Abstract

Thesis: Fractal Spaces and Bi-Lipschitz Embeddings into Banach Spaces

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In this thesis, we examine the geometry of fractals and metric spaces. We study the question of which fractal metric spaces can be embedded into Banach spaces up to a certain distortion.

Our main focus is on a metric space introduced by Urs Lang and Conrad Plaut in “Bi-Lipschitz Embeddings of Metric Spaces into Space Forms,” which we refer to as the Diamond Graph Fractal. By modifying the construction methods defined by Lang and Plaut, we develop a Generalized Diamond Graph Fractal and study whether the space converges in the Gromov-Hausdorff distance, satisfies the doubling property, and whether it can be Bi-Lipschitzly embedded into certain Banach spaces with given properties. Our approach to the Bi-Lipschitz embedding problem is to generalize the argument of Lang and Plaut, which involves the quadrilateral inequality, a property of namely Hilbert space.

In addition, we also study and explain an argument in the paper “On the Geometry of the Countably Branching Diamond Graphs” by Florent Baudier et. Al., which involves a related class of graphs and “asymptotic midpoint uniform convexity”, a property that the norm of certain Banach spaces, including Hilbert spaces, can satisfy. Our goal, by comparing these two arguments, is to better understand the properties of Banach spaces and how these properties interact with the geometry of certain fractal metric spaces.