To Be And Not To Be (Simultaneously)

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Part I

In this essay, the term "contradiction" shall mean any conjunction which says that something can both be and not be at one and the same time. For example, the proposition, "The table is simultaneously brown and not brown", is a contradiction. A contradiction is essentially captured, then, by the symbolic phrase "p and not p". For the most part, this essay concerns itself with one major question: Why is a contradiction considered to be necessarily false? In other words, why are contradictory propositions thought to be related in such a way that both cannot be true together and both cannot be false together? The latter parts of the essay concern propositions which are implicitly contradictory and related problems.

Initially we might say that we accept the contradiction as false because we accept the law of contradiction as true. This law states that something cannot both be and not be at one and the same time. If the law is accepted as true, then, in order to remain consistent, we must accept contradictions as false. The law may appear very acceptable; it may seem completely indubitable. But not to be hasty in our conclusions, we should first consider some arguments claiming to disprove the law. We shall now consider two of them.

Another way of stating the law is that a thing cannot simultaneously have and have not a particular property. However, the first argument, as we shall see, succeeds in showing that an object can both be bigger than and not bigger than another object. Consider the following two objects:
When we consider the vertical dimensions, B clearly has the property of being bigger than A. When we consider the horizontal dimensions, B clearly has the property of being not bigger than A. So, B has and has not a particular property: It is bigger than and not bigger than A. Apparently, the law of contradiction is not true; it appears that we indeed can have a true contradiction.

One might jump to the defense of the law by objecting that the term "bigger" has been misused; that "bigger" refers to area; thus, understood as area, only one object could be bigger than the other; and, it could not also be not bigger. Suppose, though, that the argument had used another term; say, "longer". Then, when we consider the horizontal dimensions, we could say that A is definitely longer than B; but vertically, A is clearly not longer than B. So A is longer than and not longer than B. But here someone may argue that if we are talking about how long something is, then we must be talking about length; and an object can only have one length. Its other dimensions must be width, or depth, or height, or whatever... but not length again. The phrase "shorter than" might be introduced, but, again, there would no doubt arise objections that shortness is technically only a vertical descrip-
For my own part, these objections do not really undermine what is substantial to the argument. Are not the objections really nothing more than semantical or grammatical quibbles? For on one hand we have someone using a term in such and such a way; and, on the other we have someone saying: "No. You can't do that." When such a problem arises, it is unlikely that any further progress can be made on the issue. In order to argue we must first have a common language---we must first agree on what certain terms mean. But the question of whether or not a word is being used correctly is highly difficult to answer. Such a realization explains the employment of word usage panels in the compilation of modern dictionaries---we feel closer to being right with a majority bias rather than a minority bias. But even if a majority of experts proclaimed that the term "bigger" was indeed properly used in the argument, we would still have those to which we could not even present the argument until we somehow overcame the language barrier. Perhaps we need a different approach.

Now the question of whether or not the term "bigger" was used properly, would for our purposes become unimportant if we were able to show that, even if we do allow such a usage of the term, the argument is invalid. And it indeed can be shown that the "bigger than argument" is doubtlessly invalid.

To begin with, we might be humored to know that one who is convinced that the law of contradiction has been refuted by the "bigger than argument" might also be persuaded to believe that some birds are Catholic. After all, it is true that all cardinals are birds and also true that all cardinals are Catholic. This argument may
be ridiculous, but it does have important similarities with the "bigger than argument". No one would accept, under ordinary conditions, that the premises are true unless it was established that two different types of cardinals are being talked about. Once that is established, the conclusion does not logically follow. Likewise, with the "bigger than argument", two different properties are actually being discussed. For the law to be proven false, one must show that a thing can have a particular property and not have that same property at one and the same time. True, B is bigger than A, but only if "bigger than" is taken to mean vertically bigger than. B is also not bigger than A, but only if we mean not horizontally bigger. So B's being bigger than and not bigger than A cannot be represented by a "p and not p" type statement since the "bigger than" in each instance represents a different property. The "bigger than argument" therefore fails to disprove the law of contradiction.

The second argument against the law of contradiction requires that we picture to ourselves a form of nothingness. It might be done like this: Form a mental picture in which there is a solid substance of some kind. The solid substance should fill the entire picture except at its very center at which there is a hole. There is nothing in this hole. Therefore, we shall refer to it as a hole of nothing (it is a form of nothingness). This hole of nothing has existence. Why? Well, we are able to refer to it. How can reference be made unless an object of reference exists? Also, that which has spatial location must exist. Our hole of nothing has spatial location----it lies at the center of the solid substance we are imagining. But, there may be some hesitation at this point: How do we know that that which has spatial location must exist? If we are to answer
this question, we must first define existence. To avoid any suspi-
cion and to be completely fair, we should choose the common meaning
of the term rather than some special definition chosen to make the
answer come out one way or another. Existence, then, it is common-
ly said, means being. Now to deal with our question: We might first-
ly observe that that which has spatial location must be somewhere.
But the admission of being somewhere, within itself, admits being.
Thus, that which has spatial location must exist. Our hole of noth-
ing, therefore, exists. But also, it does not exist because a hole
of nothing is made up entirely of nothing; that is, if it is a hole
of nothing, it must be made up entirely of nothing; and nothing is
not something; if we are going to have an existence, then we must
have something, not nothing. Upon joining our two conclusions, we
have the following: A hole of nothing can simultaneously exist and
not exist. This may be only one of the relatively few instances in
which the law of contradiction fails (in a technical sense, there are
an infinite number of failures of the law since we could give an in-
finite number of shapes and sizes to a form of nothingness); never-
theless, it seems that the law of contradiction should only be re-
garded as being for the most part true and not universally true.

One might attempt to defend the law by pointing out that a hole
of nothing has only been imagined to exist in a mental picture. I
might picture in my mind a pink polka dotted elephant which I can re-er to and perhaps spatially locate, but this is no proof that such
an elephant actually exists. The argument has shown not p (that a
hole of nothing does not exist); but, it has not shown p (that a hole
of nothing does exist). Instead, p is only imagined to be the case.
Not p has been shown, but p has not been shown; hence, a "p and not
p" type statement has not even been made, let alone shown to be true.
This defense, though, falls short of the mark. If I picture in my mind a pink polka dotted elephant, it is true that this does not necessitate the physical existence of such a beast. However, such a picture, in anyone's mind, does necessitate the mental existence of such a beast. If one pictures in his mind a hole of nothing, then that hole of nothing exists as a mental thing. The law of contradiction applies to all things—be they physical or mental. Therefore, if it is shown that a mental thing can simultaneously exist and not exist, as has apparently been done, then the law of contradiction will have been shown to be not true.

But the "hole of nothing argument" does not disprove the law of contradiction because, for one thing, the argument is logically inconsistent with itself. A hole of nothing, the argument admits, is made up entirely of nothing and is therefore itself nothing. This is sound reasoning. But on one hand the argument says that a hole of nothing, which is nothing, exists. On the other hand, it claims that if we are going to have an existence then we must have something; and nothing is not something. But clearly, we cannot assume that that which exists must be something after we have just finished showing that nothing, which is not something, also exists. This inconsistency (contradiction) keeps the argument's conclusion from validly following.

But do we have the right to declare an argument invalid because it fails to stand up to the very principle which it puts into question? The very principle which the argument attempts to destroy ends up destroying the argument. Isn't that like the defendant murdering the prosecutor so that he cannot be prosecuted? Perhaps when the law of contradiction is on trial we are inevitably forced to assume
it "innocent until proven guilty"; if we didn't, how else could we decide the validity of the trial itself? Nevertheless, even if we have shown the argument to be invalid in an unjust or unfair manner, there is still much more about the argument to be suspicious of. For instance, what grounds do we have for saying that nothing is somewhere? How do we know that there is nothing anywhere? It is true that nothing is where things leave off; but do we know that things leave off anywhere? Can we say that between the proton and the electron of a hydrogen atom there is nothing? The only grounds for such a statement would be that, thus far, we have not detected anything between this proton and this electron. But being unable to detect anything is hardly grounds for saying that there is nothing (unless by "nothing" one simply means that something cannot be detected---but this is far from what must ultimately be meant by the term).

With my own perceptual faculties, I cannot detect anything in the air around me; yet, using the proper methods of science, numerous gases----oxygen, carbon dioxide, hydrogen....etc.----could be discovered. So it seems reasonable to avoid the conclusion that there is nothing simply because it appears that there is nothing. If we have the appearance of nothing, even under our most advanced methods of detection, we must never say (in an absolute sense) that there is nothing. Modern science could hardly progress under such stagnating beliefs.

Still another problem with the "hole of nothing argument" is that, if we imagine a hole of nothing, then that hole of nothing, since it is being imagined, must be mental. But how can nothing be a mental thing? It makes no sense to say that we have a mental nothing. How would it be different from any other nothing?

There are doubtlessly other problems with the "hole of nothing
argument", but we need not get into them. We can bring this matter to an end with a most interesting observation: Even if the "hole of nothing argument" was completely sound, its conclusion would not disagree with the law of contradiction. Its conclusion says only that nothing (a hole of nothing) can simultaneously be and not be. This is no more than a restatement of the law of contradiction.

Still, there may be other reasons for doubting the truth of the law. Let's consider two more arguments which may or may not cast some doubts. The first argument asks us to consider the fact that the law is meant as a universal statement----it's meant to apply to all things. But if we are going to state a condition which is true for all things, hopefully we will have grounds for making such a statement. It would seem that if a universal statement is going to have any claim to being well-grounded, it would have to be made by someone who could station himself outside the universe in such a position that he could observe all things. Since none of us have such a perspective, how can any of us speak with any authority about all things? The law therefore reduces to an assumption or theory.

The second argument concerns the nature of ultimate reality. This is reality as it actually is----not necessarily the sum total of what we perceive and conceive. Because we are confined to perception (acquaintance by means of the five senses) and conception (mental acquaintance), we can never really be sure that we are experiencing ultimate reality. We are, so to speak, confined to view the world through a pair of sunglasses which have a color (they might be perfectly clear or even completely opaque) we are not sure of. To ask what the world is like apart from perception and conception of it is to ask the impossible. And there are other problems which add to the confusion of
the reality puzzle: Dreams and hallucinations for example. During a dream or hallucination we believe that what we are experiencing has an external physical existence. Afterwards, we most often pass off the experience as a product of the mind. The point to be made from all this is that the mere fact that we believe we are having perceptual experiences is not sufficient reason to believe that we actually are experiencing an external physical existence. With all this uncertainty, then, what right have we to think that we can say anything at all about the nature of ultimate reality? Can we say that the law of contradiction applies to ultimate reality?

The first argument suggests that we lack the ability to gather the evidence necessary to proclaim the law as being true. The second argument suggests that our means of becoming acquainted with things (our perceptual and conceptual faculties) may very well keep us from discovering evidence which would prove the law to be false. What is similar with both of these arguments is that, in one way or another, both suggest that we need a posteriori certification of the law. Neither claims to be able to disprove the law, but both demand a posteriori evidence for proof of the law. But the sort of evidence these arguments call for will never be forthcoming. That is, we will never be able to examine all things to see whether or not the law holds true for each and every one of them. Such a proof is impossible, but perhaps the next section will show that it is also unnecessary.
Part II

Let us consider what seems to be solid proof that the law of contradiction must be true. The basis of this proof does not lie in examination of all reality (of course); instead, the proof shows that the law is true by definition. "All roses are plants" is an example of a proposition which is true by definition. It asserts no more than what is already granted in the meaning of the term "roses". The law of contradiction states that if something has a property then it cannot simultaneously not have that same property. Using some simple symbols, we could state the law as follows:

If p, then not not p.

But if something is not not the case, then it is the case. Therefore, the law of contradiction says the same as the law of identity:

If p, then p.

The law of identity says only that whatever is, is. It is undeniably true because it states no more than it already grants. Since we can define anything in terms of itself—-a book is a book, a clock is a clock, an X is an X,....etc.— we can say that the law of identity is true by definition. We have shown that the law of contradiction is only a rephrasal of the law of identity. And we have also shown that the law of identity is true by definition. Our conclusion can therefore be that the law of contradiction is definitionally true.

Let's be sure that we are clear on this matter. We have analyzed the law of contradiction and found it to be true because of the meanings or definitions of the terms used to express it. More precisely, we understand the term "not" to be a symbol of negation. Thus, when something is said to be "not not", we understand the negation itself to be negated. This is the way in which we understand the symbol "not". So, by the meaning of the term "not", we were able to see that p is
the same as not not p, hence reducing the law of contradiction (if p, then not not p) to the law of identity (if p, then p). The law of identity we knew to be true by our implicit understanding of the definition of anything; that is, A is A, B is B, and so on. Thus were we able to conclude that the law of contradiction is definitionally true.

Since the law is definitionally true, it follows that contradictions must be false. But couldn't we have argued that contradictions are definitionally false----thus avoiding all the previous problems with the law of contradiction? It may be thought that we could have argued for the definitional falsity of contradictions as follows: Firstly, we know that anything is the same as itself. Secondly, our understanding of the symbol "not" tells us that if p is true, not p must be false. Thirdly, we understand the conjunct "and" in such a way that if it combines a true statement and a false statement into a complex proposition, that complex must be false. In light of these three reasons, it is clear that a contradiction is definitionally false. But, if we argue in this direction, then it should come to our attention that the second reason (and perhaps also the third) presupposes the truth of the law of contradiction: To say that from the proposition "p is true" we can conclude "not p must be false" (reason number two), is to admit that the law of contradiction must be true. So, eventually we would have had to examine the law to decide whether it is true or false.

But have we shown the law to be true beyond the shadows of any doubts? No. The problem is that the definitional basis for the law presupposes the truth of the law. The law serves as a basis for our reasoning; so, in arguing logically for the truth of the law, one inevitably finds himself presupposing its truth. To understand how the
law serves as a basis for our reasoning, we can ask ourselves this question: If we knew certain truths such as "A has the property f" and "B has the property g", what good could that information possibly be to us if we could not then exclude the possibilities that A does not have the property f and B does not have the property g? The law allows us to exclude such possibilities and this is why we reason as we do. If upon knowing \( X \) to be true we could not then know not \( X \) to be false, then we would be left with the hopeless condition of any proposition being able to be true.

Now we must answer the following question: How exactly was the truth of the law presupposed in the definitional proof that we earlier gave to it? Well, the reason we know that A is the same as not not A is that we are able to exclude not not A from not A. A Venn diagram may help us understand this:

![Venn Diagram](image)

The diagram shows that A is located within the circle; not A, we can therefore conclude, must be represented by the shaded area because the shaded area covers everything else in the universe. But if we want to find out where not not A lies, then we look for everything in the universe other than not A. As illustrated by the diagram, this exclusion shows us that not not A is the same as A. The fact that we are able to make exclusions in this manner shows that the truth of the law is already an assumption. Any logically consistent justification of
the law can only presuppose the truth of the law—making it no justificaiton at all.

Our question, then, is still at hand: Do we have a basis to say that the law is true? It seems to come down to this: We have no evidence against the law. None of us, through any of our five senses, have ever received any indication that something simultaneously has and has not a particular property. Also, none of us have ever imagined something having and having not a particular property at one and the same time. Hence, since none of us have ever perceived or conceived a violation of the law, we believe that the law must be true. Such grounds for contradictions being necessarily false may not be absolute, but a true contradiction is not even within our scope of comprehension.

A definitional proof is only as stable as the definitions themselves. Someone may argue that when we say that something is the case, we mean that that something cannot also not be the case; thus, when I say that I am sitting in my chair, I mean that I am not not sitting in my chair. This would mean that the law of contradiction is true by definition. But the problem with such a basis is that someone else comes along with different definitions which add up to a different conclusion. Who's dictating meaning? Where such fundamental matters are concerned, there is no clear cut "what we mean by such and such terminology". It rather boils down to what the individual who used the terms meant by them. The only safe approach is to abstract oneself from the linguistic problems and compare the idea itself to what we as individuals believe to be reality. As mentioned earlier, we will find no absolute with this approach since we will never be able to examine all things; not only this, but how do we examine those things we are familiar with to see whether or not they also do not have the proper-
ties that they do have? The pragmatic (practical) solution is this: Why concern ourselves with whether or not a thing also doesn't have a property that we're sure it does have? If you did find out that the chair you're sitting on was also not there, would you suddenly become unsupported and drop to the floor? So, for practical reasons, the remaining parts of this essay will treat the contradiction as being false.
There will always be obvious cases of contradictions. But sometimes we may have difficulty in deciding whether or not we really have one. For example, someone might say that Susan is at the theatre. Someone else might say that Susan is at the gym. If one assertion is true, then the other must be false since someone cannot be at two places at the same time.* However, if one assertion is false, the other assertion is not necessarily true. It may be that both assertions are false since Susan may in fact be at the library. Contradictory assertions cannot both be true together and cannot both be false together. Since both assertions may be false together we do not really have a contradiction. But implicitly we do.

We just finished saying that someone cannot be in two places at once. Surely, no thing can be in two places at once. If we accept this principle as true, then we must consider our assertions in a different light. The assertion that Susan is at the theatre ($X$) logically implies, so long as we accept the principle that something cannot be in two places at once, that Susan is not at the gym ($\neg Y$). But one assertion was that Susan is at the gym ($Y$). In our example we had $X$ and $Y$, which we said were not contradictory. However, we also know that $X$ logically implies $\neg Y$; from all this we can conclude $Y$ and $\neg Y$, which is clearly a contradiction. So, in a certain sense we can say that assertions $X$ and $Y$ are contradictory; they imply a contradiction.

This last example brings up a useful principle for deciding whether

* If the term "place" is defined as being any space of one cubic centimeter, then any person could occupy many places at once; I obviously do not have this sort of meaning in mind. If the reader puts aside the question of "How big is a place?", he will gather my meaning from context.
er or not two assertions are contradictory. That principle is that something cannot be in two places at once. This principle is widely accepted as being true. Why? Perhaps because no one has ever perceived anything to be in two places simultaneously. I have, however, met people who claim to be able to imagine a thing to be in two places at once. One person, for example, claimed that he could imagine himself to be in his car and in his house at one and the same time (his car was not also his house and his house was not also his car). Since we can have no direct acquaintance with another person's imaginations, perhaps we cannot really make any judgement about this. But setting aside any doubts, let's assume that someone can imagine that something is in two places at once. Would such an imagination mean that an object can actually be in two places at once? We might return to an earlier mentioned argument that would not allow us to here deny the necessity that an imagined object must have a mental existence. I see no way to deny that if something is being imagined then that something must mentally exist. If someone imagines a thing being in two places at once, then we have a mental thing existing in two places at once.

There may be some doubt, though, as to whether or not we can say that a mental existence has spatial location. We can point to where the table and the chair exist, but where does thought exist? After entertaining several thoughts, can we say that one thought occurred to the left of, or underneath another? It is commonly said that thought exists within someone's head. It may be that modern science can show that a significant relationship exists between the occurrence of thought and electrical activity of the brain—which would lead many to believe that thought is no more than electrical activity of the brain. If such is actually the case, then there may be nothing mental as we previously believed, but all existence would be entirely physical. For
my own part, I've yet to arrive at a satisfactory criteria for distinguishing the physical from the mental. But this subject is far too problematic to be here taken up in any detail that would do it justice. Some solution to the problem must be forthcoming, it would seem, if arguments which call for a physical-mental distinction are to be fully understood. There are arguments within this essay which call for such a distinction, but they cannot be fully discredited because of our inability to draw clear boundaries. The phrase "mental things", used in several arguments within this essay, for example, is only meant to be understood as "thoughts" or "that which is a product of the mind" (or something similar). If our thoughts are proved to be physical, then so be it, for this will not really detract from the arguments which call for the existence of "mental things"; for their points thus far have been that at least something exists (call it physical or mental) which meets such and such conditions.

But we should take a closer look at whether or not we want to allow for the possibility of someone being able to imagine an object in two places at once. To begin with, we cannot deny that if a thing is at once in two different places, then the thing in one place must be the same as the thing in the other place. All properties that the thing in one place has, the thing in the other place must have also. If the thing in one place has the property of being blue, the thing in the other place, since it is the same thing, must also have the property of being blue. But here is where we run into problems. One thing has the property of being in one particular place (place X); the other thing does not have this property since it is in a different place (place Y). Therefore, it must not be the same thing which is in each place. It is impossible, then, for one thing to be in two places at once.
But there is a problem with this last argument. From the assumption of having one thing in two different places, it comes to talk about "one thing" and "the other thing" ---as if there were two things to be dealt with. If we allow this then we will have to admit that one must be the same as two. Surely, no one will say that one is the same as two. But it is this very truth which secures our principle; in order for one thing to be simultaneously in two different places, one would have to be the same as two. If what we have argued here is sound, then any statements which assert that something is simultaneously in two places must be implicitly contradictory.
Part IV

Let us now examine a contradiction which is actually claimed to be true by a goodly number of people. Many people hold that 1) a supreme being exists who knows everything; and 2) all men have free will. Whether or not there are grounds for believing one or the other of these will be here beside the point. My point shall be that it is impossible to believe that both of these propositions are true since they are implicitly contradictory. Someone may here suggest that it is possible for someone to psychologically believe contradictory propositions to be true; but as we shall see, this really cannot be so. Firstly, let's make sure that we have a contradiction.

It is commonly believed that this supreme being knows everything in the sense that if he were to make an honest prediction of any future, he would never be wrong. In other words, any and every future event is known by this being. But also, any and every past and present occurrence is also known by this being. The long and short of it is that there is nothing which this being does not know; he knows every-thing! But, if such is true, then it is also true that all events are predetermined; this means that there exists an established plan for the universe which cannot be changed by anyone—not even the being who knows the plan. Someone may suggest that perhaps an even greater being may step in and alter the plan somehow. But, even if a greater being interferes, his interference must have already been known beforehand and hence must only be part of the great plan. With this strict determinism, it follows that nothing whatsoever can operate independently of what is predetermined. We may think that we can choose to do what we want to do, but we really are only following a predetermined plan from which we simply cannot deviate. We obviously have a contra-
diction, then, since on one hand it is said that we have free will and on the other it is implied that we do not. Or we could say that on one hand it is implied that everything is determined while on the other it is implied that everything is not determined. So we definitely have a contradiction.

Is it possible to believe contradictory propositions to be true? Not really. Take some proposition, say, "Michael is sitting in the chair". If someone is going to believe this proposition to be true, he must first be able to understand it. If someone were to say to you, "True or false?: The square of green is soft", you would probably respond by saying that you cannot believe one way or another because you don't understand what the proposition means. Now if someone were to say that he believes Michael is sitting in the chair, then we would not expect him to say that he also believes that Michael is playing football outside. If someone did claim to believe both, then we would have to assume that that person does not completely understand the propositions. We cannot say that such a person understands what the proposition "Michael is sitting in the chair" means, because if he did then he would realize that Michael could not also be outside playing football. So when someone says that they believe two propositions to be true, and those propositions are contradictory (even implicitly), we must conclude that that person must not completely understand those propositions. The person who claims to believe that "God knows everything and we are free to choose between good and evil", does not really believe this to be true because he doesn't understand what this means. He thinks that he understands the propositions, but in fact he does not; for the propositions are contradictory, and to understand their meaning would necessitate understanding the contra-
diction involved. Such a person probably has certainty about something similar to what he says he believes, but he does not have certainty about that which he says he believes.