Computer Simulations of Nonthermal Particles in Clusters of Galaxies: Application to the Coma Cluster

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Abstract

We have developed a numerical model for the temporal evolution of particle and photon spectra resulting from nonthermal processes at the shock fronts formed in merging clusters of galaxies. Fermi acceleration is approximated by injecting power-law distributions of particles during a merger event, subject to constraints on maximum particle energies. We consider synchrotron, bremsstrahlung, Compton, and Coulomb processes for the electrons, nuclear, photomeson, and Coulomb processes for the protons, and knock-on electron production during the merging process. The broadband radio through $\gamma$-ray emission radiated by nonthermal protons and primary and secondary electrons is calculated both during and after the merger event. To test the ability of the computer model to accurately calculate the nonthermal emission expected from a cluster merger event, we apply the model to the Coma cluster of galaxies, and show that the centrally located radio emission and the Hard X-ray excess observed at 40-80 keV is well fit by our model. If our model is correct, then the Coma cluster will be significantly detected with GLAST and ground-based air Cherenkov telescopes.

1 Introduction

Merger events between clusters of galaxies are extremely energetic events. The formation history of a cluster of galaxies will include several merger events (Gabici & Blasi, 2003). With typical masses for a rich cluster $\sim 10^{15} M_\odot$, dynamical estimates of the energy deposited in to the internal structure of a cluster is $\sim 10^{63} - 10^{64}$ ergs (Sarazin, 2004). Approximately 5% of the available energy is assumed to accelerate particles from the thermal pool to create a distribution of nonthermal particles. The first-order Fermi process is capable of accelerating particles to $\sim 10^{19}$ eV by the shocks that will form at the interaction boundary of two merging cluster of galaxies (Berrington & Dermer, 2003).

Optical and X-ray studies estimate that $\sim 30-40\%$ of rich clusters show signs of a current or recent merger event (Forman et al., 1981; Beers, Geller, & Huchra, 1982). Structure formation calculations estimate that a cluster will see several mergers though out its formation history (Lacey & Cole, 1993), indicating that most clusters contain a population of highly energetic nonthermal particles.

Diffuse cluster radio emission with no associated compact counterpart is observed in an ever increasing number of galaxy clusters, providing evidence for the prevalence of diffuse, highly energetic nonthermal particles in galaxy clusters (Giovannini, Tordi, & Feretti, 1999; Kempner & Sarazin, 2001). The diffuse radio emission is classified in to two categories. Diffuse emission found in the cluster center that mimics the thermal bremsstrahlung emission with random polarization is known as a radio halo. Diffuse cluster emission located on the cluster periphery characterized by
irregular shapes and strongly polarized light is known as a radio relic. These features are preferentially seen in clusters with current or recent cluster merger events, and are thought to be the observational consequences of nonthermal particles accelerated by shocks formed in a cluster merger event.

2 Models

We present results of a numerical model (Berrington & Dermer, 2003) designed to calculate the time-dependent evolution of nonthermal particle distribution functions evolving through radiative losses. The electrons and are accelerated by the first-order Fermi process and at the shock fronts formed in merging clusters of galaxies, and the resulting photon radiation is calculated. Particle injection functions \( Q_{e,p}(K_{e,p}, t) \) for electrons ("e") and protons ("p") are assumed to be described by power-law momentum spectra. In terms of kinetic energy \( K_{e,p} \), the injection functions are given by

\[
Q_{e,p}(K_{e,p}, t) = Q^{0}_{e,p}[K_{e,p}(K_{e,p} + 2m_{e,p}c^2)]^{-\frac{s+1}{2}}
\]

\[
(K_{e,p} + m_{e,p}c^2)\exp \left[ -\frac{K_{e,p}}{K_{\text{max}}} \right],
\]

where \( s \) is the injection index and \( K_{\text{max}} \) is the maximum particle energy determined by three conditions: the available time to accelerate to a given energy since the formation of the cluster merger event; the requirement that the particle Larmor radius is smaller than the size scale of the system; and the condition that the energy-gain rate through first-order Fermi acceleration is larger than the energy-loss rate due to synchrotron and Compton processes. The constant \( Q^{0}_{e,p} \) normalizes the injected particle function, and is determined by

\[
\int_{K_{\text{min}}}^{K_{\text{max}}} dK_{e,p} K_{e,p} Q_{e,p}(K_{e,p}, t) = \frac{\eta_{e,p} m_{e,p} c^3 \langle n_{\text{ICM}} \rangle}{2}
\]

(2)

where \( \langle n_{\text{ICM}} \rangle \) is the number density of the intra-cluster medium (ICM) averaged over the area \( A \) of the shock front, \( \eta_{e,p} \) is an enhancement factor to account for the ions heavier hydrogen, \( \eta_{e,p} \) is an efficiency factor taken to be 5% unless otherwise noted, and \( m_p \) is the mass of a proton. The minimum kinetic energy \( K_{\text{min}} \) is held constant at 10 keV.

The model calculates the forward \([v_1]\) and reverse \([v_2]\) shock speed from the gravitational infall velocity \( v_g \) of a merging cluster. The trajectory of the smaller merging cluster is approximated by a point mass of total mass \( M_2 \) that falls onto a dominant cluster of total mass \( M_1 \) whose density profile is described by an isothermal beta model. The velocity \( v \) of the shock fluid is calculated by solving the equation

\[
\frac{\mu_1 n_1}{\mu_2 n_2} = \frac{1 + 3M_1^{-2}}{1 + 3M_2^{-2}} \left( \frac{v_g - v}{v} \right)^2,
\]

(3)

where \( n_1 \) and \( n_2 \) are the number densities in the dominant and merging cluster, respectively. The mean atomic mass in the dominant cluster and the merging cluster are given by \( \mu_1 \) and \( \mu_2 \), respectively. Both mean atomic masses are set equal to 0.6\( m_p \).

The Mach speeds of the forward \([M_1]\) and reverse \([M_2]\) shocks are calculated by

\[
M_1 = \frac{2v}{3c_1} \left( 1 + \sqrt{1 + \frac{9c^2}{4v^2}} \right),
\]

and

\[
M_2 = \frac{2v_g - v}{3c_2} \left( 1 + \sqrt{1 + \frac{9c^2_2}{4(v_g - v)^2}} \right),
\]

(4)

where \( c_1 \) is the sound speed in the dominant cluster, and \( c_2 \) is the sound speed in the merging cluster. The Mach number is defined to be \( M_{1,2} = v_{1,2}/c_{1,2} \) for the forward and reverse shock, respectively. Equation \( 3 \) and equation \( 4 \) are derived from the shock jump conditions, by equating the energy densities of the forward- and reverse-shocked fluids at the contact discontinuity. Compression ratios \( C_1 \) (forward) and \( C_2 \) (reverse) are calculated from the equation

\[
C_{1,2} = \frac{4}{3(1 - M_{1,2}^{-2})}
\]

(5)

The time-dependent particle spectrum \( N(K, t) \) is determined from solving the Fokker-Planck equation in energy space, given by

\[
\frac{\partial N(K, t)}{\partial t} = \frac{1}{2} \frac{\partial^2}{\partial K^2} [D(K, t) N(K, t)]
\]

\[
- \frac{\partial}{\partial K} [\dot{K}(K, t) N(K, t)]
\]

\[
- \sum_{i=pp,pp',d} \frac{N(K, t)}{\tau_i(K, t)} + Q(K, t).
\]

(6)
times, the spectral power rises rapidly as the clusters
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cluster of mass
standard case of a merger event between a dominant
shown in Figures 1 & 2 using the parameters for the
Light curves at various observing frequencies are
3 Results
energy losses as the primary electrons.
tron distribution function and are subject to the same
electrons are calculated and added to the primary elec-
escape (i

Here \( \dot{K}(K, t) \) is the total kinetic-energy loss rate found
by the sum of Coulomb, synchrotron, bremsstrahlung
and Compton processes for electrons, and Coulomb
processes for protons. In addition, the protons expe-
ience catastrophic losses from proton-proton (i
 collisions, proton-\( \gamma \) (i = p\( \gamma \)) collisions, and diffusive
escape (i = d) on the timescale \( \tau_i(K, t) \). Secondary
electrons are calculated and added to the primary elec-
tron distribution function and are subject to the same
energy losses as the primary electrons.

3 Results

Light curves at various observing frequencies are
shown in Figures 1 & 2 using the parameters for the
standard case of a merger event between a dominant
cluster of mass \( M_1 = 10^{14} \text{M}_\odot \), and a merging cluster
of mass \( M_2 = 10^{15} \text{M}_\odot \) with a magnetic field strength
\( B = 1.0 \mu \text{G} \), and a beginning redshift of \( z_i = 0.3 \). The
light curves of the nonthermal radiation exhibit a
common behavior independent of frequency. At early
times, the spectral power rises rapidly as the clusters
merge. The peak emission occurs when the centers
of mass of the two clusters pass at \( t_{\text{coll}} \), after which
the emission exhibits a slow decay and approaches a
plateau at times \( t \gg t_{\text{acc}} \). The time \( t_{\text{acc}} \) is defined
as the time at which particle injection has terminated at
the forward shock (1) or reverse shock (2). The rate of
decay of the emission increases with radio frequency
due to the stronger cooling of the higher energy elec-
trons, so that the decay is slowest at lower frequen-
cies. Synchrotron emission from secondary electrons
forms the late-time plateaus at radio energies. This be-
behavior is also apparent for the hard X-ray emission,
although it is formed by primary bremsstrahlung and
both primary and secondary Compton radiation at late
times. At \( \gamma \)-ray energies, the \( \pi^0 \)-decay emission forms
a plateau of emission that dominates soon after \( t_{\text{acc}} \).
Calculations of the hardest particle injection spectral
index \( s_{\text{min}} \) formed in cluster merger shocks are shown
in Figure 3 as a function of the larger mass \( M_1 \) of the
two clusters, with a constant subcluster mass \( M_2 = 10^{14} \text{M}_\odot \).
We also assume that the onset of the merger begins at redshift \( z_i = 0.1 \); softer injection indices
are obtained for mergers at larger values of \( z_i \) because
of the smaller maximum separations for merger events
occurring at higher redshift. The mean cluster masses
are smaller at higher redshift. These lower mass clus-
3.1 Modeling the Coma Cluster

Both optical (Colless & Dunn, 1996; Edwards et al., 2002) and X-ray (Vikhlinin, Forman, & Jones, 1997; Arnaud et al., 2001) observations indicate that the dynamics of the Coma cluster is well-described by a three-body merger model. Colless & Dunn (1996) were the first to find substructure in the central region of Coma that is consistent with a recent merger event near the collision time $t_{\text{coll}}$, defined to be the time when the centers of mass of the two clusters pass through each other. Total mass estimates of the dominant cluster $M_1$ is estimated to be $0.8 \times 10^{15} M_\odot$. X-ray observations (Vikhlinin, Forman, & Jones, 1997) estimate the total mass of the merging cluster $M_2$ from the gas sheared in the merging process to be $\sim 0.1 \times 10^{15} M_\odot$. This assumes a gas fraction of 5–10%.

The ICM is well described by an isothermal beta model with core radius $r_c = 0.257$ Mpc, central electron density $\rho_e = 3.82 \times 10^{-3} \text{ cm}^{-3}$, central proton density $\rho_p = 7.43 \times 10^{-27} \text{ gm cm}^{-3}$, power-law slope $\beta = 0.705$, and a mean gas temperature $\langle T_X \rangle = 8.21 \text{ keV}$ (Mohr, Mathiesen, & Evrard, 1999). The assumed magnetic field strength is $B = 0.22 \mu \text{G}$, and an efficiency factor $\eta_{e,p} = 1\%$. Because we can never know the true gas distribution
to z

Thierbach, Klein, & Wielebinski (2003). Our models favor a primary electron source for the radio emission from the radio halo. Despite our models using a uniform density profile for the calculation of the secondary electron production, the emission from the secondary electron is an upper limit to the true secondary electron emission. However, the density in the central region is roughly constant, so that the uniform density assumption is a good approximation to the true secondary electron emission.

In Figure 5, we show a comparison of the calculated thermal and non-thermal emission with the observed Hard X-ray (HXR) emission from the central region of the Coma cluster of galaxies observed by the Phoswich Detection System (PDS) on BeppoSAX (Fusco-Femiano et al., 2004). The reported non-thermal photon emission is the total integrated emission expected within 1.5 Mpc of the cluster center. This corresponds to a field of view of \(~1^\circ7\). The PDS has a FWHM field of view of \(~1^\circ3\) which corresponds to a linear scale of \(~2.2\) Mpc. In addition we also show the OSSE 2\(\sigma\) upper limits (Rephaeli, Ulmer, & Gruber, 1994). The HXR emission observed between 20–80 keV is dominated by Compton up scattering of photons off of primary electrons.

The BeppoSAX/PDS observations include the merging cluster associated with NGC 4839, and will be contaminated with any non-thermal emission associated with shocks formed in its merging process. The expected non-thermal emission resulting from shocks for a cluster infalling at a minimum distance of \(1.6h^{-1}_{50}\) Mpc (Neumann et al., 2001) will be negligible in comparison with the emission from the merger observed in the core of Coma (Berrington & Dermer, 2003).

In Figure 6, we present the predicted \(\gamma\)-ray emission from the Coma cluster of galaxies. The observational limits are taken from Weekes et al. (2002). As seen from the figure, the predicted \(\gamma\)-ray emission falls comfortably below the predicted upper limits for the EGRET observations. Our model predicts that the space-based observatory GLAST will detect the nonthermal \(\gamma\)-rays at high significance. Furthermore, we predict that both VERITAS and HESS will have strong \(\approx 5\sigma\) detections in 50 hours of observations.

## 4 Summary and Conclusions

We describe a computer model designed to calculate the nonthermal particle distributions and photon spectra resulting from nonthermal processes produced by shocks that form between merging clusters of galaxies. Over the lifetime of a cluster merger shock \(~10^{61} - 10^{62}\) ergs will be deposited in the energy of a nonther-
We applied this model to the Coma cluster of galaxies. We show that the radio emission from the radio halo Coma C is well fit by the cluster merger model. The calculated nonthermal X-ray emission also fits the observed HXR emission observed at 40–80 keV by BeppoSAX/PDS. The γ-ray emission expected from this model is also calculated and is shown to fall below the observational limits for EGRET as reported by Reimer et al. (2003), but should be strongly detected by the space-based observatory GLAST and the ground-based air Cherenkov telescopes such as VERITAS and HESS. Even though other acceleration mechanisms or point sources could produce nonthermal emission in the core of Coma, our model of cluster merger shocks account for the entire observed radio and HXR emission.

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