

AN INTRODUCTION AND ORIGINAL PROBLEM
IN
LINEAR PROGRAMMING

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PURPOSE

The purpose of this paper is to present a quick and scanning look at the mathematical and industrial aspects of linear programming. In no way has the author encompassed the entire realm of linear programming and its applications in this paper, but rather hopes that the material presented here will serve as a tasty appetizer to encourage the reader to further fill himself with the meat of such a vast and practical field.

Many thanks go to Mr. Don Driskill, owner and manager of Driskill's I. G. A. Market in Fairmount, Indiana, for his cooperation in the release of wholesale prices on his goods and the use of his name and store in this paper.

AN INTRODUCTION AND ORIGINAL PROBLEM IN LINEAR PROGRAMMING

PART I

A basic characteristic of management is decision-making. Decision-making is the process of choosing from among a number of alternative courses of action. Decisions are made concerning the objectives of the firm: what, how much, and when to produce; the nature of personnel policies; and others. In most situations there is an enormous amount of information available to management, and the problem is to choose the relevant information and process it as quickly as possible. "Thus it is that management often makes decisions in spite of, rather than in the light of, the information available."¹

A single-plant firm produces a number of fairly similar products. Each product requires a number of machining operations, which are performed on a number of different machines. In addition, each machine requires varying lengths of time to perform any one operation. The length of time required to produce any particular product with the same set of machines is not consistent, or are the raw material requirements for each product exactly equal. Let us assume that management is interested in finding the particular combination of products

¹Eugene T. Byrne, and Thomas H. Naylor, Linear Programming (Belmont, California: Wadsworth Publishing Company, Inc., 1963) p.8.

that should be produced. There is information available concerning the process time per product per operation per machine, raw material availabilities in terms of both quantities and prices, hours of machine availability, demand prices and quantities, plus a host of other factors that may or may not have some effect on the extent to which a solution is achieved. Clearly the number of combinations that can be produced may be very large. In fact, there may be so many technically feasible combinations that it would be very difficult to decide which combinations are best. It is little wonder, then, that many management problems are solved by rules of thumb that generally tend to produce less than optimal results.

World War II and the following decade mark a period of important advances in management planning and control. One such advance was the appearance of the digital computer, and another was the application of higher mathematics and statistics to problems of industry.²

Linear programming was developed rapidly during World War II by scientists called upon to help with the analysis and solution of various tactical problems encountered by all branches of the armed forces. In the years since its conception in 1947 in connection with planning activities of the military, linear programming has come into wide use in industry. While the computer made possible rapid handling of massive

²John Crank, Mathematics and Industry (London: Oxford University Press, 1962), p.48.

amounts of data, linear programming provided a theoretical framework for the organization and examination of these data. The result is that business problems of great complexity can now be solved and made a part of the rational decisions on which the success of businesses so importantly depends.

A vital part of these advances in management is the increasing application of linear programming to solve a wide range of managerial problems. This analytical tool has become quite applicable in many industries; including petroleum, chemicals, steel, and agriculture. Exploratory studies using the linear programming approach have also been made by airlines, railroads, utilities, and financial institutions.

Interestingly enough, in spite of its wide applicability to every-day problems, linear programming was unknown before 1947.

Interest in linear programming extends beyond the world of business, however. Economists have found it helpful in studying the relation of the firm and resources. Mathematicians have through it discovered new paths of research and investigation. It is a subject that will be of considerable interest and importance to the business and academic worlds for some time to come.

The linear programming model requires a method for finding a solution to a system of simultaneous linear equations and linear inequalities which minimizes a linear formula. This central mathematical problem of linear programming was not

known to be an important one with many practical applications until the advent of linear programming in 1947. It is this which in part accounts for the lack of active interest among mathematicians in finding efficient solution techniques before that date.

"We are all familiar with methods for solving linear equation systems which start with our first course in Algebra. The literature of mathematics contains thousands of papers concerned with techniques for solving linear equation systems, with the theory of matrix algebra, with linear approximation methods, etc. On the other hand, the study of linear inequality systems excited virtually no interest until the advent of game theory in 1944 and linear programming in 1947. As evidence that mathematicians were unaware of the importance of the problem of seeking a solution to an inequality system that also minimized a linear form, we may note that none of these papers made any mention of such a problem, although there had been earlier instances in the literature."³

According to Dantzig the Russian mathematician L. V. Kantorovich should be credited with being the first to recognize that certain important broad classes of production problems had well defined mathematical structures which, he believed, were open to practical numerical evaluation and could be numerically solved.⁴

If Kantorovich's earlier efforts had been appreciated at the time they were first presented, it is possible that linear programming would be more advanced today. However, his early

³George Bernard Dantzig, Linear Programming and Extensions (Princeton, New Jersey: Princeton University Press, 1963), pp. 20-21.

⁴Ibid.

work in this field remained unknown both in the Soviet Union and elsewhere for nearly two decades while linear programming became a highly developed art. According to the New York Times,

"The scholar, Professor L. V. Kantorovich, said in a debate that Soviet economists had been inspired by a fear of mathematics that left the Soviet Union far behind the United States in applications of mathematics to economic problems. It could have been a decade ahead."⁵

In 1936, J. Neyman and E. S. Pearson explained the basic concepts for confirming statistical tests and estimating underlying parameters of a distribution from given observations. They developed a lemma, the conditions of which are the conditions that a solution to a bounded variable linear programming problem be optimal.⁶

"Credit for laying the mathematical foundations of this field goes to John von Neumann more than to any other man."⁷

He played a leading role in many fields; atomic energy and electronic computer development are two where he had great influence. In 1944, John von Neumann and Oskar Morgenstein published their work on the theory of games, a branch of mathematics that aims to analyze problems of conflict by use of models termed "games". A theory of games was first opened in 1921 by Emile Borel and was first established in 1928 by von Neumann with his Minimax theorem. The significance of this

⁵Harry Schwartz, "Russian Assails Soviet Economics," The New York Times, May 29, 1959.

⁶Dantzig, op. cit., p. 23.

⁷Ibid., p. 24

effort is that game theory, like linear programming, has its mathematical foundation in linear inequality theory.

During the summer of 1947, Leonid Hurwicz, economist associated with the Cowles Commission, worked with Dantzig on techniques for solving linear programming problems. This effort and some suggestions of T. C. Kropmans resulted in the "Simplex Method". The obvious idea of moving along edges from one vertex of a convex polyhedron to the next (which underlies the simplex method) was rejected earlier on intuitive grounds as inefficient. In a different geometry it seemed efficient and so was tested and accepted.⁸

Hence, the principal mathematical bases for linear programming, according to Dantzig and Kropmans, are the theory of linear equalities, a part of algebra, and the theory of convex polyhedra, a part of geometry. Algebraically, the calculations are similar to elimination processes for solving systems of algebraic equations.

Von Neumann, at the first meeting with Dantzig in October, 1947, was able to translate basic theorems in game theory into their equivalent statements for systems of linear inequalities. He introduced and stressed the fundamental importance of duality and proposed the equivalence of games and linear programming problems. Later he made several proposals for the numerical solution of linear programming and game problems.

A. W. Tucker took an interest in game theory and linear

⁸Dantzig, op. cit., p. 24.

programming in 1948.⁹ Since that time, Tucker and his former students (notably David Gale and Harold W. Kuhn) have been active in developing and systematizing the underlying mathematical theory of linear inequalities.

The National Bureau of Standards played an important role in the development in linear programming theory. Not only did it arrange through John H. Curtiss and Albert Cahn the important initial contacts between workers in this field, but it provided for the testing of a number of computational proposals in their laboratories. At the Institute of Numerical Analysis, Professor Theodore Motzkin proposed several computational schemes for solving linear programming problems. Alex Orden of the Air Force worked actively with the National Bureau of Standards group who prepared codes on the Standards Eastern Automatic Computer for the general simplex method and for the transportation problem. Alan J. Hoffman, with a group at the National Bureau of Standards, was instrumental in having experiments run on a number of computational methods.¹⁰

In June, 1951, the First Symposium in Linear Programming was held in Washington under the joint auspices of the Air Force and the National Bureau of Standards. By this time, interest in linear programming was widespread in government and academic circles. A. Charnes and W. W. Cooper had just begun their pioneering work on industrial applications. Aside from

⁹Dantzig, op. cit., p. 24.

¹⁰Dantzig, op. cit., p. 25.

this work, they published numerous contributions to the theory of linear programming. A two-volume treatise was published in 1961.¹¹

The special simplex method developed for the transportation problem by Dantzig was first coded for the Standards Eastern Automatic Computer in 1950 and the general simplex method in 1951 under the general direction of A. Orden of the Air Force and A. J. Hoffman of the Bureau of Standards. In 1951, W. Orchard-Hays of the RAND Corporation worked out a simplex code for the IBM-C.P.C., and for the IBM 701 and 704 in 1954 and 1956, respectively. The latter code was remarkably flexible and solved problems of two hundred equations and a thousand or more variables in five hours or so with great accuracy.¹²

The use of electronic computers by business and industry has been growing by leaps and bounds. Many of the digital computers which are commercially available have had codes of the simplex technique. In addition, there has been some interest in building analogue computers for the sole purpose of solving linear programming problems. It is possible that such computers may provide an efficient tool for the evaluation of parametric changes in a system represented by a linear programming model and may be useful when quick solutions of linear programming problems are continuously needed, as for example, in production scheduling.

¹¹Dantzig, op. cit., p. 25.

¹²Dantzig, op. cit., p. 26.

"These computers have worked well on small problems (for example twenty variables and ten equations). Because of distortion of electric signals, it does not seem practical to design analogue computers which can handle large general linear programming problems. However, it does appear very worthwhile to try to develop applications of such computers to solving large-scale systems which possess special structures."¹³

¹³Dantzig, op. cit., p. 26

BASIC ASSUMPTIONS OF LINEAR PROGRAMMING

1. The productive opportunities of an economy or economic unit, such as kilowatt, man-hour, etc., are defined by the resources and the productive processes available to it. The productive process is a physical event or series of events in which men participate purposefully in order to transform some resources into some products. The quantities of at least some of the resources are finite and so is the number of productive processes available.
2. Any productive process may be used at any positive level consistent with the supply of resources available. The consumption of resources and the output of products is proportional to the level at which the process is used.
3. Several productive processes may be used simultaneously, if the supply of resources is adequate. If this is done, the consumption of each resource is the sum of the consumptions of the individual processes used, and the output of products is the sum of the outputs of the individual processes.¹⁴

¹⁴Robert Dorfman, Application of Linear Programming to the Theory of the Firm (California: University of California Press, 1951), p. 18.

CHARACTERISTICS OF LINEAR PROGRAMMING PROBLEMS

Generally speaking, linear programming is a mathematical optimizing technique applicable to a class of problems having certain characteristics in common.

The techniques of game theory are used in the theory of linear programming.¹⁵ The interpretation of these characteristics varies in accordance with the specific content of the problem. For this reason, it is best to describe these common characteristics in mathematical terms in order to retain their generality.

(1) A linear objective function - all linear programming problems have as their objective the optimization of some explicit linear function of several variables. If this linear objective function is denoted by $F(X)$ and the appropriate variables of the problem by X_1, X_2, \dots, X_n , then the goal of a linear programming problem is always to maximize (or minimize).

$$F(X) = C_1 X_1 + C_2 X_2 + \dots + C_n X_n,$$

where C_1, \dots, C_n represent the parameters of the problem. The goal is not just to accomplish something, but to accomplish it in the "best" possible manner. Since business operations quite often are conducted with the goal of maximizing profit or minimizing cost, linear programming is found useful in business.

(2) A set of linear constraints - all linear functions by themselves have a maximum of positive infinity and a

¹⁵"Game Theory", Colliers Encyclopedia (Vol. 10), pp. 560-2.

minimum of negative infinity if no restrictions are placed on the values of their variables. Consequently, the problem of optimizing a linear function is not a mathematically meaningful problem unless the variables are restricted as to the ranges of values they may assume. In linear programming such constraints are contained in a set of linear inequalities as follows:

$$A_{11}X_1 + A_{12}X_2 + \dots + A_{1n}X_n \leq b_1$$

$$A_{21}X_1 + A_{22}X_2 + \dots + A_{2n}X_n \leq b_2$$

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$$A_{m1}X_1 + A_{m2}X_2 + \dots + A_{mn}X_n \leq b_m$$

where $A_{11}, A_{12}, \dots, A_{1n}, \dots, A_{m1}, A_{m2}, \dots, A_{mn}$ are constant coefficients and b_1, b_2, \dots, b_m simply constants.

There are three points that should be kept in mind in connection with these constraints (called structural constraints). First, they are linear. Second, they are characteristically inequalities although equality-constraints are not excluded. Finally, the number of such constraints, that is, m , is not restricted in any way except as it affects the practical problem of computation. Together these constraints define a region of acceptable values of the variables. Since this region may or may not include values up to infinity, linear programming becomes a problem of selecting a set of values (of the variables) within this region, which will yield the

maximal (or minimal) value of the objective function. In terms of the illustrative problem, the extent to which the contribution to profit and overhead could be maximized is limited, among other things, by the hours of machine availability, raw materials availability, and processing times.¹⁶

(3) The non-negativity constraints - as a direct outgrowth of its application in business and industry, all linear programming problems require that the solution values of their variables be non-negative. This is necessary because the variables in business and industry usually have no meaningful negative counterparts. Mathematically these non-negativity constraints are written as follows:

$$X_1, X_2, \dots, X_n \geq 0.$$

"It must be stipulated that $n = m$, or the problem may very well have no solution. This arises from the procedure of adding or subtracting non-negative variables to inequalities to convert them to equalities and write the constraints in the form of equations."¹⁷

BENEFITS DERIVED FROM THE USE OF LINEAR PROGRAMMING

The indication has already been made that linear programming will provide the optimum solution to a problem when certain conditions exist. In addition, linear programming:

(1) AIDS UNDERSTANDING. When management makes the decision to use linear programming, it is committing itself

¹⁶Byrne and Naylor, op. cit., p. 9.

¹⁷An-Min Chung, Linear Programming (Columbus, Ohio: Charles E. Merrill Books, Inc., 1963) pp. 3-6.

to a new way of thinking about business problems. Because of the mathematical nature of the technique, vague generalizations describing the firm's activities are no longer adequate. Management must thoroughly understand the firm's activities before it can attempt to represent them mathematically. This requires that the relationships governing the performance of an activity be fully understood and in addition the relationship of any given activity to the over-all performance of the firm must be known as well. An outgrowth of the necessity for quantifying relationships is the development of greater understanding of the firm's activities.

(2) PROVIDES INFORMATION FOR PLANNING AND CONTROL.

The use of linear programming further contributes to the fulfillment of the managerial function by providing information that is useful in planning and control. In part, the planning function is concerned with the recognition of change. This aspect of planning is aided by the susceptibility of the linear programming model to manipulation.¹⁸ Thus questions arise such as what is the effect on the present mix of products if certain raw materials' prices change? Or what is the effect of a prolonged machine breakdown or an increase in machinery downtime? Control over operations is achieved by comparing plans with results. To the extent that linear programming is useful in planning it will also contribute to the fulfillment of the managerial function of control.

¹⁸G. Hadley, Linear Programming (Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1962), Chapter 11.

(3) CONTRIBUTES TO THE DEVELOPMENT OF EXECUTIVES.

The matter of executive development is a serious problem in many firms. Training programs are valuable in this respect and their value can be increased through the use of linear programming and other mathematical models as training devices. Future executives can be given actual or fictitious problem situations in which they are required to develop the mathematical model. Their results can be easily verified without resorting to actual operations where mistakes are likely to prove quite costly. Much of the knowledge and confidence that ordinarily takes many years of experience to obtain can be gained in a significantly shorter period of time via the technique of model-building.¹⁹

It should be obvious that all of the above benefits will create a more efficient management, which will result in increased possible profit for the firm.

Before becoming completely delighted by the potential benefits of linear programming, its limitations should be considered. It cannot eliminate all of the judgment that is associated with managerial decision-making. Although linear programming may eliminate managerial judgment in such decision areas as product mix, order allocation, machine scheduling, and warehouse location, management must still decide, for example, whether the best measure of effectiveness for a particular

¹⁹Byrne and Naylor, op. cit., pp. 11-12.

activity is the contribution to profit and overhead, or whether it is best to minimize processing time. Likewise, it must make decisions on the effects of certain programs on employee morale and decide whether or not certain products must be produced so as to fill out the product line. These are elements of decision-making that management retains, and linear programming does not diminish their importance.

LINEAR PROGRAMMING APPLICATIONS

As mentioned before, linear programming has been successfully applied to a wide range of problems that are found in many different fields. The agricultural, industrial, and military sectors have made the most extensive use of linear programming. In addition, many applications can be found in the sciences, including economics and applied engineering. There is little doubt that the list of applications will continue to grow as decision makers become more familiar with the capabilities of the method. At the same time, the continuing development of high-speed computing equipment will be a major factor in the application of linear programming to those problems that fit into the framework of the technique.

In this section a few of the broader applications will be presented, with the hope that they will sufficiently indicate the general problem types that linear programming can solve. The following examples are typical of the general

problem areas in which linear programming has been employed successfully.²⁰

SCHEDULING PRODUCTION AND INVENTORY CONTROL.

Many firms produce products that are subject to seasonal sales fluctuations, e.g., oil refining companies. In such cases widely fluctuating production rates have proved extremely costly, whereas a uniform production rate builds up inventories that result in excessive costs for storing the product. Through the use of linear programming these companies can establish a production schedule that will satisfy demand when it arises as well as minimize both the costly effects of fluctuating production rates and the excessive inventory carrying charges resulting from a uniform production rate.

Further applications in this area involve finding the best uses of production capacity and the best basis for assigning production to a number of manufacturing or producing centers. Excess capacity problems involve selecting from among a group of potential products that set of products which will both utilize the capacity and maximize the contribution to profit and overhead. Assigning production to a number of producing points with due consideration to the initial location of raw materials, plant production capacities, demand requirements, and final destination of products is a problem type that is easily solved by linear programming. This is

²⁰S. Gass and V. Riley, Linear Programming and Associated Techniques (Baltimore, Maryland: The Johns Hopkins Press, 1958).

also similar to the task of assigning production to processes in a plant where the products have varying production periods and the processes are not equally efficient. In this instance linear programming will provide for the optimum allocation of products among the different processes in accordance with some objective, such as minimum processing time.

BLENDING PROBLEMS. There are numerous instances in which certain basic components are combined to produce a product that has a certain set of specifications. Examples of this type of problem are to be found in the blending of gasolines, the mixing of cattle feeds, and the mixing of meats to produce sausage or other meat products. A problem in oil refineries, for example, is to select a number of crude oil products to be blended into different grades of gasoline. The basic products have certain common characteristics, such as octane rating, vapor pressure indices, and distillation temperatures. Considering the demand for gasoline of varying specifications and the supply conditions for the crude oil products, linear programming can determine the mixture of crude products that will yield maximum profits, minimum costs, or some other optimum. The cattle-feed problem concerns minimizing the cost of producing a feed that possesses certain nutritional requirements.

PURCHASING. Processes requiring inputs that are available at different quantities, qualities, and prices present a purchasing problem that can be solved for a least-cost

objective. An oil refinery, which purchases all of its crude oil from outside sources, uses linear programming with a profit maximizing objective to determine which crudes should be purchased. The model considers output requirements and specifications, crude oils on hand, and processing costs in the evaluation of this problem.

Whether to make a product component or purchase it from an outside source is another problem area in which linear programming can be of assistance. The factors that are considered in a problem of this type are: process and product requirements, production and purchasing costs, selling costs, overhead costs, and either a profit or cost objective to be optimized.

ROUTING AND ASSIGNMENT. The route that a salesman should take in order to cover a specified territory while minimizing distance traveled is also a problem type that can be solved by linear programming. Under certain conditions the transportation method can be used for any routing problem involving an origin-destination scheme. Thus, there may be empty boxcars at various locations (origins) and requests for boxcars at various locations (destinations). Given the costs of moving empty boxcars, the transportation technique will give the optimum assignment of empty boxcars from the origins to the destinations. This technique is also useful in determining the location of additional warehousing facilities and the allocation of products to machines.

Another class of problems involves assigning facilities to jobs in a manner that embodies some performance objective. Thus, problems of assigning products to facilities in which the costs of producing each product will vary with the facility chosen can be solved by linear programming. This technique can also be used to assign workers to jobs offering a quantitative measure of the workers' effectiveness.

OTHER. Linear programming has been used with problems of optimizing executive compensation programs, location of and management of river dam projects, farm management, awarding contracts, traffic control, scheduling a military tanker fleet, minimizing trim losses in paper mills, balancing assembly-line operations, and optimizing investment allocations for both individual and institutional investors.

²¹Byrne and Naylor, op. cit., pp. 12-15.

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PART II

Part of the problem presented in this portion is based on factual information provided by Mr. Don A. Driskill, manager of Driskill's I.G.A. Market in Fairmount, Indiana. Other parts of it, however, are hypothetical in order to illustrate a practical usage of linear programming.

On his shelves Mr. Driskill usually stocks twelve different kinds of Betty Crocker cake mixes. He has 30,000 cubic inches of storage space in which to store these cake mixes. Although Mr. Driskill has weeded out those cake mixes which do not sell, the author would like to make the hypothetical assumption that customer demand varies among the different cake mixes.

Within practical limits, Mr. Driskill can order as many cases of cake mix as he desires. However, the author would make the proposition that Mr. Driskill is limited on the size of his order.

On each of ten of the twelve cake mixes, Mr. Driskill makes a profit of \$.05. A profit of \$.08 each is recognized on the remaining two. Mr. Driskill, by taking into account storage space, product demand, supply limits, and individual profit, must arrive at the number of cases of cake mix to order in order to maximize profit.

Consider the following table, noting that one case equals twelve boxes of cake mix.

CAKE MIX	ORDER AMOUNT (BOXES)	CASE SIZE (CUBIC INCHES)	PROFIT (PER BOX)	SUPPLY LIMITS (CASES)	DEMAND (BOXES)
Lemon Coconut Delight	X ₁	732	.05	2	10
Angel Food	X ₂	630	.08	2	14
Yellow	X ₃	732	.05	5	40
Devils Food	X ₄	732	.05	3	24
White	X ₅	732	.05	6	54
Honey Spice	X ₆	732	.05	3	24
Marble	X ₇	732	.05	2	18
German Chocolate	X ₈	732	.05	3	20
Dark Chocolate Fudge	X ₉	732	.05	5	50
Black Walnut	X ₁₀	732	.05	2	6
Almond Creme	X ₁₁	732	.05	2	12
Tropical Mist Angel Food	X ₁₂	630	.08	2	8

Because of advertising specials often offered by the national I.G.A., Mr. Driskill likes to reserve room for at least four cases of cake mixes in storage. The object is to maximize the profit function $F(X) = .05X_1 + .08X_2 + .05X_3 + .05X_4 + .05X_5 + .05X_6 + .05X_7 + .05X_8 + .05X_9 + .05X_{10} + .05X_{11} + .08X_{12}$ while being held to the following restrictions.

$$(1) \quad 61X_1 + 52.5X_2 + 61X_3 + 61X_4 + 61X_5 + 61X_6 + 61X_7 + 61X_8 + 61X_9 + 61X_{10} + 61X_{11} + 52.5X_{12} \leq 27,072$$

$$(2) \quad X_1 \leq 24, X_2 \leq 24, X_3 \leq 60, X_4 \leq 36, X_5 \leq 72, X_6 \leq 36, X_7 \leq 24, X_8 \leq 36, X_9 \leq 60, X_{10} \leq 24, X_{11} \leq 24, X_{12} \leq 24$$

$$(3) \quad X_1 \geq 10, X_2 \geq 14, X_3 \geq 40, X_4 \geq 24, X_5 \geq 54, X_6 \geq 24, X_7 \geq 18, X_8 \geq 20, X_9 \geq 50, X_{10} \geq 6, X_{11} \geq 12, X_{12} \geq 8$$

This situation leads to the necessity for finding values for 12 unknown quantities via the use of 26 inequalities.

If one has knowledge of the use of the computer and access to one, this would be a good method for solution of this problem.¹ However, neither the author nor Mr. Driskill had access to a computer or knowledge of the use of one. Hence this problem was tenaciously solved by an iterative method, with initial guesses for the first trial.

The results of the computation are as follows: $X_1 = 24$, $X_2 = 24$, $X_3 = 60$, $X_4 = 36$, $X_5 = 72$, $X_6 = 36$, $X_7 = 24$, $X_8 = 36$, $X_9 = 60$, $X_{10} = 24$, $X_{11} = 24$, $X_{12} = 24$, in terms of number of boxes. Complete sale of these cake mixes would maximize profit

¹The home office of a national chain could profitably rent a computer if it had hundreds of such problems to solve.

at \$23.39 using 26,676 cubic inches of storage space and leaving 3,324 cubic inches of back up storage space.

Mr. Driskill and the author discussed the use of such a tool as linear programming in his entire store. He was thrilled by the prospect, but quickly warned against using this as a set rule of thumb, due to the unpredictability of people, supply companies, weather, and similar influences. However, other grocery chains have begun to use linear programming and, except for the rare cases deviating greatly from the mean, the outcomes have been favorable to continuation of its use.² As Mr. Driskill has said, no plan will ever be able to hold constant the demand for, or even the supply of, any product. Linear programming is flexible enough to sidestep these variant occurrences in most cases, and its applicability to so many facets of the grocery store business gives promise of its becoming a valuable tool to the grocer.

²This was learned by the author from Mr. Driskill who attended a food retailers' clinic at Purdue University, April, 1966.