Mathematics: A Universal Language
An Honors Thesis (Honrs 499/Maths 498)

by

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This thesis is one particular response to the broad question, "What is mathematics?" The answer proposed within is that mathematics is a universal language. This answer is taken in the reference of a universal language meaning something which can be found to some degree in a variety of different subject areas. With that in mind, three specific areas are chosen for discussion of how mathematics is at the heart of them. These three main areas are music, art, and literature with a special emphasis on poetry. Also included are paragraphs on how mathematics relates to areas besides those dealt with in the main concentration of the discourse. The paper concludes with several famous mathematical quotations to explicitly explain the fact that mathematics can be viewed as a universal language. As a whole, this thesis provides credence to the answer of the question that many people ask of mathematicians. Which is, why do they need to learn mathematics. To which, mathematics educators answer that mathematics is an universal language through which one can gain access to a vast realm of knowledge. Mathematics--it is the universal language of all things.
Outline

Thesis statement: Mathematics is an universal language because it can be found at the heart of most if not all disciplines from archery to zoology.

I. Mathematics in music
   A. Using music to teach mathematics
      1. Patterns
      2. Ratios
      3. Fractions
      4. Symbolic notation
   B. Structure of string instruments
      1. Correlation between length and notes
      2. Correlation between length and frequency
   C. Coding theory

II. Mathematics in art
   A. Using art to teach mathematics
      1. Scaling
      2. Similarity
      3. Tessellations/tilings
      4. Ratios/proportions
   B. Mathematical pieces of art
      1. Drawing cycloids/spirals
      2. Sculptures
   C. Fractals

III. Mathematics in literature
   A. Poetry
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2. Medium of expression
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IV. Mathematics in other disciplines
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      1. Physical--biology
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      1. Home economics
      2. Architecture

V. Author opinions
   A. Opinions
   B. Supporting quotations
The question of "What is mathematics?" has been posed many times. Each time one gives an answer it is very different from someone else's answer. In fact, the same person may give different answers at different times since mathematics is an ever changing and growing science from which one can learn new ideas daily. It is these new ideas which cause a constant flux in what a person might answer in response to the question, "What is mathematics?" However, this thesis will focus on one particular answer to that question. It is the opinion of this author that mathematics is a universal language. Now, clarification is needed in respect to "universal language." In my eyes, a universal language is something which permeates all various disciplines. That is, it serves as the underlying foundation for the building of the other disciplines. In that respect, mathematics is a universal language because it can be found at the heart of most, if not all, disciplines from archery to zoology. Since showing how mathematics exists in all subjects would create an unending paper, three specific areas along with a general category for highlights from areas beyond those specifically discussed have been chosen to be the formal content of this discourse. The three specific areas are music, art, and literature (mainly poetry). These will be discussed in that order with examples of how each has mathematics at its heart.
Following those paragraphs, a section is included on highlights from other disciplines, as well as some opinions of the author.

Mathematics in Music

First, as was stated earlier, mathematics is a universal language and as such can be found in all disciplines. Yet to cover them all would be a great undertaking. So a few representatives have been chosen for discussion. The first of these is music. Many might be appalled at hearing that mathematics and music can be interrelated. However, in doing just a little research, several connections have been found. These connections relate to how music can be used to teach mathematics, the structure of string instruments, and the use of coding theory in forming music.

With respect to the first connection, music can be used to teach mathematics because of all the mathematical concepts which are used in music. Two of these are numerical patterns and ratios, which Bahna-James suggests are more easily understood in the context of music rather than in a strict mathematics classroom (479). Others include using notations/symbols to represent ideas, working with fractions, and transformations as Dienes related in his article about creating interdisciplinary lesson plans. The lesson discussed was choreographing a dance to a song. The students had to come up with symbols for the dance moves, work out moves to get to specific places on the dance floor, and figure out how to change a design created by the positions of the dancers to a new one. All of these tasks are
directly related to the aforementioned topics (Dienes 180).

Another area of mathematics which falls under music is combinatorics. Haack in his article relates the famous composition "Clapping Music" by Steve Reich as a way to discuss combinatorics since the song uses repetitions of clapping and silence in patterns that are shifted until the original pattern is obtained again. Items such as the possible combinations of claps and pauses within a certain length measure can be calculated and the number of transformations that are needed to return to the original pattern can also be counted. Both provide a way to relate mathematics to something concrete (Haack 224-225).

Considering string instruments, Pythagoras long ago noticed the mathematical connections between plucking a string and the sound it makes. Blackburn and White in their article explored this idea to its fullest bringing forth some surprising results. They stated the following relationship among notes played on a single string and the note that is heard. The whole string gave a note, say C; 8/9 the string gave a note one whole step above the first, D; and so on until 1/2 the string gave a note one octave above the first note, C'. They also showed the existence of an inversely proportional relationship among the frequencies of the notes as the fractions of the fundamental tone length string. For example, say the first note is C at 256 hertz. From the previous discussion, A would be at string length 3/5. So 256 * (5/3) is approximately 427 hertz, the frequency of A. The last relation stated in the article dealt with ratios of the fractions
from the first part. A pattern was discovered that the ratios of adjacent note lengths was either .89, .90, or .94. It was .89 or .90 for whole step note changes and .94 for half-step note changes (Blackburn and White 499-503).

Finally, an example of how mathematics and the topic of coding applies to music is in order. Lee gives the definition of code as, "A code is a scheme for accurate and efficient transfer of information from one place to another" (7). He also cites Collins Gem Dictionary for a definition of music, which is "Music is an art of expressing information by melodious and harmonious combinations of notes" (Lee 7). The most impressive story expressed by Lee is the fact that Johann Sebastian Bach translated his last name into musical notes which formed the basis for his "Art of the Fugue." Due to differences in naming of piano keys, Bach translates as B flat, A, C, and B (Lee 10). The rest of the article discusses how coding theory is actually used by computers to create new and unique compositions never heard before (Lee 10-13). All in all, these three types of examples of how mathematics is connected to music easily put down the statement made by many people, "Music is not mathematics; it's emotions!" (Bahna-James 484). See also Appendix A: Mathematical Music for "The Square of the Hypotenuse," Song A1, a song about the Pythagorean Theorem (Fadiman 241-244).

Mathematics in Art

As with music, many might think that there is no connection between mathematics and art. In this case, the feelings might
not be quite as strong. Once again, research has turned up many connections between art and mathematics. Among these are the topics of mathematics which can be taught through art, some specific art pieces, and the somewhat new topic of fractals. First, in looking at the mathematical concepts which can be taught using art, the list appears to be endless. However, a few highlights are scaling, similarity, tessellations/tilings, ratio, and proportions. With regard to scaling, Holt expresses the opinion that art is a method of reducing our experiences into compact pictures and that, in a way, mathematics does the same thing in trying to explain the rules for how things proceed in nature (17). Also, in a general sense, scaling is used because a piece of art cannot always be life-size, which explains why reference units, ratios, proportions, and similarity are used by many artists. Barratt discusses how art uses similarity at length because the goal of art is more often than not to replicate what one sees (13). He also gives extensive discourse on tessellations/tilings. In doing so, he admits that without mathematics one would not be able to explain the essence of what is behind the operations of tessellating or tiling a plane. Many art works, and especially wall paper and floor coverings, are based on tessellations in which reflections, rotations, and translations are used to create patterns that are easy to reproduce and have an attractive quality to the eye (Barratt 66-71). Finally, one last example from Barratt shows how ratios and proportions are used to create art works that have the impression of ever increasing or decreasing size out to infinity in a finite
Moving on to specific artworks, it is amazing to note the number that have been created specifically for relating a mathematical idea. A few of these will be mentioned, and, where pictures are available of good copy quality, they also will be provided. First, Walton in her article discusses the work of one particular individual, Albrecht Durer. Although known for his many woodcuts and paintings, Durer was more of a geometer at heart. He lived during the Renaissance, which allowed him first-hand experience at emerging topics of the day. Two of these were drawing cycloids/spirals and ellipses. Durer is credited with making devices to draw such curves. Another fascination of Durer was perspective drawing. Most of his woodcuts were done in some form of conical projection, creating appealing distortions in distances as related to the object being drawn (Walton 278-280). Ronald Brown brought to the forefront several artistic creations by John Robinson. Pictures of those sculptures can be found in Appendix B: Mathematical Sculptures. Immortality, Figure B1, is the first sculpture. It is a representation of a Mobius band, a surface which has only one side but the appearance of two. Next is Creation, Figure B2, in which three squares are so placed that no two are connected but the whole object cannot be taken apart. This symbolizes the concept of the whole being more than the sum of its parts. This concept is based on Borromean Rings. Another version of Borromean Rings is Genesis, Figure B3, in which three lozenge shapes, 4 sided figures, are arranged so that they cannot be taken apart while not being connected together. Finally, the
The last sculpture referenced is *Courtship Dance*, Figure B4, which is related to the mathematical idea of the ovoid (Brown 62-64).

The realm of fractals shall now be briefly explored for the last example of how art and mathematics are related. Fractals are objects which are generated by an iterative process by which one side of an object is replaced by a generator pattern over and over again. In such a process, an object which is self-similar and symmetric under magnification can be created. However, the interesting aspect of these objects is that the process can create an infinite number of different designs, all of which are quite interesting artforms (Borowski and Borwein 229). One such example using a simple equilateral triangle, Figure C1, is shown in Appendix C: Fractal (Field and Golubitsky 160).

**Mathematics in Literature**

The third major example of mathematics as a universal language involves explaining how mathematics can be found in literature, specifically, poetry. Since much poetry is written in patterns, there is little doubt that some mathematics exists behind them. One such example is the haiku, a Japanese poem of exactly seventeen words (Smith 4). However, there are many other subtleties that arise to provide interconnections between mathematics and poetry. One of these is the matter of dealing with ideas in short compact form (Buchanan 19). Smith states, "Just as mathematics, to the mathematician and to one who teaches the science, is filled with poetry, so poetry welcomes mathematics to herself, arranging her message in meter and her
sonnets with mathematical precision" (2). Others include topics such as concision, consequentiality, abstraction, symbol making, and analysis. In mathematics things are always expressed concisely. If-then statements are located throughout logic, and many have a poetic ring when stated. Many individuals refer to mathematics as being abstract; yet they see no abstraction when referring to poetry, which often is quite abstract in expressing the feelings and thoughts of the author's mind. Metaphors are used abundantly in poetry. They serve the purpose of making connections between ideas much in the same fashion as symbols in mathematics are used to relate much more than the space the symbol takes up. Finally, when a literature teacher asks students to analyze a poem, more often than not, part of that analysis involves determining the pattern used to write the poem which in turn deals with inductive reasoning and the ability to count (key mathematical abilities for success) (Robson and Wimp 2). Buchanan even refers to the theory of number(s) as being the epic (greatest in revealing the concept at hand) poem of mathematics (52). So, to summarize the section on connections between mathematics and poetry, a quote from Smith is due. That is, "Poetry and mathematics have this feature then in common--each says more and in fewer words than any other written form" (4-5).

In a slightly different vein, a few lines will be written on relating some poems about mathematics or mathematical concepts. The first of these is the excerpt of lyrics from the film, Singing in the Rain, presented by McDermot and McDermot, and
repeated below. "Moses supposes his toeses are roses, but Moses supposes erroneously. Moses he knowses his toeses aren't roses as Moses supposes his toeses to be." (McDermot 348). To the general reader, this poem probably seems a little funny and quite poetic. However, to the mathematician, this poem is a classic example of doing a proof by contradiction, which is done by supposing something, reaching a contradiction, and then concluding the negation of the original assumption. Two other examples of mathematical poems are fond in Appendix D: Mathematical Poems. They are "Poet as Mathematician," Article D1, which explains the essence of the working desire of a mathematician and "God is Zero," Article D2, which is a somewhat humorous attempt to explain the greatness of mathematics and subtlety prove the existence of God (Robson and Wimp 45 and 61).

Last, it is worth mentioning that there are many other instances of relations between general literature and mathematics. One such highlight is the work of the children's book writer, Lewis Carroll, whose real name is Charles Dodgson. He is well known for writing funny stories to relate mathematical concepts in an entertaining mode that children will enjoy so that, in a sense, they do not even know they are learning and understanding mathematics while having fun. The best known story of his containing significant mathematical content is Alice in Wonderland (Buchanan 27).

Mathematics in Other Disciplines

Now, as was outlined earlier, a few other relationships of
mathematics to other disciplines will be explored. The first of these is science. It is quite obvious to anyone who has ever taken an upper level science course that the content depends on mathematics. The mathematics often backs up the theory or even provides the medium through which the theory can be proven. One example, in particular, comes from biology. The spirals of the seeds of a sunflower form two sets, with one going clockwise and the other counterclockwise, in such a way that the number of each type is a set of consecutive Fibonacci numbers (Holt 62). To briefly highlight others, the social sciences rely heavily on trends and patterns in societies which are only discovered through collection of data and usually a statistical analysis of that data; home economics contains many applications of working with measurements and fractions for cooking and sewing; and the field of architecture uses many mathematical formulas in deciding how to construct a building or uses patterns in creating appealing landscape designs that are often symmetrical.

Finally, a few personal opinions will be stated and some famous mathematical quotations will be referenced. First, I would like to say that I chose to write on this topic because of my deep feeling that in order to survive in the world, one needs to have at least a limited knowledge of mathematics. Actually, the more one knows the better. This is my feeling because as a future educator in mathematics I have seen far too many applications of mathematics across several disciplines and after researching for this thesis, I have found many more. So this leaves only one conclusion in my mind: Mathematics has to be
some form of a universal language permeating all areas of study. Three quotes, one by Albert Einstein, another by G. H. Hardy, and a third by Bertrand Russell, give credence to this view. Albert Einstein states, "How can it be that mathematics, a product of human thought independent of experience, is so admirably adapted to the objects of reality" (464). G. H. Hardy says:

The mathematician's patterns, like the painter's or the poet's, must be "beautiful", the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test; there is no permanent place in the world for ugly mathematics . . . . It may be hard to "define" mathematical beauty, but that is just as true of beauty of any kind--we may not know quite what we mean by a beautiful poem, but that does not prevent us from recognizing one when we see it.

(4: 2027)

Bertrand Russell supposes, "Mathematics, rightly viewed, possesses not only truth, but supreme beauty--a beauty cold and austere, like that of sculpture" (Maths 498 Famous Quote Sheet). One last prime example of how mathematics is the universal language comes from Pythagoras and the Pythagoreans. It is related by the flow chart below which breaks down mathematics into parts. The parts above the dotted line are those written by Turnbull. Those below are ones which are implied or are logical conclusions of my own.

Mathematics

| The discrete | The continued |
The absolute Arithmetic Business, home economics, etc.
The relative Music
The stable Geometry Art Sciences
The moving Astronomy Sciences

Note that without much stretching, most disciplines could be put under one of the four columns, all stemming from the parent mathematics (Turnbull 1: 85).

In conclusion, it has been shown that mathematics plays an integral part in music, art, and literature, as well as briefly touching upon connections to other areas. The examples given and the quoted statements from authorities in their respective fields give credence to the goal at hand. That goal was to answer the question; "What is mathematics?", in terms of one's own personal opinions supported by examples from research. In my opinion, I believe mathematics is a universal language because mathematics can be found to drive or greatly influence most disciplines. This, I believe, I have shown to be the case for music, art, and literature (specifically poetry). However, other disciplines need not be neglected. This thesis could have easily been expanded to show relationships of mathematics in all areas from archery to zoology. Yet, in order to stay focused and restrict the amount of material to be covered, the areas of music, art, and literature were arbitrarily chosen for discussion. The connections of mathematics to other subjects could create content for another thesis. Finally, I leave the reader with the words
of Peter Hilton. He states:

"Nature speaks to us in the language of mathematics," said the physicist Richard Feynman. In writing this profound epigram, Feynman was in fact, though perhaps unknowingly, echoing the thought of Galileo, who, in *The Assayer*, referred to "the language of mathematics in which is written the book of the universe and without whose help it is humanly impossible to understand a word." (276)

As noted previously, people frequently ask mathematicians why they need to learn mathematics. My answer, which is supported by Hilton's quote, is mathematics is a universal language through which other disciplines can be explained.
Appendix A: Mathematical Music
Song A1:

The Square of the Hypotenuse

Music by Saul Chaplin
Lyrics by Johnny Mercer

The connections between music and mathematics were pointed out some time ago by the Pythagorean school. But there are very few pieces of music actually inspired by mathematics. I have unearthed only two, of which this amusing ditty, originally sung by the unique Danny Kaye, is the first. It appeared in the film Merry Andrew a few years ago.

Reprinted by permission of Commander Publications from the Sol C. Siegel-MGM production Merry Andrew. Copyright © 1958 by Commander Publications.
THE MATHEMATICAL MAGPIE

angle is equal to the sum of the squares of the

two adjacent sides. You'd not tolerate

letting your participle dangle, So

please effect the self-same respect for your geometric slides.
The Square of the Hypotenuse

Old Einstein said it,
Sure as shoot-in',
The two Wright brothers,
when he was getting nowhere Give him
when problems get in your hair Be like
before they conquered the air Like those
cred-it He was heard to declare, "Eur-ka!"
Newton Who was heard to declare, "Eur-ka!" THE SQUARE
others Orville holler, look here! Wilbur
OF THE HY-POT-ENUSE of a right tri-
THE MATHEMATICAL MAGPIE

angle is equal to the sum of the squares of the

two adjacent sides!
Appendix B: Mathematical Sculptures
Figure B1:
1) Brown, 62.

Figure B2:
2) Brown, 63.
Figure B3:

3) Brown, 64.

The Genesis

Figure B4:

4) Brown, 64.

Courtship Dance
Appendix C: Fractal
Figure 7.3 The Sierpiński triangle.

Field and Martin, 160.
Appendix D: Mathematical Poems
Poet as Mathematician

Having perceived the connexions, he seeks the proof, the clean revelation in its simplest form, never doubting that somewhere waiting in the chaos, is the unique elegance, the precise, airy structure, defined, swift-lined, and indestructible.

1) Robson and Wimp, 45.
Article D2:

**ERNEST ROBSON**

god is zero

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**GOD IS ZERO**

GOD IS THE FIRST OF ALL NATURAL NUMBERS

BECUSE

GOD IS ZERO

\[ n^0 = 1 \]

ONLY GOD CAN ANNULATE ANY NUMBER ANY ON

BECUSE

GOD IS ZERO

\[ 0 \times 0 = 0 \]

ONLY GOD HAS THE POWER TO BECOME

FINITE SMALL

BECUSE

GOD IS ZERO

\[ \frac{0}{0} = \infty \]

ONLY GOD CAN BE DERIVED FROM THE GOD BY

THE ELIMINATION GOD

BECUSE

GOD IS ZERO

\[ \infty \]

2) Robson and Wimp, 61.
Works Cited


Haack, Joel K. "Clapping Music--A Combinatorical Problem."


Bibliography


