such as this the students have an opportunity to do something and see a visual representation

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of what is being discussed. The geometric ideas of translation, reflection, and rotation can be introduced through slides, flips, and turns of concrete shapes and objects. A concrete example of an abstract idea works wonders for the student who is struggling to understand. If a student understands something, he is more likely to have an interest in learning something about it.

Although this is only a very brief collection of ideas to make mathematics more enjoyable, some careful thought and a little imagination could produce a seemingly endless amount of activities. There is little hope for any teacher who takes the position that motivation is a completely self-generated phenomenon contained totally within the student. The teacher is probably the most important factor in motivation. No one can deny the fact that students are much more efficient if they have a desire to learn what is being taught. It is a teacher's duty, therefore, to bring out this desire and help to provide the needed motivation.
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Burt, Bruce C. "Drawing Conclusions From Samples." The Arithmetic Teacher, 16 (1969), 539-541.


