Application of Collective Risk Theory to Certain Reinsurance Contracts

An Honors Thesis (ID 499)

by

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Table of Contents

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Section I: Introduction

My interest in the specific stop-loss area of reinsurance began during my summer internship with a large midwestern mutual life insurance company. Reinsurance was an area of insurance that I knew very little about when the summer began. As the summer progressed I gained a working knowledge of reinsurance and began to realize its importance in the field of insurance. One project that I worked on throughout the summer dealt with specific stop-loss. The project consisted of considering the basic factors involved in specific stop-loss and then compiling a Lotus worksheet that would provide a set of rates given various input factors. I became interested in this particular branch of reinsurance and wanted to pursue further my understanding of this subject. I hope to apply the things that I learned this summer to the use of collective risk theory in order to expand my knowledge of specific stop-loss reinsurance.

Section II: Reinsurance and Retention Limits

The basic idea of reinsurance is that one insurance company insures one or more portfolios of policies issued by another insurance company. According to Kenneth Black and Harold Skipper, authors of Life Insurance, "Reinsurance may be defined as a device by which a life insurance company transfers all or a portion of its exposure under a life insurance policy to another company. It is insurance for the insurer" (p.431). The insurance company that is receiving the coverage is referred to as the ceding company while the company taking on the risk is the reinsurer.

The first step in the basic reinsurance process is for an insurance company to decide on an upper limit to the amount of money that they can pay on a claim, or the retention limit. In his paper entitled "Limits of Retention for Ordinary Life Insurance", Irving Rosenthal pointed out, "The primary purpose, although not the exclusive purpose, of a schedule of retention limits is to obtain a stabilization of mortality experience" (p.6). However, in order to stabilize mortality experience it is necessary to understand the variations that may occur in mortality. According to Irving Rosenthal, these variations include chance variation, secular variation, catastrophic variation, cyclical variation, and variation due to incorrect classification of risk or insufficient knowledge of basic mortality.

Chance variation involves the use of a probabilistic mathematical approach. The assumption here is that there is a correct theoretical mortality rate and any variation from that rate is caused from the sample size being too small. The assumption is that if the sample size were large enough, the mortality rate would be completely accurate.

This is the most important variation to consider when deciding upon a retention limit.

Secular variation refers to the mortality rates changing slowly but permanently over a long period of time as a result of improved health care and economic conditions. As medical technology has become more advanced and people have become more aware of the importance of eating right and exercising, over the years mortality experience has improved. This change in mortality experience is a result of secular variation. Catastrophic variation is a type of secular variation that is more sudden and less permanent.

Cyclical variation deals with changes in mortality rates that vary according to changes in the economic or social cycle and return back to normal when the economic or social situation returns to normal. It has been shown that mortality experience during a depression is higher than during regular economic conditions. According to Irving Rosenthal, "This, it is supposed, results from the combination of the 'boom-period' relaxation of underwriting standards and the severe economic strains suffered by many large policyholders during depression periods" (p. 14). Cyclical variation therefore takes into account the effect of the cycles of the economy on mortality experience.

Another possible cause of a variation in mortality experience results from insufficient knowledge of basic mortality. This situation is prominent in smaller companies that do not have enough experience to have sufficient information concerning mortality rates, or even in a larger company that is experimenting with a new form of coverage. It is important to consider all of these factors that may contribute to fluctuations in mortality in order to set realistic retention limits.

A good example of some of the factors involved in actually selecting a suitable retention limit can be found in Dr. John A. Beekman's Two Stochastic Processes (pp. 55-56). In Dr. Beekman's example, a company is considering raising its retention limit from $20,000 to $50,000. The current distribution has weights of .3, .2, .3, and .2 at $2,000, $5,000, $10,000, and $20,000. The example states that a $50,000 retention level would spread the $20,000 policies evenly among $20,000, $30,000, $40,000, and $50,000 face amounts. The following represents these two distributions:

$$P_1[X \le z] = \begin{cases} 0 & \text{for } z<2 \\ .3, & 2 \le z<5 \\ .5, & 5 \le z<10 \\ .8, & 10 \le z<20 \\ 1.0, & 20 \le z \end{cases}$$

$$P_2[X \le z] = \begin{cases} 0, & z<2 \\ .3, & 2 \le z<5 \\ .5, & 5 \le z<10 \\ .8, & 10 \le z<20 \\ .85, & 20 \le z<30 \\ .9, & 30 \le z<40 \\ .95, & 40 \le z<50 \\ 1.00, & 50 \le z \end{cases}$$

As in the example, consider the initial amount of capital, u, needed to hold the probability of ruin below .01. For the first distribution the initial amount of capital needed would be $125,000 to keep the probability of ruin below .01. For the second distribution, this initial amount would go up to $250,000. This means that increasing the retention limit from $20,000 to $50,000 would require $125,000 more of initial capital. It is evident from this example that the retention limit is an important factor and must be carefully considered.

Once the retention limit has been set, then any policies which would require a payment that is more than this amount must be reinsured to the amount in excess of the retention limit. For example, an insurance company may decide that their upper limit for any claim will be $700,000. If someone wants to buy a policy for $1,000,000 then the insurance company will need to obtain $300,000 of reinsurance. If a claim should occur, the ceding company would pay $700,000 while the reinsurer would pay the other $300,000. This is the basic idea of reinsurance in its simplest form.

While reinsurance provides additional business to the reinsurer, there are also several advantages of reinsurance to the ceding company. The most basic advantage is to relieve the company of the possibility of financial devastation caused by a particular claim or group of claims. Catastrophes in which a large number of people are killed at one time could have a harmful effect on the financial status of an insurance company that is not reinsured to some degree. The recent crash of Flight 103 is an example of a catastrophe that had a major effect on certain insurance companies.

Another major advantage of reinsurance to the ceding company is the guidance and advice that the reinsurer provides them. Since the reinsurer is obligated to pay part of the claims of the ceding company, the reinsurer has a definite interest in the underwriting and history of the ceding company as well as the basic components of the policy. In working together with the reinsurer, the ceding company gains from the knowledge and experience of the reinsurer. This could prove to be helpful to smaller companies, new companies, or companies that wish to experiment with a new type of coverage.

Section III: Nonproportional Reinsurance

Proportional reinsurance is the most common type of reinsurance, especially in the area of life insurance. In a proportional reinsurance contract, it is decided in advance that the reinsurer will pay a certain percentage of each claim and the ceding company will pay the rest. An example would be if the reinsurer agreed to pay 40% of the claim, leaving the ceding company to pay 60%. So if a claim amounted to $5,000, the reinsurer would pay $2,000 and the ceding company would pay $3,000. If the claim amount was $200,000, the reinsurer would be required to pay $80,000 and the ceding company would pay $120,000.

In nonproportional reinsurance, no proportion is determined in advance. A nonproportional contract may state that the reinsurer will pay any amount in excess of $50,000. If a claim amount was $5,000, then the reinsurer would pay nothing, or 0%, and the ceding company would pay the remaining $5,000, or 100%. However, if the claim was in the amount of $200,000, the reinsurer would pay $150,000, or 75%, and the ceding company would pay the remaining $50,000, or 25%. In his article entitled "Introduction to Nonproportional Reinsurance", Herbert L. Feay pointed out, "The kind of reinsurance that protects against all fortuitous variations in claim rates and claim costs is complete nonproportional reinsurance" (p. 25).

Nonproportional reinsurance commonly takes the form of catastrophe reinsurance, excess-of-loss, or stop-loss reinsurance. Catastrophe reinsurance, as illustrated in John C. Wooddy's "Risk Theory and Reinsurance", involves setting a limit to the amount that a ceding company would have to pay for claims that arise from a single accident. Usually the contract sets a maximum amount per policy and per accident that the ceding company will have to pay.

Consider, for example, a catastrophe policy that specifies that the ceding company will pay up to $100,000 per life and up to $1,000,000 per event. The reinsurer will therefore pay any claims in excess of these amounts. Consider an example where four actuaries, all policyholders with the ceding company, were traveling by train to an actuarial exam seminar. The train derailed, killing all of the passengers. Each actuary had $100,000 of life insurance and $50,000 of accidental death benefit. The ceding company would therefore be required to pay $400,000, since $100,000 is the maximum amount per life that the ceding company would have to pay. The reinsurer would then be responsible for the remaining $200,000. If each of the actuaries had had $100,000 of accidental death benefit rather that $50,000, the ceding company would still only have to pay $400,000, since this is the maximum amount per life as specified in the contract, and the reinsurer would be responsible for the remaining $400,000. If twenty actuaries had been on board, each with $100,000 of life insurance and $100,000 of accidental death benefit, the ceding company would have been required to pay $1,000,000, which is the maximum amount per event specified in the contract. The reinsurer would then be responsible for the remaining $3,000,000 in claims.

Excess-of-loss reinsurance involves setting an amount in advance concerning a single claim above which the reinsurer agrees to pay. Stop-loss is a similar idea, but it concerns aggregate claims. Stop-loss is, however, the most difficult of the three to consider. Catastrophic reinsurance represents a small line of reinsurance contracts, and excess-of-loss, dealing with a single policy, can be adjusted fairly easily. However, this is not the case for stop-loss. John C. Wooddy, the author of "Risk Theory and Reinsurance", explains, "The principal company will normally purchase stop-loss in contemplation of an increase of sizable proportions in its retention limit. Cancellation of the stop-loss cover, therefore, could have an awkward result" (p.5). Stop-loss reinsurance is therefore a line of insurance that must be considered carefully.

Section IV: Specific Stop-Loss

Specific stop-loss provides reinsurance to a company that is providing group insurance to a set of employees. In a standard reinsurance agreement, the ceding company pays a certain amount of an entire claim and the reinsurer must pay the rest. However, in a specific stop-loss agreement, the ceding company agrees to pay a certain amount of claims incurred during a specified period of time while the reinsurer pays the rest. The basic difference between standard reinsurance and specific stop-loss then is that specific stop-loss sets a specified period of time in which the claims must be incurred in order to be covered by the reinsurer. Specific stop-loss considers the accumulated amount of claims incurred during a specified period of time.

Consider an example where the ceding company agrees to pay up to $100,000 in medical expenses incurred in a given year for each employee. In that year, if the medical expenses for the entire year totaled less than this amount, the ceding company would pay all claims. If an employee had a few common medical expenses throughout the year that totaled $10,000, the ceding company would be expected to pay for all of the claims for that year. If the $100,000 amount was exceeded, however, the reinsurer would pay any amount in excess of the $100,000. If an employee had to have a kidney transplant that cost $250,000, then the ceding company would pay $100,000 while the reinsurer would have to pay the remaining $150,000.

Section V: Risk Theory Applications

Collective risk theory is a vital concept in considering reinsurance in that the purpose of risk theory is to examine fluctuations that occur as a result of claim amounts and claim distributions. John C. Wooddy commented on this relationship between risk theory and reinsurance when he wrote, "Reinsurance and risk theory are very closely related in that the purpose of reinsurance is to guard against the effects of risks that cause adverse fluctuations while risk theory is the mathematical analysis of these random fluctuations" (p.1). Collective risk theory deals with both the distribution function relating to the aggregate claims in a portfolio and the theory of ruin.

Paul M. Kahn pointed out in his article "An Introduction to Collective Risk Theory and Its Application to Stop-Loss Reinsurance", "Stop-loss reinsurance presents a natural application for collective risk theory, for such a reinsurance treaty covers the total claims, or a percentage thereof, above a certain fixed amount arising on a portfolio" (p. 400). While it is clear that a stop-loss contract covers any amount above a certain amount agreed upon in advance, it is important to consider this from a mathematical standpoint. The following notation was adapted from the text book Actuarial Mathematics, by Newton Bowers, Hans Gerber, James Hickman, Donald Jones, and Cecil Nesbitt. Consider a deductible amount d, above which the reinsurer will pay all claims, and an amount I_d, which is the amount that the reinsurer will pay. Then

$$I_d = \begin{cases} 0, & \text{for } S \leq d \\ S-d, & \text{for } S > d \end{cases}$$

where S represents the total claims in a given period. Therefore, the amount retained by the ceding company would be $S - I_d$ or

$$S - I_d = \begin{cases} S, & \text{for } S \leq d \\ d, & \text{for } S > d \end{cases}$$

Since d is the maximum amount that the ceding company would have to pay, the loss is stopped at this point, and thus the name stop-loss is appropriate.

Consider the case where F(x) is the distribution function for S, the total claims in the given period, and f(x) is the probability density function of S. These functions can then be used to calculate the expected value of the claims that the reinsurer will have to pay, which is the net stop-loss premium assuming a deductible of d. The following four formulas can be used to calculate this net premium:

Formula I: $E[I_d] = \int_d^\infty (x-d)f(x)dx$

Formula II: $E[I_d] = E[S] - d + \int_0^d (d-x)f(x)dx$

Formula III: $E[I_d] = \int_d^\infty [1-F(x)]dx$

Formula IV: $E[I_d] = E[S] - \int_0^d [1-F(x)]dx$

For discrete distribution functions, the integrals in the above formulas must be replaced by summations. Several other formulas that may be of interest include

$E[I_{d+1}] = E[I_d] - [1-F(d)]$ where $E[I_0] = E[S]$

$Var[I_d] = E[I_d^2] - \{E[I_d]\}^2$.

It is important to remember that the expected values of the claims of the reinsurer represent only a lower bound to the premium. It will be necessary to include a loading to cover expenses and fluctuations.

To illustrate these points, consider the following applications.

Example I: The amount of aggregate claims is $100,000 and the deductible amount is $75,000. Then the amount paid by the reinsurer, I_d, would be $25,000 and the amount paid be the ceding company,

$S-I_d$, would be $75,000. If the deductible amount were $200,000, then the reinsurer would pay nothing, and the ceding company would pay the entire $100,000.

Example II: Consider an example with the following distribution:

x	f(x)
0	.2000
1	.2500
2	.1500
3	.1750
4	.1000
5	.0625
6	.0625

If the deductible were set at d = 2, then the net premium would be calculated as follows:

$$E[I_2] = \sum_{x=d+1}^{\infty} (x-d)f(x) = \sum_{x=3}^{6} (x-2)f(x)$$
$$= 1f(3) + 2f(4) + 3f(5) + 4f(6)$$
$$= 1(.175) + 2(.1) + 3(.0625) + 4(.0625) = .8125$$

This amount is the lower bound for the net premium. It is necessary to consider the variance to take into account loadings that will be necessary.

$$Var[I_2] = E[I_2^2] - \{E[I_2]\}^2$$
$$E[I_2^2] = \sum_{x=3}^{6} (x-2)^2 f(x) = 1^2 f(3) + 2^2 f(4) + 3^2 f(5) + 4^2 f(6)$$
$$= 1(.175) + 4(.1) + 9(.0625) + 16(.0625) = 2.1375$$

$$Var[l_2] = 2.1375 - (.8125)^2 = 1.47734375$$

It is interesting to consider the relationship between stop-loss reinsurance and experience rating. When considering expenses for a portfolio of group accounts, a general expense for the experience of the entire portfolio is decided upon and charged to the entire portfolio regardless of the individual experience of the group. An experience rating, on the other hand, takes into account the experience of the individual groups and allows for a premium refund. Hans Ammeter describes experience rating in the following manner in his article "Stop Loss Cover and Experience Rating": "The method of Experience Rating provides a second tarification of each single group, leading to a retroactive partial premium refund, depending on the individual claim experience realized in each single group" (p.1). Experience rating therefore encourages the group to have lower claims because if their experience is good they will have lower premiums, or higher dividends.

The following notation is adapted from Actuarial Mathematics. Let the premium refund be denoted by D, the aggregate claims by S, and the gross premium by G. Then the dividend would equal the excess of a fraction of the premium over the claims, or

$$D = \begin{cases} kG - S & \text{for } S<kG \\ 0 & \text{for } S>kG \end{cases}$$

and $$E[D] = \int_0^{kG} (kG - x)f(x)dx$$

Consider the following equations in illustrating the relationship between experience rating and stop-loss reinsurance:

$$S + D = kG + I_{kG}$$

$$S + D - G = I_{kG} - (1 - k)G$$

This is interpreted in Actuarial Mathematics as, "The balance of the claim payments and dividends received over the premium paid is the same as the corresponding balance for a stop-loss contract with deductible kG and stop-loss premium (1 - k)G" (p. 387). It is therefore evident that the theory of experience rating is closely related to the theory of stop-loss reinsurance.

Section VI: My Summer Experience With Stop-Loss: A Realistic Application

This summer during my internship with a large midwestern mutual life insurance company I was presented with a project dealing with specific stop-loss reinsurance. The project consisted of studying a manual in which rates for a variety of situations had already been determined and setting up a Lotus spreadsheet that would take into account all of the various situations and output a premium. At that point in my college career I had not yet taken any courses in life contingencies or risk theory. This perspective is therefore less mathematical and deals with stop-loss from an applied standpoint. One reason that I was constructing this program was so that people with a relatively limited knowledge of insurance and rate making could gather the relevant information from a client, input it into the program, and quote a premium to the prospective client. Appendix I, found at the end of the paper, gives an example of all of the information needed in order to find the gross premium. This shows how the net premiums that are figured from the expected values are adjusted for the different situations and used to calculate the gross premium. After the basic rates have been set, there are adjustments that must be made to those rates depending on certain factors concerning the client. My program was designed to take all of these factors into account.

There are a variety of factors that must be taken into account when determining the rates to be charged for each individual employee's coverage under specific stop-loss. One of the main considerations that comes to mind is the amount of specific stop-loss or the deductible amount. The lower the deductible amount, the greater the chance that the reinsurer will have to pay a claim and the higher the premium per employee would be. There is a much greater chance that a particular employee's medical expenses would exceed $10,000 in one year than that they would exceed $250,000 in one year. In addition to deciding the point at which the reinsurer will start paying for the claims, it is important to consider the maximum amount for which the reinsurer will be responsible. One million dollars would be a reasonable maximum benefit. These factors deal with clarifying how much of a claim will be paid by the ceding company and how much will be paid by the reinsurer.

Another major factor to consider is the payment period specification. Since specific stop-loss deals with the amount of claims accumulated over a specific time period, it is important to decide on that time period so that it is clear as to who is responsible for paying the claim.

Consider a situation in which the deductible amount is $100,000 and the specified period is one year, beginning on January 1, 1989. If a claim for $60,000 is incurred on June 1,1989 and a claim for $50,000 is incurred on December 31, 1989, the ceding company would be responsible for paying $100,000 while the reinsurer would have to pay $10,000. If the second claim had been incurred on January 1, 1990, however, the amount of the claims for the year of 1989 would have totaled only $60,000, which is not in excess of the $100,000 deductible amount, and the reinsurer would not have to pay out any money in that particular year. It is therefore very important to specify the exact period of time involved.

Some common examples of payment periods include Incurred in Twelve Months and Paid in Twelve Months, Incurred in Twelve Months and Paid in Fifteen Months, Incurred in Twelve Months and Paid in Eighteen Months, and Incurred in Twelve Months and Paid in Twenty-four Months. In these cases, "incurred" refers to the date of service rather than the date of disability or admission. In most cases, the claims must be incurred in a one year period, but the claim does not have to be paid right away. Usually a period of between three months to a year is provided in which to pay the claim in order to provide a reasonable amount of time to process the claim.

Certain factors about the employees must be considered in determining the rates for specific stop-loss. One factor involves whether the employee wants just employee coverage or composite dependent coverage. The rates would be more if the employee wished to cover his or her entire family under the plan rather than just himself or herself. Another factor to consider concerning the employee is the maximum amount that the employee would have to pay out of his or her own pocket before any coverage would start. The lower this amount, the higher the specific stop-loss rates would be. A reasonable out-of-pocket maximum would be $1000.

Some factors concerning characteristics of the ceding company that is offering this group insurance to its employees must also be considered. The area in which the company is located must be considered in determining rates. In certain areas of the country, medical expenses are much higher than others. Medical expenses in New York City would be expected to be much higher than those in Muncie, Indiana. The rates charged for employees of a business in a certain area should reflect the relative degree of medical expenses in that area. Also, the age and sex distributions of the employees of the company must be considered. Underwriting for group insurance is generally not as specific as that for individual insurance, but the overall age and sex distribution can be a consideration. In general, men and older people tend to have a greater number of large medical expenses. However, the age of the employee does not affect the dependent rate since the risk of the dependent offsets the risk of the employee at different ages. In other words, when the employee is at an age in which the risks are statistically low, typically the dependent is at an age in which the risks are statistically high, and vice versa. The reinsurer should also consider whether the ceding company employs managed care. A company that uses managed care rather than traditional care takes the time to check and follow up on claims to ensure that they are valid. Managed care involves the use of pre-admission certification and concurrent review. The use of managed care would therefore reduce the net rates because of the effect on the length of hospital stays.

Various provisions offered by the ceding company to the employees may also affect the rates. One such provision is the actively at work provision. Under this provision, a person that is not at work because of disability at the time the policy begins will not be covered. However, this condition is sometimes waived in order to attract business, especially for large group cases. This provision must therefore be taken into account since some companies choose to waive it. Another provision to consider is if the ceding company offers extended benefits to its employees. If the company offers extended benefits, it would be necessary to increase the net rates. This provides continued coverage during the next policy year for disabled employees after the group has ended. Therefore it would be an added expense even after the termination of the reinsurance.

Several other miscellaneous factors must also be considered. Since all policies will not have the same starting date, it is important to take into account trend factors. Trend factors take into account that adjustments must be made for various starting dates in order to take into account changes in medical costs and technology. Since it is probable that all policies will not begin on January 1, it is important to consider the trend factors involved with other starting dates. There must also be some consideration given to loadings for expenses incurred by the reinsurer in writing the policy for the ceding company. The following is an example of an equation that can be used for this purpose:

Gross Premium = (Net Premium + Fixed Expenses) / (1-%Premium Expenses)

Several examples will help to clarify this point. Consider an example where the net premium is calculated to be $300 and the fixed expense for that individual is $70. The percent of premium expense for this individual is shown to be 40%. The gross premium for this employee would be calculated as follows:

$$\text{Gross Premium} = (300+70)/(1-.4) = (370)/(.6) = \$616.67$$

Consider another individual with a net premium of $300 and a fixed expense of $70, but with a percent of premium expense of only 10%. The gross premium for this individual would be \$411.11, as shown in the following calculation:

$$\text{Gross Premium} = (300+70)/(1-.1) = (370)/(.9) = \$411.11$$

Another individual with a \$300 premium, a \$10 fixed expense, and a percent of premium expense of 5% would have a gross premium of only \$326.32, as shown in the following calculation:

$$\text{Gross Premium} = (300+10)/(1-.05) = (310)/(.95) = \$326.32$$

The fixed expenses and percent of premium expenses will vary depending on the characteristics of the policy and the individual insurance company. Fixed expenses include expenses that remain fixed no matter how much business is written, and do not vary by the amount of the policy. Examples of fixed expenses include rent, heating, and maintenance expenses. The percent of premium expense, on the other hand, is directly related to the expenses incurred by writing a particular policy. Examples of factors affecting the percent of premium expense would be the administrative expenses and commissions relating to writing a particular policy.

A final factor to consider is whether or not there is a partial first year. Usually, stop-loss periods are easiest to record and keep track of if they follow the calender year. This means that they would start on January first of the year in question. However, sometimes a company will want to start coverage later in the year for the first year and then, when January first of the next year arrives, they will want to start the stop-loss from that date. For example, a company may decide that they need stop-loss coverage in June of a particular year. However, in January they may want to obtain a policy of coverage that begins on January first of that year so that the coverage corresponds to the calender year. The coverage from June until January is considered a partial first year. The net premium would be lower for a partial first year because the chance of claims reaching the deductible amount is less for a partial year than for a full year.

Once all of these factors have been considered, it is possible to input this information into the program and receive a page of output that contains all of the relevant information and the final premium rate. Appendix II is an example of the output page that resulted from running this Lotus program.

Section VII: Conclusion

The purpose of this paper was to take the information that I had gained this summer, and broaden my knowledge of stop-loss reinsurance in light of risk theory. I first considered the basic theory of reinsurance, retention limits, and nonproportional reinsurance. I then examined the mathematics of stop-loss reinsurance and how stop-loss premiums are calculated. Only at this point was I able to see how my experience with stop-loss this summer fit into this whole idea by adjusting these net premiums to come up with a final premium rate to quote to a client.

My working with stop-loss this summer is analogous to starting a book by reading the final chapter. I found the final chapter so interesting that I worked throughout the year to start at the beginning of the book and fill in the rest of the information. Writing this paper has been a satisfying learning experience since I now feel that the entire book fits together. Hopefully sometime in the future I can continue my study of stop-loss reinsurance and add a chapter of my own.

Appendix I

SPECIFIC STOP LOSS RATE CALCULATION WORKSHEET

		Cell Address
Client Name:	________	A6
Prepared by: (your name)	________	A10
Area: (A, B, C, D, E, F, G, H, I, J, K, or L)	________	A15
Actively at Work Provision: (1 = In Effect, 2 = Waiver)	________	A20
Amount of Specific Stop Loss - Deductible Amount: Please enter one of these values without commas. (5,000; 10,000; 15,000; 20,000; 25,000; 30,000; 40,000; 50,000; 60,000; 75,000; 100,000; 125,000; 150,000; 200,000; or 250,000)	________	A26
Payment Period: (1 = Incurred in 12, Paid in 12) (2 = Paid in 12 - Only valid when Actively at Work Provision is Waived) (3 = Incurred in 12, Paid in 15) (4 = Incurred in 12, Paid in 18) (5 = Incurred in 12, Paid in 24) (6 = Incurred in 12, Unlimited)	________	A36
Out-of-Pocket Maximum: (Base Rate - $1,200)	________	A41
Maximum Benefit: (Base Rate - $1,000,000) Please enter one of these values without commas. (250,000; 500,000; 1,000,000; or 2,000,000)	________	A46
Are extended benefits offered (Y or N):	________	A51

Age/Sex Distribution:

Age	Number of Lives Male	Number of Lives Female	Cell Address
<30	____	____	C60 D60
30-34	____	____	C61 D61
35-39	____	____	C62 D62
40-44	____	____	C63 D63
45-49	____	____	C64 D64
50-54	____	____	C65 D65
55-59	____	____	C66 D66
60-64	____	____	C67 D67
65-69	____	____	C68 D68
70 & over	____	____	C69 D69

		Cell Address
Month of Starting Date: (January - December)	________	A74
Is there a partial first year (Y or N):	________	A78
Traditional vs. Managed Care Plan: (1 = Traditional Plan, 2 = Managed Care Plan)	________	A82
Amount of Fixed Expenses:	________	A86
Percentage of Premium Expenses (Decimal Form):	________	A90

Appendix II

SPECIFIC STOP LOSS RATE CALCULATION

EMPLOYERS NATIONAL

		EMPLOYEE	COMPOSITE DEPENDENT
BASE RATE			
AREA =	A		
ACTIVELY AT WORK PROVISION	IN EFFECT		
AMOUNT OF SPECIFIC STOP LOSS (DEDUCTIBLE AMOUNT)	5000		
PAYMENT PERIOD (12/15 MUST BE USED TO CALCULATE THE BASE RATE FOR 12/18, 12/24, AND UNLIMITED)	12/12		
NO PARTIAL FIRST YEAR			
BASE RATE		16.01	21.49
ADJUSTMENTS			
OUT-OF-POCKET MAXIMUM	1,200	0.00	0.00
MAXIMUM BENEFIT	1,000,000	0.00	0.00
ADJUSTED RATE #1 (BASE RATE)+(OUT-OF-POCKET MAXIMUM ADJUSTMENT) +(MAXIMUM BENEFIT ADJUSTMENT)		16.01	21.49
EXTENDED BENEFITS FACTOR		1.000	1.000
AGE/SEX FACTOR		1.000	1.000
PAYMENT PERIOD FACTOR FOR	12/12	1.000	1.000
TREND FACTOR - STARTING MONTH -	JULY YEAR - 1989	1.000	1.000
ADJUSTMENT FOR	TRADITIONAL PLAN	1.000	1.000
ADJUSTED RATE #2 (ADJUSTED RATE #1)x(EXTENDED BENEFIT FACTOR)x(AGE/SEX FACTOR)x (PAYMENT PERIOD FACTOR)x(TREND FACTOR)x TRADITIONAL/MANAGED CARE PLAN FACTOR)		16.01	21.49
FIXED EXPENSES	0		
%PREMIUM EXPENSE	0.0%		
FINAL RATE (ADJ RATE #2+FIX EXP)/(1-%PREM EXP)		16.01	21.49

Prepared by PATTI WARDER
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