Here is an example of a 4-inch tangram.
CREATING POLYGONS FROM TANGRAMS

Label your tangram created in "Constructing A Tangram" as follows:

- two largest triangles, 1 and 2
- two smallest triangles, 3 and 5
- the remaining triangle, 7
- the square, 4
- the parallelogram, 6

Using your tans, create an example of each polygon below. Form the figures on blank white paper and then trace around each tan and label it in the polygon.

triangle  quadrilateral  pentagon
hexagon  heptagon  octagon
**DIAGONALS OF A RECTANGLE**

Materials: Squared paper

Mark out different size rectangles on the squared paper where the length \( l \) and width \( w \) is a whole number of squares. Now draw in a diagonal of each rectangle and note the number of squares which it crosses, \( d \).

<table>
<thead>
<tr>
<th>( l )</th>
<th>( w )</th>
<th>( d )</th>
</tr>
</thead>
</table>

Fill in the above table. Find the relationship between \( l \), \( w \) and \( d \).

**ANSWER**

If \( h \) is the highest common factor of \( l \) and \( w \) then the relationship is
\[
d = l + w - h.
\]

Example: \( l = 15 \), \( w = 10 \), then \( h = 5 \) so \( 15 + 10 - 5 = 20 \).
There are two major types of polygons, concave and convex.

Concave: bends or turns inward
Convex: does not bend or turn inward

Label the parts of your tangram as follows:
- two largest triangles, 1 and 2
- two smallest triangles, 3 and 5
- the remaining triangle, 7
- the square, 4
- the parallelogram, 6

Create the following polygons from your tans. Sketch each polygon and indicate the position of each tan.

1. Use tans 1, 3, 4, 5 to form a convex quadrilateral.
2. Use tans 1, 3, 4, 5 to form a convex pentagon.
3. Use tans 1, 3, 4, 5 to form a concave hexagon.
4. Use tans 1, 3, 4, 5, 7 to form a convex pentagon.
5. Use tans 1, 2, 3, 4, 5, 7 to form a concave hexagon.
6. Use tans 1, 3, 4, 5, 7 to form a concave pentagon.
7. Use tans 2, 3, 6 to form a convex quadrilateral.
8. Use tans 1 and 7 to form a concave pentagon.
9. Use tans 5, 6, 7 to form a concave quadrilateral.
10. Use tans 2, 3, 5, 6 to form a convex pentagon.
11. Use all tans to form a concave pentagon.
12. Use all tans to form a triangle. (All triangles are convex.)
CONCAVE AND CONVEX POLYGONS - ANSWERS

Answers may vary. Following is a possible answer for each.

1.  
   1  
   5  
   3

2.  
   1  
   3  
   4  
   5

3.  
   4  
   5  
   1

4.  
   1  
   3  
   5  
   7

5.  
   1  
   3  
   5  
   7  
   2

6.  
   6  
   5  
   3  
   7  
   1

7.  
   2  
   6  
   3

8.  
   7  
   1

9.  
   5  
   6  
   7

10.  
   6  
   5  
   3

11.  
   7  
   3  
   4  
   5  
   6  
   1

12.  
   1  
   2  
   7  
   4  
   6  
   5
A BIG QUADRILATERAL

For the quadrilateral below you are given information about the measures of some of the angles. At the bottom of the page you are asked to find the measures of certain angles. It might be helpful to record all measure that you know even if they are not asked for because they may help you find the angles that are asked for. **Do not use a ruler or protractor.** Caution: Double-check your measures because one incorrect measure can lead to further miscalculations.

Given:

- $\angle EAB = 95$
- $\angle BED = 80$
- $\angle 1 = 83$
- $\angle 3 = 101$
- $\angle 5 = 46$
- $\angle 7 = 35$
- $\angle ABC = 104$
- $\angle 2 = 62$
- $\angle 4 = 104$
- $\angle 6 = 70$
- $\overline{AC} \perp \overline{CD}$

Find:

- $\angle 8$
- $\angle 9$
- $\angle 10$
- $\angle 11$
- $\angle 12$
- $\angle 13$
- $\angle 14$
- $\angle 15$
- $\angle 16$
- $\angle 17$
- $\angle 18$
- $\angle 19$
A BIG QUADRILATERAL - ANSWERS

Find:

\[ \angle 8 = 18 \quad \angle 9 = 115 \]
\[ \angle 10 = 50 \quad \angle 11 = 82 \]
\[ \angle 12 = 100 \quad \angle 13 = 97 \]
\[ \angle 14 = 93 \quad \angle 15 = 75 \]
\[ \angle 16 = 35 \quad \angle 17 = 17 \]
\[ \angle 18 = 32 \quad \angle 19 = 14 \]
Below is an isosceles trapezoid. Record the given information and then use your knowledge of triangles and quadrilaterals to fill in the missing angle measures. It might be helpful to record all measures that you know even if they are not asked for because they could help you find the angles asked for. Do not use a ruler or protractor. Caution: Double-check your measures because one incorrect measure can lead to further miscalculations.

**Given:**
- Quadrilateral ABCD is an isosceles trapezoid with base $\overline{DC}$
- $\overline{AB} \parallel \overline{EF} \parallel \overline{DC}$
- $\overline{AE} \cong \overline{EH}$
- $\angle AHE \cong \angle EHG$
- $\angle BGH$ is a right angle
- $m \angle BGF = 40$
- $m \angle BEF = 22$

**Find:**
- $m \angle 1$
- $m \angle 2$
- $m \angle 3$
- $m \angle 4$
- $m \angle 5$
- $m \angle 6$
- $m \angle 7$
- $m \angle 8$
- $m \angle 9$
- $m \angle 10$
- $m \angle 11$
- $m \angle 12$
- $m \angle 13$
- $m \angle 14$
- $m \angle 15$
A BIGGER QUADRILATERAL - ANSWERS

Find:

\[
\begin{align*}
\text{m} \angle 1 &= 50 & \text{m} \angle 2 &= 22 \\
\text{m} \angle 3 &= 108 & \text{m} \angle 4 &= 50 \\
\text{m} \angle 5 &= 65 & \text{m} \angle 6 &= 50 \\
\text{m} \angle 7 &= 43 & \text{m} \angle 8 &= 65 \\
\text{m} \angle 9 &= 75 & \text{m} \angle 10 &= 18 \\
\text{m} \angle 11 &= 25 & \text{m} \angle 12 &= 40 \\
\text{m} \angle 13 &= 115 & \text{m} \angle 14 &= 65 \\
\text{m} \angle 15 &= 115 & \\
\end{align*}
\]
IDENTIFYING ATTRIBUTES OF QUADRILATERALS

Examine the quadrilaterals in each row and determine which one does not belong with the other three. On a separate sheet of paper write the letter of the quadrilateral that does not belong with the others in its row. Provide a reason for your choice.

<table>
<thead>
<tr>
<th>Quadrilaterals</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>---</td>
<td>----</td>
<td>----</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>6</td>
<td><img src="diamond.png" alt="Diamond" /></td>
<td><img src="rectangle.png" alt="Rectangle" /></td>
<td><img src="parallelogram.png" alt="Parallelogram" /></td>
<td><img src="trapezoid.png" alt="Trapezoid" /></td>
</tr>
<tr>
<td>7</td>
<td><img src="square.png" alt="Square" /></td>
<td><img src="rectangle.png" alt="Rectangle" /></td>
<td><img src="parallelogram.png" alt="Parallelogram" /></td>
<td><img src="trapezoid.png" alt="Trapezoid" /></td>
</tr>
<tr>
<td>8</td>
<td><img src="quadrilateral_angles.png" alt="Quadrilateral with angles" /></td>
<td><img src="quadrilateral_angles.png" alt="Quadrilateral with angles" /></td>
<td><img src="quadrilateral_angles.png" alt="Quadrilateral with angles" /></td>
<td><img src="quadrilateral_angles.png" alt="Quadrilateral with angles" /></td>
</tr>
<tr>
<td>9</td>
<td><img src="quadrilateral_arrows.png" alt="Quadrilateral with arrows" /></td>
<td><img src="quadrilateral_arrows.png" alt="Quadrilateral with arrows" /></td>
<td><img src="quadrilateral_arrows.png" alt="Quadrilateral with arrows" /></td>
<td><img src="quadrilateral_arrows.png" alt="Quadrilateral with arrows" /></td>
</tr>
<tr>
<td>10</td>
<td><img src="quadrilateral.png" alt="Quadrilateral" /></td>
<td><img src="quadrilateral.png" alt="Quadrilateral" /></td>
<td><img src="quadrilateral.png" alt="Quadrilateral" /></td>
<td><img src="quadrilateral.png" alt="Quadrilateral" /></td>
</tr>
</tbody>
</table>
IDENTIFYING ATTRIBUTES OF QUADRILATERALS - ANSWERS

1. All are parallelograms except D.
2. All are rhombi except B.
3. All base angles in each figure are congruent except A.
4. All are convex quadrilaterals except C.
5. All are trapezoids except D.
6. All are irregular quadrilaterals except A.
7. All are rectangles except D.
8. All measures of the angles are correct except B.
9. All sides are correctly marked except A.
10. All diagonals are congruent except A.
PROGRAMMING QUADRILATERALS

OVERVIEW:

This lesson uses a flow sheet format to characterize and form relationships between quadrilaterals. Students will work in pairs or small groups, depending on class size and available materials. Each group will be given examples of quadrilaterals and will have to document their progress through the flow sheet for each quadrilateral. As a final product, the students will have each quadrilateral characterized and have formed a definition for that quadrilateral. Each group will also have to theorize how each quadrilateral is related to the others. As a second activity, students will work on Geometer’s Sketchpad in pairs to further explore quadrilaterals and the relationships between them. The pairs will then regroup into the original groups and each group will create a visual chart of their design to demonstrate the relationships between the quadrilaterals. These charts will be presented to the class and hung around the room for display. This lesson would be one in a larger unit addressing the study of polygons. This lesson is designed to last three days with 45 minutes of class time each day.

PURPOSE:

The purpose of this lesson is to:
• help students develop their critical thinking skills.
• continue the development of the students’ deductive reasoning skills and methods.
• encourage group work and interactions between students.
• create useful information that can go into student portfolios as references and examples of their work.

LEARNING OBJECTIVES:

Students will
• have an understanding of how quadrilaterals fit into the general term polynomial.
• have an understanding of the general term of quadrilateral and what it can mean in different contexts.
• will be able to categorize the special quadrilaterals into five main terms; square, rectangle, rhombus, parallelogram, and trapezoid.
• understand the specific qualities of each of the five special quadrilaterals.
• form their own definitions for the five special quadrilaterals.
• form an understanding of the relationships between the five special quadrilaterals.

**VOCABULARY:**

Before lesson:
- Congruent sides
- Parallel sides
- Polygon
- Quadrilateral
- Right angle
- Vertices

Will form during lesson:
- Parallelogram
- Rectangle
- Rhombus
- Square
- Trapezoid

**RESOURCES / MATERIALS:**

- Computer lab access
- Geometer’s Sketchpad
- Geometer’s Sketchpad activity on computers
- Geometer’s Sketchpad Handout
- Handout of shapes the students will categorize.
- Handout of the flow sheet.
- Large paper
- Markers
- Pencil
- Personal disk, either 3 ½ floppy or zip disk
- Protractor
- Ruler

**PREPARATORY ACTIVITIES/ ASSUMED KNOWLEDGE:**

Students will need to know how to
- measure the length of the side of a polygon.
- to measure the angle of a polygon.
- determine equivalency between lengths of the sides of polygons.
• determine equivalency between two or more angles in a polygon. In Geometer’s Sketchpad, students will need to know how to
  • Measure lengths/distances
  • Measure angles
  • Manipulate figures
  • General knowledge of program

MAIN ACTIVITIES:

Day 1
Introduction: (10 minutes)
Have students get into their predetermined groups (4 students/group).
  Review Previous Material and Introduce New Material:
    Yesterday we studied triangles. **What general term does a triangle fall into?** (polygon)
    **So a triangle is a polygon with how many sides?** (3)
    **If we studied a polygon with 3 sides, what kind of polygon do you think we are going to study next?** (a 4 sided figure, quadrilateral)
  
    Class Discussion:
    **Do you think all quadrilaterals are going to look alike or similar?**
    (put student input onto the board, hopefully with a reason)

Activity: (20 minutes)
Give the handouts to the students.
Students will work on the handouts in their groups.
Go to the groups and check on progress and answer questions.

When a group is done with the handout:
  Make sure each group have some sort of definition for each special quadrilateral.
  Write down any relationships between the quadrilaterals that you notice.

Conclusion: (15 minutes)
Students need to stay with their groups. **Keep all of these papers. You will need some of the information that you have written down on them for tomorrow.**

Review: **What are the characteristics of the quadrilaterals?** (student input)
- **SQUARE**: 4 sides of equal length, 4 right angles, 2 pairs of parallel sides
- **RECTANGLE**: 2 pairs of parallel sides, 4 right angles, each pair of sides are of different length
- **RHOMBUS**: 4 sides of equal length, 2 pairs of parallel sides, do not have to have right angles
- **PARALLELOGRAM**: 2 pairs of parallel sides, do not contain right angles, sides do not have to be the same length
- **TRAPEZOID**: only 1 pair of parallel sides, can contain right angles, but does not have to have them

Class Discussion:

*Now that you have done the activity, do you still think that all quadrilaterals look alike? Why or why not? How has your answer changed? What made you change your answer?*

**Day 2**

Introduction: (5 minutes)

*At the end of class yesterday, each group had made a list of the characteristics of the quadrilaterals and of the relationships that you might have noticed between them. Call on groups to share answers and write them on the board.*

Activity (Day 2): (40 minutes)

Each group will break into two pairs and share computers next to each other, so that they can share the previous work done by the group.

Give students handout directions:

*(Write the names of the files on the board so students can see what files are supposed to be there and so they know which ones they have used; Quad1, Quad2, Quad3, & Quad4)*

**Day 3**

Introduction: (3 minutes)

*Yesterday you worked in Geometer’s Sketchpad to change one quadrilateral into another to help you see how they are related. Get back into your groups once again.*

(Pass out paper and markers while students are organizing themselves.)

Activity (Day 3):

Group Work: (20 minutes)
Using the information that you and your group have gathered over the past two days, I want you to make a visual representation of how the quadrilaterals are related to each other. Make sure you make it self-explanatory. In about 20 minutes you will have to share what you learned and your poster with the class.

Presentations: (10 minutes)

Each group will get time to explain how they think the quadrilaterals are related and give an explanation.

Conclusion: (12 minutes)

Review:
Make a flow chart on the board as part of the review of how the quadrilaterals are related. Have students help fill in the chart.

Start with a square and a rectangle. Ask students how they are related. Work your way through the flow chart below.

Are there any questions concerning quadrilaterals?
**ASSESSMENT:**

Informal:
- Observation of activities in groups.
- Completion of Day 1 handout.
- Completion of Geometer's Sketchpad activity.

Formal:
- Accuracy of relationships between the quadrilaterals.
Directions: Pick a shape from the handouts and follow the steps from the beginning for each quadrilateral. Then write in the name of each quadrilateral and any properties that are noticeable from the activity to help you form a usable definition.

Are there two pairs of parallel sides?

- **YES**
  - Does the shape have ONLY right angles?
    - **YES**
      - Are all 4 sides of equal length?
        - **YES**  <---- SQUARE
        - **NO**  <---- RECTANGLE
    - **NO**  <---- TRAPEZOID
  - **NO**  <---- PARALLELOGRAM

- **NO**  
  - Are all 4 sides of equal length?
    - **YES**  <---- RHOMBUS
    - **NO**
Name:

Properties:

Name:

Properties:
Quadrilaterals and Geometer’s Sketchpad

During this activity, you should think about how each of the five quadrilaterals that we discussed yesterday are related to each other. Follow the directions below to use Geometer’s Sketchpad to transform one quadrilateral into another.

- Open the folder Quadrilaterals found on your desktop.
- In this folder there will be 4 saved quadrilaterals.
- The order that you use these quadrilaterals does not matter, just as long as you use all of them.
- For each of the saved quadrilaterals, fill out the information below.
- Each time you successfully form a new quadrilateral SAVE IT.
- Make sure that you do not save over the file on the computer but go to SAVE AS and save it on your own disk.
- Once all four group members has successfully used every file, work together to form a chart of the relationships between each of the five quadrilaterals. (Each group will get a large piece of paper and markers to draw their chart so that it can be hung in the classroom.)

Hints for using each new quadrilateral:
- Create labels at each of the vertices.
- Find the distance or length between the vertices.
- Find the angles between the sides of the quadrilateral.

File used:
Starting Quadrilateral:

Quadrilateral Formed:

File used:
Starting Quadrilateral:

Quadrilateral Formed:
File used:

Starting Quadrilateral:

Quadrilateral Formed:

File used:

Starting Quadrilateral:

Quadrilateral Formed:

File used:

Starting Quadrilateral:

Quadrilateral Formed:

File used:

Starting Quadrilateral:

Quadrilateral Formed:

File used:

Starting Quadrilateral:

Quadrilateral Formed:
Screenshots of the Program Files From Geometer’s Sketchpad

The blue lines must always be parallel. Remember that a trapezoid has ONLY ONE pair of parallel lines.
CIRCLES

- Magic Circles
  - problem solving involving knowledge of diameters
- Circles And Symmetry
  - practice symmetry pertaining to circles
- Tangent Circles And Lines
  - practice tangency with circles
- Big Circle Puzzle
  - properties of angle measures
  - properties of circles
- Bigger Circle Puzzle
  - properties of angle measures
  - properties of circles
- Biggest Circle Puzzle
  - properties of angle measures
  - properties of circles
- Reasoning With Circles
  - practice logical thinking with circles
- Graphing Circles On The Coordinate System
  - practice using radius, diameter, center, and tangency to graph circles on the coordinate system
- Practicing Circle Vocabulary
  - practice circle vocabulary skills
- Creating An Ellipse From String
  - investigate lengths between the perimeter and the foci
- Stretching A Circle
  - expanding knowledge of circles to ellipses
- Creating An Ellipse From Repeated Folds
  - expanding knowledge of circles to ellipses
CIRCLES BIBLIOGRAPHY


MAGIC CIRCLES

A magic circle is a circular array of numbers arranged in a special manner such that when the numbers along each diameter are added their sums are equal.

1. Use the numbers 1 to 11 to fill in the magic circle below so that the diameters add to be 18. Each number will only be used once.

2. Use the numbers from 1 to 13 to fill in the magic circles below so that the diameters of the small circle add to be 12 and the diameters of the large circle is 33. Each number will only be used once.
MAGIC CIRCLES - ANSWERS

Answers may vary.

1.

2.

9

3

2

1

7

10

5

11

8

8

3

7

6

12

13
CIRCL£S AND SYMMETRY

Materials: Need 4 different colors of colored pencils.

Circles can contain three types of symmetry, horizontal, vertical, or rotational.

<table>
<thead>
<tr>
<th>VERTICAL SYMMETRY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shade 1, 2, 3, 4, 7, 8: vertical line of symmetry, from top to bottom.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HORIZONTAL SYMMETRY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shade 1, 2, 3, 4, 5, 6: horizontal line of symmetry, from left to right.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ROTATIONAL SYMMETRY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shade 1, 4, 5, 8: rotational symmetry of order 2, meaning the sectors must be rotated twice to end in their original positions.</td>
</tr>
</tbody>
</table>
Color two sectors of the circle so that:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>the circle has one horizontal line of symmetry</td>
<td></td>
</tr>
<tr>
<td><img src="image1.png" alt="Circle 1" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>the circle has one vertical line of symmetry</td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Circle 2" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td>the circle has a rotational symmetry of degree 2</td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Circle 3" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Color four sectors so that the circle has:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>4.</td>
<td>one horizontal line of symmetry and no vertical line of symmetry</td>
<td></td>
</tr>
<tr>
<td><img src="image4.png" alt="Circle 4" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>5.</td>
<td>one vertical line of symmetry and no horizontal line of symmetry</td>
<td></td>
</tr>
<tr>
<td><img src="image5.png" alt="Circle 5" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
</tr>
</tbody>
</table>
| 6. | one horizontal line of symmetry  
one vertical line of symmetry |
|   | ![Diagram](image1.png) |
| 7. | two lines of symmetry, but no  
horizontal or vertical lines of  
symmetry |
|   | ![Diagram](image2.png) |
| 8. | rotational symmetry of degree 2 |
|   | ![Diagram](image3.png) |
| 9. | a horizontal line of symmetry, a  
vertical line of symmetry, and  
rotational symmetry of degree 2 |
|   | ![Diagram](image4.png) |
| 10. | rotational symmetry of degree 4  |
|    | ![Diagram](image5.png) |
CIRCLES AND SYMMETRY - ANSWERS

Answers may vary.

Shade the following sectors for each circle.

1. 2 and 5
2. 1 and 2
3. 1 and 5
4. 2, 3, 4, 5
5. 1, 2, 3, 8
6. 1, 2, 5, 6
7. 1, 4, 5, 8
8. 1, 4, 5, 8
9. 1, 2, 5, 6
10. 1, 3, 5, 7
A tangent to a circle that lies in the plane can be a circle or line that intersects or “touches” the circle at one and only one point.

A common tangent is a line that is tangent to two or more circles.

Circles may also be tangent to each other.

Example of a common tangent.

Example of a common tangent and two circles that are tangent.

Draw circles and tangent lines so that the following conditions are met.

1. Two circles with no common tangents.
2. Two circles that have one common tangent.
3. Two circles that have two common tangents.
4. Two circles that have four common tangents.
5. Three circles that have one common tangent.
6. Three tangent circles.
7. Three circles so that each one is tangent to the other two.
8. Three circles so that two intersect and the other one is tangent to the other two.
TANGENT CIRCLES AND LINES - ANSWERS

Here are some possible answers.

1.

2.

3.

4.

5.

6.

7.

8.
BIG CIRCLE PUZZLE

Given the following conditions, fill in the measures on the circle diagram below. Make sure you double-check yourself often.

\[ \overline{DF} \text{ and } \overline{CA} \text{ are diameters of Circle } O. \]

\[ m\overline{AB} = 10 \quad m\overline{HI} = 6 \]
\[ m\angle HIO = 90 \quad m\overline{OF} = 10 \]
\[ \angle DOA = \angle BOF \]

1. \( m\overline{CA} = \)  
2. \( m\overline{OB} = \) 
3. \( m\overline{OA} = \) 
4. \( m\overline{AB} = \) 
5. \( m\angle DOA = \) 
6. \( m\angle FOB = \) 
7. \( m\overline{DB} = \) 
8. \( m\overline{ADB} = \) 
9. \( m\overline{GI} = \) 
10. \( m\overline{GH} = \) 
11. \( m\overline{OI} = \) 
12. \( m\overline{IC} = \) 
13. \( m\overline{CD} = \) 
14. \( m\overline{CF} = \) 
15. \( m\angle GIC = \) 
16. \( m\overline{FDB} = \) 
17. \( m\angle COF = \) 
18. \( m\angle COD = \)
1. 20
2. 10
3. 10
4. 60
5. 60
6. 60
7. 120
8. 300
9. 6
10. 12
11. 8
12. 2
13. 120
14. 60
15. 90
16. 300
17. 60
18. 120
**BIGGER CIRCLE PUZZLE**

Given the following conditions, fill in the measures on the circle diagram below. Make sure you double-check yourself often.

- $\overrightarrow{GF}$ is tangent to Circle $O$ at $D$.
- $\angle DFE = 122$°
- $\angle MAC = 20$°
- $\angle DCE = 90$°
1. \( \angle EDF = \) 
2. \( \angle DOE = \) 
3. \( \angle OED = \) 
4. \( \angle ODE = \) 
5. \( \angle BDG = \) 
6. \( \angle BCD = \) 
7. \( \angle ADC = \) 
8. \( \angle CDG = \) 
9. \( \widehat{AB} = \) 
10. \( \angle BFE = \) 
11. \( \angle ADB = \) 
12. \( \angle DAE = \) 
13. \( \angle AED = \) 
14. \( \angle BOE = \) 
15. \( \angle BDF = \) 
16. \( \angle CDB = \) 
17. \( \angle AIB = \) 
18. \( \angle CHD = \) 
19. \( \angle DHB = \) 
20. \( \angle AIC = \) 
21. \( \angle BIJ = \) 
22. \( \angle AJB = \) 
23. \( \angle EJD = \) 
24. \( \angle AJD = \) 
25. \( \angle JEO = \)
1. 61
2. 122
3. 29
4. 29
5. 90
6. 90
7. 10
8. 45
9. 70
10. 58
11. 35
12. 61
13. 55
14. 58
15. 90
16. 45
17. 141
18. 80
19. 100
20. 39
21. 39
22. 96
23. 96
24. 84
25. 26
BIGGEST CIRCLE PUZZLE

Given the following conditions, fill in the measures on the circle diagram below. Make sure you double-check yourself often.

O is the center of the circle.
AB, CD, and EF are diameters.

\[
\begin{align*}
\text{m}\angle GB &= 26 \\
\text{m}\angle OAH &= 50 \\
\text{m}\angle IGA &= 24 \\
\Pi &= 4.5 \\
\text{CM} &= 10 \\
\text{hm} &= 7.5
\end{align*}
\]

1. \(\text{m}\angle CB = \) 
2. \(\text{m}\angle AJC = \) 
3. \(\text{m}\angle CG = \) 
4. \(\text{m}\angle FOB = \) 
5. \(\text{m}\angle FG = \) 
6. \(\text{m}\angle FPC = \) 
7. \(\text{m}\angle FIG = \) 
8. \(\text{m}\angle FI = \) 
9. \(\text{m}\angle IKF = \) 
10. \(\text{m}\angle AE = \) 
11. \(\text{m}\angle ELD = \) 
12. \(\text{m}\angle CD = \) 
13. \(\text{m}\angle COF = \) 
14. \(\text{m}\angle FM = \) 
15. \(\text{m}\angle FNB = \)
LD and LE are tangent to the circle.
BIGGEST CIRCLE PUZZLE - ANSWERS

1. 150
2. 43
3. 124
4. 50
5. 24
6. 31
7. 12
8. 82
9. 119
10. 50
11. 80
12. 16
13. 100
14. 8
15. 78
Below are facts pertaining to circles. For each statement provide a logical reasoning.

<table>
<thead>
<tr>
<th>DIAGRAM</th>
<th>STATEMENT</th>
<th>YOUR REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td>A circle with a radius of 5 inches is congruent to a circle with a diameter of 10 inches.</td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
<td>The diameter of a circle is the longest chord.</td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td>All circles are similar.</td>
<td></td>
</tr>
<tr>
<td><img src="image4.png" alt="Diagram" /></td>
<td>ΔAOB is an isosceles triangle</td>
<td></td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram" /></td>
<td>ΔCDE is a right triangle.</td>
<td></td>
</tr>
<tr>
<td>DIAGRAM</td>
<td>STATEMENT</td>
<td>YOUR REASON</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>-------------</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td>ΔFGH is an isosceles triangle.</td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
<td>ΔOIJ is a right triangle.</td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td>ΔOIJ ≅ ΔOKJ</td>
<td></td>
</tr>
</tbody>
</table>
Below are facts pertaining to circles. For each statement provide a logical reasoning. Explanations may vary.

<table>
<thead>
<tr>
<th>DIAGRAM</th>
<th>STATEMENT</th>
<th>YOUR REASON</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td>A circle with a radius of 5 inches is congruent to a circle with a diameter of 10 inches.</td>
<td>A diameter is twice the radius, therefore the circles have the same size.</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram" /></td>
<td>The diameter of a circle is the longest chord.</td>
<td>The length of the diameter is twice the length of the radius, and the radius is the distance from the center to the circle. Therefore the diameter is the longest chord.</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td>All circles are similar.</td>
<td>All circles have the same shape and are therefore similar.</td>
</tr>
<tr>
<td><img src="image4.png" alt="Diagram" /></td>
<td>ΔAOB is an isosceles triangle</td>
<td>OA and OB are radii and are therefore congruent</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram" /></td>
<td>ΔCDE is a right triangle.</td>
<td>Angle C is a right angle because it is inscribe in a semicircle.</td>
</tr>
<tr>
<td>DIAGRAM</td>
<td>STATEMENT</td>
<td>YOUR REASON</td>
</tr>
<tr>
<td>---------</td>
<td>-----------</td>
<td>-------------</td>
</tr>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td>$\triangle FGH$ is an isosceles triangle.</td>
<td>$FG$ and $GH$ are tangent segments from the same external point and are therefore congruent.</td>
</tr>
<tr>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td>$\triangle OIJ$ is a right triangle.</td>
<td>Angle $OIJ$ is a right angle because the radius and tangent segment form a right angle at the point of tangency.</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram 3" /></td>
<td>$\triangle OIJ \cong \triangle OKJ$</td>
<td>The radii are congruent. Tangent segments are congruent. $OJ$ is congruent to itself, therefore the triangles are congruent by SSS.</td>
</tr>
</tbody>
</table>
GRAPHING CIRCLES ON THE COORDINATE SYSTEM

Graph each of the following circles on the coordinate system with the information given.

1. The center is (1, 2) and the radius is 2. Label C₁.

2. (-4, 4) is the center and (-2, 4) is a point on the circle. Label C₂.

3. (0, -3) and (-6, -3) are the endpoints of the diameter. Label C₃.

4. The center is (4, -1) and the circle is tangent to x = 3. Label C₄.

5. The center is (-2, 1) and the circle is congruent to C₄. Label C₅.

6. The center is (-3, -2) and the circle is tangent to the x-axis. Label C₆.

7. The circle is in the interior of C₁ and is tangent to the x-axis. The radius is half of the radius of C₁. Label C₇.

8. The radius is 3 and the circle is tangent to C₃ and the y-axis. Label C₈.
Graph each of the following circles on the coordinate system with the information given.

1. The center is $(1, 2)$ and the radius is $2$. Label $C_1$.
2. $(-4, 4)$ is the center and $(-2, 4)$ is a point on the circle. Label $C_2$.
3. $(0, -3)$ and $(-6, -3)$ are the endpoints of the diameter. Label $C_3$.
4. The center is $(4, -1)$ and the circle is tangent to $x = 3$. Label $C_4$.
5. The center is $(-2, 1)$ and the circle is congruent to $C_4$. Label $C_5$.
6. The center is $(-3, -2)$ and the circle is tangent to the $x$-axis. Label $C_6$.
7. The circle is in the interior of $C_1$ and is tangent to the $x$-axis. The radius is half of the radius of $C_1$. Label $C_7$.
8. The radius is $3$ and the circle is tangent to $C_3$ and the $y$-axis. Label $C_8$. 
PRACTICING CIRCLE VOCABULARY

Use the circle below to complete the fill in the blank below. When finished, write your answers down the column into the phrase below and you will know the next 13 digits of the approximation of $\pi$!!

Pi is approximately equal to 3. __ __ __ __ __ __ __ __ __ __ __ __ __ __ __

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| $$
\overrightarrow{BDC}$$ | 1. right angle |
| $$BA$$ | 2. tangent line |
| $$\angle OAE$$ | 3. semicircle |
| $$\angle ABD$$ | 4. chord (not the diameter) |
| $$OA$$ | 5. inscribed angle |
| $$\leftrightarrow AE$$ | 6. minor arc |
| $$\widehat{AC}$$ | 7. central angle |
| $$\angle BCD$$ | 8. diameter |
| $$\widehat{BC}$$ | 9. radius |
| $$\angle CBD$$ |   |
| $$\overline{BC}$$ |   |
| $$\overline{OC}$$ |   |
| $$\angle AOC$$ |   |
O is the center of the circle.
Pi is approximately equal to 3.1415926535897......
**CREATING AN ELLIPSE FROM STRING**

1. Attach a piece of paper to corkboard.

2. Put two pushpins into the paper approximately 10 cm apart. Mark one A and the other B.

3. Take a piece of string approximately 14 cm long and tie one end to A and one end to B.

4. Using a pencil, pull the string taut. Keeping it taut, draw an ellipse around the two *foci*.

5. Repeat this process with different lengths of string. What are the effects of different lengths of string?

6. For any of the ellipses, what can be said about the length $AP + BP$, where $P$ is any point on the perimeter?

7. For any point $Q$ inside an ellipse, what can be said about $AQ + BQ$?
1. Draw a circle with a radius of 5 cm. in the center of a piece of paper.

2. Draw in a diameter AOB as shown.

3. Draw in diameter COD perpendicular to AB and extend it across the page.

4. Draw a set of lines across the paper that are parallel to CD and are 1 cm. apart.

5. The circle can now be stretched to double its length in the directions of the parallel lines by marking off points such as P' and Q' whose distance from AB is twice that of P and Q from AB.
6. The new shape formed by drawing a smooth curve through the points \( P', Q', \) etc. is called an ellipse.

7. Measure the perimeter of the ellipse using a piece of string.

What is the relationship between the perimeter of the ellipse compared to the circumference of the original circle?

What is the relationship between the areas of the ellipse and the circle?

8. Now draw an ellipse that is formed by stretching the circle by three times in the direction of the parallel lines.

What is the relationship between its perimeter and the circumference of the circle?

What about the relationship between the areas?

9. What happens when the circle is shrunk by half in the direction of the parallel lines?
CREATING AN ELLIPSE FROM REPEATED FOLDS

1. On a piece of white paper draw a circle with a diameter of 16 cm, mark center C, and cut out the circle.

2. Mark point A approximately 2 cm. from the edge of the circle. (diagram 1)

3. Fold the circle along PQ so that the edge of the circle touches point A. (diagram 2)

4. Unfold the circle and draw in the fold line. (diagram 3)

5. Repeat this process numerous times (at least 10 times). You should start to see an ellipse surrounded by the fold lines. (diagram 4) These lines are called tangents to the ellipse. A and C are known as foci of the ellipse.

6. Now repeat this process but move A closer to the center of the circle. What happens to the ellipse?

7. What happens when A coincides with the center of the circle?
SOLIDS

- Euler’s Relation
  - investigates the number of vertices, edges and faces of solids
- Ready To Use Polyhedron Models
  - ready for student use
  - minimum assembly required
- Polyhedron Construction Kit
  - allows students to see how triangles, squares, pentagons, and hexagons combine to form polyhedrons
- Go Fly A Kite!
  - Creating kites from tetrahedrons


**Euler's Relation**

<table>
<thead>
<tr>
<th>Polyhedron</th>
<th>Name</th>
<th>Vertices (V)</th>
<th>Edges (E)</th>
<th>Faces (F)</th>
<th>V - E + F</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Cube Icon]</td>
<td>Cube</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Tetrahedron Icon]</td>
<td>Tetrahedron</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Pyramid Icon]</td>
<td>Pyramid</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Triangular Prism Icon]</td>
<td>Triangular Prism</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Octahedron Icon]</td>
<td>Octahedron</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>![Hexagonal Prism Icon]</td>
<td>Hexagonal Prism</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now state Euler’s Relation.
Euler’s relation can be stated as $V - E + F = 2$, where $V$ is the number of vertices, $E$ is the number of edges and $F$ is the number of faces of a polyhedron.
Ready to cut our polyhedron models. Includes:

- Triangular prism
- Pyramid with a square base
- Cube
- Cylinder
- Prism with a rectangular base
- Prism with a hexagonal base
- Icosahedron
- Dodecahedron
- Octahedron
TRIANGULAR PYRAMID
PYRAMID WITH A SQUARE BASE
CUBE
PRISM WITH A RECTANGULAR BASE
PRISM WITH A HEXAGONAL BASE
DODECAHEDRON
POLYHEDRON CONSTRUCTION KIT

Using an equilateral triangle and a square, you can create numerous polyhedrons. Start by using the templates below. In the 4cm template you will need to add $\frac{1}{2}$ cm tabs to each side of the figure. For the 8 cm template, you will need to add 1 cm tabs to each side of the figure. (See a) Using the templates with tabs, copy each figure onto stiff paper (card stock works well). Create a crease at the sides of the tabs. Use rubber bands to connect the pieces together as shown below. (See b) For more permanent polyhedrons you can staple the tabs together.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{polyhedron_construction}
\caption{4 cm figures}
\end{figure}
Examples of Polyhedrons You Can Create With This Kit

Octahedron

Tetrahedron

Triangular prism joined to a tetrahedron

Square-based pyramid
Cube

Triangular prism

Cuboctahedron

Snub cube
Rhomicuboctahedron

Icosahedron

Square anti-prism

**One square above another square turned 45° and linked by a ring of eight triangles.**
More Elaborate Polyhedrons Using Hexagons And Pentagons

For these polyhedrons you will have to use hexagons and pentagons.

Dodecahedron

Rhombicosidodecahedron
4 cm Pentagon

4 cm Hexagon
8 cm Pentagon
8 cm Hexagon
GO FLY A KITE!

Materials:

- 24 straws
- 8 meters of fishing line
- Glue
- Scissors
- 2 sheets of tissue paper
- Kite pattern

Preparation:

- Trace the kite pattern on folded tissue paper and cut around the pattern and open it out.
- Each tetrahedron needs two sets of tissue paper cut in the shape of the pattern. Each kite needs four tetrahedrons.

Directions:

- To make one kite, you will need four tetrahedrons, so if you have a partner, each of you will need to make two tetrahedrons.

- To make one tetrahedron, you will need six straws. First feed the fishing line through three straws and form a triangle by tying a knot. Keep one end of the string as long as possible.

- Feed the long end of the string through two more straws to make another triangle. Tie a knot.
• Feed the long end of the string back through one straw and attach the sixth straw to form a tetrahedron.

• Lay one triangular face on the center of a tissue pattern. Fold the flaps over and paste.

• Paste another piece of tissue onto a second side. One of the flaps will overlap the first triangle.

• Once you have made your four tetrahedrons, line up the bases of three of them. Make sure the covered sides all face exactly the same way. Tie them together.

• Tie the fourth tetrahedron on the top. Again, make sure the two covered sides face the same direction.

• Attach a loose line of string along the edge where two covered sides meet. Tie the kite string to this loop.

• Go fly a kite!!
TETRAHEDRON KITE LESSON
PROJECTS

• Pop-Up Books
  o creative project to help students master/review concepts
• It's Only Natural
  o help students make connections to geometric objects in the everyday world
• PROVE
  o BINGO with a geometric twist
  o can be used as a review
• Castles
  o fun project to review solids
PROJECTS BIBLIOGRAPHY


These books can be used throughout any chapter to help students review and understand concepts. It is easiest if you give the students a rubric to act as a guideline. About one or two concepts per page is a good guideline to follow. Ask students to get creative and have fun with these. They can even be used as a reference during tests!

**POP-UP BOOKS**

Dinah’s Rule for Pop-Up Books:
Always cut on a fold, never glue on a fold.

1. Fold a sheet of paper (8 1/2" x 11") in half like a hamburger.

2. Beginning at the fold, or mountain top, cut one or more tabs.

3. Fold the tabs back and forth several times until there is a good fold line formed.

4. Partially open the hamburger fold and push the tabs through to the inside.

5. With one small dot of glue, place figures for the Pop-Up Book to the front of each tab. Allow the glue to dry before going on to the next step.

6. To make a cover for the book, fold a sheet of construction paper in half like a hamburger.

7. Place glue around the outside edges of the Pop-Up Book and firmly press inside the construction paper hamburger.
IT'S ONLY NATURAL

There are many examples of geometric patterns and shapes in the world around us. Ask students to choose from the topics below and find examples from nature. If available, the Internet is a very helpful tool to help students' research and find examples for their project.

- Spirals
- Motion
- Cracks
- Fibonacci numbers
- Spheres
- Symmetry
- Packing

Students can then be asked to create a poster that answers the following questions.

- Where is it found in the world?
- How does it affect people, animals and the world?
- How does it happen?
- What causes it?
- What makes it?
- What does it do to us?
- Why does it occur in the world?
- Why does it occur in nature?
- When does it occur?

**If there are examples of these in your textbook it might be helpful to give students page numbers where they can see an example before starting their research.**
PROVE

Played just like BINGO, PROVE can be used as a review of material before quizzes or tests. Students fill in the spaces with answers provided beforehand. When a question is read, the students solve the problem and then find the corresponding answer on their game card.
CASTLES

Break the class into groups. Each group will create a castle from solid objects that they can either find from home or create. This is a fun project to help them review solids. You can decide upon guidelines about the size and the required number of solids that must be incorporated into the project. It is also helpful if you require the students to label each aspect of the castle.
MISCELLANEOUS RESOURCES AVAILABLE

- Geometer's Sketchpad: Distributed by Curriculum Press.
  - Many blackline activity masters available
  - Can help students masters difficult drawing concepts by removing the coordination required that some students lack

  - Easy to use toolbars
  - Able to change to different geometric views for extension exercises: Euclidean, Hyperbolic, Spherical, Polar Euclidean, and Polar Spherical

Screenshot of the program.
Screenshot of the program when using a hyperbolic viewing window.

- **Paper Folding: A Fun and Effective Method for Learning Math:** By Robert Jones and distributed by LWCD inc.
  - Comes with a student workbook.
  - Relates the five main theorems of Euclidean geometry to five main folds and uses these as the base for activities.

- **TI - 92**
  - Offers a portable device to explore Euclidean geometry.
  - Offers an alternative for schools without large computer labs, but with the financial resources to supply technology enhancements in the classroom.
BIBLIOGRAPHY


