

Activities, Games, and Motivational Devices  
for the Secondary Mathematics Classroom

A Creative Project (ID 499)

by

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## FOREWORD

This collection of activities, games, and motivational devices is intended for use in secondary mathematics classes. It is my intention and hope that such items will add variety to my daily lessons and encourage enthusiasm for learning on the part of my students.

This compilation is by no means complete but is rather a sampling of ideas upon which I hope to build throughout my years as an educator. Its present finiteness is due to the time limit imposed. I do intend, however, for its length and scope to increase directly as my time and experience as a teacher.

Ann J. Wickersham

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MIDDLE SCHOOL MATH

OR

HIGH SCHOOL GENERAL MATH

(3)

### FRACTION "GO FISH"

This card game is an adaptation of a traditional card game to help encourage students to review work with fractions.

The deck consists of any number of cards desired. These can be made easily by cutting ordinary 3 x 5 inch index cards in half. On each card, write one common fraction such as  $1/2$ ,  $1/4$ ,  $3/4$ ,  $6/8$ ,  $5/10$ ,  $1/3$ ,  $4/12$ . Be careful not to reduce any of the fractions to lowest terms. Have as many pairs of equivalent fractions in the deck as possible. Be certain that every fraction has at least one equivalent form somewhere in the deck. Provide each student with scrap paper and a pencil for computation.

#### Instructions

Each pupil playing the game should receive either five or seven cards, depending upon the number of players. Limit the number of participants to four. The balance of the deck is placed face down in the center of the playing area.

All players begin by discarding any pairs of equivalent fractions they may have in their hands. These are placed face up in front of the players.

The first player (to the left of the dealer) asks any other specific player if he has a particular card to enable him to make a pair. For example, player number one might have a card with  $15/20$  on it. He might ask player number three, "Do you have a card with  $3/4$  on it?" If player number three has a card with any equivalent form of  $3/4$  on it, he must give it to player number one who asked for it. This gives him a pair, which he can discard. He then may continue in a similar manner until he asks for a card that a player does not have.

When the player asked does not have a requested card, the player asking must "go fish" from the deck in the center, picking the top card. If this card makes a pair with one card in his hand, he may discard the pair, but his turn does not continue. Play passes to the player to the left.

The winner of the game is the first to discard all the cards in his hand by throwing out pairs of equivalent fractions. If no one succeeds in discarding all his cards, the winner of the game is the player with the fewest cards remaining in his hand when the deck in the center is used up.

#### Extensions

A more difficult variation of this game is to have students discard only pairs of cards whose sum is one. Thus, a player might discard a card having  $15/20$  on it if he could find one with  $1/4$  on it.

(3)

### FRACTION "WAR"

Fraction "War" is a card game that can be used successfully to review and strengthen work in fractions, especially in comparing their size. It involves finding a common denominator and changing the given fractions to equivalent fractions. The game can be played by two, three, or four students at one time. (And do not expect a silent classroom!)

The deck consists of 66 cards, each approximately three inches by two and one-half inches in size. (These can easily be made by cutting ordinary 3 x 5 inch index cards in half.) On each card, write one of the following common fractions, being careful not to reduce to lowest terms:

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$
$\frac{5}{6}$	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{4}{8}$	$\frac{5}{8}$	$\frac{6}{8}$	$\frac{7}{8}$
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$
$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{1}{11}$	$\frac{2}{11}$	$\frac{3}{11}$	$\frac{4}{11}$	$\frac{5}{11}$	$\frac{6}{11}$	$\frac{7}{11}$	$\frac{8}{11}$	$\frac{9}{11}$	$\frac{10}{11}$	$\frac{1}{12}$
$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$				

Each player also has some scrap paper and a pencil with which to make his/her computations.

### Instructions

The deck is shuffled and dealt face down to the players in turn, until all the cards have been distributed.

Each player keeps his cards face down in front of him.

Each player turns his top card face up. The players must then decide which card shows the fraction having the greatest value. The scrap paper is used to do this.

The player whose card has the fraction with the highest value takes all the cards that were turned up and places them face down at the bottom of his pack of cards.

In the event of a tie (two equivalent fractions are turned up, such as  $\frac{2}{3}$  or  $\frac{8}{12}$ ) a "war" is declared. Each of the players involved in the war places the next three cards from his pack face down. He turns over the fourth card. The person with the fraction of highest value now showing takes all the cards involved in the "war".

(FRACTION "WAR")

Play continues in a similar manner until one player has lost all his cards or until time is called. In either case, the winner is the player with the most cards at the end of the game.

(6)

THE FORTUNE OF CARMIDES

Name \_\_\_\_\_

Medical practice in ancient Rome was partially based on religion and magic. Later, Greek doctors came to Rome seeking their fame and fortune. Many Greeks who came used herbs, oils, and drugs. Under the practices of Asclepiades, Galen, and Xenophon, Roman medicine improved and earned respect and support from the Roman emperors. Soon there were many doctors to treat every level of society. One group of doctors cared for the army and navy, another for the wealthy, and a different group for the poor.

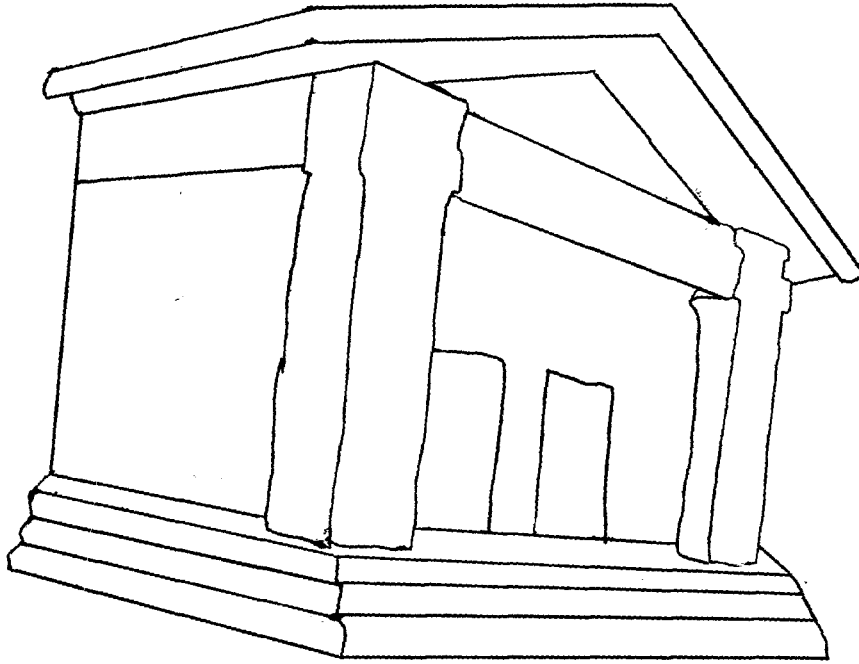
To learn about one doctor, Carmides, who made a quick fortune in Rome, work the problems below.

FIRST, solve the problems and locate each answer in the table. SECOND, place the letter by the answer in the blank next to the problem.

THIRD, match the numbers by the picture with the corresponding letters.

- |  |  |
|--|--|
| 1. _____ $\frac{1}{2}$ of 4                          | 11. _____ $\frac{11}{12}$ of 9                                       |
| 2. _____ product of $\frac{1}{16}$ and 4             | 12. _____ divisor of 9 63  |
| 3. _____ $\frac{1}{4}$ divided by $\frac{1}{32}$     | 13. _____ product of $\frac{2}{3}$ , $\frac{7}{8}$ , & $\frac{4}{5}$ |
| 4. _____ remainder of 8 5                            | 14. _____ 119 7  |
| 5. _____ $3\frac{1}{2}$ times $\frac{4}{5}$          | 15. _____ $\frac{1}{32}$ divided by $\frac{1}{4}$                    |
| 6. _____ $\frac{4}{5}$ of 25                         | 16. _____ remainder of 63 16   |
| 7. _____ 119 17                                      | 17. _____ $\frac{5}{12}$ of $9\frac{3}{5}$                           |
| 8. _____ product of $\frac{3}{8}$ and $1\frac{2}{3}$ | 18. _____ product of $2\frac{1}{2}$ and $\frac{2}{5}$                |
| 9. _____ remainder of 119 7                          | 19. _____ $\frac{3}{16}$ of 24                                       |
| 10. _____ dividend of 119 7                          |  |

A = 3	E = $2\frac{4}{5}$	L = 4	P = $\frac{1}{4}$	V = 15
B = $\frac{7}{15}$	F = $4\frac{1}{2}$	M = 20	R = 63	W = 7
C = $\frac{1}{8}$	H = 17	N = 119	S = $8\frac{1}{4}$	Y = 1
D = $\frac{5}{8}$	I = 0	O = 2	T = 8	



14-5 14-4-8 1-10-17-18 1-10-5 3-12-5-4-3-6-5-10-3:  
4-10 9-15-5 15-1-17-8 13-4-3-14 5-16-5-10 9-10  
3-14-5 15-1-17-8-5-11-3 2-4-12-3 1-19 7-9-10-3-5-12.

(2)

### 50-METER FIELD GOAL

Recently, professional football owners, players, and officials have been discussing the role of the field goal in professional football since the field goal, as a means of scoring points, is dominating the other ways to score. The controversy can be used to motivate the consideration of a metric football field.

#### Instructions

If possible, let the entire class visit a football field because many students have never had the first-hand experience of standing on a football field to get a feeling of its size. Kicking a field goal looks easy when you are in the stands, but looking at the goal posts from the 30- or 40-yard line gives a person a deeper respect for the field goal kicker's ability.

A discussion of the field goal controversy before this activity is begun would be helpful. Many boys realize that the hash marks on the field were moved in through a recent rules change. This means that the angle of the field goal has been narrowed; thus, the field goal is more likely to be successful. Also, many football "experts" feel that if defensive alignments such as the zone pass defense were outlawed the importance of the field goal would be reduced because more touchdowns would be scored. Finally, the goal posts, in professional football, are on the goal line. Some people would like to see the goal posts moved to the back of the end zone to increase the distance that field goals would be kicked from any given yard marker. All of this discussion will naturally lead you to interject the necessity of someday changing the 100-yard football field to metric measures.

50-METER FIELD GOAL

Name \_\_\_\_\_

1. What is the length of a football field using metric measure?
2. What is the width of a football field using metric measure?
3. 10 yards = \_\_\_\_\_ meters
4. The depth of the end zone from the goal line to the back of the end zone is how many meters?
5. At the football field, measure the length and width of the area inside the stadium. Would it hold a football field which measures 50 meters wide and 100 meters long plus 10-meter end zones on both ends?
6. If your area would not allow changing to a 100-meter field, which would be best, a 90-meter field with 10-meter end zones or an 80-meter field with 10-meter end zones?

Why?

7. If you could change the rules of football, how would you alter the rules to reduce the importance of the field goal? Remember, changing to the metric system should play a part in your answer.

(2)

### STANDING METRIC JUMP

This activity allows the student to measure length in the metric system in a recreational activity. This may be a noisy activity, and for safety purposes requires a 15-meter by 10-meter clear area. Thus, it is often an outdoor activity.

#### Instructions

Three meter sticks and a record chart will be needed. Place the three meter sticks on the floor or ground end to end. Have the students place both feet at the beginning of the first meter stick with toes even with the end of the stick. The student is asked to jump in a line parallel to the meter sticks by taking off from both feet at once. The student then records his/her jump distance on the metric chart.

#### Extensions

This activity may be included in a "Metric Olympics" by combining it with Test of Metric Strength, How Many Cubes?, and Pour Metric.

(2)

### TEST OF METRIC STRENGTH

This activity allows the student to measure area in the metric system in a recreational activity. Since this activity is noisy, the students will work best on the floor or outdoors on a smooth flat surface.

#### Instructions

Materials needed are modeling clay and centimeter graph paper. The students will first roll the clay into a smooth spherical ball, then place the ball in the center of the graph paper. The students will each do this and then hit the ball one time as hard as they can. With a pencil, they will draw an outline of the shape of the flattened clay, and peel the clay from the paper. Next, they must count the number of squares which were covered by the flattened clay, estimating for the partially covered squares.

If a contest is desired, it is best to separate the results of the boys and the girls to be fair.

#### Extensions

This activity can be a part of the "Metric Olympics" to motivate the study of the metric system by using it with Standing Metric Jump, How Many Cubes?, and Pour Metric.

(2)

### HOW MANY CUBES?

The students will estimate the number of centimeter cubes needed to fill a given container (similar to guessing how many beans are in a jar.)

#### Instructions

Materials needed are a handful of centimeter cubes, a box (from a grocery item), and a box into which students will place guesses.

Place the box for which the volume will be estimated on a table in the classroom, with the centimeter cubes near the box in a pile. Do not, however, let the students "measure" with them. Have each student place his/her estimate of how many cubes would be needed to fill the box in the "guess box."

At the end of a designated period of time, allow the students to fill the box with centimeter cubes to determine the volume. Select the closest estimate, announce the winner, and award a prize.

#### Extensions

This activity could be included in a "Metric Olympics" for those students who are not highly motivated by physical activities. If this is done, use it with Test of Metric Strength, Standing Metric Jump, and Pour Metric.

(2)

### POUR METRIC

This activity is designed as a competitive task which will motivate and introduce liquid measure in the metric system.

#### Instructions

Divide the class into teams of four or five students each. Give each team three containers or beakers with capacities of 800 milliliters, 500 milliliters, and 300 milliliters. Fill the 800 ml container with water. The students' task is to pour the water into the other containers so that one container has 400 ml in it. But when they pour the water, they must either empty the container from which they are pouring or fill the container into which they are pouring. In other words, they may not estimate.

The students should record their steps as follows:

(800 ml., 0, 0)

(0, 500 ml., 300 ml.)

The answer with the fewest number of steps wins.

#### Extensions

If this activity is used with Standing Metric Jump, Test of Metric Strength, and How Many Cubes?, it may become a part of the "Metric Olympics."

If an alternate problem is desired, try using 600 ml., 500 ml., and 300 ml. beakers to achieve 400 ml. in one beaker. (Of course, other examples may be tried.)

(2)

### METRIC OLYMPICS

Students who are apprehensive about learning a new system of measurement need to be introduced to the metric system in a non-threatening manner. This activity will allow students to become familiar with the metric system of linear, square, cubic, and liquid measurement.

#### Instructions

Divide the class into five teams and set forth the following rules:

1. Every team must enter every event.
2. Every team member must participate in at least one event.
3. The scoring will be 6 points for first place in an event, 4 points for second place, 3 points for third place, 2 points for fourth, and 1 point for fifth.

The order of events will be:

1. Standing Metric Jump (two team members and use successive jumps)
2. Test of Metric Strength (two team members with one hit each)
3. How Many Cubes? (two team members consult for one answer)
4. Pour Metric (team event)

Two hours of class time should be allowed to complete the entire Metric Olympics.

(2)

### GUESS YOUR MASS

In this recreational lesson the students will have the opportunity to estimate and measure mass in metric units.

#### Instructions

A one kilogram mass and a bathroom scale will be needed for this activity. Have each student lift the kilogram mass, and let each record his/her estimate of his/her weight.

After all students have estimated their weights, let them all weigh themselves (allow for some privacy here so that students are not embarrassed.) Also, the dial on the scale needs to be replaced with one which will give the students their weight in kilograms. The students will record their actual weight on the paper with their estimates.

The papers can be turned in (without students' names) and the data used in making graphs.

#### Extensions

This activity could become a part of the "Metric Olympics," but care should be taken not to embarrass any of the students.

(2)

### SPEED TRAP

The students will develop and use a speed trap, the principle of which is similar to those used by the Highway Patrol.

#### Instructions

The students will work in pairs in this out-of-class activity. Each pair will need a tape measure or yard stick, a stop watch, pencil, paper, and a stretch of road with suitable traffic. Each student will receive a work sheet to record data and make conclusions.

Make sure to caution students as to the necessary care to be exercised when near the highway.

SPEED TRAP

Name \_\_\_\_\_

Partner \_\_\_\_\_

To establish your speed trap, first decide how long it will be (at least 100 yards long.) Select a straight stretch of road, measure your distance, and mark the beginning and end of the trap.

To operate the trap, a person should be at each end ready to signal when a car enters and leaves the trap. One person needs a stopwatch to time the car. When considering the car's entrance and exit from the speed trap, be sure to use the same part of the car. For example, use the front bumper.

Time at least 15 cars and record your data below.

CAR	TIME WITHIN TRAP	RATE OF SPEED
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

How long is your speed trap? \_\_\_\_\_

What is the formula for finding distance? \_\_\_\_\_

Rewrite this equation to find the rate of speed given the distance and the time.  
\_\_\_\_\_

(SPEED TRAP)

Convert the times in seconds to times in hours.

For example:

$$4 \text{ seconds} = 4 \text{ seconds} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{4}{3600} \text{ hours}$$

$$\text{so } 4 \text{ seconds} = \frac{1}{900} \text{ hours}$$

Convert your distance in yards to distance in miles.

For example:

$$100 \text{ yards} = 100 \text{ yards} \cdot \frac{1 \text{ mile}}{1760 \text{ yards}} = \frac{100}{1760} \text{ miles}$$

Find the rate of speed for each car.

What is the average speed of the cars that pass through your speed trap?

(2)

### POUR IT ON

This activity, involving pouring water or sand, is designed to show a way to generate a variety of equations for one problem.

#### Instructions

The needed materials are four graduated cylinders, water or sand, and a dittoed sheet with directions for the students.

The student is, through some combination of the given amounts in containers A, B, and C, to arrive at the amount listed under the desired result and then write an equation for the way he arrived at the result. Comparison of equations by different students will show that there are many ways of obtaining the desired solution from the given amounts. Actually pouring materials should encourage getting the most direct solution.

It is possible that a student would follow a pattern rather than look for another solution; this is not always the best thing to do. For instance, examples 2, 3, and 4 are all solvable with the equation  $B - 2A + C = R$ , a usable pattern in example 5. This pattern does work in number 5, but a much easier one would be  $2C = R$ .

One advantage of providing a set of examples all solvable by the same equation is that the slower student has a "built-in" help that permits him to do all the problems correctly.

#### Example problem

<u>GIVEN AMOUNTS</u>		<u>DESIRED RESULTS</u>	
<u>A</u>	<u>B</u>	<u>C</u>	<u>R</u>
24	2	5	17

Possible solutions:

$$A - B - C = R$$

$$5C - 4B = R$$

$$6B + C = R$$

$$B + 3C = R$$

POUR IT ON

Name \_\_\_\_\_

Often there are many different ways of getting an answer to a problem. In this activity you are to try to find a number of ways of getting a given result and write an equation for each way that works.

PROBLEM	GIVEN AMOUNTS			DESIRED RESULTS
	A	B	C	R
1	28	2	5	19
2	7	24	3	13
3	19	61	5	28
4	20	29	24	13
5	23	49	3	6
6	8	27	4	15
7	4	3	2	0
8	8	5	10	6
9	11	4	6	13

(2)

PIECES OF %

This activity allows students to determine "parts of one hundred" on the concrete level.

Instructions

Each student will need one piece of ten by ten centimeter graph paper and several smaller pieces of graph paper which can be cut to represent various percentages.

Each student should obtain the six sizes of paper indicated and number them as below:

- 1 - 10 x 10 square
- 2 - 1 x 1 square
- 3 - 10 x 10 square with one 1 x 1 square removed
- 4 - 1 x 4 rectangle
- 5 - 5 x 5 square
- 6 - 3 x 10 rectangle

The students will use piece #1 as their standard shape. Through manipulating the other pieces on top of this first piece, they will fill in the chart on the student activity sheet.

PIECES OF %

Name \_\_\_\_\_

Obtain six pieces of graph paper which are the sizes indicated below and number them accordingly.

- # 1 - 10 x 10 square
- 2 - 1 x 1 square
- 3 - 10 x 10 square with one 1 x 1 square removed
- 4 - 1 x 4 rectangle
- 5 - 5 x 5 square
- 6 - 3 x 10 rectangle

Use piece # 1 as your standard shape. Place the smaller pieces of graph paper on top of this paper one by one and fill in the chart below.

Piece	Number of Squares	No. of Squares in Piece #1	Fraction of #1 Covered
2			
3			
4			
5			
6			

There is another way to write fractions with a denominator of 100. The sign, %, means part of 100 and is read "percent." So, piece 2 represents  $1/100$  or 1% of piece 1.

Write the fractions for pieces 3, 4, 5, and 6 as percents.

- Piece 3 99/100 \_\_\_\_\_
- Piece 4 \_\_\_\_\_
- Piece 5 \_\_\_\_\_
- Piece 6 \_\_\_\_\_

Now using a clean sheet of graph paper, make pieces of paper that show the following:

- 11%, 23%, 85%, 7%, 45%, 36%, and 66%

(2)

## BUY RIGHT

Students need to become more aware of the uses of mathematics in the world of the consumer. Although stores often list comparative prices, many people do not use them, are not aware of them, or do not know how they are derived.

### Instructions

The students need to realize that they are getting a price for one unit (usually ounce) for each size and then compare those prices to see which is the best value.

When the student considers the best value, he needs to look at related influencing factors. For example, perhaps the larger size is less expensive per unit but, if it is not all used before spoiling, is it practical to purchase the larger size?

Although this activity can be done either as an individual activity or as a group, the motivation for this project could be enhanced by visiting a store. Newspaper advertisements could be used if a store trip is not practical. Newspapers could also be used to make comparisons between different stores that list the same brand but different sizes.

Each student will need a dittoed activity sheet.

BUY RIGHT

Name \_\_\_\_\_

Do you consider yourself a smart shopper? Your answers to the questions below should help you decide.

Is the largest size of an article always the best buy? \_\_\_\_\_

Select an item which comes in at least three sizes and note the price of each. Also, record the net weight of the product.

Product	A	B	C
Price			
Net Weight			

To find the cost per ounce, divide the cost by the number of ounces (net weight).

	A	B	C
Cost per ounce			

Which is the best buy according to cost per ounce?

Which would be the best buy for your family, considering other factors such as spoilage?

(\*)

METRIC LINGO

Name \_\_\_\_\_

The words listed at the bottom of the page are hidden in the letters below. Circle all of the ones you can find, then give a short definition by the listed words.

N R S T B A L K E H P F M E  
B K O L Y V O L U M E D Y R  
A P K I R Z A K T S B C A J  
R G M E T E R D P U N I T G  
C D A K J O E S E D F R A O  
K E R N M C A P A C I T Y V  
H F G Z Y E E R I T I E L I  
V S Q U I N T L S P J M H S  
E A R C O T L X E Z E C F J  
W T E E N I B D S N L P B C  
T J T V M R U E E R G E D U  
O L I K B A C K M B X T O M  
C E L S I U S A W E I G H T

LENGTH  
CAPACITY  
WEIGHT  
AREA  
VOLUME  
METER  
LITER  
GRAM  
KILO

HECTO  
DEKA  
UNIT  
DECI  
CENTI  
MILLI  
CELSIUS  
DEGREE  
METRIC

(\*)

DOES IT MEASURE UP?

Name \_\_\_\_\_

Hidden in the letters below are the words listed at the bottom of the page. First, circle all of the ones you can find, then give a brief definition by the listed word.

S E C O N D Q U M W V N E B R V T  
T C Y X H T N O M U T O N Z P I L  
P A D A Y O S D I F G K L T N H C  
I T R B R N E M L W L Q E U S T R  
N F A H R E N H E I T B D O M I A  
T I Y G A P O L I K F Z T I M E S  
C A K M R H L E N G T H N O R F U  
H P G U P I L F C D G U S A J E D  
F O O T Q N A E H I T R A V C X M  
B H S U F L G K E E W P S N A V O  
L C I T N R Z W S T R A U Q T P N  
K D Q U N D E G R E E O H Y E A R

HOUR  
LENGTH  
LIQUID  
WEIGHT  
TIME  
AREA  
FOOT  
INCH  
GALLON  
QUART  
POUND  
OUNCE  
MINUTE

FAHRENHEIT  
DEGREE  
WEEK  
VOLUME  
YARD  
DAY  
YEAR  
UNIT  
SECOND  
PINT  
MONTH  
TON  
MILE

PROBABILITY

(2)

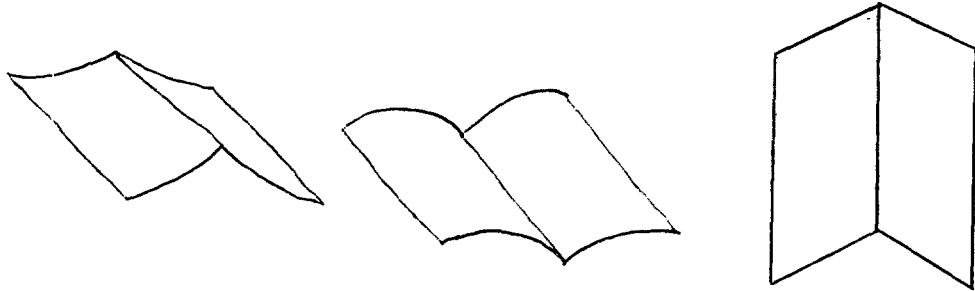
## PAPER DROP

In this activity a piece of paper or an index card is used in two ways to allow the student to experiment with probability.

### Instructions

Divide the class into groups of four or five students each. Each group will need a piece of paper about  $5\frac{1}{2}$ " x  $8\frac{1}{2}$ " and a piece of tape. Each student will need a pencil and a dittoed student activity sheet.

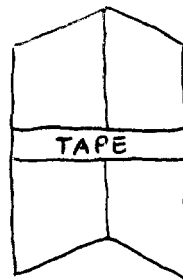
Have each group fold their piece of paper in half so that the edges form approximately a 90 degree angle (an angle formed by two planes is called a dihedral angle.) They will then drop the paper from a height of about two meters. There are three feasible outcomes for how the paper will land, each of which is illustrated below.



If the paper lands on edge, the crease line will be perpendicular to the surface. Have the students drop the paper 20 times and record the results.

Using the results from dropping the paper 20 times, have the students predict the probable outcome for each possibility if the paper were dropped 10 times and record those predictions. They will then check these predictions by dropping the paper 10 more times, using the same procedure as before and recording the results.

For the second part of the experiment, place a piece of tape as shown below on the paper. The tape should hold the paper at a right angle. Ask the students if they think the tape will change the probability of the paper landing a certain way and if this will bias the results. Drop the paper 20 times and record the results.



(2)

## SHAKE 'EM UP

This is a probability activity that gives the students experience in predicting and testing probabilities.

### Instructions

The materials needed are a shoe box (with a lid) and 12 markers or chips--three red, four white, and five blue. Also, each student should have a duplicated activity sheet.

To begin, the students will estimate how many of each color of the chips will be drawn and record these estimates. If it would be expected that three reds would be drawn out of 12 tries, then three times three or nine reds would be expected if 36 draws are to be made, since 36 is 12 times three.

All of the chips should be placed in the box and shaken. Hold the box above the head of the person who will draw a chip out of the box. Once the chip is drawn, record the color of the chip and replace the marker.

### Extensions

Variations of this activity could include drawing without replacement and considering how the probability will change for each color as the individual color numbers and the totals change. Consideration could also be given to drawing two chips at a time with replacements and the probability of drawing two of the same color for each color as well as drawing two different colors for all possibilities without paying attention to order.

Better students could be challenged by a situation involving drawing two chips where order is significant. Note that this activity can easily be done by individuals.

SHAKE 'EM UP

Name \_\_\_\_\_

1. Of the 12 markers or chips, what fractional part of the total number is red?
2. What part is white?
3. What part is blue?
4. What is the sum of these three fractions?

If 36 draws are to be made out of the box, in the table below estimate how many of each color will be drawn.

COLOR OF MARKER	ESTIMATED RESULTS IN 36 DRAWS	OBSERVED RESULTS IN 36 DRAWS
RED		
WHITE		
BLUE		

Record the actual results of the drawings in the right column above.

5. How close did your experimental results come to the predicted results?
6. What do you suppose the results would be if the number of drawings were increased more and more? Why?

(2)

## PAPER CUP PROBABILITY

Frequently students are asked to predict outcomes of situations in which the results are "known" because of the objects used. In this activity, the students will be confronted with a situation in which the expected outcome is an unknown and must be established.

### Instructions

Divide the class into groups of four or five students each. Each group will need a paper cup, and each student will need an activity sheet.

The paper cup is to be dropped from about 4 feet above the floor and the manner in which it lands is to be recorded. When the cup is dropped, there are three realistic ways it can land (outcomes): top, bottom, and sideways. Each drop is called a trial. After all trials are completed, the tally marks for each position are counted and this total is called the frequency for that landing position.

### Extensions

Results can be compared for different cups. For example, is the probability of a McDonald's cup landing on its side greater than, less than, or equal to the probability of a Burger King cup landing on its side?

After the initial trials, permit the students to alter the cups by cutting off the top or bottom or by cutting holes in the side to get different results.

PAPER CUP PROBABILITY

Name \_\_\_\_\_

Complete the chart below for 30 trials of dropping a paper cup.

OUTCOME	TALLY	FREQUENCY	PART OF TOTAL TRIALS
TOP			
BOTTOM			
SIDE			

Compare your results with those of others. Are the results similar?

Why did your comparison come out the way it did? Did the manner in which you dropped the cup influence the outcome?

Using your first trials as data, predict what would happen if you repeated the experiment.

In \_\_\_\_\_ out of 30 tosses, the paper cup would land on top.

In \_\_\_\_\_ out of 30 tosses, the paper cup would land on the bottom.

In \_\_\_\_\_ out of 30 tosses, the paper cup would land on its side.

Test your prediction using the same procedure as above.

OUTCOME	TALLY	FREQUENCY	PART OF TOTAL TRIALS
TOP			
BOTTOM			
SIDE			

(2)

## DRAW

Rather than discussing probabilities and when they occur, students should have the opportunity to experience the concept development. In this activity, the student will draw objects out of a container, knowing the contents of the container. However, there will be a structured sequence to be followed as to what objects will be in the container.

### Instructions

Materials needed are five objects of each of three colors but the same size and shape, an opaque container, and dittoed student activity sheets.

I. Place five red objects in the container. Have students determine that the probability of selecting a red object is one and the probability of selecting a white object is zero. (Zero probability comes from an impossible situation.)

II. Put one white object and one red object in the container. Since two objects can be drawn, this is the number of possible outcomes. Have the students determine the probability of drawing a red object here ( $\frac{1}{2}$ ). Do the same for the white object ( $\frac{1}{2}$ ). Have the students make 20 drawings to see how close to the expected probability they come.

III. Place five red and five white objects in the container. The probability of a red is  $\frac{5}{10}$  or  $\frac{1}{2}$ , and the same is true for the white object. Draw with replacement one object at a time from the container 30 times to see how closely they can get to a probability of  $\frac{1}{2}$  for red objects.

DRAW

Name \_\_\_\_\_

I. Five red objects in the container.

1. What is the probability of selecting a red object? \_\_\_\_\_
2. Of selecting a white object? \_\_\_\_\_

II. One white object and one red object in the container.

3. What is the number of possible outcomes? \_\_\_\_\_
4. What is the probability of drawing a red? \_\_\_\_\_
5. Of drawing a white? \_\_\_\_\_

Draw	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Color Drawn																					

6. What was the ratio of red objects to total trials? \_\_\_\_\_
7. White objects to total trials? \_\_\_\_\_

III. Five red and five white objects in the container.

8. What is the probability of choosing a red? \_\_\_\_\_
9. Of choosing a white? \_\_\_\_\_

Draw	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Color Drawn															

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30

(5)

### COIN TOSSING

What is the probability that when two coins are tossed, both fall heads? Repeated tosses emphasize the notion of probability in the long run. As more trials are performed the computed experimental probabilities tend to level off at the correct value. This is vividly illustrated by the line graph and by the cumulative fractions and percents in the table.

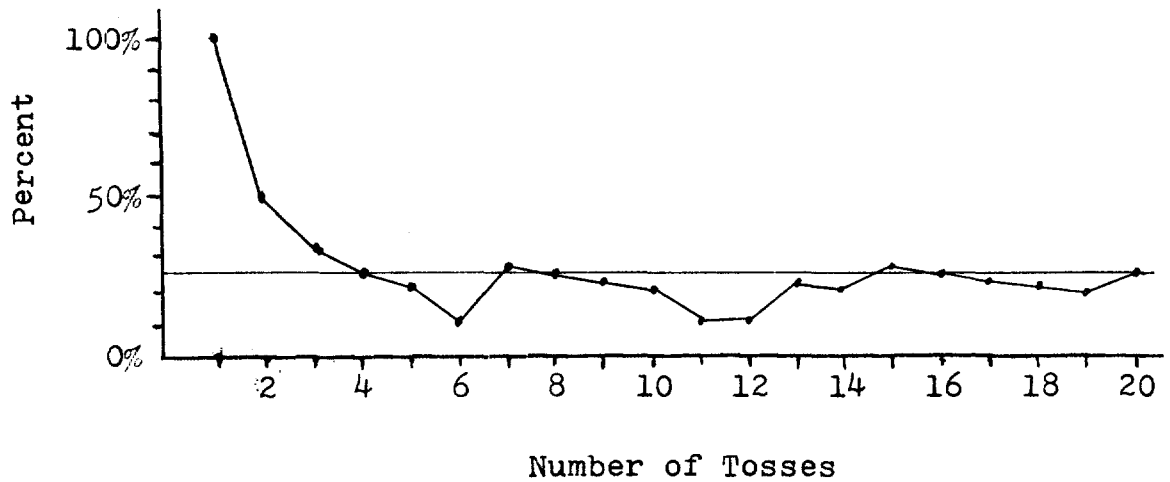
#### Instructions

Students first guess at the answers. Then tossing two coins at a time, they compute the ratio of successes (2 heads) to trials and the corresponding cumulative percents. These percents are then plotted on a graph for each of 20 successive trials. Typical results might look as here:

Toss	1	2	3	4	5	6	7	8	9	10
Two Heads	X						X			
Not Two Heads		X	X	X	X	X		X	X	X
Rate of Successes To Total	$\frac{1}{1}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{2}{7}$	$\frac{2}{8}$	$\frac{2}{9}$	$\frac{2}{10}$
Percent of Successes	.000	.500	.333	.250	.200	.100	.285	.250	.222	.200

11	12	13	14	15	16	17	18	19	20
		X		X					X
X	X		X		X	X	X	X	
$\frac{2}{11}$	$\frac{2}{12}$	$\frac{3}{13}$	$\frac{3}{14}$	$\frac{4}{15}$	$\frac{4}{16}$	$\frac{4}{17}$	$\frac{4}{18}$	$\frac{4}{19}$	$\frac{5}{20}$
.181	.166	.230	.214	.266	.250	.235	.222	.210	.250

(COIN TOSSING)



The original guesses can then be compared with the collected data and modified, based upon the experimental evidence.

Analysis

Students may first guess the probability to be  $1/3$ , arguing that the coins can fall 2 heads, 1 head, or 0 heads. However, the correct answer is  $1/4$ , since only one of these four equally likely possible outcomes is a success.

HH HT TH TT

The greater the number of repetitions, the more likely the graph will tend toward this value.

ALGEBRA

(5)

## NIM

Although Nim may be used purely as recreation at any time, it is best presented after the student has some knowledge of the binary system of numeration, thus providing an interesting application of that topic. The game is played by two students at a time and is probably best introduced by having the teacher challenge a student to a match.

### Instructions

Many versions of the game can be played. One of the easiest starts with three piles of chips. Pile A contains 3 chips, pile B contains 4 chips, and pile C contains 5 chips. At a particular player's turn he selects one pile of chips and removes as many chips as he wishes from that one pile. He must, however, remove at least one chip. Players alternate, and the player who picks up the last chip on the board is the winner.

### Strategy

Students may develop informal strategies for winning. The formal winning strategy is quite complex and consists of writing the number of chips in each pile in binary notation. In order to win, one must be certain that the sum of the digits in each binary place is even after his move. To illustrate this principle, here is an example of a game where the first player wins:

Beginning set up:

A	XXX	In binary notation:	11
B	XXXX		100
C	XXXXX		101

First player takes two chips from pile A.

A	X	In binary notation:	1
B	XXXX		100
C	XXXXX		101

Second player takes three chips from pile C.

A	X	In binary notation:	1
B	XXXX		100
C	XX		10

(NIM)

First player takes one chip from pile B.

A	X	In binary notation:	1
B	XXX		11
C	XX		10

Second player takes both chips in pile C.

A	X	In binary notation:	1
B	XXX		11

First player takes two chips from pile B.

A	X	In binary notation:	1
B	X		1

Second player takes one of the chips, and the first player wins on his next move by taking the remaining chip.

Note in the above game that the first player always made a move that left the second player with an array of chips whose number in binary notation had an even number of 1's in each column

### Extensions

The game can also be played using the rule that the one who is forced to take the last chip is the loser. It can also be played with an indefinite number of chips in each of the original three piles, although the strategy for winning remains the same.

(5)

### TAC TIX

Tac Tix is a game designed for two players and is a variation of the game of Nim.

#### Instructions

Arrange 16 coins or chips, as in the figure. These are numbered here for ease of reference.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

Players alternate removing any number of chips from any single row or column. However, as an additional constraint, only adjacent chips may be removed. For example, if player A removes chips 14 and 15 on his first move, player B may not take 13 and 16 in one move. The player who is forced to take the last chip is the loser.

#### Strategy

It is necessary to play this game so that the one who takes the last chip loses because otherwise the first player would always win. On a 3 x 3 board, using nine counters and following the rules stated above, the first player can win by taking the center chip, or a corner chip, or all of a central row or column. There is no known strategy for winning the game with 16 chips as described above.

#### Extensions

It has been suggested that one can best gain an introduction to this game by solving specific problems. Thus, it seems worthwhile to present Tac Tix problems for students to solve before actually playing the game with an opponent. In the two problems below, find a move that will guarantee a win.

			4
	6	7	8
9	10	11	12
13	14	15	16

	2		
	6		
9	10	11	12
		15	

(5)

### TOWER OF HANOI

This activity requires three disks of decreasing size and three pegs or three areas. The three disks are stacked on one peg or area with the largest item on the bottom and the smallest on top.

#### Instructions

The object of the game is to transfer the disks to one of the other pegs or areas following these conditions:

- (a) Move only one disk at a time.
- (b) No disk may be placed on top of one smaller than itself.
- (c) Use the fewest possible moves.

(With three objects, only seven moves are needed.)

#### Extensions

After students are successful at completing the game using three objects in seven moves, let them attempt the game using four objects. This can be tried using a quarter, nickel, penny, and dime. For four objects, and using the same rules, 15 steps are necessary. For five objects, 31 steps are needed. In general, for  $n$  objects,  $2^n - 1$  steps are needed.

Have the students try to find this general equation on their own. Once they do, the following should prove most interesting:

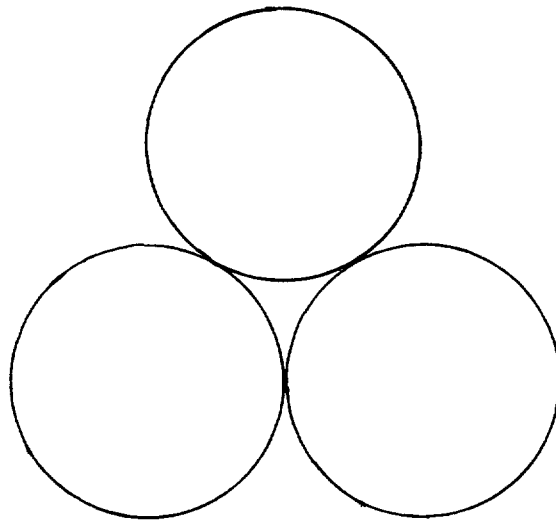
An ancient Hindu legend states that Brahma placed 64 disks of gold in the temple at Benares and called this the tower of Brahma. The priests were told to work continuously to transfer the disks from one pile to another in accordance with the rules set forth earlier. The legend states that the world would vanish when the last move was made. The minimum number of moves to complete this task is:

$$2^{64} - 1 = 18,446,744,073,709,551,615$$

Ask students to estimate this figure before revealing it. Then give the actual number and have students estimate how long this would take at a rate of one move per second. (The world seems safe from destruction by vanishing!)

TOWER OF HANOI

Name \_\_\_\_\_



Number of objects (n)

Steps required (s)

Write a general equation using  $n$  and  $s$  which will tell you how many steps are needed for any number of objects.

(3)

### PEG GAME

The Peg Game is an example of an individual strategy game that can be used with an algebra class. The game consists of a playing surface with eleven congruent square regions.



Colored pieces of construction paper (called FREEBELS) are used on this surface. If the square regions are large enough, coins may be used, such as dimes and pennies, or even small chips of two different colors. For illustration purposes the FREEBELS shall be represented here as:



Red FREEBELS



Blue FREEBELS

Each student should receive a dittoed sheet with the playing surface on it, plus a set of ten FREEBELS, five of each color.

### Instructions

To begin the game, five red FREEBELS are placed in the five squares at one end of the playing surface, and five blue FREEBELS are placed in the five squares at the other end. The middle square region is left open.



The object of the game is to interchange the positions of the red and blue FREEBELS in a minimum of moves. A FREEBEL can be moved in two ways:

- (a) forward, from one square region to the adjacent one.
- (b) by "jumping" over a FREEBEL of the opposite color.

Moves backward are not allowed (it would no longer be a minimum number of moves if we reversed moves.) Jumping a FREEBEL does not remove it from the playing surface.

(PEG GAME)

After a suitable "frustration period," suggest that the game be tried with one red FREEBEL and one blue FREEBEL. The playing surface should be set up this way:



After the pupils have completed the game with one red and one blue FREEBEL, the game should be replayed with two red and two blue FREEBELS and the results should be recorded. The game should be played with three of each color, and then with four of each. The results will give the following pattern:

<u>Number of red FREEBELS (p)</u>	<u>Number of moves required (n)</u>
1	3
2	8
3	15
4	24
5	?

At the same time, a pattern of play should also emerge; this is to keep the FREEBELS in as much of an alternating color pattern as possible throughout the moves. The students should, on the basis of the collected data, then be asked to estimate the minimum number of moves for the full complement of five red FREEBELS. (The answer is 35.) They also should be encouraged to write a mathematical equation using  $p$  and  $n$ , where  $p$  = the number of red FREEBELS used and  $n$  = the minimum number of moves. (The answer is  $p^2 + 2p = n$  or  $p(p + 2) = n$ ).

PEG GAME

Name \_\_\_\_\_

--	--	--	--	--	--	--	--	--	--	--

Number of red  
FREEBELS (p)

Number of moves  
required (n)

Write a general equation using  $p$  and  $n$  which will tell you the minimum number of moves required for however many red FREEBELS are used.

(5)

## THE GAME OF 50

The Game of 50 is a game designed for two individuals. An effective way to introduce the game is for the teacher to announce that he is the world champion Game of 50 player, and challenge a student to try to win. Thereafter, students can play against one another.

### Instructions

The game is played using the numbers 1, 2, 3, 4, 5, and 6. The two players alternate in selecting numbers, and the first to reach 50 wins. As each new number is selected, it is added to the sum of the previously selected numbers. For example, if the student goes first and selects 3, the teacher might then select 6, to give a sum of 9. If the student then selects 5, the total is 14, and it becomes the teacher's turn to go. The game continues in this manner until one player becomes the winner by reaching 50.

### Strategy

Analysis of the game shows that you can always reach 50 if you first reach 43. (Regardless of what number your opponent selects, you can choose a number to obtain 50.) Working backward, you can reach 43 if you can get to 36. Continuing in this way, the following "winning numbers" are obtained:

1, 8, 15, 22, 29, 36, 43, 50

Thus the strategy for winning is to go first and begin with 1. Thereafter, select the complement of your opponent's number relative to 7. That is, if he picks 4, you choose 3, if he picks 2 you choose 5, and so forth. If your opponent goes first, work to one of the winning numbers as soon as you can.

### Extensions

Many variations of this game are possible. A similar game is played by using a set of 16 cards consisting of the four aces, four 2's, four 3's, and four 4's. Players alternate selecting one card at a time from the pile of 16 cards, without replacements. As before cumulative sums are kept. The winner is the first person to select a card that brings the total to exactly 22, or forces his opponent to go over 22. The set of numbers here is {2, 7, 12, 17, 22}. The strategy for winning is to go first and begin with 2. Thereafter, select the complement of your opponent's number relative to 5. That is, if he picks 4, you choose 1, and so forth. However, this is not a fool-proof strategy because the number of cards is limited. Suppose that your opponent repeatedly chooses 3. This would

(THE GAME OF 50)

force you to repeatedly choose 2 and you would run out of 2's prior to reaching the objective number of 22. Can you consider alternative strategies for such a situation?

SPROUTS

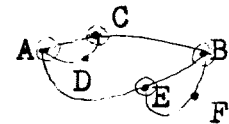
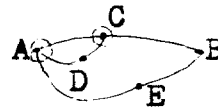
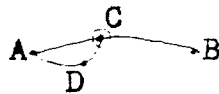
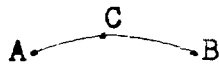
The game of Sprouts is a game for two that is most effectively presented by dividing the class into pairs to play against one another. The one who wins two out of three games can then be declared the winner, and the winners paired against one another again until a class champion emerges.

Instructions

The game starts with two points, labeled A and B in the figure and called spots. Each player takes a turn drawing an arc from one spot to another or to the same spot. He then places a new spot on his arc. For example, here are two possible moves that the first player might make:



The two basic rules of play are that no arc may cross itself or pass through another arc or spot, and that no spot may have more than three arcs from that point. The winner is the last person who is able to draw an arc. Here is an example of a game played; a circle around a spot indicated that there are three arcs at the point and the point is thus no longer in play:



1st player

2nd player

1st player

2nd player

In the game illustrated, the second player wins. The first player cannot draw an arc from D to F because it would pass through another arc. Also, he cannot draw arcs from D to itself (or from F to itself) because then there would be four arcs at the point.

Strategy

Note that each spot becomes a dead spot once it is used three times as the endpoint of an arc. Therefore, the game begins with six possible "lives." The first player uses up two lives, but adds one when he adds a spot to his arc. Therefore, after the first play the game has five lives left. In a similar manner, there are four lives left after the second move, and only one life left after the fifth move. With only one arc available, the game must be at an end. Therefore, the game has a maximum of five possible moves, a small enough number for a student to be able to draw the set of all possible games.

(SPROUTS)

Here is an example of a game that utilizes the maximum number of moves, resulting in a win for the first player:



1st player



2nd player



1st player



2nd player



1st player

Extensions

The game can be played using any number of spots initially. Consider playing the game using three spots, in which case there will be a maximum of eight possible moves. The maximum number of moves possible in four-spot sprouts is 11.

(4)

## "ZERO"

"Zero" is a card game that can be played by any middle or high school student who has received initial instruction in the addition of integers. Besides giving a student practice in the addition of integers, the game also develops the concept of "opposite" and helps students improve their problem-solving strategies.

The game is played much like rummy, using a special deck of integer cards. The object of "Zero" is for a player to get three cards whose numbers have an algebraic sum of zero. Two to four students can play the game with one deck of cards.

### Instructions

Prepare a deck of cards using 42 3" x 5" index cards. Two cards are prepared for all integers from -10 to 10, including 0.

To start play, each player draws a card from the deck, and the player who draws the highest card is designated the dealer. He shuffles the deck and deals three cards, one at a time, to each player. Those cards not dealt out are placed face down in the middle of the playing area to make a blind deck. The top card of the blind deck is turned face up next to it. Play begins with the player on the dealer's left.

If players do not have three cards adding up to zero, they draw either the face-up card or the top card from the blind deck. They then discard a card face up. If left with three cards having a sum of zero, a player says, "Zero" and lays down his cards. If the player does not have zero, play continues to the left until a player gets zero.

Players can draw only the top card from either of the stacks. Zero must be achieved with exactly three cards.

When a player has zero, the play continues to the dealer so that all players have the same number of turns.

Play ends for a hand at the end of a round in which one player has a "Zero." At the end of the round, all players add up their score, and the deal for the next hand passes to the person on the previous dealer's left.

There are two ways to calculate the score:

- I. The winner of the hand receives a score of 0. The other players take the absolute value of the three cards in their hand; their score is the sum of these three.
- II. The winner of the hand receives a score of 0. The other players find the algebraic sum of

("ZERO")

the three cards; the score is the absolute value of this sum.

The game can end either when a time limit is reached, such as two minutes before the end of the class, or when any player has reached a predetermined score, such as 50. The winner is the player whose score is closest to zero when the game ends.

(2)

### DROP 'EM

Often students are encouraged to conjecture, estimate, and predict. Yet, seldom are they provided with immediate data to check their endeavors. This activity will provide the opportunity for rapid verification of estimations.

#### Instructions

Materials needed are a tennis ball, golf ball, handball, or basketball. This activity requires a smooth level surface next to a vertical wall. The students will drop a ball from a designated altitude and then, using successive horizontal sightings, estimate the height of each bounce. The number of bounces used can vary, but at least five should be used.

Different students should be assigned the task of mentally noting the height of the bounces and then, after the last recorded bounce, marking the height of their respective bounces. After a few trial drops the student will know about where his point will be. To eliminate distortion, the students should be encouraged to get on their knees, sit, or squat to provide a horizontal line of sight with the peak of each bounce. More than one recording trial should be made to check the accuracy of the results.

#### Extensions

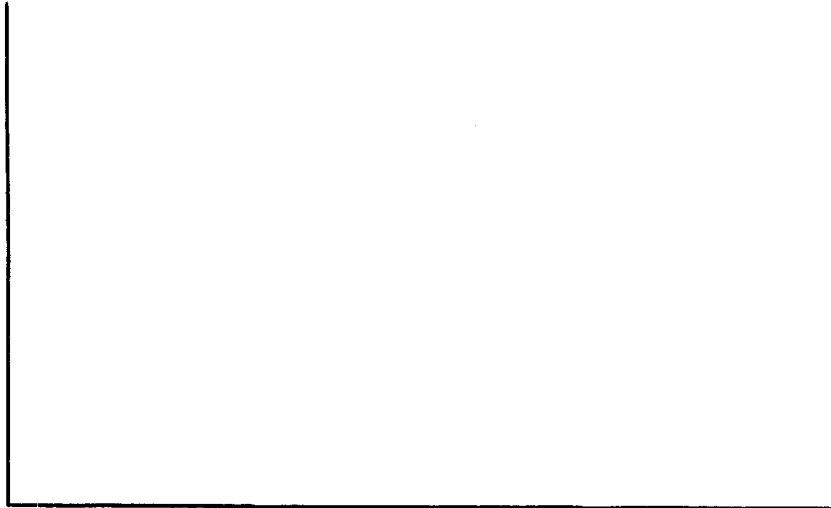
One interesting related activity is to record bounce heights for different types of balls and then compare the results. Can bouncing of different balls be related? Is there a constant ratio between bounce heights one and two for the different balls? Does a constant ratio exist between bounce heights two and three for different balls?

For second year Algebra students, you may wish to have them develop an equation which describes the curve generated by the points on the graph(s).

DROP 'EM

Name \_\_\_\_\_

Graph the results of the bounce heights.



1. Compare the ratio between the height of the first and second bounce with the ratio of the height of the second and third bounce, etc. Is there a constant ratio that exists between the different bounces?
2. Using the information gathered from the first five bounce heights, try to predict the height of the sixth bounce.
3. If the ball were dropped from a point twice as high, would the successive bounces be twice as high? Try it.
4. Is the ratio between the successive bounce heights the same when the ball is dropped from different altitudes?

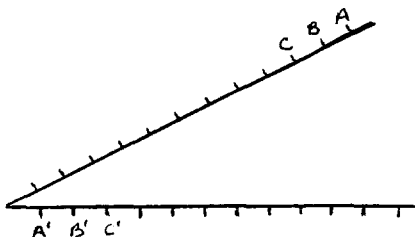
(5)

## CURVE STITCHING

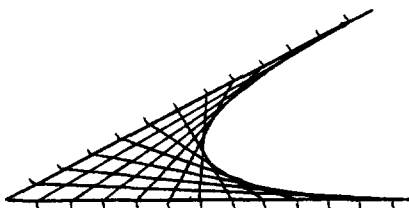
Curve stitching is an enrichment topic of particular interest because it seems to capture the attention of students of varying levels of ability. It is an effective topic to use before a holiday, and can serve as the basis for very dramatic bulletin board displays.

### Instructions

Before allowing students to complete designs of their own, it is well to first have everyone construct one together under the teacher's guidance. A basic one with which to start begins by drawing an angle and marking off the same number of equally spaced units on each side of the angle. In the below figure, 12 units are located and marked.



Next connect point A to point A', B to B', C to C', and so on. The line segments drawn will appear to form a curve called a parabola.



Many different variations are possible merely by changing the angle between the line segments, the distance between points, and by combining several curves.

This activity is called "curve stitching" because the figures can be made by using colored thread and stitching through cardboard. Push up through each point from the back, and then stitch between points in the same manner as you would draw line segments.

### Extensions

It is worthwhile to prepare a school exhibit consisting of a variety of curve-stitching designs made by individual students. A contest can be held with viewers asked to vote for the most attractive as well as the most original design created.

(2)

### RISE AND RUN

This activity explores the concept of slope in a setting familiar to the students.

#### Instructions

This activity is best conducted as an individual project or as an independent study for small groups of students since large groups in the halls are often disturbing to others in the school. Also, in a large group not every student will participate actively in the project.

Each student will need a dittoed activity sheet. The individual students or small groups are to be directed to a specified set of stairs where they will measure using a ruler the rise and the run of the stairs.

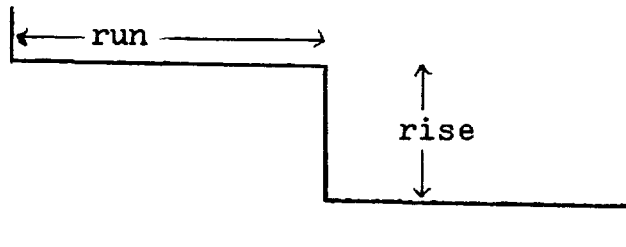
RISE AND RUN

Name \_\_\_\_\_

Have you ever noticed that climbing some stairs is more difficult than others? Why is this so? List your reasons below.

1. \_\_\_\_\_
2. \_\_\_\_\_
3. \_\_\_\_\_
4. \_\_\_\_\_
5. \_\_\_\_\_

Go to the stairs that your teacher has selected, and measure as shown below.



The vertical distance is called the rise of the stair and the horizontal distance is the run of the stair.

Look at other stairs in the school or in your home. Measure the rise and run of these stairs. Do all stairs seem to have the same rise and run?

What would happen to the stairs if the rise were increased and the run held constant?

What would happen if the rise were held constant and the run were increased?

(RISE AND RUN)

Consider the following ratios of rise to run:

$$\frac{\text{rise}}{\text{run}} = \frac{12 \text{ in.}}{12 \text{ in.}} = \frac{24 \text{ in.}}{24 \text{ in.}} = \frac{36 \text{ in.}}{36 \text{ in.}}$$

Which would describe the best stairs? \_\_\_\_\_

Why? \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

What ratio of rise to run do you think makes the best stairs?

\_\_\_\_\_. Why do you think so? \_\_\_\_\_

\_\_\_\_\_

(3)

### NUMBER GRIDS

Many people are familiar with letter grids in which words are hidden in an array of letters. This idea is easily extended to a mathematics grid that can be used to stimulate thought and enjoyment on the part of students. At the same time, it provides a review drill in the fundamental operations of arithmetic. These grids can be challenging at any level, if carefully designed.

#### Instructions

The grid is simply an array of numbers. These should be carefully thought out, so as to provide many possible combinations for the students to recognize.

Each student should be given a copy of the grid which has previously been duplicated. The student is to discover any true mathematical statements within the grid, by putting in the operational symbols and the equality symbol. Parentheses may also be used. Numbers may be used more than once. Statements can be made horizontally, vertically, and diagonally. Students using the grid will often notice involved mathematical concepts such as inverse operations ( $7 \times 3 = 21$  or  $7 = 21 \div 3$ ), commutative, associative, distributive laws, and so forth.

(3)

MAKE AN EQUATION

9	6	3	81	42	7	6	19	23
45	2	3	27	8	6	8	25	8
5	7	18	3	5	1	14	7	15
21	4	4	9	6	14	3	11	5
58	7	4	28	2	15	2	9	75
3	18	7	2	5	10	8	3	12
36	9	49	14	2	28	42	7	63
7	6	7	36	9	4	50	3	11
32	54	6	9	8	3	6	5	12

(2)

Name \_\_\_\_\_

By inserting  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $=$ , and parentheses between appropriate numbers, equations can be found in vertical, horizontal, and diagonal form. How many different equations can you find?

23	76	25	56	28	6	20	8	12
4	28	1	28	28	5	4	7	5
53	3	23	28	9	2	3	1	6
2	8	7	5	3	6	5	6	5
9	2	4	41	7	24	36	3	2
7	36	3	4	2	30	1	6	7
63	6	5	4	4	3	21	4	14
5	6	42	32	23	9	14	8	6
9	6	3	81	8	3	3	42	5