THE THEORY OF NONGRADED MATHEMATICS
AND ITS APPLICATION IN CLASSROOMS OF A GRADED SCHOOL
ANDREA TAYLOR NEGANGARD
ID 499
BALL STATE UNIVERSITY
DR. ROY MC CORMICK
MAY 1, 1969
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The Nongraded School

Mark, seven, and Lynn, five, were discussing what they would like to be. Mark was going to be a sailor so he could go fishing whenever he liked. Announced Lynn, "I'm going to be a school teacher. All you have to do is buy some chalk and write on the chalkboard." 1

Although this is only an anecdote, too often this is precisely the image teachers project to their students. Today chalkboard and lecture teaching is finally being recognized as outdated. The emphasis of progressive trends in education is individualized instruction which brings the teacher from the head of the class into the midst of activity. The role of the modern teacher in the teaching-learning process necessitates a diagnosis of the individual student by the teacher, and the learner becomes directly and personally involved in the learning process. 2 No longer is the student merely a tape

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1 The Instructor, Vol. 76, November, 1966.

recorder repeating the knowledge he is fed. Education is becoming a mutual, cooperative effort of teacher and pupil.

The epitomy of such individualized education is probably best illustrated by the nongraded school. Contrary to the common belief, nongraded does not in any sense mean no evaluation grades of pupil progress. It does mean, however, the elimination of grade levels one through twelve which are now determined mostly by chronological age. In the ungraded system each student is permitted to progress in various areas of study at the rate his abilities, needs, and interest enable him. More formally:

A nongraded school is a school which denies the limitations of grade structure and is organized so that the individual student may develop his academic and creative talents as rapidly or as slowly as his abilities permit.\(^1\)

It is an "educational continuum". (See chart on page 3)

The basic philosophy of education is to permit each student to develop his potential to the fullest extent of his capabilities. The degree to which the graded system works against this philosophy is evidenced by the tables on page 4. In addition test have shown that the abilities of the pupils in an average first grade class vary five years and by the sixth grade this difference doubles.\(^2\)

\(^1\)Edward Q. Buffee, "Historical Perspective", *Nongraded Schools in Action*, p.21.
\(^2\)Dr. Maurie Hillson, "A Dynamic Concept", *Nongraded Schools in Action*, p. 50ff.
An Educational Continuum

Innovation, Institute for Educational Activities, Florida
ACHIEVEMENT LEVELS FOR A FOURTH GRADE CLASS IN
BLOOMINGTON, INDIANA

<table>
<thead>
<tr>
<th></th>
<th>Reading</th>
<th>Language</th>
<th>Arithmetic</th>
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<tr>
<td>High</td>
<td>8.7</td>
<td>7.9</td>
<td>4.9</td>
</tr>
<tr>
<td>75%</td>
<td>6.4</td>
<td>6.6</td>
<td>4.3</td>
</tr>
<tr>
<td>Median</td>
<td>4.7</td>
<td>4.6</td>
<td>3.9</td>
</tr>
<tr>
<td>25%</td>
<td>3.9</td>
<td>4.0</td>
<td>3.7</td>
</tr>
<tr>
<td>Low</td>
<td>2.9</td>
<td>2.7</td>
<td>2.8</td>
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I. Q. DATA FOR THREE FOURTH GRADE CLASSES

<table>
<thead>
<tr>
<th></th>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>122</td>
<td>145</td>
<td>129</td>
</tr>
<tr>
<td>75%</td>
<td>109</td>
<td>125</td>
<td>103</td>
</tr>
<tr>
<td>Median</td>
<td>104</td>
<td>117</td>
<td>92</td>
</tr>
<tr>
<td>25%</td>
<td>100</td>
<td>106</td>
<td>82</td>
</tr>
<tr>
<td>Low</td>
<td>91</td>
<td>96</td>
<td>70</td>
</tr>
</tbody>
</table>

Classes in this table were located in three different schools, all within the same community.

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1 Edward Buffie, Nongraded Schools in Action, p.23.
The tables on page 6 list the achievement test data for five school children. From merely looking at this table one might guess that child one was in the fourth grade, child two—the seventh, child three—the second, child four—the fourth, and child five the third. In only one instance is this correct, child three was in the second grade of an urban school. However, child one was in the third grade of a small town school, child two was in the fifth grade of a suburban school, child four was in the sixth grade of a semi rural school, and child five was in the fifth grade in the same school as child two. All five of these pupils were selected at random from achievement distributions of four American schools. These inconsistencies are also evidence of the insufficiency of the traditional, lock-step graded structure.¹

The nongraded school accepts six basic educational tenets: (1) Every group of learners has a wide variety of interests and abilities; (2) Undesirable results occur more frequently with pupils not promoted than with slow learners who are promoted; (3) Each child should be judged by the best which he can do; (4) No child should be evaluated by the average performance of a nonselected group; (5) No evaluation should be based solely on the chronological age of the child; and (6) No child should

Achievement Data for Five School Children

<table>
<thead>
<tr>
<th></th>
<th>Paragraph</th>
<th>Word</th>
<th>Average</th>
<th>Spelling</th>
<th>Arithmetic</th>
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<tbody>
<tr>
<td>1.</td>
<td>4.6</td>
<td>5.3</td>
<td>5.0</td>
<td>4.2</td>
<td>2.7</td>
</tr>
<tr>
<td>2.</td>
<td>9.4</td>
<td>9.4</td>
<td>9.4</td>
<td>7.4</td>
<td>6.1</td>
</tr>
<tr>
<td>3.</td>
<td>2.9</td>
<td>3.1</td>
<td>3.0</td>
<td>1.8</td>
<td>2.3</td>
</tr>
<tr>
<td>4.</td>
<td>3.6</td>
<td>4.2</td>
<td>3.9</td>
<td>5.2</td>
<td>5.6</td>
</tr>
<tr>
<td>5.</td>
<td>4.1</td>
<td>3.5</td>
<td>3.8</td>
<td>3.3</td>
<td>3.4</td>
</tr>
</tbody>
</table>

These children were selected at random from achievement distributions of four American Schools.

be judged on a grade standard inconsistent with the principles of child growth and development.\(^1\) As stated by Paul F. Bandwein:

There is no reason why any given concept within a conceptual scheme should be assigned to a given grade or a given age level. There is no reason why different youngsters cannot be at different rungs in the conceptual ladder.\(^2\)

(Comparisons between the graded and nongraded schools are listed on page 8)

A very important feature of nongrading is grouping. The criteria most often used include chronological age with emphasis on the behavioral aspects, achievement test, IQ tests, social maturity, interest, needs, and physical maturation primarily in terms of motor skills.\(^3\)

It is a generally accepted rule of thumb that it is better to underestimate a child and then advance him if necessary, than to overestimate him and have to place him in a lower level later.

Previously most assignments in grouping have been made on the basis of reading achievement. The correlation of reading ability with success in most academic endeavors is evident, but this concept presupposes the existence of what could be termed a "pole of

\(^{1}\)Hillson, "A Dynamic Concept", p. 30.

\(^{2}\)Institute for Educational Activities, Innovation.

\(^{3}\)Hillson, p. 40.
Comparisons between the Graded and Nongraded School

<table>
<thead>
<tr>
<th>Graded School</th>
<th>Nongraded School</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>STUDENTS</strong></td>
<td></td>
</tr>
<tr>
<td>1. Classified by grades determined by year in school.</td>
<td>1. Not classified by years in school.</td>
</tr>
<tr>
<td>2. Spend most of their day in class groups.</td>
<td>2. Spend most of the day in individual study.</td>
</tr>
<tr>
<td>3. Are quasi-passive for much of the school day.</td>
<td>3. Are active most of the school day.</td>
</tr>
<tr>
<td>4. Are evaluated according to group norms.</td>
<td>4. Are evaluated according to personal progress.</td>
</tr>
<tr>
<td>5. Study four or five courses each year.</td>
<td>5. Study unlimited numbers of subjects.</td>
</tr>
<tr>
<td>6. Perform tasks selected by teachers</td>
<td>6. Perform learning tasks selected by themselves</td>
</tr>
</tbody>
</table>

**Teachers**

<table>
<thead>
<tr>
<th>Graded School</th>
<th>Nongraded School</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Work mostly with groups of students.</td>
<td>1. Work mostly with individual students.</td>
</tr>
<tr>
<td>2. Spend large portion of the day explaining and directing.</td>
<td>2. Spend a large portion of the school day listening and advising.</td>
</tr>
<tr>
<td>3. Gear instruction to group interests and abilities.</td>
<td>3. Gear instruction to individual interest and ability.</td>
</tr>
<tr>
<td>4. Work most often along with student.</td>
<td>4. Work as a member of a teaching team.</td>
</tr>
</tbody>
</table>

**Content**

<table>
<thead>
<tr>
<th>Graded School</th>
<th>Nongraded School</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Is usually the same for all students.</td>
<td>1. Is varied depending on the student’s abilities and interests.</td>
</tr>
<tr>
<td>2. Is determined essentially by the textbook.</td>
<td>2. Is determined by faculty decisions.</td>
</tr>
</tbody>
</table>

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intellectuality*. It is also evident, however, that such a "pole of intellectually" also exists for mathematics.\textsuperscript{1}

A look again at the table on page 5 illustrates this fairly well. This discrepancy between mathematical ability and other scholastic talents is often overlooked by educators, yet the need for grouping based on mathematics achievement definitely exists. Increased effort should be made in this direction.

Once the pupils have been grouped they must be placed in levels or phases. Levels are used primarily in elementary and the chart on page 10 illustrates a typical levels plan. The second chart shows a mathematics levels plan using a basal textbook. B.F. Brown in a book on nongraded high schools divides the four basic intellectual disciplines—English, history, mathematics, and science—into five phases. The first is a remedial phase for those needing intense help. The second phase entails activities emphasizing the skills of the basic subject area. Phase three deals with the material of the subject involved. An in depth treatment of the subject matter is encompassed by the fourth phase and the final phase consists of independent study on the part of the student.\textsuperscript{2}

\textsuperscript{1}Hillson, p. 36.

\textsuperscript{2}Ibid, p. 51ff.
General Levels Plan\(^1\)

<table>
<thead>
<tr>
<th>Pools</th>
<th>Initial Assignments (Tentative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Able</td>
<td>Levels II, III, OR IV and continuing</td>
</tr>
<tr>
<td>Able</td>
<td>Levels II or III (Low or High)</td>
</tr>
<tr>
<td>Solid Average</td>
<td>Level I (High)</td>
</tr>
<tr>
<td>Average</td>
<td>Level I (Middle)</td>
</tr>
<tr>
<td></td>
<td>Level I (Low)</td>
</tr>
<tr>
<td>Low Average</td>
<td>Level I (Low)</td>
</tr>
<tr>
<td>Extended Readiness</td>
<td>Level I (Low and continuing readiness)</td>
</tr>
</tbody>
</table>

Elementary Mathematics Levels Plans Using a Basal Text\(^2\)

<table>
<thead>
<tr>
<th>Level I</th>
<th>Level II</th>
<th>Level III</th>
<th>Level IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Readiness</td>
<td>Book 1</td>
<td>Book 2</td>
<td>Book 3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level I</th>
<th>Level II</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Readiness</td>
<td>Book I Ch. 1-5</td>
<td>Book I Ch. 6-10</td>
<td>Book II Ch. 1-5</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Level V</th>
<th>Level VI</th>
<th>Level VII</th>
<th>Level VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book II Ch. 6-10</td>
<td>Book III Ch. 1-5</td>
<td>Book III Ch. 6-10</td>
<td>Book III Ch. 11-15</td>
</tr>
</tbody>
</table>

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1Hillson, p.46
2Ibid p.45
Within the phases or levels of instruction a vast variety of techniques, materials, and facilities are combined to provide an education adaptable to the needs, interests, and abilities of each student. Increased student freedom is made possible by flexible modular scheduling, a broad selection of learning activities, and independent study. Team teaching, large group presentations and small group discussions enhance teacher instruction. Programmed materials, and well-equipped resource centers augment diversity and the opportunities for individual work. In addition new ideas are being introduced and tried continuously. (A typical student's day is illustrated on a chart on page 12)

Ungrading has removed the shackles previously placed on education and innovation. The unlimited implications and potentialities are not difficult to see, but the implementation is not so simple.

Since its introduction in 1930, ungrading has progressed very slowly. It has only been during the last few years that the nongraded school has attracted widespread interest. Future progress is dependent upon the extensive support of parents, teachers, administrators, and educators on all levels. Although administrators are the prime figures as agents of change, teachers can also initiate changes toward more meaningful instruction, and it is primarily the teachers that make administrative innovations work. Introducing nongrading into one area of the curriculum may be a good way to introduce it into the
The Student's Day

system. Teachers thinking of moving in this direction may begin with organizing teaching teams and using flexible scheduling, and there also several directions individual teachers may take within their own classrooms.¹

Implications for the Mathematics Classroom

As stated previously, there exists a "pole of intellectuality" in mathematics. The tables on pages 4 and 6 illustrated the wide range of abilities in mathematics. Thus mathematics holds a prominent position in any comprehensive nongraded program.

The Nova Nongraded High School has been accredited with one of the best programs of high school mathematics yet developed. It was this fact that led the writer to investigate the ungraded movement. The approach at Nova encompasses instruction, comprehension, manipulation, and dexterity. Study progresses from incipient ideas to advanced and abstract concepts. Each child is placed in the proper instructional level for his abilities, needs, interests, and achievement, independent of the year level. The program allows for movement within the groups appropriate to the pupils rate of understanding. Materials
used include the Addison-Wesley series, teacher prepared materials, and academic games.¹

The crux of this program, and all programs at Nova, is the Learning Activity Package. The Learning Activity Package (LAP) is a "...management system for learning which allows the student, through a multi-media opportunity to become involved in a diversity of learning experiences." It consists of a rationale, specific performance objectives, a provision for self assessment, options for in depth study, and a definitive teacher evaluation. Interrelated subunits of important information help keep attention directed constantly toward the main concept which is to be assimilated during such co-directed learning activities.²

Although Nova had developed LAP's for its entire curriculum, it has not released them for use in other schools. The reasoning here is that these should be developed individually for each school and each curriculum. The task of constructing these Learning Activity Packages is left practically entirely to the teacher. A flow chart on page 16 is used as a basis for all LAP's in order to maintain some uniformity.³

The main portion of this project is the development of a study plan modeled after these packages. However,

¹Innovation
²The l.a.p. at Nova
³Ibid.
FLOWCHART FOR THE LEARNING ACTIVITY PACKAGE
Developed by the Nova Staff November 1, 1966

The L. A. P. at Nova
instead of constructing this for a nongraded situation, it will be designed as a guide for individualizing instruction in a graded algebra class, consisting almost entirely of high school freshmen approximately fourteen or fifteen years of age. This Learning Activity Package will serve as a supplement to the basic text. The instruction will follow a basic unit of study and the pupils will be given a pretest to find what skills and understandings they already possess. After an introduction of a unit, the more able students branch off and participate in enrichment work and perhaps even go ahead to more advanced study. Other pupils will be directed in their study and activities according to their ability and interest.

The experimenter hopes to demonstrate by this project that mathematics can best be taught by a type of ungrading. Aware of the complications in nongrading a school, the writer thus hopes to show the relative ease with which mathematics lends itself to a nongraded approach and plausibility of using such an approach within a graded structure.
A Student Study

Directory

for

Modern Algebra: Structure & Method

Book I

by

Dolciani, Berman, & Freilich

(Includes Chapters 9, 10, and 11)
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<td></td>
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<tr>
<td>CHAPTER X (continued)</td>
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<td>43</td>
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<td>45</td>
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<tr>
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<td>46</td>
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CHAPTER IX
GRAPHS

Educational Objectives

The student should be able to

1. Set up a coordinate system, plot points, and define abscissa, ordinate, quadrants, axis, ordered pair, and origin in relationship to a coordinate system;
2. Define and be able to graph linear equations in two variables;
3. Determine the slope of a line and its x and y intercepts;
4. Determine the equation of a line using slope-intercept method;
5. Graph an inequality in two variables;
6. Graph a parabola and relate its shape to a quadratic equation; and
7. Construct a statistical graph.
Section I
Objectives 1 and 2

Ordered Pairs of Numbers and Points in a Plane
Linear Equations and Straight Lines (Does not include the slope of a line and its uses)

Pre-Test

This Pre-Test is to aid you and the teacher in establishing your study guide. It will not affect your grade, but it is very important. If you are successful you will be able to go on to more advanced material. You will grade the test yourself and mark the incorrect problems. Take your time, work carefully, and be fair to yourself.

Part I Review

What is the solution set of the following equations?
1. \(3t+6=24, \ t \in \{1, 3, 6, 9\}\)
2. \(5(t-2)=0, \ t \in \{0, 5\}\)
3. \(3/4 \ k-3/4=24, \ k \in \{\text{all numbers}\}\)
4. \(9d+3(5+2d)=10(d+1), \ t \in \{\text{all numbers}\}\)

Graph the roots of the above equations, if possible.

Solve the following inequalities \(n \in \{0, 1, 2, 3\}\):
1. \(4n+1 \geq 6\)
2. \(n(5n) < n5nn\)
3. \(2(n+7) > 14\)
Part II Objective 2

Find the solution set for the following equations. (In 1 and 4 find just three members of the solution set)

1. \( x + y = 10 \), \( x \) & \( y \in \) all numbers
2. \( 5x + 3y = 15 \), \( x \) & \( y \in \) \( \{0, 1, 2, 3, 4, 5\} \)
3. \( 2x = y + 15 \), \( x \) & \( y \in \) \( \{2, 5, 7, 9\} \)
4. \( x = y \), \( x \) & \( y \in \) all numbers

Part III Objective 1 & 2

Plot the following points on a graph.

1. \((2, 4)\) 4. \((2, -2)\)
2. \((5, 7)\) 5. \((-5, 1)\)
3. \((0, 2)\) 6. \((4, -1)\)

Graph equations 1 and 4 in Part II
PRE-TEST ANSWER KEY

Part I
1. $6 \triangleleft 3$
2. $\triangleleft 3$
3. $\triangleleft 3$
4. $\triangleleft -1$

Graphs
1. <2, 1, 6, 3, 4, 5, 6>
2. Not Possible
1. $2, 3 \triangleleft$
2. $2, 3, 2 -1, 0, 1, 2, 3$

Part II
1. $\vec{1}, (2, 3), (3, 7), ...$
2. $\vec{2}, (0, 5), (1, 6)$
3. $\vec{2}, (2, 3)$
4. $\vec{1}, (1, 1), (2, 2), (3, 3), ...$

Part III
Answer Key continued

Part: After you have graded your Pre-Test, follow the instructions below:

1. If you missed more than one in Part I circle I in the chart below.

2. If you missed more than in Part II circle II in the chart below.

3. If you missed more than one in Part III circle III in the chart below.

Based on the results of the Pre-Test, I need work in the areas covered in the part circled below.

I II III

If no objectives are circled go on to Quiz I A.
STUDENT CONTRACT I

Turn to the next page and carefully read the
Section I Study Guide and based on the parts of the Pre-
Test in which you need additional work decide on your
time schedule. Repeat for Section II.

Do not exceed the eight days maximum time allowed
for each section.

Estimated Time:

I will complete this section in ____ days.

____ days.

(date started) (date to be completed)

(date started) (date to be completed)

(student) (teacher)

Actual Time:

Quiz I A ___________________ date taken ___ grade ___

Quiz I B ___________________ date taken ___ grade ___
SECTION I STUDY GUIDE

Part I Review

Study pp. 44-46
Exercises p. 47
Do 1-10, Find the solution set and graph.
Exercises p. 307
Do 2, 4, 6, 7, 8, 10, Graph all problems.

Part II Objective 2

Study pp. 333-336
Exercises pp. 336-337
Do 1-6, 7, 9, 11, 13, 16, 19, 22, 24, 28.
Supplementary problems may be found on pp. 364-368
in Ninth Year Mathematics by Isidore Dressler.

Part III Objective 1 & 2

Study pp. 337-340
Exercises p. 340
Do 13, 16, 18, 19 and work sheet of picture graphs.
Supplementary problems may be found on pp. 360-364 in
Ninth Year Mathematics.
Study pp. 340-341
Oral Exercises p. 341
Draw graphs for 1-9 and determine difference in linear
and non-linear graphs.
Exercises pp. 342-343
Do 1, 4, 7, 9, 12, 13, 15, 18, 19, 22, 24, 26.

Take Quiz I A
QUIZ I A

1. If \((3a, 2b-1)=(a-1, 3b)\), find the values of \(a\) and \(b\).

2. The expression \((x,y)\) is a or an \underline{__________________}.

3. If \(x = -1, 0, 1\) and \(y = 0, 1\), give the solution set of \(2x+y=3\).

Set up a rectangular coordinate system in a plane and label the following:

4. Horizontal axis
5. Origin
6. Vertical axis
7. I quadrant
8. II quadrant
9. III quadrant
10. IV quadrant

11-16. On a coordinate system plot the following points:
\((-4,0), (1,-1), (0,5), (-1,-2), (1,0), (-3,1)\).

17. The \underline{__________________} of the point A is (-3).

18. The \underline{__________________} of the point A is 2.

19. The \underline{__________________} of the point A are (-3,2).

The graph below is for problems 17-19.

Solve for \(y\), if possible, and graph the following linear equations.

20. \(y=4x\)
21. \(2x-3y=6\)
22. \(x^2+y^2=4\)
23. \(x=4\)
24. \(y=4-3x\)
25. \(xy=2\)
Section II
Objectives 3 through 7

Linear Equations and Straight Lines (Slope, slope-intercept, and determining the equation of a line)
Inequalities and Special Graphs
Statistical Graphs

Pre-Test

This Pre-Test is to aid you and the teacher in establishing your study guide. It will not affect your grade, but it is very important. If you are successful you will be able to go on to more advanced material. You will grade the test yourself and mark the incorrect problems. Take your time, work carefully, and be fair to yourself.

Part I Objective 3
Find the slope of the line containing each pair of points below:
1. (6,1), (10,3) 2. (-1,5), (-1,10) 3. (1,-2), (4,-6)

Write a linear equation with integral coefficients and the given slope and y-intercept below.
1. m=3, b=1 2. m=1/4, b=0 3. m=1, b=-1

Now graph these equations using the slope and y-intercept only.
Part II Objective 4
Find an equation of the line through the given point, having the given slope.
1. (-3,-2), 3  
2. (0,0), -\frac{1}{2} 
3. (-2,3), 0

Find an equation of the line through the given points.
1. (1,4), (3,6) 
2. (5,-3), (0,0) 
3. (0,-1), (4,-3)

Find an equation of the line parallel to the given line, through the given point.
1. x+y=5, (3,4) 
2. x-3y=6, (-6,-2)

Part III Objective 5
Solve the following inequalities for $y$ and graph on a coordinate system.
1. $y \geq 2x-1$ 
2. $x+y < 3$ 
3. $x-4 \leq y+2$

Part IV Objective 6
Solve and graph the following quadratic equations.
1. $y=x^2+2$ 
2. $3y=x^2$ 
3. $y-x^2=2x+1$
PRE-TEST ANSWER KEY

Part I
1. \( m = \frac{1}{2} \)  
2. no slope  
3. \( m = -\frac{4}{3} \)
1. \( y = \frac{3}{2}x + 1 \)  
2. \( 4y = x \)  
3. \( y = x - 1 \)

Graphs
1. 
2. 
3.

Part II
1. \( y = 3x + 7 \)  
2. \( y = -\frac{1}{4} \)  
3. \( y = 3 \)
1. \( y = x + 3 \)  
2. \( y = -\frac{3}{5}x \)  
3. \( y = -\frac{1}{2}x - 1 \)
1. \( x + y = 7 \)  
2. \( x - 3y = 0 \)

Part III
1. \( y \geq 2x - 1 \)  
2. \( y \leq -x + 3 \)
3. \( y \geq x - 6 \)
Answer Key continued

Part IV

1. \( y = x^2 + 2 \)
2. \( y = \frac{1}{3} x^2 \)
3. \( y = x^2 + 2x + 1 \)

After you have graded your Pre-Test, follow the instructions below:

1. If you missed more than one in Part I circle I in the chart below.
2. If you missed more than one in Part II circle II in the chart below.
3. If you missed more than one in Part III circle III in the chart below.
4. If you missed more than one in Part IV circle IV in the chart below.

Based on the results of the Pre-Test, I need work in the areas covered in the part circled below.

I II III IV

If no objectives are circled go on to Quiz I B or Objective 7 in the Study Guide.
SECTION II STUDY GUIDE

Part I Objective 3

Study pp. 343-345
Exercises pp. 345-346
Do a minimum of 2 problems from 1-6 and two from 7-12. Do more if you do not understand them.
Do 13-18 also.

Study pp. 346-348
Exercises p. 348
Do a minimum of four problems from 1-10 and four from 11-18. Do additional problems if you do not understand or if you need more practice.
Supplementary work may be found in Ninth Year Mathematics pp. 375-384.

Part II Objective 4

Study pp. 349-350
Exercises p. 350
Do a minimum of two problems in each group: 1-9, 10-18, 19-22, 23-26, 27-30. Do additional problems if you feel you need practice.

Part III Objective 5

Study pp. 350-352 and pp. 6-1
Exercises pp. 352-353
Do 1, 3, 8, 12, 13, 15, 18, 19, 22, 24.
Study Guide continued

Part IV Objective 6

Study pp. 353-354
Exercises p. 354
Do 1, 6, 9, 12, 14, 17, 20.

Objective 7 (Optional)

Study pp. 354-358
Choose some area to investigate and construct a statistical graph of your findings.
Examples: Record the number of traffic tickets distributed per hour on some day. Record the absences of several students and compare their success in school. More advanced students could perhaps construct a control chart for a local industry. The decision is up to you.

Take Quiz I B

Study Chapter Summary p. 359
Do Chapter Test pp. 360-361
Grade and if you miss four or less you may take final test for this section. If not go back and work additional problems like those you missed.

Enrichment

Do problems 1 and 2 in *Suprise Attack in Mathematics*. 
QUIZ I B

Find the slope of the following equations. (Also find the y-intercept for 1-3)

1. $2x + y = 4$
2. $x - 2y = 6$
3. $y = 1$
4. A line passing through points $(0,0)$ and $(2,2)$
5. A line passing through points $(-1,3)$ and $(2,6)$

Determine the equation for the conditions stated below.

6. $m = -1/2$, $b = 3$
7. $m = 5$ and passes through $(1,6)$
8. Passes through $(0,2)$ and $(-2,-4)$
9. Parallel to $x = 4$ and passes through $(0,5)$
10. Parallel to $2x + y = 1$ and passes through $(1,1)$

Determine which of the following are quadratic equations and graph.

11. $x + 2y = 5$
12. $y = x^2 + 2$
13. $y = x/2$
14. $y = x^2 - 9$
15. $4y = x^2$

BONUS

Graph the following inequalities.

1. $5x + y \geq -1$
2. $x \geq 0$
3. $|y| > x + 2$
4. $y - 3x \leq -1$
5. $x > 4 \& y < -4$
CHAPTER X

SENTENCES IN TWO VARIABLES

Educational Objectives

The student should be able to

1. Solve simultaneous equations graphically, by addition and subtraction, and using substitution;

2. Define the terms inconsistent, dependent, and independent in relation to simultaneous equations;

3. Be able to graph pairs of linear inequalities;

4. Apply knowledge of simultaneous equations in solutions of digit, motion, and age problems.
Section I
Objectives 1 through 3

Solving Systems of Linear Open Sentences

Pre-Test

This Pre-Test is to aid you and teacher in establishing your study guide. It will not affect your grade, but it is very important. If you are successful you will be able to go on to more advanced material. You will grade the test yourself and mark the incorrect problems. Take your time, work carefully, and be fair to yourself.

Part I Review
Graph the following equations.
1. \(x+y=5\), \(x=7\); Put on the same graph
2. \(x+2y=6\), \(y=-\frac{1}{2}x-3\); Put on the same graph
3. \(3x=y+2\), \(6x-2y=4\); Put on the same graph

Part II Objective 1
Find the common roots for the following equations graphically, if they exist.
1. \(y=3-x\) \hspace{1cm} 2. \(x=y\) \hspace{1cm} 3. \(x+2y=6\)
   \(y=1+x\) \hspace{1cm} \(3y=0+3x\) \hspace{1cm} \(y-2x=5\)
Part III Objective 1

Find the common roots of these equations by subtracting or adding the following pairs of equations. Use multiplication if necessary.

1. \( x+y=7 \)
   \( x-y=16 \)

2. \( 7x+10y=6 \)
   \( 3x+y=23 \)

3. \( 3s+2t=17 \)

4. \( s+2t=5 \)

Part IV Objective 1

Solve each pair of equations using substitution.

1. \( x+2y=5 \)
   \( x=3y \)

2. \( 3x+2y=33 \)
   \( y-2x=-8 \)

3. \( w-2z/3=5 \)

4. \( w+z=9 \)

5. \( s-t=8 \)

6. \( s/3-t/2=1 \)

Part V Objective 3

Graph the following pairs of inequalities. Shade in the intersections of the solution sets for those pairs.

1. \( y<2x \)
   \( x>1 \)

2. \( y<x-1 \)
   \( y>1-x \)

3. \( x\geq1 \)

4. \( 2x-y\leq2 \)
   \( 2x-y\geq0 \)
PRE-TEST ANSWER KEY

Part I
1. 

Part II
1. 
2. 
3. 

Part III
1. \( x=45, y=29 \)
2. \( x=10, y=-7 \)
3. \( s=6, t=-\frac{3}{2} \)
4. \( r=4, t=3 \)
Answer Key continued

Part IV
1. $y=1, x=3$
2. $y=6, x=7$

Part V
1. $z=3, w=6$
2. $s=18, t=10$

After you have graded your Pre-Test, follow the instructions below:

1. If you missed more than one in Part I circle I in the chart below.
2. If you missed more than one in Part II circle II in the chart below.
3. If you missed more than one in Part III circle III in the chart below.