Teaching Fractions in Elementary School

An Honors Thesis (HONR 499)

by

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Abstract

Fractions are a significant struggle for many students. Students are typically taught to memorize algorithms to use when working with fractions. They have little to no understanding of how or why these algorithms work. This turns math into memorization rather than a way of thinking. Allowing students to develop their own strategies for working with fractions will increase their understanding so that they can successfully build on their knowledge and apply it to real-life situations. Through lessons in which students develop their own strategies for solving fraction problems, students will gain a conceptual understanding of fractions and recognize that mathematics is not memorization, but a way of thinking.

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Teaching Fractions in Elementary School

Fractions are poorly understood in today's classrooms. Students are taught to memorize methods and formulas to use when working with fractions, but typically are not taught why these methods and formulas are used or how they work. As a result, students have only a very surface level understanding of fractions. I believe that this lack of understanding is responsible for the difficulties many students experience when using fractions.

Traditionally, fractions are taught using algorithms. When dividing a fraction, students are told to flip and multiply. When adding fractions, students are told to add the numerators but keep the denominators the same. Because of this, mathematics becomes a process of memorization rather than a way of thinking. Consequently, students obtain a false impression of what mathematics truly is. Burns (1994) states, "Imposing the standard arithmetic algorithms on children interferes with their learning, and it can give students the idea that mathematics is a collection of mysterious and often magical rules and procedures that must be memorized and practiced" (p. 472). Students need to recognize that mathematics is much more than simply memorizing rules and algorithms.

When students are taught to solve fraction problems by using an algorithm with no understanding, mathematics becomes following directions and the teacher becomes only the giver of directions. Rowan and Robles (1998) explain, "The teacher's role is more that of a facilitator to encourage children to build on what they already know than that of a giver of information and procedures to get answers" (p. 505). Rather than telling students to add the numerator and keep the
denominator the same when adding fractions, teachers should guide the students to understand why they add the numerator and not the denominator.

Students must understand basic concepts or they will struggle to advance to more complicated topics. Mathematics is constantly building on prior knowledge. Because of this, it is crucial that students have a firm understanding of each mathematical concept and not just a memorized algorithm. Students must recognize a fraction as more than simply one number over another number before they can understand what it means to add, subtract, multiply, or divide fractions. Without this understanding, students will have difficulty applying information about fractions to real life.

In order to gain a true understanding of fractions, students must be encouraged to experiment with concepts themselves. As Huinker (1998) says, "If students are to truly believe that mathematics, and fractions in particular, makes sense, instruction must allow students to invent their own ways to operate on fractions rather than memorizing and practicing the procedures imposed by the teacher or textbook" (p.180). Instead of telling students how to solve a mathematical problem, students should be given a problem and allowed to explore on their own. This gives them the opportunity to practice using their own reasoning and thinking and will result in a deeper understanding of mathematics and fractions.

As students work on solving the problem, teachers should ask questions that direct their thinking and help them focus on understanding what they are doing. Rowan and Robles (1998) explain, "Using questions and prompts to cause children
to make sense of, reason about, predict from, solve, and apply mathematics is a powerful teaching strategy and may be the key to fostering self-reliance and success" (p. 504-505). As students share their answers, teachers should ask them why they solved the problem that way, if there are any other ways to solve that problem, if they noticed any patterns as they solved the problem, etc. For example, as students work on figuring out how to solve fraction addition problems, the teacher might ask them if they notice any patterns involving the numerator and denominator and why these patterns exist. These questions make mathematics about thinking rather than following directions.

Using the problem solving method to allow students to develop their own algorithms has many benefits and results in a deeper and firmer understanding of the concepts being taught.

Huinker (1998) lists the following advantages of the problem solving method:

Students developed an interest in solving and posing world problems with fractions. Students became flexible in their choice of strategy for solving fraction word problems and computation exercises. Students became more proficient in translating among real-world, concrete, pictorial, oral language, and symbolic representations. Students became accustomed to communicating and justifying their thinking and reasoning. (p. 181)

Teaching fractions following the problem solving method will equip students with the thinking and reasoning skills they need in order to be successful in mathematics. If students understand why a certain method is used and why it works, they will
better be able to extend their understanding of mathematics to real life and to more advanced material.

In the following section, I created lesson plans that can be used to teach fractions in the elementary grades. These lesson plans address the Common Core Standards regarding fractions for grades three through six. Each lesson plan is broken into three stages. During the launch stage, students review what they already know about the material to be covered during the lesson. During the explore stage, students are given a problem that they must work in partners to figure out how to solve. Throughout this stage, the teacher will observe the students and ask them questions focused on understanding. During the final stage, the summarize stage, the teacher will guide a discussion in which the students share their thinking. The teacher will ask questions and help the students to summarize the new information they learned from the lesson.

The lessons I wrote focus on more than the students merely knowing the algorithms. A variety of models are used to help students recognize all the difference concepts fractions can represent. As Van de Walle, Karp, and Bay-Williams (2013) say, "Understanding fractions means understanding all the possible concepts that fractions can represent" (p. 291). Area models, length models, and set models are used throughout the lessons so that students can understand a fraction in flexible ways. I use story problems continually throughout the lessons to help students make connections between symbols and real-world examples. The lessons also teach students to use "benchmark" fractions to increase their number sense of fractions. Students should be able to have an intuitive feel for fractions and have an
idea of their relative size (Van de Walle, Karp, & Bay-Williams, 2013, p. 303).

Through lessons built on the problem-solving method, students will gain a true understanding of fractions and will learn to see mathematics as a way of thinking instead of memorization.
References


CCSS.MATH.CONTENT.3.NF.A.1
Understand a fraction \( \frac{1}{b} \) as the quantity formed by 1 part when a whole is partitioned into \( b \) equal parts; understand a fraction \( \frac{a}{b} \) as the quantity formed by \( a \) parts of size \( \frac{1}{b} \).

Goal: Students will model fair-sharing problems in order to gain an understanding of fractions.

Materials: worksheets

Gearing Up:
- Ask the students if they have ever shared a cookie or pizza with a friend.
- Ask the students how they would divide the cookie or pizza if they were going to share it fairly.

Launch:
- Draw a picture of the cookie or pizza on the board.
- Have a student show how they would divide it if they were going to share it fairly with a friend.
- Point out that in order to share the cookie or pizza equally, you must divide it equally.

Explore:
- Tell the students that now they are going to practice dividing pizzas fairly between different numbers of people.
- Place the students in pairs. Give each pair a piece of paper with descriptions of situations such as: "share two cookies with four people" and "share one pizza with three people".
- Tell the students to draw pictures to model each situation.
- Have the students describe in words the amount each person will get below each picture.
- Observe the students as they work and remind them that to share something fairly, they must divide it into equal parts.

Summarize:
- Have the students share their answers and draw their pictures on the board.
- Ask them explain their pictures and descriptions.
- Introduce labeling the parts as one-half, three-fourths, etc. Discuss what this language means.
- Describe that in three-fourths, the fourths are how many parts the pizza/cookie is divided into, and the three is how many fourths there are.
Gearing Down:

- Revisit the question at the beginning in which you asked the students to draw a picture of how you would divide a cookie/pizza to share it with a friend.
- Ask the students how you would label the parts in this question and why.
CCSS.MATH.CONTENT.3.NF.A.1
Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.

Goal: Students will learn what a fraction is. They will understand that the denominator represents the number of parts that make a whole and that the numerator represents the number of parts.

Materials: poster boards, circles cut out of construction paper

Gearing Up:
- Ask the students what they know about fractions.
- Have several students explain what they think a fraction is.
- Tell the students that today they are going to learn what a fraction is and what the numbers in a fraction mean.

Launch:
- Tell the students that they are going to be working in small groups to make fraction posters.
- Explain that they are going to divide circles into fractions to put on their poster.
- Take one circle and draw a line dividing it in half. Explain that each side is one half and write $\frac{1}{2}$ on each side.
- Repeat the same steps, but divide the circle into four parts.
- Tell the students that now they are going to make more examples of fractions and glue them to their poster.

Explore:
- Divide the students into groups of three or four. Give each group a poster board and some circle cut out of construction paper.
- Tell the students to use your example to draw more fractions to add to their poster.
- Remind the students to think of similarities, differences, and patterns in their fraction models.
- As the students work, go around to each group to make sure they understand. Talk to each group about the similarities, differences, and patterns they are noticing.

Summarize:
- Ask the students to share the similarities, differences, and patterns they noticed regarding the fraction models that they put on their poster.
- Point out that each circle is divided into equal parts.
- Ask the students if they notice anything about the bottom number of the fraction.
• Direct the students to realize that the bottom number refers to the number of parts the circle is divided into. Explain that this is called the denominator. It tells what is being counted.
• Ask the students if they notice anything about the top number of the fraction.
• Explain that this number is called the numerator and represents how many is/are being counted.
• Write a fraction on the chalkboard and have each group shade the corresponding amount on their poster.

Gearing Down:
• Review the denominator and numerator with the students. Remind them that the numerator counts, and the denominator tells what is being counted.
• Ask the students if they can think of any time they might use fractions in real life. For example, cutting a cake at a birthday party.

References:

CCSS.MATH.CONTENT.3.NF.A.1
Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by $a$ parts of size $\frac{1}{b}$.

Goal: Students will use set models to broaden their understanding of fractions.

Materials: counters, dry-erase boards and markers, list of problems

Gearing Up:
- Ask the students what they remember about fractions. Encourage them to model what they remember on the board.
- Write the fraction $\frac{3}{4}$ on the board. Ask a student to model that number using an area model.
- Ask the students to identify the numerator and the denominator and what they signify.

Launch:
- Show the students a set of 20 counters. Ask the students if they can think of a way to model $\frac{3}{4}$ using the counters. Have them draw their model on their dry-erase boards.
- Model the correct answer: Twenty counters represent the whole. The denominator is 4 so we must divide the counters into 4 equal parts, or fourths. Since the numerator is 3, we must take 3 of these fourths to represent $\frac{3}{4}$.
- Repeat this process to determine what fraction of the class is made up of boys.

Explore:
- Tell the students that now they are going to use counters to figure out similar problems.
- Put the students in pairs. Give each pair some counters and a list of problems such as: What fraction of the class has the letter $T$ in their name?, What fraction of the girls is wearing tennis shoes?, etc.
- Encourage students to think about the numerator, denominator, and what makes up a whole as they complete the problems.
- If students finish quickly, have them write and solve their own problems following this pattern.

Summarize:
- Have the students share their answers to the problems and explain how they got those answers.
- Emphasize that the set of students, girls, boys, etc. represents the whole or 1.
- Review the meaning of the numerator and the denominator in these problems.
Gearing Down:
- Ask the students to brainstorm more examples of fractions in the real world. For example, if you have completed \( \frac{1}{3} \) of the six chores you have each day, how many have you completed?
- Have each student take out a sheet of paper and model the fraction \( \frac{2}{3} \) using both an area model and a set model.

References:

CCSS.MATH.CONTENT.3.NF.A.1
Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

Goal: Students will recognize that shapes must be partitioned equally when using fractions.

Materials: tangram pieces, paper

Gearing Up:
- Draw several shapes on the chalkboard. Divide each shape into four parts, but divide some equally and some unequally.
- Ask the students which shapes are correctly divided into fourths and why.
- Remind the students that the shape must be equally divided.

Launch:
- Tell the students that today they are going to use fractions to design different sizes of playgrounds.
- Draw a shape on the board by tracing the small square and the small triangle twice. (See example on right.)
- Tell the students that this shape represents one whole playground. Ask them how you could create a playground that is 1/2 this size.

Explore:
- Have students get in partners and give each pair some tangram pieces.
- Tell the students that now they are going to figure out designs for playgrounds that are 3/4, 1/3, 5/6, and 3/2 this size.
- Have them trace the tangram pieces on their paper to draw their playground designs.
- As the students work, remind them to think about what represents the whole and that the shape must be divided equally into parts.

Summarize:
- Go through each fraction and have students share and explain their answers.
- Emphasize that the four shapes shown in the original drawing are not equal so the squares had to be split in half in order to create six equal parts.
- Reinforce the ideas of the numerator and denominator and what they signify.

Gearing Down:
- Ask the students why the shape must be divided into equal parts.
- Give the example of sharing a cookie with a friend. If the cookie was not split in half equally, each person would not truly have half a cookie.
- Ask the students if they can think of any more examples.
References:

CCSS.MATH.CONTENT.3.NF.A.2.A
Represent a fraction l/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size l/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

Goal: Students will use length models to increase their understanding of fractions. Students will learn to represent fractions on a number line.

Materials: paper, pencils

Gearing Up:
- Ask the students what they remember about fractions.
- Write the fraction \( \frac{3}{4} \) on the board.
- Have students come up and model this problem in different ways.
- Tell them that today they are going to learn another way to think of fractions.

Launch:
- Tell the students that Dillon walked to school today and saw many different animals on the way. When he was halfway to school, he saw a rabbit.
- Model this on the board by drawing a school and house with a line in between.
- Ask a student to identify where he saw the rabbit. Draw a rabbit at that spot.
- Ask the student to explain how he/she knew where he saw the rabbit.

Explore:
- Place the students in pairs. Give each pair a sheet of paper and have them draw a similar sketch of a line with a house and a school.
- Write a list on the board of the animals Dillon saw and at what distances. For example: "When Dillon had walked \( \frac{1}{4} \) of the way, he saw a turtle," and "When Dillon had walked \( \frac{5}{8} \) of the way, he saw a dog."
- Tell the students to label where Dillon saw each animal on their paper.
- As the students work, encourage them to think about what represents a whole in this context and how they need to divide up the distance to answer each question.

Summarize:
- After the students have finished, have the pairs take turns adding an animal/label to the line on the board. Have them explain their work.
- Divide the line so as to show where each animal should be. For example, if he saw the turtle at \( \frac{1}{4} \) of the way, you must divide the line into fourths.
- Tell the students to think of this line as a number line.
- Ask students to label 0 and 1 on the number line.
- Label the fraction that goes along with each animal.
Gearing Down:
- Ask the students what the similarities were between area models, set models, and length models.
- Point out that in each case, the students had to divide the line, set, or area into equal parts to determine what represented $\frac{1}{4}$, $\frac{5}{8}$, etc.

References:

CCSS.MATH.CONTENT.3.NF.A.2.B
Represent a fraction $a/b$ on a number line diagram by marking off a length $1/b$ from 0. Recognize that the resulting interval has size $a/b$ and that its endpoint locates the number $a/b$ on the number line.

Goal: Students will represent fractions on a number line and understand the size of the distance between the beginning and the fraction.

Materials: yarn, scissors, paper, tape

Gearing Up:
- Draw a number line on the board. Label 0 and 1.
- Ask a student to come up and label $\frac{1}{4}$ on the number line.
- Have them explain how they knew where to put $\frac{1}{4}$.

Launch:
- Tell the students that now we are going to use a piece of yarn as the number line.
- Measure a piece of yarn that is the length of a desk. Explain that this piece of yarn is 1 desk long.
- Tape the piece of yarn to the board. Have students come up and label 0, $\frac{1}{2}$, and 1 on the yarn.
- Tell the students that now they are going to use fractions to measure different lengths of the yarn.

Explore:
- Place the students in pairs. Give each pair some yarn, a pair of scissors, tape, and a piece of paper.
- Have the paper labeled with fractions such as 1 desk, $\frac{1}{2}$ desk, $\frac{2}{5}$ desk, $\frac{3}{4}$ desk, etc. Tell the students to cut pieces of yarn that are the specified length and tape them to the piece of paper.
- Watch them as they work and make sure they correctly understand how to find these fractions.

Summarize:
- After the students have finished, have students share their work and explain how they decided how long to make each piece of yarn.
- Recall the students' attention to the piece of yarn on the chalkboard. Ask a student to come up and show how much of that piece of yarn would be $\frac{1}{2}$ of a desk.
- Label $\frac{3}{4}$ on the yarn/number line. Ask the students how much yarn is represented by the point that marks 0 and the point that marks $\frac{3}{4}$.
- Help students make the connection that the segment of a number line from 0 to $a/b$ is the length $a/b$. 19
Gearing Down:

- Tell the students that it is 1 mile from your house to a playground. Ask them, “If I walked $\frac{1}{2}$ of the way to the playground, how far have I walked?”
- Ask the students if anyone else can think of similar examples and have them share them with the class.
CCSS.MATH.CONTENT.3.NF.A.3.A
Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

Goal: Students will use a number line to recognize that fractions such as 2/3 and 4/6 are equivalent.

Materials: paper, pencils

Gearing Up:
- Draw a number line from 0 to 2 on the board.
- Ask a student to come up and label 3/2 on the number line.
- Ask the students if they think another fraction could be located at the same place on the number line. Have them explain why or why not.

Launch:
- Tell the students that today they are going to work on a problem to figure out if fractions can be located at the same place on a number line.
- Tell them that Kirk, Kyle, Jon, Connor, Jacob, and Ryan are having a race. Kirk is 7/8 of the way there, Kyle is 2/4 of the way, Jon is 3/4 of the way, Connor is ½ of the way, Jacob is 2/8 of the way, and Ryan is 6/8 of the way.
- Ask the students to take out a sheet of paper and write down who they think is in 1st place, 2nd place, etc.

Explore:
- Tell the students that now they are going to use a number line to check their guesses.
- Put the students in pairs. Give each pair a sheet of paper and have them draw a number line from 0 to 1.
- Tell the students to place and label each of these fractions on the number line.
- Observe the students as they work.

Summarize:
- Have the students share their work.
- Ask them how close their guesses were to the correct answer.
- Have them explain why they guessed correctly or incorrectly.
- Ask them if it surprised them that some runners were at the same point.
- Discuss why it is possible to describe an equivalent distance using different fractions.

Gearing Down:
- Revisit the number line on the board.
- Ask the students again if another fraction could be located at the same point as 3/2 on the number line.
• Ask the students what this fraction or fractions would be.
• If the students need a hint, mark fourths on the number line.

References:

CCSS.MATH.CONTENT.3.NF.A.3.B
Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Goal: Students will divide area models into parts to come up with equivalent fractions.

Materials: dry-erase boards & markers, paper

Gearing Up:
- Write the fractions ½, 2/4, and 4/6 on the board.
- Ask the students if any of these fractions are equivalent.
- Have a student draw a number line on the board to check their answer.

Launch:
- Tell the students that today they are going to learn more about equivalent fractions.
- Explain that two neighbors share a field. They are growing corn in ¾ of their field.
- Draw a rectangle on the chalkboard to represent the field and divide it into fourths using horizontal lines. Have a student come up and shade the portion that corn will be grown in.
- Explain that the farmers will equally share the corn. Draw a vertical line to represent dividing the corn in half.
- Now ask students what fraction of the newly divided field is corn.
- Help the students recognize that these two fractions (3/4 and 6/8) are equivalent.

Explore:
- Tell the students that now they are going to divide the field in different ways to find more equivalent fractions.
- Put the students in pairs and give them dry-erase boards and paper.
- Tell them to draw fields on their dry-erase marker and divide them in different ways to discover more equivalent fractions. Have them list the equivalent fractions they found on the paper.
- Encourage them to try using different shapes for their fields like circles or triangles.

Summarize:
- Have the students share and explain their answers.
- Ask them to explain how they found the equivalent fractions.
- Ask them how this is similar or different to how they found equivalent fractions on a number line.
Gearing Down:

- To review, tell the students that a mother was giving some of a cake to her two sons. She told one son that she was giving him $\frac{1}{4}$ of the cake and the other son that she was giving him $\frac{3}{12}$ of the cake.
- Ask the students how they would explain to the boys that they are each getting the same amount of cake. Encourage them to draw pictures to model their explanation.

References:

CCSS.MATH.CONTENT.3.NF.A.3.B
Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Goal: Students will use set models to determine whether or not fractions are equivalent.

Materials: counters, worksheets

Gearing Up:
- Ask students what they know about equivalent fractions.
- Have a student name a pair of equivalent fractions.
- Call on several students to come to the board and draw different models to explain that these fractions are equivalent.

Launch:
- Give each student some counters.
- Tell the students to use the counters to model 2/3.
- Have a student come to the board and draw their model.
- Ask students if anyone else modeled it differently and have them draw their model on the board.
- Point out that you can model the same fraction in different ways. For example, using six counters to make three groups of two, twelve counters to make three groups of four, etc. and selecting two of these groups.

Explore:
- Put the students in pairs.
- Give each pair a worksheet that lists different pairs of fractions, some of which are equivalent.
- Tell the students that their job is to use their counters to figure out which fractions are equivalent.
- Encourage the students to look for similarities between the equivalent fractions as they work.

Summarize:
- Have the students share and explain their answers.
- If students disagree on an answer, have them draw number line models or area models to determine the correct answer.
- Ask them if they noticed any similarities between the pairs of equivalent fractions.

Gearing Down:
- To review, tell the students that Zack and Zoe are both eating some M&Ms. Zack told Zoe that he only ate ¼ of the bag. Zoe knows that she ate 10/40 of the bag.
• Ask the students who ate more M&Ms or if they ate the same amount.
• Have them explain their answer.

References:

CCSS.MATH.CONTENT.3.NF.A.3.C
Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.

Goal: Students will use area models to come up with different ways to represent whole numbers on a number line.

Materials: fraction pie piece models, paper w/chart

Gearing Up:
- Draw a circle on the board and divide it onto fourths.
- Ask a student to come up and shade a half of the circle.
- Ask students how they would write a fraction to represent this. See if they can come up with equivalent fractions.
- Remind the students that these are equivalent fractions and that they still represent the same portion of the shaded circle and are located at the same spot on a number line.

Launch:
- Now shade in the entire circle on the board.
- Ask the students how much of the circle is shaded now.
- Put the students in pairs and brainstorm different ways they could say the circle is shaded (one whole, four fourths, etc.)
- Have them share and explain their answers.
- Draw a number line on the board. Mark 1 on the number line and underneath write the other ways they came up with to express this.

Explore:
- Have the students get back in their pairs.
- Give each pair a set of fraction pie piece models and a sheet of paper.
- Add other whole numbers to the number line (2, 3, etc.) and have the students copy the number line from the board on their paper.
- Explain that their challenge is to come up with fractions that can express these other whole numbers and add those to their number line.
- Encourage the students to use their fraction pie piece models to figure out different representations.
- As they work, remind them to look for patterns between their answers.

Summarize:
- Have the students share and explain their answers.
- Ask the students what patterns they noticed.
- Prompt them to recognize that 2 halves, 3 thirds, 4 fourths, etc. always equals one. Have students draw models to explain why this is.
• Make sure students also recognize that 1 can be the denominator of a fraction meaning that the shape is only divided into one part. Help them notice the pattern of representing whole numbers by writing how many wholes as the numerator and 1 as the denominator. Draw models to explain this.

Gearing Down:
• To review, revisit the circle on the board at the beginning of the lesson.
• Ask them if any of the patterns they noticed help them come up with more ways to write fractions for the shaded portion.
CCSS.MATH.CONTENT.3.NF.A.3.D
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Goal: Students will draw models to compare fractions with the same numerator or the same denominator.

Materials: worksheets, paper, fraction cards

Gearing Up:
• Draw two squares on the board.
• Shade 1/2 of one square and 1/4 of the other square.
• Ask the students to think about how they would describe what portion of each shape is shaded.
• Have students come up and label the shapes and explain why they think that portion of the shape is shaded.

Launch:
• Now ask the students which shape has the larger portion shaded in.
• Ask the students if they remember what symbol they could use to show that one shape is greater than the other.
• Have a student come up and place the appropriate symbol between the fractions.
• Tell the students that now they are going to figure out more ways to determine which fractions are larger.

Explore:
• Put the students in pairs. Give each pair a sheet of paper that lists pairs of fractions with the same denominator or numerator with a space in between for a comparison symbol.
• Tell the students that their job is to draw pictures or other models to determine which fraction is larger.
• As they work, encourage them to look for patterns and similarities.

Summarize:
• Have the students share their work and explanations with each other.
• Ask the students if they noticed and patterns and/or similarities.
• Prompt them to notice that when there are two fractions with the same denominator, the fraction with the larger numerator is always the greater fraction.
• Have them draw models to understand this.
• Prompt them to notice that when there are two fractions with the same numerator, the fraction with the smaller denominator is always the greater fraction.
• Have them draw models to understand this.

Gearing Down:
• To review, divide the students into two groups.
• Give each student in one group a fraction card with the same denominator, but different numerators.
• Give each student in the other group a fraction card with the same numerator, but different denominators.
• Have them order their cards from least to greatest.
• Afterwards, have them share their answers and explain how this is an example of the patterns discussed earlier.

References:

CCSS.MATH.CONTENT.3.NF.A.3.D
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Goal: Students will learn that fraction size is not always the same and depends on the size of the whole.

Materials: paper, Cuisenaire rods

Launch:
- Write the fraction 3/5 on the board in fraction notation and words.
- Ask a student to come to the board and draw a model of the fraction.
- Ask the students if they can think of any other ways to model the fraction. Have them draw their models on the board.
- Tell the students that Caroline and Annie each got some of a cake. Caroline got 3/5 of a chocolate cake, and Annie 3/5 of a vanilla cake. Caroline is complaining because she says that Annie got more cake than her.
- Ask the students whether or not they think Caroline is right.
- Tell them that she is right, and their job today is to figure out how that could be.

Explore:
- Put the students in pairs. Give them each some paper.
- Tell them to work together to figure out how Caroline could be right.
- Tell them to explain their answer in words and pictures.
- If students get stuck, encourage them to think about the size of one whole cake.
- As the students begin to figure out the solution, ask them if one third of a pizza would always be greater than one half of a pizza.
- Also, ask them when one-third of a pizza would be the same as one-third of another pizza.
- Tell them to think about what is important to know when comparing fractions.

Summarize:
- Have the students share their answers with each other. Have the students share their pictures with each other. If a student doesn't understand or agree with another student's picture, prompt them to discuss what they don't understand or think differently about.
- Ask the students if they were really hungry and were offered 1/3 of a pizza or ½ of a pizza, which they would choose. Ask them if there are any questions they should ask the person giving them the pizza before they make their choice.
Give the students Cuisenaire rods. Ask the students if they can use the Cuisenaire rods to explain why 1/3 of a pizza might be more pizza than 1/2 of a pizza.

Ask the students what is important to know when comparing fractions.

Help them conclude that when comparing fractions, you must make sure you are comparing fractions of the same whole.

Ask the students what a fraction tells us then if it doesn’t tell us the size of the whole or the size of the parts.

Discuss this and help them conclude that a fraction tells us the relationship between the part and the whole.

References:

CCSS.MATH.CONTENT.4.NF.A.1
Explain why a fraction \( \frac{a}{b} \) is equivalent to a fraction \( \frac{n \times a}{n \times b} \) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Goal: Students will draw pictures to realize that multiplying a fraction by \( \frac{n}{n} \) will result in an equivalent fraction.

Materials: worksheets

Launch:
- Draw a square on the board and shade 1/3 of it by dividing it into three parts with vertical lines.
- Ask the students what fraction of the square is shaded.
- Ask them if they can think of an equivalent fraction to 1/3. Have them explain their answer and model it on the board.
- If the students are stuck, divide the square in half with a horizontal line, and ask the students again if they can come up with an equivalent fraction.
- Write 1/3=2/6 on the board.
- Tell the students that now they are going to draw pictures to solve similar problems.

Explore:
- Put the students in pairs.
- Give them each a worksheet that gives pairs of equivalent fractions, but with one number missing. For example, \( \frac{3}{4}=?/8 \).
- Tell the students to draw pictures to explain their answers.
- As they work, ask the students if they notice any patterns in the answers and pictures.
- Ask them if they can think of a formula or algorithm that would help them solve these problems.

Summarize:
- Have the students share their answers, pictures, and explanations with the class.
- Discuss any answers the students got different answers for.
- Ask them if they noticed any patterns in the answers and/or pictures.
- Ask the students if they figured out any formulas or algorithms that would help them find equivalent fractions.
- Draw a square on the board and divide it into fifths using vertical lines. Divide it into fifths using horizontal lines. Shade four fifths of the square.
- Use a paper to cover most of the square except the two sides.
- Ask the students what equivalent fractions this square models.
• Ask them how they can count all the fractional parts when they can't see them all.
• Show the students that you can multiply the sides in order to count all the fractional parts: \(4 \times 5 = 20\) and \(5 \times 5 = 25\).
• Show the students how you can write this as an algorithm to find equivalent fractions: \(\frac{4}{5} \times \frac{5}{5} = \frac{20}{25}\).
• Ask the students why you can multiply the fraction by \(\frac{5}{5}\) and not change the value. Help them realize that \(\frac{5}{5}\) is the same as 1 and multiplying a number by 1 does not change the number.
• Ask the students how they could apply this example to find equivalent other equivalent fractions. Help them realize that multiplying a fraction by \(n/n\) will result in an equivalent fraction.
• Have several students come to the board and draw models to show more examples of this.

References:

CCSS.MATH.CONTENT.4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Goal: Students will learn to compare two fractions by comparing them to benchmarks.

Materials: worksheets, blank paper

Launch:
- Ask the students if 2/5 or 4/5 is greater. Have them explain their answer in words and/or pictures.
- Ask the students if ¼ or 1/6 is greater. Have them explain their answer in words and/or pictures.
- Ask the students if 7/8 or 4/10 is greater. Have them explain their answer.
- Check their answer by drawing a number line on the board. Mark 0, ½, and 1 on the number line.
- Have a student label the approximate location of each number on the number line.
- Tell the students that now they are going to use a number line to compare more fractions.

Explore:
- Put the students in pairs.
- Give each pair a worksheet that lists different pairs of fractions that can be compared using benchmarks such as 0, ¼, ½, etc. Also, give each pair some blank paper to draw number lines.
- As the students work, remind them to simply estimate the location of the fraction on the timeline.
- Tell the students to think about which fractions are useful to label on the number line in order to estimate the location of the fractions they are comparing.

Summarize:
- Have the students share their answers and number lines with the class.
- Discuss any answers the students got different answers for.
- Ask the students what fractions they found useful to label on their number line in order to estimate the location of other fractions.
- Ask them how these fractions could be useful when comparing other fractions.
- Write the fractions 4/9 and 1/7 on the board. Ask them which fraction is greater and how they know.
• Ask them which fractions they used as benchmarks to figure out the answer.

References:

CCSS.MATH.CONTENT.4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Goal: Students will review previous fraction comparison strategies in order to help them recognize that finding common denominators or numerators will help them compare fractions.

Materials: worksheets

Launch:
- Write the fractions 1/5 and 3/5 on the board. Have a student explain which one is greater and why.
- Write the fractions 2/8 and 2/6 on the board. Have a student explain which one is greater and why.
- Write the fractions 8/9 and 3/7 on the board. Have a student explain which one is greater and why.
- Tell the students that now they are going to practice comparing more fractions and see if they can come up with any new strategies.

Explore:
- Put the students in pairs.
- Give each pair a paper that lists pairs of fractions without common numerators or common denominators that cannot be easily compared using benchmarks.
- Tell them to explain each comparison in words and/or pictures.
- As the students work, remind them of the strategies learned previously and ask them if they can apply them in any way to these fractions.
- If the students get stuck, ask them if they think equivalent fractions could help them solve the problem.

Summarize:
- Have the students share their answers with the class.
- Discuss any answers the students got different answers for.
- Ask the students what new strategies the students used.
- Point out that some students found common denominators. Review an example of this.
- Point out that some students found common numerators. Review an example of this.
- Ask the students how they know the new fraction the students found is still equivalent to the first fraction. Have the students explain this using words and pictures.
References:

CCSS.MATH.CONTENT.4.NF.B.3.A
Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

**Goal:** Students will be introduced to the idea of adding fractions through story problems.

**Materials:** worksheet

**Launch:**
- Tell the students that you have two red marbles, three green marbles, and five blue marbles. Ask them how many marbles you have all together. Ask a student to write the equation that represents this story problem.
- Tell the students that today they are going to practice combining fractions.
- Tell the students that they are going to read some simple stories involving fractions and draw pictures to answer the questions.

**Explore:**
- Put the students in pairs and give them each a worksheet with questions like the ones below.
  - Bill walks $\frac{1}{4}$ of a mile every day of the week. How many miles will he have walked in three days?
  - Each of the five children ate $\frac{1}{6}$ of the pie. How much of the pie did they eat all together?
- As the students work, tell them to think about what math operation they are doing. Ask them if they can write the problems as equations.
- Ask them to think about how this is similar or different to adding whole numbers.
- As they write the equations that represent the story problems, ask them what they notice about the numerators and the denominators.

**Summarize:**
- Have the students share their equations and pictures with the class.
- Discuss any answers the students got different answers for.
- Ask the students what math operation they were doing when they solved the story problems.
- Look at the equations the students wrote. Ask them if they noticed anything about the numerators and denominators.
- Point out that when adding fractions when the denominator is the same, you just add the numerators.
- Ask the students why you don't add the denominator.
- Have them draw a picture to explain their answer.
References:

CCSS.MATH.CONTENT.4.NF.B.3.B
Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
Examples: \(\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}; \frac{3}{8} = \frac{1}{8} + \frac{2}{8}; \frac{21}{8} = 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}\).

Goal: Students will learn to use addition equations to model breaking a fraction into parts.

Materials: paper, whiteboards & dry-erase markers

Launch:
- Write the expression \(\frac{2}{4} + \frac{1}{4}\) on the board.
- Ask the students how they would solve this problem.
- Have students come to the board and model solutions for this problem using both an area model and a number line.
- Tell the students that now they are going to practice using fractions in math equations on their own.
- Tell the students: "Pretend you are a farmer. Your field is divided into eight sections. You can plant corn, tomatoes, soybeans, carrots, lettuce, broccoli, pumpkins, and/or potatoes. Choose what crops you want to plant in your field and how much of each crop you want to plant. You are going to draw a picture of your field and write an equation that explains the different amounts of each crop in your field."

Explore:
- Put the students in pairs and give them paper to model their fields.
- Have them write the equation represented by the field below each picture.
- As the students work, suggest that they might want more of one crop than another or none of another crop. Ask them how they would model this.
- As they finish, tell them to then pretend that half of the field is flooded and cannot be used to plant crops. Decide what you would plant in the remaining part of your field. Draw a picture and write an equation to represent this.
- As the students finish, ask them what other options they can think of for planting the crops in their field.
- Ask them to see how many different equations they can write to represent different options.

Summarize:
- Have the students share their pictures and equations with the class.
- Discuss any responses the students think are incorrect.
- As the students share their equations, list them on the board.
- Ask the students what they notice about the list of equations.
- Draw a number line and ask the students to model several of the equations on the number line.
• Give each student a whiteboard and a dry-erase marker.
• Write the fraction 3/5 on the chalkboard.
• Tell the students to see how many addition equations they can write to equal 3/5.
• Ask a student to explain in words and/or pictures what the equation means.
• Repeat the same exercise with 4/4.
CCSS.MATH.CONTENT.4.NF.B.3.C
Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

CCSS.MATH.CONTENT.4.NF.B.3.D
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Goal: Students will explore different ways to solve story problems involving mixed numbers with like denominators.

Materials: worksheets w/questions, rulers

Launch:
- Write the equation $\frac{4}{5} + \frac{2}{5}$ on the board.
- Ask the students how they would solve this problem.
- Ask a student to draw a picture to explain the answer.
- Ask another student to use a number line to explain the answer.
- Tell the students that now they are going to practice doing some problems like this on their own.

Explore:
- Put the students in pairs. Give them a worksheet that lists story problems involving added fractions like the ones below. Include problems that involve area, linear, and set models.
  - Megan gathered 4 pounds of walnuts, and Annie gathered $2 \frac{3}{7}$ pounds. How much did they gather altogether?
  - Lauren walked 1 $\frac{3}{4}$ miles on Monday and 2 $\frac{1}{4}$ miles on Tuesday. How much did she walk altogether?
  - Rachel bought 2 $\frac{2}{3}$ dozen donuts, and Caroline bought 1 $\frac{2}{3}$ dozen donuts. How many dozens of donuts did they buy altogether?
- Tell the students to write the equation represented by each problem and to figure out a way to solve it.
- As they work, ask them if they can think of a way to solve the problems that they hadn’t already used.
- Ask them what they notice about the numerators, denominators, and whole numbers when adding the fractions.
- If students finish quickly, ask them to model the problems using a number line if they haven’t already.

Summarize:
- Have the students share their models and equations with the class.
- Have them explain how they solved each problem.
- Discuss any problems that the students got different answers for.
• Ask the students what happens to the denominators when you add fractions.
• Ask them to explain why you don't add the denominators.
• Ask the students what happens to the numerators when you add fractions.
• Ask them to explain why you add the numerators.
• Ask them what happens to the whole numbers when you add mixed numbers.
• Ask them to explain why this is.
• To conclude the lesson, use rulers as a number line to model solving the problems.

References:

CCSS.MATH.CONTENT.4.NF.B.3.C
Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

CCSS.MATH.CONTENT.4.NF.B.3.D
Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Goal: The students will solve story problems requiring them to subtract mixed number fractions with like denominators. They will use models to these problems to understand how to rewrite the mixed numbers as improper fractions to solve the problem.

Materials: worksheets w/questions, rulers

Launch:
- Write the equation 1 4/5 + 3 2/5 on the board.
- Ask the students how they would solve this problem.
- Ask a student to draw a picture to explain the answer.
- Give the students rulers, and have a student explain solving this problem with a ruler.
- Now tell the students they are going to try to solve some more fraction problems.

Explore:
- Put the students in pairs. Give them a worksheet that lists story problems involving subtracting fractions like the ones below. Include problems that involve area, linear, and set models.
  - Sarah had 3 cakes. Her friends ate 1 ¼ cakes. How much cake is left?
  - It is 1 2/5 to get from Emily's house to the school. She has already walked 4/5 of a mile. How much more does she have to walk?
  - Dillon bought 2 2/3 dozen donuts. 1 1/3 of the donuts have chocolate frosting and the rest have strawberry frosting. How many dozen donuts have strawberry frosting.
- Tell the students to write the equation represented by each problem and to figure out a way to solve it.
- As they work, ask them if they can think of a way to solve the problems that they hadn't already used.
- Ask them what they notice about the numerators, denominators, and whole numbers when subtracting the fractions.
- Ask the students if they can think of any other way to write the equation that makes it easier to subtract.
Summarize:
- Have the students share their models and equations with the class.
- Have them explain how they solved each problem.
- Discuss any problems that the students got different answers for.
- Ask the students what happens to the denominators when you subtract fractions.
- Ask them to explain why you don’t subtract the denominators.
- Ask the students what happens to the numerators when you subtract fractions.
- Ask them to explain why you subtract the numerators.
- Ask them what happens to the whole numbers when you subtract mixed numbers.
- Ask them to explain why this is.
- Ask the students if they came up with any other ways to write the subtraction equation.
- If the students need help, revisit the problem 1 2/5 – 4/5. Ask a student to model this with an area model.
- Now ask if there’s any other way to write 1 2/5 that would make it easier to subtract 4/5 from.
- Point out that in modeling the subtraction problem, the students divided the 1 into fifths. Show that 1 2/5 can be written as 7/5. Write the problem as 7/5 – 4/5.
- Ask the students to show how you would use this method to solve some of the other problems.
- Revisit the addition problem (1 4/5 + 3 2/5) from the beginning of the lesson, and ask the students if they can rewrite this problem in a similar way to make it easier to solve.

References:

CCSS.MATH.CONTENT.4.NF.B.4.A
Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).

Goal: Students will use different representations of multiplication problems and fractions to recognize that a fraction a/b can be written as the multiplication expression a x (1/b).

Materials: paper

Launch:
• Write the expression 3x4 on the board.
• Tell the students to get with a partner and brainstorm as many ways to solve this problem as possible using words and/or pictures.
• Have the students share their solutions with the class.
• Point out how 3x4 can mean 4+4+4, 4 three times, etc.
• Tell the students that now they are going to think about how multiplication relates to fractions.

Explore:
• Write the fraction 3/5 on the board.
• Put the students in pairs.
• Tell them that they have several jobs.
• First, tell them to come up with as many ways as possible to model and explain 3/5 using words and/or pictures.
• As they work on this, ask them to also think of addition expressions to represent 3/5.
• Eventually, have the students use their models of the multiplication problem at the beginning to think of a way to represent 3/5 using multiplication.
• If the students finish this, ask them to try expressing other fractions as multiplication problems in the same way.

Summarize:
• Have the students share with the class the multiplication expression they used to represent 3/5.
• Ask them to explain why they decided to express 3/5 in this way. Encourage them to use pictures in their explanation.
• Ask them if they thought of an addition problem that helps explain the multiplication expression.
• Write the expression 1/5+1/5+1/5 on the board. Compare this to the expression 4+4+4 from the beginning of the lesson.
• Explain that just like 4 three times is 3x4, 1/5 three times is 3x(1/5).
• Ask the students to share other examples of this using other fractions.
References:

CCSS.MATH.CONTENT.4.NF.B.4.B
Understand a multiple of \(a/b\) as a multiple of \(1/b\), and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express \(3 \times (2/5)\) as \(6 \times (1/5)\), recognizing this product as \(6/5\). (In general, \(n \times (a/b) = (n \times a)/b\).)

Goal: Students will find different multiplication expressions that equal a fraction in order to recognize that \(n \times (a/b) = (n \times a)/b\).

Materials: dry-erase boards & markers, paper

Launch:
- Write the expression \(6 \times (1/8)\) on the board.
- Have a student explain how you would find the answer to this problem.
- See how many different representations of this the students can come up with.
- Write the number 16 on the board and model it with 16 dots.
- Ask the students to think of a multiplication expression that equals 16.
- Model their expression by drawing circles around the dots to create groups. (For example, if a student says \(8 \times 2\) draw circles around groups of two dots to create 8 groups of 2.)
- Ask the students if they can think of any other expressions. Model this expression as well.
- Next, tell the students that they are going to try doing the same thing with the fraction \(6/8\).

Explore:
- Put the students in pairs.
- Tell the students that their job is to find other multiplication expressions that equal \(6/8\).
- Give each student paper and tell them to write the expression and draw a picture that represents that expression.
- If the students need help getting started, ask them how they would model the expression \(6 \times (1/8)\). Suggest drawing six \(1/8\)ths.
- If the students are stuck, tell them to try creating groups like you did when modeling the expressions that equal 16.
- As the students work, ask them what they notice about the numerator and the denominator.
- If students finish quickly, tell them to try to find different multiplication expressions that equal \(12/5\).

Summarize:
- Have the students share their models and expressions with the class.
- Write the expressions on the board.
- Give the students dry-erase boards and markers.
• Write the fraction 12/5 on the board.
• Tell the students to come up with a multiplication expression that equals this fraction.
• After they finish, compile a list of the different equations on the board.
• Ask the students what they notice about the numerator and the denominator.
• Ask them why the denominator is not multiplied by the whole number. Help them realize that the denominator is not multiplied because it represents the size of the parts.
• Ask them why the numerator is multiplied by the whole number. Help them realize that the numerator is multiplied because it represents the number of parts.
CCSS.MATH.CONTENT.4.NF.B.4.C
Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?

Goal: Students will solve story problems that require them to multiply a fraction by a whole number. They will figure out why multiplication is used in these story problems.

Materials: worksheets

Launch:
- Ask the students what they remember about multiplying fractions.
- Write the expression 1/8 x 5 on the board.
- Ask the students to solve the problem and come up with different ways to model it.
- Tell the students that now they are going to work on some story problems that use multiplication.

Explore:
- Put the students in pairs.
- Give each student a worksheet with the following problems:
  o Each person will eat 3/8 pounds of chicken at the party. There will be 5 people at the party. How many pounds of chicken will be needed?
  o The walk from home to school takes 15 minutes. Anna has gone 2/3 of the way. How many minutes has she walked? (Assume she is walking at a constant rate.)
  o Dillon has 20 cars in his matchbox car collection. 1/5 of the cars are blue. How many of the cars are blue?
  o Amber filled 5 glasses of with 3/4 c of milk in each. How much milk did she use?
- Tell the students to draw pictures to solve the problem and write a multiplication expression that represents the problem.
- As the students work, ask them to what steps they are taking to solve each problem.
- Ask the students why a multiplication expression can be used to represent the problem.
- Ask the students what steps they are taking to solve the problems.
- Ask the students between what two whole numbers their answers lie.
- Ask students who finish quickly if they can come up with any story problems of their own that require you to multiply fractions.
Summarize:
- Have the students share their answers and multiplication expressions with the class.
- Have them explain how they got these answers and the steps they took.
- Discuss any answers the students got different answers for.
- Ask the students why multiplication expressions can be used to solve each problem.
- Ask the students, “If 15 is a whole, how much is 1/3 of the whole?”
- Ask them why multiplying 15 and 1/3 will find the answer.
- Help them recognize that multiplication is used because you are finding what 15 is 1/3 times. Compare this to 15x2 and that this is finding what 15 is 2 times.
- Ask the students if they can think of any other story problems with fractions that you could use multiplication to solve.

References:

CCSS.MATH.CONTENT.5.NF.A.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

Goal: Students will estimate the answers to addition and subtraction problems involving fractions with unlike denominators by using benchmark fractions and thinking of the relative size of fractions.

Materials: worksheets

Launch:
- Write the expression $1/5 + 3 2/5$ on the board.
- Ask the students how they would solve this problem and why.
- Ask the students to explain why they only add the numerators and not the denominators.
- Have the students solve the problem. Ask them if the answer they came up with makes sense. Ask them how they know this.
- Tell the students that now they are going to practice estimating the answers to more problems with fractions.

Explore:
- Put the students in pairs.
- Give each student a worksheet that contains addition and subtraction problems using fractions with unlike denominators (for example, $7/8 + 1/10$). Include a number line below each problem.
- Tell the students that their job is to mark the approximate location of each answer on the number line.
- As the students work, ask them to think whether numbers are close to benchmark numbers such as 0, ½, or 1. Ask them how this could help them to estimate the answer to the problem.
- Also, remind the students to think about the relative size of a fraction. For example, in the problem $7/8 + 1/10$, ask the students how the size of $1/8$ compares to the size of $1/10$.

Summarize:
- After the students finish, have them share their answers with the class.
- As you go through each problem, have the students explain the reasoning behind their estimate.
- Ask the students if there were any benchmark fractions that they found useful to think of when figuring out their estimates.
• Ask the students how thinking about the relative size of a fraction part helped them figure out an estimate.
• Next, tell the students that you are going to do a little activity to practice the strategies that were just talked about.
• Write an addition or subtraction problem with fractions on the board.
• Have the students show you a "thumbs up" if the answer is greater than 1 and a "thumbs down" if the answer is less than 1.
• Discuss what strategies they used to estimate the answer.

References:
CCSS.MATH.CONTENT.5.NF.A.2
Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.

Goal: Students will solve word problems involving addition and subtraction of fractions with unlike denominators by using pictures.

Materials: worksheets

Launch:
- Write the expression $\frac{7}{8} + \frac{1}{10}$ on the board.
- Ask the students to estimate the answer to this problem.
- Have a student explain his/her answer.
- Tell the students that now they are going to solve some story problems by adding or subtracting fractions and use estimating to check their work.

Explore:
- Put the students in pairs.
- Give each student a worksheet that contains addition and subtraction word problems using fractions with unlike denominators such as the problems below.
  - Yesterday, Peter ran $1\frac{1}{2}$ miles. Today Peter ran $2\frac{3}{4}$ miles. How many miles did he run all together?
  - $\frac{1}{5}$ of the class walked to school today. $\frac{1}{3}$ of the class took the bus. How much of the class walked or took the bus?
  - Shannon has $3\frac{1}{2}$ pizzas. His friends ate $\frac{7}{8}$ of one pizza. How much pizza does he have left?
- Include a number line below each problem.
- Tell the students to first estimate the answer to the problem and mark it on the number line. After that, tell the students to figure out the exact answer to the problem.
- Encourage them to draw pictures or models to help them figure out the answer.
- After they solve the problem, tell them to check their answer with their estimate.
- As the students work, ask them what strategies they are using to solve the problems.
- Ask the students how they know that they are getting the right answer.
- If students finish quickly, ask them to come up with their own word problem and solve it.
Summarize:

- After the students finish, have them share their answers and pictures/models with the class.
- If there is a disagreement on an answer, ask the students to explain their estimates to figure out which answer is most likely right. Then discuss the problem to determine which answer is right and why the other answer(s) is/are wrong.
- Ask the students which strategies worked well for figuring out the answer.
- Model these different strategies on the board.
- Go back and try using different strategies to solve the same problem.
- Ask the students why they chose to model the problems as they did.
- Ask them if they can think of any other ways to model the problem.
- Practice modeling the same problem in different ways.

References:

CCSS.MATH.CONTENT.5.NF.A.1
Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

Goal: The students will use pictures to rewrite fractions in addition and subtraction problems so that they have a like denominator and understand why this helps them solve the problems.

Materials: worksheets

Launch:
- Write the expression 1/4 + 3/8 on the board.
- Ask a student to draw a picture to solve the problem. Have the student explain the picture he/she drew.
- Draw a number line on the board and mark the number ½ on it.
- Ask the students if there are any other ways to write the number ½ and how they know this.
- Tell the students that now they are going to solve more fraction problems. Tell them to also try to think of other ways to write the problem that may help them solve it.

Explore:
- Put the students in pairs.
- Give each student a worksheet with various addition and subtraction problems using fractions with unlike denominators.
- As the students work, ask them if they can think of other ways to write each fraction in the equation.
- If they are stuck, have them draw a model to solve the problem. Then ask them how they are dividing the model in order to solve the problem. Ask them if they can write the problem in the same way that they drew it.
- Ask the students how they know that writing the problem in this different way will still get them the right answer.

Summarize:
- After the students finish, have them share their answers and how they solved the problems.
- Discuss any problems the students got different answers for.
- Make a list of the original equation and the changed equations that the students came up with to help them solve the problem.
- Ask the students how they know that the changed equation will still provide the same answer as the original equation.
- Ask the students what is similar about the changed equations.
• Ask them why writing the fractions so that they have the same denominator makes it easier to solve the problem.

References:

CCSS.MATH.CONTENT.5.NF.B.3
Interpret a fraction as division of the numerator by the denominator \((a/b = a ÷ b)\).
Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret \(\frac{3}{4}\) as the result of dividing 3 by 4, noting that \(\frac{3}{4}\) multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size \(\frac{3}{4}\). If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Goal: Students will solve word problems to discover that \(a/b\) means the same as \(a ÷ b\) and to notice the relationship between fractions and division.

Materials: worksheets

Launch:
- Draw a circle on the board.
- Ask a student to divide it into fourths.
- Shade one of the fourths.
- Ask the students how much of the circle is shaded.
- Have a student write the corresponding fraction on the board.
- Ask the students to explain the meaning of the numerator and the denominator.
- Tell the students that now they are going to practice more dividing.

Explore:
- Put the students in pairs.
- Give each pair a worksheet with questions like the following:
  - Three people are sharing one pizza. How much pizza will each person get?
  - Four people are sharing one pizza. How much pizza will each person get?
  - Five people are sharing two pizzas. How much pizza will each person get?
  - Ten people are sharing three pizzas. How much pizza will each person get?
  - Two people are sharing three pizzas. How much pizza will each person get?
  - Three people are sharing five pizzas. How much pizza will each person get?
- Tell the students to draw pictures to solve the problems.
- After they draw the pictures, ask them to figure out the equation represented by their pictures.
- As the students work, ask them if they are noticing any patterns.
- Ask the students to think about how a fraction represents a division problem.
• If students finish quickly, give them a fraction and ask them to write the fraction as a division problem.

Summarize:
• After the students finish, have them share their pictures and answers with the class.
• Discuss any problems the students got different answers for.
• Make a list of the equation expressed by each problem on the board.
• Ask the students to look at the equations and see if they notice any patterns. Have them explain the patterns they notice.
• Ask the students, “If one hundred people are sharing thirty-seven pizzas, how much pizza would each person get?”
• See if they can use the patterns they noticed to write an equation for this problem. Add that equation to the list.
• Write the fraction 5/8 on the board. Ask the students how they could write this fraction as a division problem.
• Write the fraction 7/4 on the board. Ask the students how they could write this fraction as a division problem.
• Ask the students why fractions can be written as a division problem.
• Discuss the relationship between fractions and division.
CCSS.MATH.CONTENT.5.NF.B.4.A
Interpret the product \((a/b) \times q\) as a parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\). For example, use a visual fraction model to show \((2/3) \times 4 = 8/3\), and create a story context for this equation. Do the same with \((2/3) \times (4/5) = 8/15\). (In general, \((a/b) \times (c/d) = ac/bd\).)

CCSS.MATH.CONTENT.5.NF.B.6
Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Goal: Students will solve story problems to recognize that \((a/b) \times c\) is the same as \(c \div b \times a\).

Materials: worksheet

Launch:
- Tell the students this story problem: "The shirt was originally $18. However, it is on sale for half off. How much is the shirt now?"
- Ask the students what the question is asking. Have them describe the problem in words. (For example, the question is asking us to find \(\frac{1}{2}\) of 18.)
- Ask the students if they can think of an expression that represents this question.
- Have the students explain how they would solve this problem.
- Tell the students that now they are going to work on some more problems on their own.

Explore:
- Put the students in pairs.
- Give each pair a worksheet with these three problems.
  - There are 25 students in the class. \(\frac{3}{5}\) of the students are girls. How many girls are in the class?
  - The soccer ball costs $21. Peter has \(\frac{4}{7}\) of the money he would need to buy the soccer ball. How much money does he have?
  - It is 9 miles from the school to the library. Shannon has travelled \(\frac{2}{3}\) of the way. How far has she travelled?
- Tell the students to solve the problems by drawing pictures and to write the multiplication problem represented by the problem.
- Tell the students to describe in words the steps they take to solve the problem.
- After this, ask them if they can write an equation to describe the steps.
- If the students are stuck, ask them to explain how they drew their picture. Then ask if they can break this down into steps.
- Ask the students, how multiplication and division relate to these problems.
- Ask the students why dividing is a part of multiplying fractions.
Summarize:

- After the students finish, have them share their responses to each problem and how they came up with their response.
- Discuss any problems the students got different responses for.
- As they share their multiplication problems, write them on the board. (For example, 3/5 x 25, 21 x 4/7, etc.)
- Next, write the equations expressed by their steps next to the original equation. (For example, 25 ÷ 5 x 3, 21 ÷ 7 x 4, etc.)
- Ask the students why division is used in these problems.
- Write the expression 5/6 x 18 on the board. Ask them to explain the steps they could use to solve the problem.
- Revisit the problem from the beginning of the lesson (1/2 x 18). Ask the students to explain how they would use the steps discussed previously to solve this problem.
CCSS.MATH.CONTENT.5.NF.B.4.A
Interpret the product \((a/b) \times q\) as a parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q ÷ b\). For example, use a visual fraction model to show \((2/3) \times 4 = 8/3\), and create a story context for this equation. Do the same with \((2/3) \times (4/5) = 8/15\). (In general, \((a/b) \times (c/d) = ac/bd\).)

CCSS.MATH.CONTENT.5.NF.B.6
Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Goal: Students will use pictures to solve story problems that require you to multiply a fraction by a fraction.

Materials: paper, worksheet

Launch:
- Write the expression \(6 \times 2/3\) on the board.
- Give each student a blank sheet of paper. Tell them to write down, draw, and/or describe all the different ways they can think of to solve this problem.
- Ask the students to explain what this problem is asking.
- Review that it is asking you what \(2/3\) is \(6\) times or what \(6\) is \(2/3\) times.
- Have the students share their methods of solving the problem with the class.
- Tell the students that now they are going to try to solve some more fraction problems.

Explore:
- Put the students in pairs.
- Give each pair a worksheet with these three problems.
  - There is \(3/4\) of a cake left. You give \(1/3\) of the leftovers to your friend. How much of a whole cake did your friend get?
  - Someone ate \(1/10\) of the loaf of bread, leaving \(9/10\). If you use \(2/3\) of the loaf of bread to make sandwiches, how much of the whole loaf will you have used?
  - Bailey used \(2 \frac{1}{2}\) tubes of paint for her painting. Each tube holds \(4/5\) ounce of paint. How many ounces of paint did Bailey use?
- Tell the students to draw pictures to solve these problems.
- Ask the students to describe in words the problem they are trying to solve. (For example, I am trying to find \(1/3\) of \(3/4\).)
- As they work, ask the students how they could write the problem using symbols. Ask them which math operation they are using and how they know that.

Summarize:
- After the students finish, have them share their picture and answers with the class.
• Discuss any problems the students got different answers for or don’t understand.
• Ask the students what these questions are asking.
• If they are stuck, point out that in the first problem, you are trying to find 1/3 of the leftovers. The leftovers are 3/4 cake so you need to find 1/3 of 3/4.
• Ask the students how you would use this wording to describe the other problems.
• Ask them if they can write these problems using a math expression. If they are stuck, direct them to the last problem they solved during the launch phase of the lesson. Help them realize that these problems are asking you to solve a multiplication problem.
• With the students’ help, write the corresponding multiplication problem of each problem.
• Ask the students how they would explain to someone that these problems are asking you to multiply.
• Write “3/4 of 8” on the board. Ask the students what math operation this problem is asking you to perform and how they know that.
• Revisit the problem from the beginning of the lesson (6 x 2/3). Ask them if this lesson helped them think of any other ways they could describe what this expression is asking.

References:

CCSS.MATH.CONTENT.5.NF.B.4.A
Interpret the product \((a/b) \times q\) as a parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\). For example, use a visual fraction model to show \((2/3) \times 4 = 8/3\), and create a story context for this equation. Do the same with \((2/3) \times (4/5) = 8/15\). (In general, \((a/b) \times (c/d) = ac/bd\).)

CCSS.MATH.CONTENT.5.NF.B.4.B
Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

**Goal:** Students will show how counters can be positioned in a rectangle to model multiplying a fraction by a fraction.

**Materials:** counters, yarn

**Launch:**
- Write the expression \(3/5 \times 2/3\) on the board. Tell the students to draw a picture to solve this problem.
- Have a student share his/her answer and picture with the class.
- Have them explain how their picture models the problem.
- Tell the students that now they are going to try to use counters to solve the same problem.

**Explore:**
- Put the students in pairs.
- Give each pair some counters and some yarn.
- Tell the students to work with their partner to model \(3/5 \times 2/3\) using the counters.
- If the students need help getting started, tell them to make a rectangle that is 3x5 counters. Tell them to use this rectangle to model the multiplication problem.
- If the students are stuck, ask them to first show you \(2/3\) of the rectangle. Then, suggest that they use the yarn to show \(3/5\) of this.
- As the students work, ask them what the sides of rectangle represent.
- Ask them why the sides of the rectangle are the same as the denominators.
- Tell the students to think about how their counter model is similar to the picture used to model the problem at the beginning of the lesson.

**Summarize:**
- After the students finish, have a student share their model by drawing it on the board.
- Have the student explain the steps he/she took to model the problem.
  - First, you turn over 2 of the 3 rows of counters to show \(2/3\).
Next, you use the yarn to put a circle around 3/5 of the counters that are turned over.

- You now have a circle around 6 of the 15 counters. So 6/15 is the answer.

- Ask the students what the sides of the rectangle represent. Point out that the sides of the rectangle are the same as the denominators.
- Ask the students why the sides of the rectangle are the same as the denominators. Discuss that this makes it easy to divide the rectangle into thirds and fifths.
- Discuss any questions the students have.
- Ask them how the counter model of the multiplication problem is similar to the picture drawn to model this problem at the beginning of the lesson.
- Next, have them model \( \frac{3}{4} \times \frac{1}{2} \) to check if they understand. Have them explain their model.

References:

Interpret the product \((a/b) \times q\) as a parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\). For example, use a visual fraction model to show \((2/3) \times 4 = 8/3\), and create a story context for this equation. Do the same with \((2/3) \times (4/5) = 8/15\). (In general, \((a/b) \times (c/d) = ac/bd\).)

**Goal:** Students will solve fraction multiplication problems in order to recognize that when multiplying fractions the numerators and the denominators are multiplied \((a/b \times c/d = ac/bd)\).

**Materials:** counters, yarn, paper

**Launch:**
- Write the expression \(4/5 \times 2/3\) on the board.
- Ask the students to describe in words what the problem is asking and estimate the answer.
- Give the students counters and yarn.
- Have the students model the problem with the counters and yarn.
- Ask them to explain their model and give the answer.
- Tell the students that now they are going to solve some more multiplication problems and look for patterns in the answers.

**Explore:**
- Put the students in pairs.
- Write the expressions \(5/6 \times 1/2\), \(3/4 \times 1/5\), and \(1/3 \times 4/7\) on the board.
- Tell the students to solve each problem by modeling it with the counters. Have them draw their models on a sheet of paper.
- As they work, tell them to look for patterns in how the answers are found.
- Ask them how they figured out the denominator of the answer. Ask them if they can find a pattern that is true for the other problems.
- Ask them how they figured out the numerator of the answer. Ask them if they can find a pattern that is true for the other problems.

**Summarize:**
- After the students finish, have the students share their models and answers. Draw the model for each problem on the board.
- Ask the students how they found the denominator of the answer.
- Ask them if they noticed any patterns in how the denominators are determined.
- If the students need help, point to the first model and ask the students how they knew the denominator would be twelfths. Help them realize that there are twelve total counters.
- Ask them if this pattern applies to the other examples.
- Write the expression \(2/3 \times 4/5\) on the board. Ask the students to use the pattern to figure out what the denominator of the answer would be. Help the
students realize that since the model using the counters would have sides of 3 and 5, you can multiply 3 and 5 to find the denominator of the answer.

• Now, ask the students how they found the numerator of the answer.
• Ask them if they noticed any patterns in how the numerators are determined.
• If the students need help, point to the first model and ask the students how they knew the numerator would be five. Help them realize that there are five counters in the circled part of the model.
• Ask them if this pattern applies to the other examples.
• Refer back to the expression 2/3 x 4/5. Ask the students to use the pattern to figure out what the numerator of the answer would be. Help the students realize that since the circled rectangle of counters in the model would have sides of 2 and 4, you can multiply 2 and 4 to find the numerator of the answer.
• Write the expression 5/9 x 4/7 on the board. First, have the students explain what the problem is asking and estimate the answer. Next, tell them to use the pattern to find the answer. Repeat these steps with several other problems.

References:

CCSS.MATH.CONTENT.5.NF.B.4.B
Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Goal: Students will learn how to find areas of rectangles with fractional side lengths.

Materials: dry-erase boards & markers, paper with squares

Launch:
- Write the expression 2/3 x 2/5 on the board.
- Give each student a dry-erase board and marker. Tell the students to model this problem on their board.
- Have a students explain their picture to the class. Ask the students if anyone drew a different picture. Have them explain their pictures.
- Draw counters on the board to model this problem.
- Ask the students to talk with a partner about how this model relates to finding the area of a square.
- Have the students share their ideas with the class.
- Now, tell the students that they are going to solve a problem using this idea.

Explore:
- Put the students in pairs.
- Give them the following problem: Emily and Ben are each building a pen for their pet rabbit inside a square of grass that is 1 yard by 1 yard. Emily's pen is ¼ yard by 2/3 yard. Ben's pen is 5/6 yard by ½ yard. Which pen is larger?
- Give each partner a paper with two squares that are marked 1 yard by 1 yard. Tell them to shade the portion of the square that will make up the pen in order to solve the problem.
- As the students work, ask them how they know what portion of the square to shade.
- Ask them what the area is of each pen.
- Ask them how this problem is similar to and/or different from finding the area of a rectangle with lengths that are whole numbers.
- As the students are finishing, ask them how they can prove that their answer is correct.

Summarize:
- After the students finish, ask a student to share which pen is bigger and how they know that. Have the student draw their model on the board.
- Ask the students if anyone solved the problem in a different way. Have them share their explanations with the class as well.
• As the students give their explanations, ask them how they decided how much of the square to shade. Show the students that you can divide each length into unit fractions. For example, for the first pen, they can divide the square into thirds horizontally and fourths vertically.
• Ask the students how they knew for sure that Emily’s pen is larger. Help them recognize that each square is divided into twelfths. 6/12 of Emily’s square is shaded, while only 5/12 of Ben’s square is shaded.
• Ask the students what the area of each pen is and how they know this.
• Ask them how this is similar to or different from finding the area of a rectangle with lengths that are whole numbers.
• Using their pictures, help them realize that in both cases you multiply the length by the width.
• Draw a rectangle on the board and label one side 2/3 and the other side 2/5. Ask the students what the area of this rectangle is and how they know that. Prove their answer by dividing the rectangle into smaller rectangles with unit fraction side lengths.

References:

CCSS.MATH.CONTENT.5.NF.B.5.A
Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Goal: Students will use pieces of yarn to show that multiplication can be interpreted as resizing and to estimate the answer to a multiplication problem.

Materials: yarn, scissors, chart, tape

Launch:
- Write the expression $6 \times 2/3$ on the board.
- Ask the students to draw a picture to show what this problem is asking.
- Have the students share their answers.
- Tell the students that now they are going to think about another meaning of multiplication.

Explore:
- Put the students in partners.
- Give each pair a piece of yarn that is 12 inches long and some extra yarn that they can cut different lengths from. Give them a chart that is divided into different columns. Label the columns with the following titles:
  o Twice the length of the original yarn
  o Half the length of the original yarn
  o One-third of the length of the original yarn
  o Three-fourths of the length of the original yarn
- Tell the students that their job is to cut new pieces of yarn that match each description. Have them tape the new piece of yarn under the appropriate description.
- As the students work, ask them how what they are doing relates to multiplication.
- Ask them what multiplication problem is being performed in each example. Ask them to write this multiplication problem in the appropriate column on their chart.
- Tell the students that their original piece of yarn is 12 inches long. Ask them to estimate how long they think each other piece of yarn is and to add this to their chart.

Summarize:
- After the students finish, ask the class how what they did is connected to multiplication. Help the students realize that multiplication can be interpreted as resizing something.
- Have the students share the multiplication problems they came up with for each example. Ask them to explain how they came up with each multiplication problem.
• Next, go through the examples and have the students share their estimates. Ask them to explain how they got their estimates.
• Revisit the problem from the beginning of the lesson (6 x 2/3) and ask the students to give an example of how this problem can be seen as resizing something. Draw pictures on the board to model their examples. Have the students estimate the answer to this problem using their examples.

References:

**Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence \( \frac{a}{b} = \frac{(n \times a)}{(n \times b)} \) to the effect of multiplying \( \frac{a}{b} \) by 1.**

**Goal:** Students will solve multiplication problems in order to find patterns between the answer and the factor a number is multiplied by.

**Materials:** worksheet

**Launch:**
- Write the expressions 12 x 2 and 12 x \( \frac{1}{2} \) on the board.
- Ask the students to describe what these problems are asking.
- Review how the first problem is asking for twelve 2 times and the second problem is asking for twelve \( \frac{1}{2} \) times.
- Ask the students what the answers to the problems are and how they figured the answers out.
- Tell the students that now they are going to solve some more problems like this.

**Explore:**
- Put the students in partners.
- Give each pair a worksheet with the following problems: 24 x 10, 24 x 2, 24 x \( \frac{4}{3} \), 24 x \( \frac{3}{4} \), 24 x \( \frac{1}{2} \), 24 x \( \frac{1}{3} \), 24 x \( \frac{7}{7} \) and 24 x \( \frac{5}{6} \).
- Tell the students to first estimate the answer to each problem and then find the exact answer.
- Tell the students to think about how the number they multiply 24 by affects the answer. Ask them if they see any patterns.
- As the students work, ask them to explain their thinking as they estimate the answers.
- If the students are having trouble finding the pattern, suggest that they organize the problems according to the number they multiply 24 by.
- Ask the students how multiplying a number by a number smaller than 1 is similar and different to multiplying a number by a number greater than 1.

**Summarize:**
- After the students finish, draw three columns on the board. Label the columns "factor less than 1", "factor equal to 1", and "factor greater than 1".
- Tell the students that as you discuss each problem, they will put the problem in the appropriate column according to the number they multiplied 24 by.
• Go through each problem. Have the students share their estimates, the answer, and which column the problem should be placed in.

• After this, ask the students if they notice any similarities and/or differences between the columns.

• Ask the students what is the same about all the answers in the “factor equal to 1” column. Help the students realize that all the answers are 24 or the same as the number you multiplied. Ask them if they can think of why this is. Help them realize that this is because the problem is asking for 1 of 24.

• Ask the students what is the same about all the answers in the “factor greater than 1” column. Help the students realize that all the answers are greater than 24 or greater than the number you multiplied. Ask them if they can think of why this is. Help them realize that this is because the problem is asking for more than 1 of 24.

• Ask the students what is the same about all the answers in the “factor less than 1” column. Help the students realize that all the answers are less than 24 or less than the number you multiplied. Ask them if they can think of why this is. Help them realize that this is because the problem is asking for less than 1 of 24.

• Tell the students that now you are going to practice this. Tell them that you are going to multiply the number 7 by different numbers. Write a problem on the board (7 x 8/5, 7 x 1/3, 7 x 4/4, etc.), and tell the students to put their thumbs up if the answer is greater than 7, thumbs sideways if the answer is equal to 7, and thumbs down if the answer is less than 7. Repeat this with different problems.

• Revisit the expressions at the beginning of the lesson. Ask the students to explain why the answers would be greater than or less than 12.
CCSS.MATH.CONTENT.5.NF.B.7.A
Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for \((1/3) \div 4\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \((1/3) \div 4 = 1/12\) because \((1/12) \times 4 = 1/3\).

CCSS.MATH.CONTENT.5.NF.B.7.C
Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Goal: Students will solve story problems using a measurement interpretation of division in order to understand what it means to divide a fraction by a whole number.

Materials: worksheet

Launch:
- Tell the students that you have 16 cookies for a birthday party you are planning. You are going to divide the cookies evenly among 8 guests. How many cookies will each guest get?
- Ask the students how they would solve this problem. Encourage them to draw pictures.
- Have a student share how they would solve the problem with the class.
- Ask the students if they can think of any equations that would represent this problem.
- Tell the students that now they are going to solve some more problems, but with fractions.

Explore:
- Put the students in partners.
- Give each pair a worksheet with the following problems:
  - Amy has 1/3 of a yard of ribbon to make bows for 2 presents. How much ribbon should she use for each bow if she wants to use the same length for each bow?
  - There is 1/4 of a cake left. This needs to be divided evenly between 3 people. How much of the whole cake will each person get?
  - Mark has 1/2 hour to finish the last 5 problems of his math homework. If he spends the same amount of time on each problem, how much time should he spend on each problem?
- Tell the students to draw pictures to solve the problems.
- Afterward, tell them to try to write an equation for each problem.
- As the students work, ask them to explain their pictures to you.
• Ask them how the problems they are solving are similar to division.
• Ask them how the problems relate to multiplication.
• If students finish quickly, ask them if they can come up with any similar story problems and solve them.

Summarize:
• After the students finish, have the students share their answers and pictures with the class.
• Discuss any problems the students got different answers to in order to figure out which answer is the correct one.
• Ask the students what equations they came up with to represent the problems. Ask them how they came up with this equation.
• Ask the students how the problems they solved can be interpreted as a division problem.
• Explain that you are dividing 1/3 into 2 equal parts, ¼ into 3 equal parts, and ½ into 5 equal parts. Connect this to the problem at the beginning of the lesson in which you divided 16 into 8 equal parts. Write the division problems for each question on the board.
• Ask the students if these problems relate to multiplication in any way.
• If the students are stuck, tell them to think of the problem from the beginning of the lesson (16 ÷ 8 = 2).
• Point out that since 16 ÷ 8 = 2, we know that 2 x 8 = 16.
• Ask them to explain why this is true. Use pictures to model this.
• Next, ask them how this connects to the fraction problems. Explain that since 1/3 ÷ 2 = 1/6, we also know that 1/6 x 2 = 1/3.
• Ask them to explain why this is true using pictures.
• Finally, ask the students if they can come up with any story problems that would require them to divide a whole number by a fraction.

References:
CCSS.MATH.CONTENT.5.NF.B.7.B
Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.

CCSS.MATH.CONTENT.5.NF.B.7.C
Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$-cup servings are in 2 cups of raisins?

Goal: Students will solve story problems in order to understand what it means to divide a whole number by a fraction.

Materials: worksheet

Launch:
• Tell the students that you have 16 cookies for a birthday party you are planning. You are going to give each guest 2 cookies. Ask them how many guests you could serve.
• Ask the students how they would solve this problem. Encourage them to draw pictures.
• Have a student share how they would solve the problem with the class.
• Ask the students if they can think of any equations that would represent this problem.
• Tell the students that now they are going to solve some more problems, but with fractions.

Explore:
• Put the students in partners.
• Give each pair a worksheet with the following problems:
  o A serving is $\frac{1}{2}$ of a cookie. How many servings can be made from 3 cookies?
  o Each person will eat $\frac{1}{4}$ of a pizza. How many people will 5 pizzas feed?
  o You have 6 pints of ice cream for your birthday party. You will serve each guest $\frac{1}{3}$ of a pint of ice cream. How many guests can you serve?
• Tell the students to draw pictures to solve the problems.
• Afterward, tell them to try to write an equation for each problem.
• As the students work, ask them to explain their pictures to you.
• Ask them how the problems they are solving are similar to division.
• Ask them how the problems relate to multiplication.
• If students finish quickly, ask them if they can come up with any similar story problems and solve them.

Summarize:
• After the students finish, have the students share their answers and pictures with the class.
• Discuss any problems the students got different answers to in order to figure out which answer is the correct one.
• Ask the students what equations they came up with to represent the problems. Ask them how they came up with this equation.
• Ask the students how the problems they solved can be interpreted as a division problem.
• Explain that you are figuring out how many $\frac{1}{2}$s are in 3, how many $\frac{1}{4}$s are in 5, and how many $\frac{1}{3}$s are in 6. Connect this to the problem at the beginning of the lesson in which you figured out how many 2s are in 16. Write the division problems for each question on the board.
• Ask the students if these problems relate to multiplication in any way.
• If the students are stuck, tell them to think of the problem from the beginning of the lesson ($16 \div 2 = 8$).
• Point out that since $16 \div 2 = 8$, we know that $8 \times 2 = 16$.
• Ask them to explain why this is true. Use pictures to model this.
• Next, ask them how this connects to the fraction problems. Explain that since $3 \div \frac{1}{2} = 6$, we also know that $6 \times \frac{1}{2} = 3$.
• Ask them to explain why this is true using pictures.
• Finally, ask the students if they can come up with any story problems that would require them to divide a whole number by a fraction.

References:
CCSS.MATH.CONTENT.6.NS.A.1
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for \((2/3) \div (3/4)\) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that \((2/3) \div (3/4) = 8/9\) because 3/4 of 8/9 is 2/3. (In general, \((a/b) \div (c/d) = ad/bc\).) How much chocolate will each person get if 3 people share \(1/2\) lb of chocolate equally? How many \(3/4\)-cup servings are in \(2/3\) of a cup of yogurt? How wide is a rectangular strip of land with length \(3/4\) mi and area \(1/2\) square mi?.

**Goal:** Students will use pictures to solve story problems involving dividing fractions by fractions.

**Materials:** worksheet

**Launch:**
- Write the equation \(12 \div 4 = 3\) on the board.
- Ask the students to explain what this equation is saying.
- Ask them what different pictures they could draw to model this problem.
- Encourage the students to think of the problem in different ways. For example, the equation is saying that you can measure out three 4’s in 12 or that if you divide 12 into 4 equal parts, each part will be 3.
- Tell the students to keep this in mind as they solve some story problems in which they will divide fractions.

**Explore:**
- Put the students in partners.
- Give each pair a worksheet with the following problems:
  - How much chocolate will each person get if 3 people share \(3/4\) pound of chocolate equally?
  - How many \(3/4\)-cup serving are in \(2/3\) of a cup of yogurt?
  - If you have \(2/3\) of a cake left and must divide it into 2 containers, how much cake would be in each container?
  - If \(1/3\) of a cake fills up \(1/2\) of a container, how much cake would fill up a whole container?
  - Brittany can walk \(2 \frac{1}{2}\) miles in \(3/4\) of an hour. How fast is she walking in miles per hour?
  - Dan paid \$3 for a \(3/4\) pound of cereal. How much is that per pound?
  - How wide is a rectangular strip of land with a length of \(3/4\) mi and an area of \(1/2\) square miles?
- Tell the students to draw pictures to solve each problem.
- Then have them write the equation the problem is asking them to solve.
- As the students work, ask them to explain what each question is asking.
- Ask them to explain their pictures help them to solve the problems.
• Ask them how they know what division problem the question is asking them to solve.

**Summarize:**
• After the students finish, have the students share their answers and pictures with the class.
• Discuss any problems the students got different answers to in order to figure out which answer is the correct one.
• Ask the students what equations they came up with to represent the problems. Ask them how they came up with this equation.
• Go through each problem and ask the students to explain why this represents a division problem.
• Compile a list on the board of the different situations in which division is used.
• Ask the students to come up with another problem with fractions using each situation.
• Solve these problems as a class and discuss why each of these problems is asking you to do a division problem.

**References:**


CCSS.MATH.CONTENT.6.NS.A.1
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for \( \frac{2}{3} \div \frac{3}{4} \) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that \( \frac{2}{3} \div \frac{3}{4} = \frac{8}{9} \) because \( \frac{3}{4} \) of \( \frac{8}{9} \) is \( \frac{2}{3} \). (In general, \( \frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc} \).) How much chocolate will each person get if 3 people share \( \frac{1}{2} \) lb of chocolate equally? How many \( \frac{3}{4} \)-cup servings are in \( \frac{2}{3} \) of a cup of yogurt? How wide is a rectangular strip of land with length \( \frac{3}{4} \) mi and area \( \frac{1}{2} \) square mi?

Goal: Students will solve fraction division problems that guide them to learn the common-denominator algorithm.

Materials: worksheet

Launch:
- Tell the students that 5 cookies make up a serving, and you have 20 cookies. Ask them how many servings of cookies you have.
- Ask the students how they solved this problem. Ask them what math operation they are performing when they solve this problem. Ask them to explain why this is a division problem.
- Now tell the students that a serving is \( \frac{1}{2} \) of a cookie, and you have 8 cookies. Ask them how many servings of cookies you have.
- Ask the students how they solved this problem. Ask them what math operation they are performing when they solve this problem. Ask them to explain why this is a division problem.
- Tell the students that now they are going to solve some similar problems with fractions.

Explore:
- Put the students in partners.
- Give each pair a worksheet with the following problems:
  - A serving is 2 cookies. How many servings are in 3 cookies?
  - A serving is \( \frac{2}{5} \) cookie. How many servings are in \( \frac{3}{5} \) cookie?
  - A serving is \( \frac{2}{9} \) cookie. How many servings are in \( \frac{3}{9} \) cookie?
  - A serving is 3 cookies. How many servings are in 7 cookies?
  - A serving is \( \frac{3}{8} \) cookie. How many servings are in \( \frac{7}{8} \) cookie?
  - A serving is \( \frac{3}{11} \) cookie. How many cookies are in \( \frac{7}{11} \) cookie?
- Tell the students to draw pictures to solve each problem. Tell them to write the division problem below each picture.
- As the students work, ask them if they notice any patterns.
- Ask them if they can explain these patterns.
- Ask the students what the numerators and the denominators represent in the problems.
• Ask them how the numerator and denominator affect the answer and why this is.

Summarize:
• After the students finish, have the students share their answers, pictures, and equations with the class.
• Ask the students what the denominator and the numerator represent in this problem. Remind the students that the numerator represents the number of parts and the denominator represents the size of the parts.
• Ask the students what patterns they noticed in these problems. Help the students realize that the denominator doesn't matter if the denominator of the size of the serving and the given amount are the same.
• Ask them if they can figure out why this is. Remind them to look back at their pictures to help them and to think about what the denominator represents.
• Help them realize that since the denominator refers to the size of the parts, this doesn’t affect our answer when the parts don’t change sizes. Compare this to the first problem. If the 20 cookies and the 5 cookies get larger or smaller, our answer won’t change.
• Ask the students how they could use this new information to solve the problem 6/17 ÷ 2/17. Show them that since the parts are the same size, the answer is the same as the answer to 6 ÷ 2.
• Ask the students how they could use common denominators to solve 2/3 ÷ 1/2. Show them that we can rewrite this as 4/6 ÷ 3/6. This would be the same as the answer to 4 ÷ 3.

References:

CCSS.MATH.CONTENT.6.NS.A.1
Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for \(\frac{2}{3} \div \frac{3}{4}\) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that \(\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}\) because \(\frac{3}{4}\) of \(\frac{8}{9}\) is \(\frac{2}{3}\). (In general, \(\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}\).) How much chocolate will each person get if 3 people share \(\frac{1}{2}\) lb of chocolate equally? How many \(\frac{3}{4}\)-cup servings are in \(\frac{2}{3}\) of a cup of yogurt? How wide is a rectangular strip of land with length \(\frac{3}{4}\) mi and area \(\frac{1}{2}\) square mi?

Goal: Students will solve fraction division problems that guide them to learn the flip and multiply algorithm.

Materials: worksheet

Launch:
- Tell the students that you have 3 cakes that you need to divide between 6 different containers.
- Ask them how they would draw a picture to solve this problem.
- Next, ask them what equation they could write to solve this problem.
- Ask them to explain why this is a division problem.
- Tell the students that now they are going to solve some similar problems with fractions.

Explore:
- Put the students in partners.
- Give each pair a worksheet with the following problems:
  - If I have \(\frac{1}{2}\) of a cake and divide it evenly between 3 containers, how much cake would I put in each container?
  - If I have \(\frac{1}{4}\) of a cake and divide it evenly between 5 containers, how much cake would I put in each container?
  - If I have \(\frac{1}{5}\) of a cake and divide it evenly between 8 containers, how much cake would I put in each container?
  - If I have \(\frac{2}{3}\) of a cake and divide it evenly between 5 containers, how much cake would I put in each container?
- Tell the students to draw pictures to solve each problem. Tell them to write the division problem below each picture.
- As the students work, ask them if they notice any patterns.
- Ask them if they notice any patterns between the denominator and the number of containers.
- Point to their pictures, and ask them how the size of the parts changed when they divide the cake between the containers.
Summarize:

- After the students finish, go through each problem and have the students share their answers, pictures, and equations.
- Write the pictures, equations, and answers to each problem on the board.
- Ask the students what patterns they noticed.
- Ask them how the size of the parts changed when they divide the cake between the containers. Point to the pictures and help them notice that they divide the size of the parts according to the number of containers. For example, when they divide a half into three, they become sixths.
- Ask the students how this relates to multiplication. Direct them to the pictures. For example, when you divide fourths into 5 parts each, you now have 20 total parts.
- Ask them what operation they perform between the number of parts and the denominator. Ask them how they could rewrite the first three problems to show this. For example, they can write the first problem as \( \frac{1}{2} \times \frac{1}{3} \) because they know they need to multiply the denominator of the fraction of cake by the number of containers.
- Now, ask the students how the numerators affect the answer.
- Tell them to look at the picture for the last problem while paying attention to the number of parts.
- Ask them how they could rewrite this problem in the same way as the other problems.
- Write the problem \( \frac{2}{3} \div \frac{3}{4} \) on the board. Ask them to use the pattern they just learned to solve the problem. Ask them to draw pictures to explain why the pattern works.

References:
