STOCHASTIC APPROACHES TO INTEREST RATES

An Honors Thesis by

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Abstract

This synopsis of stochastic interest rates is here to provide the future actuary with some idea of the process involved in determining assumed interest rates. This is important in a couple of respects. For the actuary to earn the professional designation of Fellow in the Society of Actuaries, there is an exam that covers this topic. This exam will be required to become an Associate by the end of 1995.

More importantly perhaps is the amount of interest rate manipulation that the actuary is called on to do for a company. The actuary must be well knowledged about interest rate derivation. This includes what can and cannot be assumed and also what is and is not allowed by the government. Sometimes what is acceptable by the government is more important than what could normally be assumed by prevailing rates of interest. These considerations must be weighed when determining the correct interest rate to use in any financial model.

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I. Introduction

Stochastic approaches to interest are very important in the actuarial science field. If one can find a model for the determination of interest rates, then reserving for insurance companies becomes much less unwieldy. It also allows for more precise pricing of products, since the time value of money is a crucial part of the insurance and relies heavily on the rate of interest credited.

The study of interest is also a crucial part of the professional designation for an actuary. In order to get through the series of exams, one must pass an associate exam dealing entirely with interest. There also is a Fellowship exam that covers stochastic material presented in Kellison's The Theory of Interest. Both exams are required in order to obtain the designation of Fellow from the Society of Actuaries, the governing body of life insurance actuaries.

There are three main approaches that can be taken when studying interest rates. Each has a certain area in the stochastic realm where it generates the best results. The model most desired depends on if you assume independent or dependent rates of interest. It also depends on if you want to look at securities or just guaranteed interest rates. It even depends on the kind of security you are interested in.

The random walk approach works best when trying to forecast the future price of a security. This assumes that the future price of the security is purely random. The binomial lattice structure is very important if we assume further that the future price has only two options. This randomness would become very difficult once the lattice extended very far into the future. However, with computers even one with thousands of future possibilities can be handled in minutes.

The asset pricing model is more focused towards future values of assets. This was developed to explain variance between different investments. This leads to the notion that the riskier the investment, the higher the interest rate has to be for the investment to be worthwhile. This is widely used in determining where to invest a corporation's profits. It also helps to determine if investing in the company will produce the best yield or if outside sources will generate the smartest return.

The option pricing model has a different function. It determines what a stock option is worth and how much of a return it will generate. It is a very specific model that helps in terms of hedging against future risk. Options hedge against future risk by allowing the investor to guarantee the price he can buy or sell a stock regardless of where the market price is at the time of the option. The extra money charged for the option lowers potential profit, but also reduces the amount of risk inherent in purchasing securities.

All three models are important in deciding how to invest money. That is an important part of the insurance function. When you are dealing with billions of dollars of future retirement benefits you need to generate the largest rate of return you can. If you cannot get them the largest guaranteed return, they will invest with someone who can.

All the models have something in common; they all are based on probability functions. This assumes that interest rates can be explained as random variables. By presuming that interest rates can be defined by random variables they become much more manageable. Now applications can be done to determine what future rates might be based on the random variable.

Before the individual models can be looked at we must consider whether interest rates are independent of one another. If they are independent, they may follow the random walk model. However, they need not be independent. In fact, a look at historical trends in interest rates shows a drift in interest rates. They appear to drift upwards for a time and downwards for a time. But even in those drift patterns, there seems to be some evidence that might support a theory of independent rates.

II. Independent Rates of Interest

While it certainly appears that in the long run interest rates are not independent, fluctuations from period to period often appear to jump randomly about the previous rate. Often the reason an interest rate goes up or down in a given week is based on confidence in what the future holds for monetary vehicles. This confidence is so hard to predict that assuming the decision is random approximates the logic closely.

Given that interest rates appear to move in a random manner on the short term, it would be nice to determine what the expected interest rate would be and what amount would be expected to accumulate over time. The expected rate of interest and how much it would accumulate, is often quite different than the expected amount of accumulation and the rate of interest that would give you that amount.

Example 2.1: Assume that the future rate of interest for the next five years is equally likely to be 5%, 7.5% and 10%. The expected rate of interest would be 7.5%. This would accumulate one dollar to:

$$(1.075)^5 = 1.43563.$$

However the expected value of accumulation based on elementary statistics would be:

$$[(1.05)^5 + (1.075)^5 + (1.10)^5]/3 = 1.44081$$

which gives us i = .07577. This kind of seemingly minor fluctuation is used often in politics when trying to hide debt. Observe the difference when considering the current four trillion dollar deficit facing our country.

$$\$4,000,000,000,000 * (1.43563) = \$5,742,520,000,000$$

but, $\$4,000,000,000,000 * (1.44081) = \$5,763,240,000,000$

for a difference of 20.72 billion dollars. No wonder the government has found it so easy to hide its debt!

If interest rates are allowed to fluctuate from period to period, from the same distribution and totally independent then the accumulated value will be determined by the expected interest rate.

Formula 10.1:

$$E[a(n)] = E\left[\prod_{t=1}^{n}(1+i_t)\right] = \prod_{t=1}^{n}E[1+i_t] \text{ \{from independence\} } = (1+i)^n$$

(Kellison, 338)

This result is also true for present values, although the rate for the present value will be different than the rate for the accumulated value.

To determine what range of values the accumulated value might fall within, the variance needs to be determined. This is just as critical as finding the expected value, since it will tell us how spread out the values are. This also informs us of how probable the actual accumulation will be "near" the expected value. The larger the variance, the less likely the two values will be close.

$$var[a(n)] = E[a^2(n)] - \{E[a(n)]\}^2 = E[a^2(n)] - (1 + i)^{2n}$$

To determine the second moment about the origin of a(n) assume the i_t's have variance s^2.

$$E[a^2(n)] = E[\prod_{t=1}^{n}(1 + i_t)^2] = \prod_{t=1}^{n} E[(1 + i_t)^2] \quad \text{from independence}$$

$$= \prod_{t=1}^{n} E[1 + 2i_t + i_t^2] = (1 + 2i + i^2 + s^2)^n$$

Thus the variance is given by

$$var\ [a(n)] = (1 + 2i + i^2 + s^2)^n - (1 + i)^{2n}$$

This bases the variance of the accumulated value entirely on the assumed interest rate and variance which are determined by the random variable distribution selected to model the interest rate.

If the first two moments of $1 + i_t$ and $(1 + i_t)^{-1}$ are known, then the mean and variance of accumulated or present values can be determined for single amounts or annuities. However, even with these values known, the probability density function generally cannot be determined. This leads to the common practice of simulation. Simulation is usually handled by a five step approach:

1. Make appropriate assumptions about the p.d.f. for i_t.

2. Generate a series of random numbers to conduct the test. For *m* trials, *mn* random numbers are necessary.

3. Using standard techniques, the random numbers are used to compute the *m* sets of values for the i_t's.

4. For each of the *m* sets of i_t's compute the financial function (accumulated value, present value, etc.).

5. The *m* outcomes can be used to develop an approximate probability density function for that financial function. Probabilities for different outcomes can be determined using the *m* trial values.

III. Dependent Rates of Interest

Now consider interest rates as dependent. This seems to have more intuitive appeal than assuming independent rates. The rate of interest this month seems to be closely associated with last month's rate. It also may relate to some long term rate, one that is the average rate over some considerable length of time. In general, if the current rate is higher than the average rate then the following interest rate will also tend to be higher than the average interest rate. The current rate has a certain amount of influence over short term future rates.

Historically this seems to be born out by most security rates offered in both private and public forums. In the early 1980s after Reagan was elected President, it took a long time for interest rates to gradually fall. In 1982 some certificates of deposit were offering up to fourteen percent per year on terms of two or more years. By 1985 rates had gradually lowered to eleven percent. In 1987 they were down to nine percent. Today four or five percent is a decent rate to offer. This was a gradual process. No institution woke up one day and said, "Ok. Let's drop our rates by ten percentage points." Rather it was a quarter of a percent here, a half a point there, until rates fell to the recent historical lows that they are today.

This idea of dependent interest rates can be modeled in many ways. Two of these models are borrowed from statistics. It uses time series analysis, a concept which is tested on in the Society of Actuaries Exam 120. This fifteen credit exam is optional and is covered in the regression analysis statistics course here at Ball State University.

The two primary models used for interest rates are moving average (MA) models and autoregressive (AR) models. These have been used both separately and in combination to determine interest rates. Experience shows that simple autoregressive models have been more successful in modelling interest rates than have moving average models.

Consider uniformly distributed interest rates with a mean of i. If we assume that successive rates can be linked by the recursion formula

$$i_t = i + k(i_{t-1} - i), \qquad \text{where } 0 < k < 1.$$

This dependency assumes that the rates are uniformly distributed with the center being i_{t-1} for each value of t-1.

The constant k is the relative weighting between the long term average rate and the most recent interest rate. The larger k is, the more weighting lent towards the recent rate. The smaller k is, the more it appears as if interest rates are independent and results become similar to section two. This is an example of an autoregressive process of order one, or AR(1) for short.

Now apply the AR(1) process more fully where the long-term average force of interest of $\partial_{[t]} = \partial$ in the expectated function. Assuming that $\partial_{[t]}$ is based on both the long term average force and the most recent period's force of interest yields the form:

$$\partial_{[t]} = \partial + k(\partial_{[t-1]} - \partial) + e(t).$$

This is very similar to the earlier formula involving k. The expression e(t) is the error term which is independent and identically distributed. Its normal distribution has a mean of zero and variance σ^2.

The variance of $\partial_{[t]}$ is given by:

$$var[\partial_{[t]}] = \sigma^2/(1-k^2)$$

and the covariance by

$$cov[\partial_{[s]}, \partial_{[t]}] = \sigma^2 * k^{t-s}/(1-k^2) \qquad \text{for } t>s \text{ and } |k| < 1.$$

Notice that if k=0 then the rates are independent and the results from part two are applicable.

The autoregressive process of order two is a bit more sophisticated, although it uses similar logic to the AR(1). This uses the long-term average force of interest as well as the two most recent forces. It takes the form:

$$\partial_{[t]} = \partial + k_1(\partial_{[t-1]} - \partial) + k_2(\partial_{[t-2]} - \partial) + e(t)$$

where the symbols are defined in a similar manner to the AR(1) process.

The variance of $\partial_{[t]}$ is given by:

$$var[\partial_{[t]}] = (1-k_2)*\sigma^2/\{(1+k_2)*[(1-k_2)^2 - k_1^2]\}.$$

The covariance is given by:

$$\operatorname{cov}[\partial_{[s]},\partial_{[t]}] = \operatorname{var}[\partial_{[t]}][\tau\, g_1^{t-s} + (1-\tau)g_2^{t-s}] \quad \text{for } t>s,$$

where

$$\tau = g_1(1-g_2^2)/[(g_1-g_2)^*(1+g_1g_2)]$$

and where g_1 and g_2 are the reciprocals of the roots of the characteristic equation $f(x) = 1 - k_1x - k_2x^2 = 0$

provided

$$k_1 + k_2 < 1$$

$$k_2 - k_1 < 1$$

$$-1 < k_2 < 1.$$

Again if $k_1 = k_2 = 0$, then there is independence and part two holds true.

These approaches are very analytical. Another way to approach the data is by using simulation. Using simulation further models can be developed. These models need to be tested as well to see how they handle actual data in comparison with known results.

IV. The Capital Asset Pricing Model

One of the most widely used models in all of finance is the Capital Asset Pricing Model. This was developed to try and explain why there is a variation in yield rates for many different types of investments. For example a typical Treasury bill issued by the United States government produces a nominal yield rate of 3.5% while a bond issued by a corporation will usually produce a yield of 5.1%. Even better results can be obtained in the common stock market, where it is quite common to have a yield rate of 12%.

Presumably the reason that Treasury bills have the lowest yield and common stocks are the highest in return is the relative amount of risk involved. The Treasury bill is considered risk-free. The only way they would not be paid is if the government collapsed. That would devastate so much of the financial world that relies on money. Since money itself is backed by the U.S. government, our economic system would make little sense without assuming the continued success of the government. So we allow Treasury bills to be assumed as the base rate of interest. No investment has less risk, so the excess interest is derived from what is called the risk premium. In the case of corporate bonds it would be 1.6%, for common stock 8.5%.

Another premise in the Capital Asset Pricing Model is that there are two types of risk involved in an investment. There is unsystematic (sometimes called unique) risk. This reflects price movements which cannot be explained by the collective market behavior. This should be eliminated by a diversified portfolio.

The second type of risk is referred to as systematic (or market) risk. This reflects movements by the whole market. This cannot be eliminated by a diversified strategy. This is the risk that is being modeled in the Capital Asset Pricing Model.

This model was developed to explain the variation of yield rates on different common stock. In the process it was discovered that it modeled various other investments quite well. It seemed to provide help finding an appropriate rate to discount money in cash flow analysis. The rate of interest used in present value techniques now reflected the degree of risk in each investment and estimated the uncertainty in the cash flows.

The formula is given by:

$$E[r_k] = r_f + \beta_k(E[r_p] - r_f)$$

where the following definitions are true:

r_k = yield rate on a specific security k

r_f = risk-free rate of interest

r_p = yield rate on the market portfolio

β_k = a measure of systematic risk for security k

Thus the formula states that the expected yield rate on a specific security is equal to the risk-free rate plus a multiple of the excess of the expected yield rate of the portfolio over the risk-free rate.

$\beta_k > 0$ is a multiple for the measure of systematic risk. The closer β_k is to 0, the less risk is involved. When $\beta_k = 1$, then the security has the same level of systematic risk as the market portfolio. If $\beta_k > 1$, then more risk is involved than the market portfolio.

Risk premium for the market portfolio is that amount of the interest rate that is in addition to the risk-free rate. In the formula it is determined by subtracting r_f from $E[r_p]$. When the risk premium for the market portfolio is multiplied by the multiple for systematic risk, the risk premium for the particular security is the result.

The Capital Asset Pricing Model has been studied under extensive empirical tests. Even though the results have performed reasonably well, it has been shown that other factors do influence yield rates. Not only have seasonal and size of investment factors affected the rates, but the systematic risk has varied over time as well.

Yet the model works well enough for many practical applications. It is used often enough that values of β_k are periodically published for a large number of common stocks. They can be obtained from most brokerage firms.

V. An Option Pricing Model

Options are financial vehicles sold in the public market which allow purchasers to buy or sell a security at a guaranteed price. This price is promised for some future date in time when the option can actually be exercised and the security is sold or bought at the stated price. Options have developed into a good way to hedge against a particular security rising or falling in price. This allows the owner of the option to guarantee a certain price in case the market has moved the price of the security in a way that would otherwise be detrimental to the particular investor.

Consider investor A who wants to sell his 10 shares of stock six months from today. Right now his stock is worth $50 per share. He does not want to sell for less than that so he buys a put that costs $5/share. The put guarantees him that six months from now he can get $50/share. If the stock falls below $50 per share he can exercise the option and save some money. If the stock rises above $50 per share he can disregard the option and collect the excess that he can get for his shares. The option solidified his position so he could not lose too much (or gain too much either).

The same can be thought of investor B. He wants to buy stock in six months, but only for $45 a share. He can buy a call option for $4 per share. In this case if the stock drops below his option of $45 he would not exercise the option. Instead he would purchase the stock at its current price. The option protects him in case the price were to rise above his target. He will not pay more than $45 per share since that is the most he would have to pay. The option has insulated him from extreme negative results.

There is a formula that relates the value of a put and a call to the price on a given stock.

$$P + S = C + v^n E \qquad (5.1)$$

where

P = value of put

C = value of call

S = current stock price

E = exercise stock price

n = time remaining until expiry

This relationship is known as a put-call parity. The interest rate is the risk-free rate on Treasury securities.

Options were initially created in connection with common stocks. They have proved to be so useful in a hedging strategy that they are now available in many different financial vehicles. Foreign currency exchange, commodities, bonds, and future interest rate options are all sold as identifiable financial instruments.

Furthermore, many securities now contain options that are part of the security. The callable bond is a good example of this. The noncallable bond will sell at a higher price than the callable bond since bond purchasers are willing to pay more for bonds which cannot be called early. This is due to the relationship

$$B_{nc} = B_c + C$$

where

B_{nc} = value of the noncallable bond

B_c = value of the callable bond

C = value of the call option

When an investor buys a callable bond he is in essence buying a noncallable bond and selling a call option to the bond issuer.

One popular way of determining the value of an option is the Black-Scholes Option Pricing Model. This model develops a formula for the price of a European call option. This option can only be exercised at the expiry date. The value of a call is given by

$$C = S\,N(d_1) - Ee^{-\partial n}N(d_2)$$

where

C = value of the European call

S = current stock price (current value of asset)

E = exercise price

∂ = risk-free force of interest for n periods

n = time remaining until expiry

N = cumulative distribution function for the standard normal distribution

$d_1 = \{ \ln(S/E) + (\partial + \sigma^2)n\}/\sigma n^{.5}$

$d_2 = \{ \ln(S/E) + (\partial - 1/\sigma^2)n\}/\sigma n^{.5}$

σ = standard deviation of the force of interest on the asset

This assumes that the asset does not pay dividends prior to the expiry date. The value for a put assumes the same definitions with P being the value of the put and is defined as

$$P = Ee^{-\partial n}[1-N(d_2)] - S[1-N(d_1)]$$

Empirical tests of the Black-Scholes formula imply that the formula works well under certain conditions. Errors tend to develop in the following situations:

1. When the exercise price is far from the current market price.

2. For extremely volatile or involatile securities (σ quite high or low).

3. For large values of n.

Example 5.1 The value of a European call expiring in one year with an exercise price of $200 for a stock currently selling for $175 can be determined using the Black-Scholes method. The standard deviation for this stock's continuous rate of return is .5 and the force of interest that is risk free is 6%.

First determine d_1 and d_2.

$$d_1 = [\ln(175/200) + (.06 + .25/2)(1)]/(.5*1) = .103$$

$$d_2 = [\ln(175/200) + (.06 - .25/2)(1)]/(.5*1) = -.397$$

Now from the standard normal tables

$$N(.103) = .54$$

$$N(-.397) = .35$$

Now using the Black-Scholes formula returns

$$C = 175(.54) - 200(e^{-.06})(.35) = \$28.58$$

VI. A Random Walk Model

The binomial model is based on a random approach to interest rates. In the random walk the probabilities of going up or down in price from period to period remain unchanged. Furthermore, the outcome of each period (or trial) is completely independent of any previous outcome.There seems to be considerable evidence that would imply that securities follow a random walk model.

Look at the $175 stock from the previous example. If it follows a random walk it may have a 50-50 chance of being either $200 or $160 in one year. Now the price of the call (with an exercise price of $180) will be different than the earlier example. Assume the interest rate is 6%.

If the stock falls the call is not exercised since the price is lower. If it rises to $200 then the call is worth $20. Compare this to the results when one buys a share of stock and borrows $150.94. If the stock falls to $160, it can be sold and the loan can be paid off for a net gain of zero. If the stock rises we can sell it for $200, pay off the loan, and still realize a net gain of $40. Since the results of both transactions are equivalent, we have

value of 2 calls = value of stock - bank loan

= $175 - $150.94 = $24.06

Thus the value of one call is $12.03.

This illustration demonstrates that two seemingly unrelated transactions produce the same exact outcomes. Transactions that do so are called replicating transactions. Many sophisticated investment techniques involving options use replicating strategies that are quite complex.

An alternative approach to determine the value of the call can be used if the investor is assumed to be indifferent about risk. If this is so then the investor in the earlier example would expect a return of 6%, since that is what he could get by loaning his money elsewhere. Let p be the probability that the stock rises and 1-p that the stock falls. If it rises from $175 to $200, the yield rate will be 14.3%. If the stock price falls to $160, the yield rate will be -8.6%. Assuming risk-neutrality produces

$$.143p + (-.086)(1-p) = .06$$

which yield p = .6375.

Now the expected present value of the call returns

$$[.6375(20) + .3625(0)]/1.06 = \$12.03$$

which is the same answer as before. The probabilities of the two different outcomes were determined by the requirement to eliminate risk-free arbitrage, so risk-neutrality must be assumed.

This model can be generalized beyond one period. If we assume that the movement upwards of the security moves the price by a factor of 1+k and the downwards move causes a change in price of $(1+k)^{-1}$, the resulting outcomes and their respective probabilities are represented by a binomial probability distribution.

This leads to a generalized formula for n periods. Let the exercise price be E. The value of the call at expiry for each trial is the greater of zero and the excess of the stock price at expiry over the exercise price. Thus

$$C = (1/(1+i)^n)\sum_{t=1}^{n}\left[\begin{smallmatrix} n \\ t \end{smallmatrix}\right]p^{n-t}(1-p)^t \max[0, S(1+k)^{n-2t}-E]$$

is an expression for the value of a European call. The value of p is determined by the risk-neutral approach. This formula can be handled quite well on the computer.

To determine the value of C requires an appropriate value for k. An excellent choice for k is given by

$$k = e^{\sigma\sqrt{h}} - 1$$

where σ is the same as in the Black-Scholes formula and h is the length of time between successive steps. Those securities with higher variances will have higher values for k.

Example 6.1 Use the binomial method to find the value of a call expiring in one year with an exercise price of \$50. The stock currently sells for \$45, has a standard deviation of .1, and a force of interest that is also .1 allowing for a risk factor of zero. Use a quarterly basis.

$$k = e^{.1\sqrt{.25}} - 1 = .05127$$

To determine p first find the upside and downside values.

$$\text{(upside value)} = 45(1+k) = 47.31$$

$$\text{(downside value)} = 45(1+k)^{-1} = 42.81$$

Thus p is

$$42.81(1-p) + 47.31p = 45e^{.25(.1)} = 46.14$$

p = .74.

There are 5 possible outcomes from these trials:

$$45(1+k)^{4} = 54.963$$

$$45(1+k)^{2} = 49.733$$

$$45 = 45$$

$$45(1+k)^{-2} = 40.718$$

$$45(1+k)^{-4} = 36.843$$

Apply this to the binomial formula to yield:

$$C = e^{-.1}(5)(.74)^{4}(54.963 - 50) = \$6.73$$

Notice that the more periods that are involved, the closer the answer produced by the binomial method will be to one determined by the Black-Scholes method.

VII. Scenario Testing

Currently there is a popular technique for practical applications called scenario testing. This is a type of simulation that uses extensive computer capability to compute a large number of calculations. To apply this approach, a different pattern of future interest rates is used to determine each "scenario."

Most companies try to match their assets and liabilities to determine whether their future cash inflows will be enough for the outflows of cash. These cash flows are fairly uncertain as to the exact timing and amount of payment. The scenario can also be affected by the level of interest assumed. The interest rate affects both flows of cash; the higher the interest rate, the less money available on both sides of the ledger.

The amount of surplus is computed for each scenario whether positive or negative. This set of values can be used in a statistical analysis to determine various measures which can be quite useful in making budgets for upcoming periods. For example, if the mean of the trials seems like a good estimate for surplus, that may be what is used as the target surplus for the budget.

There are three general methods used in determining the interest rate paths in scenario testing. Under the preset method the analyst specifies all the interest rates in all the various paths. This method allows the analyst the most flexibility in scenario choices. This usually includes some pessimistic, optimistic, and midrange paths with one usually on either extreme end. Interest paths that are both highly volatile and not so volatile are included. So are paths that trend up and those that trend down. Extreme case scenarios contain rates that reflect hyperinflation or a severe depression.

One common mistake in forming conclusions is assuming that all scenarios are equally likely. Probabilities need to be assigned to the various paths in order to make statistical measures a true reflection of what is likely to occur. Assuming that hyperinflation or a severe depression is just as likely as interest rates remaining close to current rates is a very unsound practice.

The preset method has two main flaws. One weakness is that the method is quite time consuming. The analyst must specify all the various rates of interest, which requires a lot of man hours to determine the various paths. Also the analyst must be the one to determine what rates are likely and what probabilities should be assigned to each path. This can be very tricky, even for the well seasoned analyst.

Another method that can be used is the probabilistic method. A common method is to use the binomial lattice approach. This assumes that the rate of interest follows a random walk. Thus the probability of an upward movement is p for each period and 1-p for downward movement.

The third method is a stochastic method. The most common of these is to assume that successive values of $1+i_t$ follow a lognormal distribution. The advantage of this approach is the ability to adjust for the volatility of interest rates.

The problem with both the stochastic and probabilistic methods is that interest rates may become too high or too low after several periods. Adjustments can be made by setting the volatility assumed lower or by setting upper and lower bounds. Neither solution is ideal however.

It is difficult to describe the ways to do scenario testing, since there are probably as many ways to test as there are analysts that do it. However, these are some of the more basic approaches to begin testing with which explain some of the various considerations involved.

Scenario testing can be very important under certain insurance regulations. In Canada the regulations that govern what reserve requirements are necessary are very flexible. Reserves are allowed to be set at any justifiable level. Since reserves are very interest sensitive they are reviewed by an independent actuarial audit. The company whose actuary uses scenario testing to justify his interest rates (that determine the reserves) has a better tool to explain his choice to the audit than if some other technique is used.

Example 7.1

Insurance companies can sell guaranteed investment contracts (known as GICs) which are similar to CD's. Company ABC Mutual sells a one-year contract guaranteed at a rate of interest of 9%. ABC Mutual invests in instruments that are convertible quarterly with a current rate of 8.8%. The probability of upward movement is .6. Fluctuations each quarter are .16% up or down. Find the probability of a loss on the GIC.

There are eight possible paths. Since rates are quarterly, all rates must be divided by fourths in order to get the quarterly rate.

Number	Path	Probability	Accumulated value
1	UUU	.216	(1.022)(1.0224)(1.0228)(1.0232) = 1.0935
2	UUD	.144	(1.022)(1.0224)(1.0228)(1.0224) = 1.0927
3	UDU	.144	(1.022)(1.0224)(1.022)(1.0224) = 1.0918
4	UDD	.096	(1.022)(1.0224)(1.022)(1.0216) = 1.0909
5	DUU	.144	(1.022)(1.0216)(1.022)(1.0224) = 1.0909

6	DUD	.096	(1.022)(1.0216)(1.022)(1.0216) = 1.0901
7	DDU	.096	(1.022)(1.0216)(1.0212)(1.0216) = 1.0892
8	DDD	.064	(1.022)(1.0216)(1.0212)(1.0208) = 1.0884

Thus the only two paths where the company would lose money are paths seven and eight. The probability of loss is .096 + .064 = .16.

VIII. Conclusion

There are many, many approaches to determining the appropriate interest rate to assume for an investment. The actuary is needed to determine what is appropriate for the given situation. These rates of interest greatly influence the company's overall performance.

One of the key elements in reserving is to determine what rate of interest to base the reserves on. Not only do you want the one that makes the company appear to be earning money, there are also other factors that must be considered. The government regulates what interest rates you can assume, if you assume too high of rates (in order to lower the amount of money in the reserves) the insurance commissioner can order your company to lower the rates to a more reasonable level. By the same token the rating agencies that rate all insurance companies may take your assumed interest rates into consideration when they issue your rating. These ratings are given a lot of importance by many potential insured as well as insurance brokers, who may only deal with companies that are rated at a certain level.

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