Building Three Dimensional Objects Evaluation Form

Name:______________________________ Period: ____ Date: ________________
Group Members: ____________________________
____

Mark your reaction to the following statements by placing an *X* on the scale closest to the area which most matches your feelings.

1. This activity was fun.

5-----------4---------3-----------2-----------1
| |
agree disagree

2. This activity was challenging.

5-----------4---------3-----------2-----------1
| |
agree disagree

3. I would like to do this activity or activities like this again.

5-----------4---------3-----------2-----------1
| |
agree disagree

Answer the following questions in complete sentences. Use correct grammar, punctuation, and spelling.

4. What do you feel was the goal of this activity? Why do you think this was the goal?

5. What was the most challenging part of this activity? Why?

6. If you encountered problems in this activity where did you encounter them? What did you do to solve the problem?

Title: Follow Directions Carefully--A Listening Activity for Geometry

Grade Level: 5 - 12

Time Required: 25-30 minutes

Objectives:
1. Students will use correct mathematical terminology to communicate with each other.
2. Students will estimate distances and angle measures in order to communicate about and draw geometric figures.

Materials Needed:
___ tape recorders for each group of students (optional)

Notes to the Teacher:
This activity is an excellent way to assess students knowledge of geometric shapes and terms. By using the tape recorder AND having students record their statements on their own the teacher has a surefire way to determine if students understand terminology they were to have learned and can use this terminology appropriately.
If this activity is used more than once you may want to have a "Communication" bulletin board in the room which shows students' previous attempts at the activity and improvements on future attempts.
A suggested rubric for assessment of this activity follows:
A team receives an A if
- terminology used was mathematical
- terminology was used appropriately
- finished drawings look almost identical

A team receives a B if
- terminology used was mathematical but some of these terms were used inappropriately.
- terminology used was appropriate but drawings are seriously different

A team receives a C if
- some terminology used was mathematically appropriate but an equal amount was inappropriate
- drawings are quite different from one another

A team receives a D if
- the majority of terminology used was inappropriate to the description.
- drawings do not resemble one another at all.

A team receives an F if
- no effort was made to complete the activity.

Procedure:
Divide students into teams of two. (You may use groups of three - in which the third student records the specific instructions given by the first student. In this case, tape recorders really would not be necessary except as an additional means of assessment.)

Students should sit back to back. The first student draws a geometric figure on a piece of paper. This person then gives their partner instructions on how to draw the figure. As they give the instructions they should record the exact instructions given. The other person draws the figure according to the directions given to them. Students should switch roles and repeat.

Journal topics to consider from this activity are how this activity made them feel, what they thought the objective of this activity was, or why they think their drawings did or did not match.

Extensions:
Set conditions for how instructions should be given such as:

1. You can not tell how many sides the figure has.
2. Give side lengths only in metric units.

Adapted From:
Tietze, Martha. "A Core Curriculum in Geometry." *Mathematics Teacher.* 85 (April 1992): 300-304.

Title: Enlarging learning opportunities

Grade Level: 7 - 12

Time Required: two to three class periods

Objectives:

1. Students will work cooperatively to enlarge a drawing.
2. Students will develop ideas of projective geometry from a real world application such as using a copy machine.

Materials Needed:

___ picture which can be gridded and cut apart
___ Square sheet of drawing paper for each student or group of students.
___ Rulers
___ Drawing/Art Supplies

Procedures:

Before beginning the teacher should select a picture for the class to enlarge. If you have an artistic student or students in your class you might hold a mini art show and contest to select the picture.

To begin the activity divide the picture into squares so that there are enough squares for each group to have one. Number the squares in some sort of order so it easy to put the finished picture back together. Give students the square of paper they will put their finished product on and the piece of the picture to be enlarged. Engage students in a discussion about what the scaling factor should be and how the task should be completed.

Students should then work together in small groups to enlarge their piece of the picture. The teacher should provide assistance only when absolutely necessary.

Once a group has transferred and enlarged their portion of the picture they should color their portion. If students have scene the completed drawing prior to the project you may want to have students come up with a color/decorating scheme for the entire project or you may wish to let each individual group decorate their portion of the picture.

Display the finished enlargement on a bulletin board or in a hallway.

Adapted From:

Tietze, Martha. "A Core Curriculum in Geometry." *Mathematics Teacher.* 85(April 1992): 300-303.

Title: Patterns and Proofs

Grade Level: 5 - 9

Time Required: One or two class periods

Objectives:

1. Students will make conjectures about the number of sides in a new polygon depending on the cut made in an original convex polygon.
2. Students will collect data to support their conjectures.
3. Students will verify their conjectures.

Materials Needed:

___ construction paper
___ scissors
___ observation sheet
___ record sheet

Procedure:

Have students begin by cutting several convex polygons out of construction paper. They should cut several different polygons all about 3" to 4". Demonstrate to students how to count the number of sides in their original polygon, make a cut in the polygon, and record the number of sides in the two new polygons (figure 1).

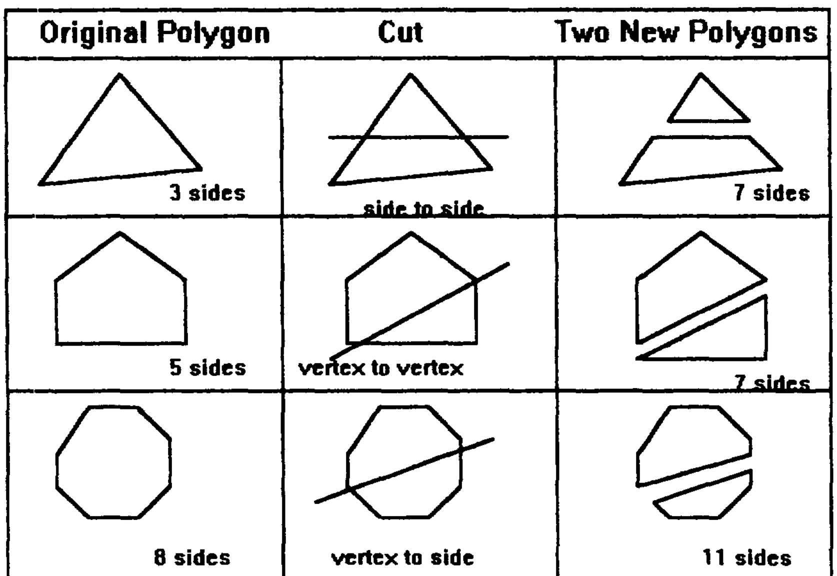


Figure 1

Each student should cut six to eight different polygons. As they cut have students record the data on the record sheet. Once all students have gathered a data set have students get into groups of three or four. Each group should have between 24 to 32 polygons. The groups should discuss the activity, organize the data, and look for patterns. Pass out observation sheet to students while in their groups.

Bring the class back together. Direct them to the conclusion that new polygons have exactly two, three, or four more sides. Encourage students to look for any other possibilities. Once they are sufficiently satisfied that these are the only possibilities proceed.

Have students classify their original cut polygons into groups. You may need to help students to see that three classifications exist in how the polygons have been cut: vertex to vertex, side to side, and vertex to side. Students should look for correlations between these groupings and their conclusion that the polygons gain two, three, or four more sides when cut. Let students record their observations on the final worksheet.

Adapted From:
Sconyers, James M. On My Mind: Proof and the Middle School Mathematics Student. *Mathematics Teaching in the Middle School.* 1(7): 516-518.

Polygons and Proofs Record Sheet

Record the shape of your original polygon, how you cut it, and the two new polygons formed in the space provided. Also indicate the number of sides in the original and new polygon.

Original Polygon	Cut	Two New Polygons
Number of Sides ____		**Number of Sides** ___
Number of Sides ____		**Number of Sides** ___
Number of Sides ____		**Number of Sides** ___
Number of Sides ____		**Number of Sides** ___
Number of Sides ____		**Number of Sides** ___
Number of Sides ____		**Number of Sides** ___

Number of Sides ___		Number of Sides ___
Number of Sides ___		Number of Sides ___
Number of Sides ___		Number of Sides ___

Do any patterns exist in the data collected? Discuss these patterns here.

Polygons and Proofs

Finalize the observations we have discussed in class today by finishing each of the following statements correctly. Be sure to use proper mathematical terminology.

1. If a polygon is cut from vertex to vertex, then

2. If a polygon is cut from side to vertex, then

3. If a polygon is cut from side to side, then

Title: Investigating Symmetry

Grade Level: 8 - 12

Time Required: One or two 50 minute class periods.

Objectives:

1. Students will conjecture about properties of symmetry in polygons.
2. Students will investigate the validity of their conjectures.
3. Students will use correct mathematical language to describe their investigations.

Materials Needed:

___ construction paper
___ rulers
___ protractors
___ "Investigating Symmetry" activity
___ scissors
___ compasses

Notes to the Teacher:

You can see the aesthetic aspects of mathematics everywhere, especially when you consider the property of symmetry. A good introduction to this activity is to take students on a mathematical walk through your school or neighborhood. While on this walk have them look for objects that are symmetric in some respect. For example, stop signs (octagons) are symmetric. Have students look for arrangements of objects symmetrically such as trophies in a trophy case, pictures hanging on a wall, etc. Students may write in their journals on why people might hang a group of pictures in an arrangements which is symmetric in some respect or on what makes symmetric objects pleasing to the eye.

Procedure:

After students have discussed, investigated, and reflected on the various properties of symmetry in regular polygons they should extend this knowledge and make connections to non-regular polygons. This activity would accompany the lesson on polygons and proofs very well.

After completing the activity sheet assign students to write a short report (1 - 3 pp.) including diagrams and cutout examples summarizing their findings about symmetry. If you have done several activities on symmetry this is an excellent assessment idea with which to conclude the unit.

Adapted From:

Shilgalis, Thomas W. *Symmetries of Irregular Polygons.* Mathematics Teacher. 85(5): 342 - 344.

Investigating Symmetry in Non-Regular and Regular Polygons

Investigate each of the following questions using the construction paper, scissors, compasses, protractors, etc. You may also need to refer to other activities we have done on symmetry which should be in your notebook. Record your observations below each question. Be sure to save and label all of your figures so that you may use them in your report on symmetry in regular and non-regular polygons.

1. How many lines of symmetry can a triangle have?

2. A Quadrilateral ?

3. A Pentagon?

4. A Hexagon?

5. A Septagon?

6. An Octagon?

7. A Nonagon?

8. What do you notice about symmetry in a polygon with an even number of sides?

9. What do you notice about symmetry in a polygon with an odd number of sides?

Title: Using Geoboards with Secondary Students

Grade Level: 9 - 12

Time Required: one class period

Objectives:
1. Students will manipulate figures on a geoboard.
2. Students will generate several figures which meet a set of given conditions.

Materials Needed:
___ Geoboards, rubber bands, and dot paper for each student or group of students
___ Activity Sheet

Procedure:
Pass out geoboards, bands, and dot paper to students. Allow them time to "play" by making figures in one or more of the following categories on the geoboard and recording them on the dot paper:

1. Something that moves.
2. Something that flies.
3. A Letter of the alphabet
4. A number

Move on to having students find figures meeting conditions you (or they) set forth. You may insist these figures be purely geometric or you may want to encourage students to be more creative and find any kind of shape meeting a condition or set of conditions. Students should record their designs on dot paper.
A few suggested conditions:
1. Figure having given area
2. Figure having given perimeter
3. Figure having given area and perimeter
4. Figure having given number of sides
5. Figure having given number of sides and given perimeter
6. Figure having given number of sides and given area
7. Figure having given number of sides, area, and perimeter.

Adapted From:
Tietze, Martha. "A Core Curriculum in Geometry." *Mathematics Teacher.* 85 (April 1992): 300-304.

Title: Locus of Points Treasure Hunt

Grade Level: 9 - 12

Time Required: One class period

Objectives:
1. Students will work in cooperative groups.
2. Students will use problem solving skills and geometric knowledge to locate hidden objects.

Notes to the Teacher:
Mathematics, as those who love the subject know, is not the boring drill and practice/memorization activity many students perceive it as. Mathematics is everywhere and in everything. For that reason, students in a mathematics class should see not a single room but the whole world as their classroom. Teachers of mathematics need to plan activities, such as the one described her, which will help students form these connections between the mathematics classroom and the outside world.

The idea of a mathematical scavenger hunt is certainly not limited to the geometric topics discussed here. For younger students you can have them find examples of numbers, shapes, etc. Algebra students could factor and solve equations whose solutions were the distances between objects, or they could locate objects using clues based on linear and quadratic equations.

Materials Needed:
___ 2 - 50 foot (20 meter) measuring tapes for each group of four or five students
___ several 4" nails with flat heads to hide in the lawn as "buried treasures"
___ clue sheets
___ index cards for each group to mark treasures
___ reward for finding a treasure or treasures

Procedure:
Sometime prior to class go into the school yard and label several points or landmarks in the school yard (trees, signs, etc.) with letter cards (A, B, C, etc.). Construct several clues such as those in Fig. 1. The clues for Fig. 1 are based on the locations marked in Fig. 2. Hide nails according to the clues you have made.

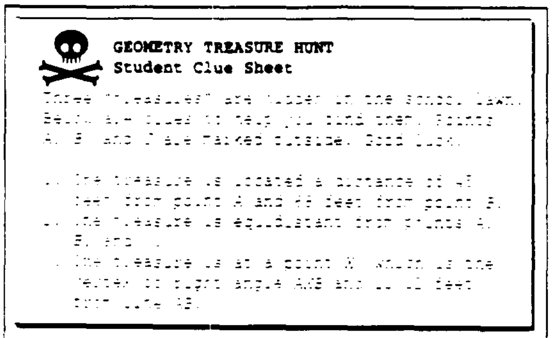


Fig 1

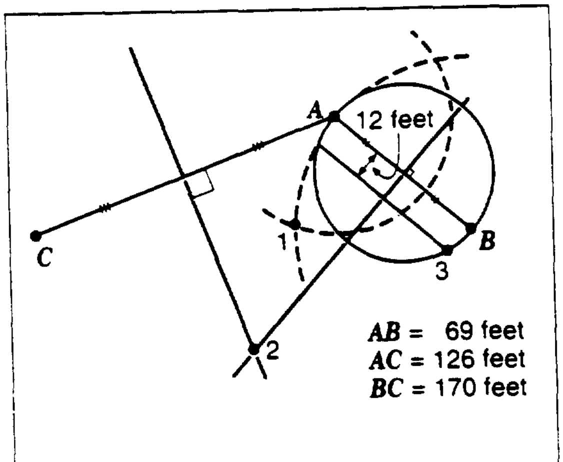


Fig 2
Placement of nails 1, 2 and 3 corresponds with sample clues 1, 2 and 3 respectively

Once the class arrives, have them get into their groups. Explain the activity to the class. Students should work together using the measuring tapes to locate the nails. Nails should be pounded into the ground so that they are not obvious. When a group locates a nail they should "tag" the nail by placing an index card with the names of the group members and the number of the clue that corresponds to that nail.

Once back in the classroom students should redeem tags for real "treasures". They should then work in groups to make a "map" of where the "treasures" were located.

<u>Source</u>:

Hayek, Linda. "Sharing Teaching Ideas: Using a Treasure Hunt to Teach Locus of Points." *Mathematics Teacher*. 86 (February 1993): 133 - 134.

Title: Can you believe what you heard?

Grade Level: 10 - 12

Time Required: 30-50 minutes

Objectives:
1. Students will use listening skills to assess whether a statement is correct or not.
2. Students will reflect on the need to evaluate carefully what you hear.

Materials Needed:
___ Wizard of Oz videotape
___ Scarecrow worksheet

Notes to the Teacher:
Learning to communicate mathematically includes being a scrutinizing listener when presented with mathematical information. This lesson gives students the chance to see whether or not the scarecrow in the Wizard of Oz really "got a brain" or whether he got swindled.

Procedure:
Cue up videotape to the final scene when the wizard is exposed and he is granting the promised rewards.

Ask students if they remember what the four travelers received for defeating the wicked witch.

Dorothy-a trip back to Kansas
Lion-Courage
Tin Man-A heart
Scarecrow-A brain

Have students view the scene concentrating on what the scarecrow receives. After viewing the tape ask students what the scarecrow said after receiving his diploma. Many students will say they heard the Pythagorean theorem. Play the clip again. Ask the same question of students. Many will probably say that it sounds like the Pythagorean theorem, but it is not exactly the Pythagorean theorem. Pass out the Scarecrow worksheet and allow students to prove or disprove this statement in cooperative groups.

Adapted From:
Petrone, Frank. "Reader Reflections: The Scarecrow Meets Pythagoras." *Mathematics Teacher.* 88 (December 1995): 725.

Scarecrow Worksheet

After receiving his brain, the Scarecrow in The Wizard of Oz makes the following statement.

> *The sum of the square roots of any two sides of an isosceles triangle is equal to the square root of the remaining side.*

Prove or disprove this statement. Write your proof in full sentence, paragraph form and provide any diagrams necessary to illustrate your conclusions.

Procedure:

Students should work in small groups during this project.

At the beginning have students describe the objects they brought to class; especially why they think they are interesting from a geometric standpoint.

Have students complete numbers 1 and 2 on the Project sheet. Most students should get an answer very close to 180^{o}. This result should not surprise students much. Discuss whether or not students think this result will hold in a triangle on the curved surfaces they have brought to class.

If time allows have students construct "flappy" triangles or pass out premade triangles to the class along with instructions for their use. Students should complete numbers 3, 4, and 5 on the Project sheet.

Have students list on the chalkboard or overhead projector the object or objects whose triangle sum they measured and the measurement they obtained. Have students look for patterns in this data to see if the objects can be classified into groups. Discuss with students the different interpretations of Euclid's fifth postulate which led to non-Euclidean geometries and some of the basic ideas of non-Euclidean geometry.

Students should complete numbers 6, 7, 8, and 9 on the Project sheet on their own for homework or you may want to simply discuss these as a class.

Adapted From:

Casey, James. *Using a Surface Triangle to Explore Curvature.* Mathematics Teacher. 87(2): 69 -77.

Constructing a "Flappy" Triangle

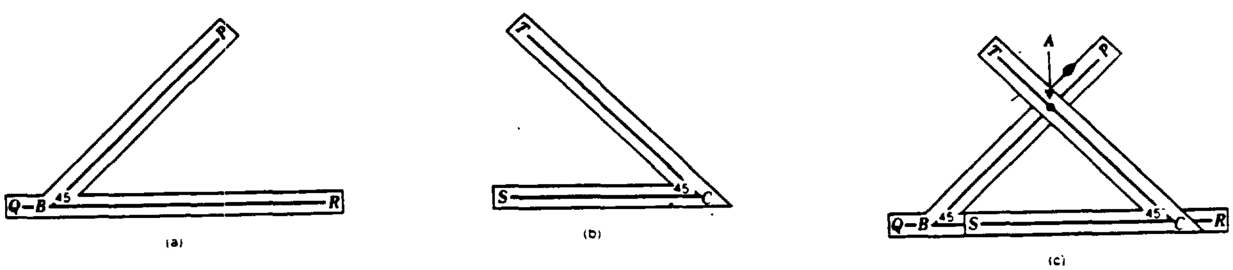
1. On a sheet of paper of medium weight, draw two straight line segments QR and BP intersecting at 45 degrees at B (Fig. 2a). Choose the dimensions to match the size of the object whose surface is to be examined.

2. Draw two other straight line segments SC and CT also meeting at 45 degrees (Fig. 2b).

3. Cut out the strips of paper containing the two angles.

4. Carefully lay the line segment SC along QR, and tape the two strips together along this base (Fig. 2c). Do not tape the flap BP to CT. Mark both flaps at the point A where the line segments BP and CT cross.

Fig 2
Making a "flappy" triangle



Check to be sure your flappy triangle is properly constructed:
1. Place the triangle on a flat surface.
2. Lightly tape the figure at Q, run a finger along QR, and tape the base at R.
3. Repeat this procedure with each of the flaps, taping them close to their point of intersection A.
4. Measure angle BAC. It should be very close to 90 degrees.

Using a "Flappy" Triangle

Choose an object you would like to examine.

1. Tape the tab at Q to the surface.
2. Run a finger along QR to fit the strip QR smoothly to the surface. Make sure no wrinkles are present. Tape the base midway and again at R.
3. Similarly, lay the strip BP smoothly on the surface and tape it at its end.
4. Lay the strip CT and tape it at its end. In general, the strips no longer cross at the point at which they intersected on the plane (A). Let the point at which the curves BP and CT now intersect be denoted by Z.

You should have something that resembles figure 3.

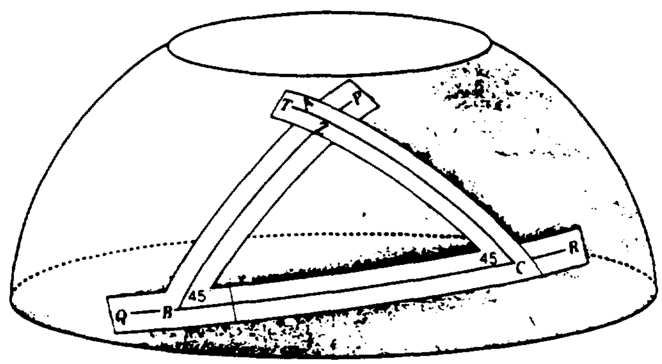


Fig. 3
A geodesic triangle on a bowl

To determine the measure of angle BZC:

1. Cut two narrow rectangular strips from a lightweight lined sheet of paper.
2. Place one strip over the angle you wish to measure in such a way that a line on the strip falls along BP. Tape the strip lightly to the surface.
3. Place the other strip along the side CT of the angle, hold it in place, and carefully tape the two rectangular strips together where they cross one another. Darken the lines of the strips so that the interior angle can be measured as accurately as possible.
4. Remove the joined strips fro the surface and tape them to a flat page. Draw rays on the page to extend those appearing on the taped strips.
5. Measure the angle on the page.

Title: Discovering properties of non-Euclidean geometry

Grade Level: 9 - 12 Geometry

Time Required: 1 or more class periods

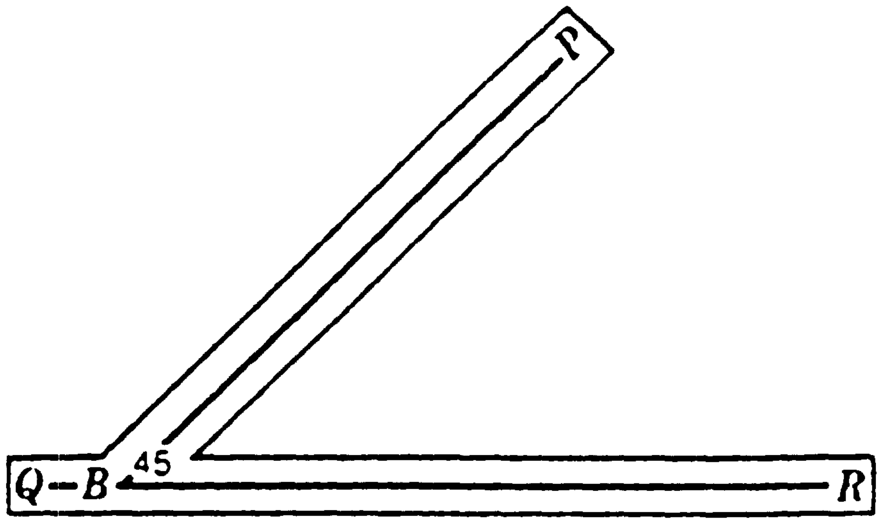
Objectives:
1. Students will discover the sum of the angle measure inside a triangle on several curved surfaces.
2. Students will explain the various interpretations of Euclid's fifth postulate.
3. Students will classify curved objects into three categories: Basically Euclidean, Lobachevskian, or Riemannian.

Materials Needed:
___ Several interestingly curved surfaces provided by both teacher and students such as cans, balls, cones, fruits and vegetables, mixing bowls, etc.
___ Copies of *Project: Measuring the Angle Sum of a Surface Triangle* (From: Casey, James. *Using a Surface Triangle to Explore Curvature.* Mathematics Teacher. 87(2): 69-77.
___ "Flappy" triangles in several sizes.
___ Scotch tape
___ Protractors
___ Scissors

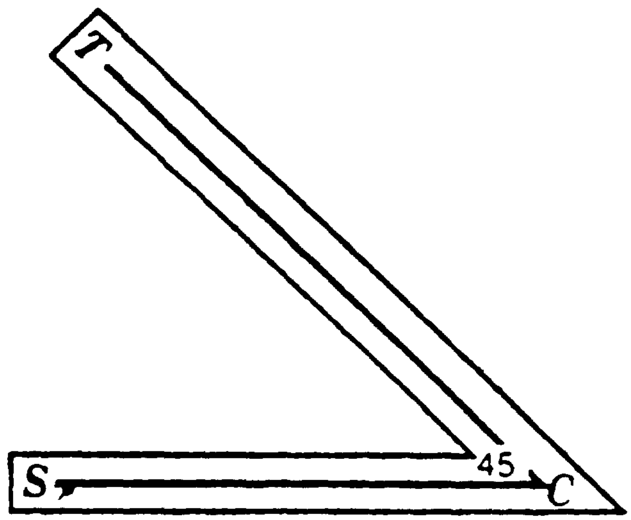
Notes to the Teacher:
In the *Curriculum and Evaluation Standards for School Mathematics* the National Council of Teachers of Mathematics states:

> In grades 9 - 12, the mathematics curriculum should include the continued study of the geometry of two and three dimensions so that all students can-
>
> - interpret and draw three-dimensional objects;
> - represent problem situations with geometric models and apply properties of figures;
> - classify figures in terms of congruence and similarity and apply these relationships;
> - deduce properties of, and relationships between, figures from given assumptions;
>
> and so that, in addition, college-intending students can-
>
> - develop and understanding of an axiomatic system through investigating and comparing various geometries.

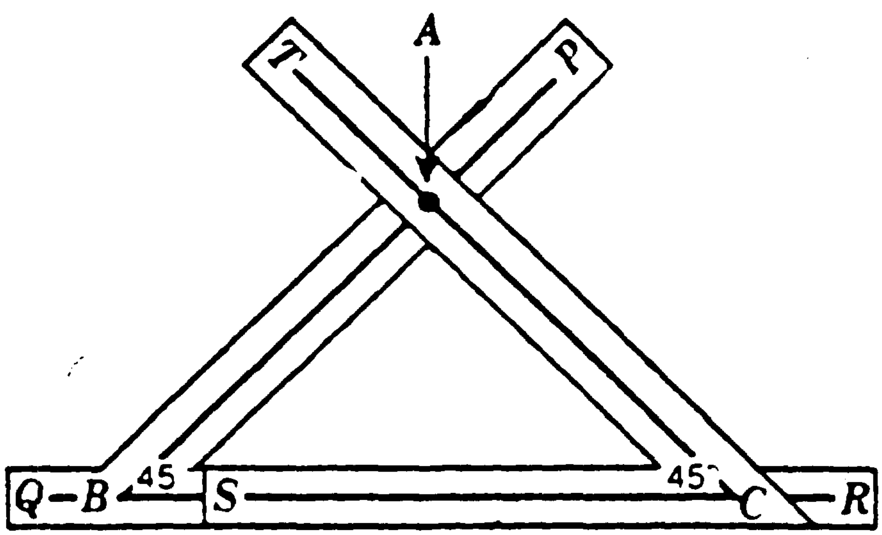
This lesson addresses several of these goals in the case of three dimensional geometries. Students are often given opportunities to explore properties such as these for two dimensional geometry, however the opportunity to explore geometries is often reserved only for the most advanced students. This lesson is appropriate for all students in a high school geometry course.



(a)



(b)



Project: Measuring the Angle Sum of a Surface Triangle

Name ______________________

1. Construct any plane triangle and measure its interior angles with a protractor.

   Angle sum in plane = ______________________ (a)

2. Determine the angle sum for a triangle on the lateral surface of a cylinder.

   Angle sum on cylinder = ______________________ (b)

3. Determine the angle sum for a triangle on an arbitrary curved surface.

   What surface did you use? ______________________

   Sketch the triangle and the surface:

   Angle *BZC* = ______________________ degrees

   Angle sum on surface = ______________________ (c)

4. Give reasons for the similarities and differences among the results (a), (b), and (c).

From the *Mathematics Teacher*, February 1994

Project: Measuring the Angle Sum of a Surface Triangle

Name ______________________

5. Were you surprised by any of the results? Explain.

6. What did you learn from this lesson?

7. Would you like to learn more about surface geometry?

8. Did you feel some reluctance to explore geometrical results physically? Explain how you felt.

9. List any comments or questions you may have on the material.

From the *Mathematics Teacher*, February 1994

Title: Using three-dimensional tic-tac-toe to graph and build a solid

Grade Level: 11 - 12

Time Required: two or three class periods

Objectives:

1. Students will visualize three dimensional concepts using a hands-on model.
2. Students will discover a very important relationship between the volume of tetrahedrons formed from a triangular prism.
3. Students will use measurement, algebra and geometry skills to construct three dimensional models.

Materials Needed:

___ three dimensional tic-tac-toe board and/or three dimensional tic-tac-toe game sheet
___ three dimensional graph paper
___ transparencies of 3 dimensional tic-tac-toe board and three dimensional graph paper
___ overhead projector and markers
___ centimeter graph paper
___ compass and straightedge
___ 3 different color crayons for each students
___ scissors
___ tape

Notes to the Teacher:

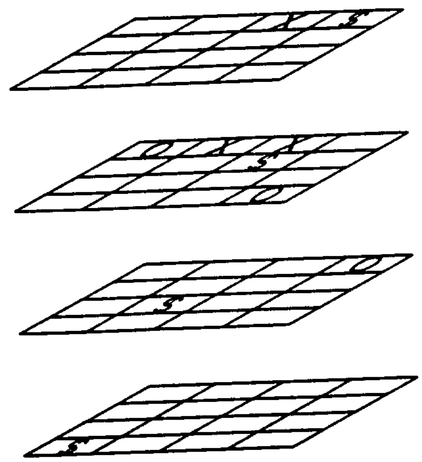
Standard 7 of the *Curriculum and Evaluation Standards for School Mathematics* call for students in grades 9 - 12 to "interpret and draw three-dimensional shapes." Furthermore, college-intending students should "develop an understanding of an axiomatic system through investigating and comparing various geometries." The topics presented in this lesson will aid the teacher in meeting these standards.

Ideas from non-Euclidean geometries are often left to the end of the textbook and therefore are often rushed through at the last minute or not even covered due to time constraints. However, these are ideas which are fundamental to understanding what really happens in the three dimensional world students live in.

Procedure:

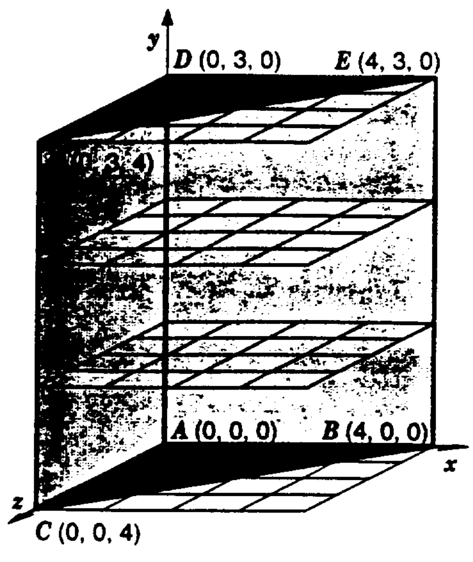
If you have a three dimensional tic tac toe board call two or three volunteers to the front of the room. (If you do not have a board then simply use the overhead of the game board.) Have students play a game of three dimensional tic-tac-toe.

Wins occur when a student gets four collinear symbols or markers. In the figure below the person using the $ symbol has won.



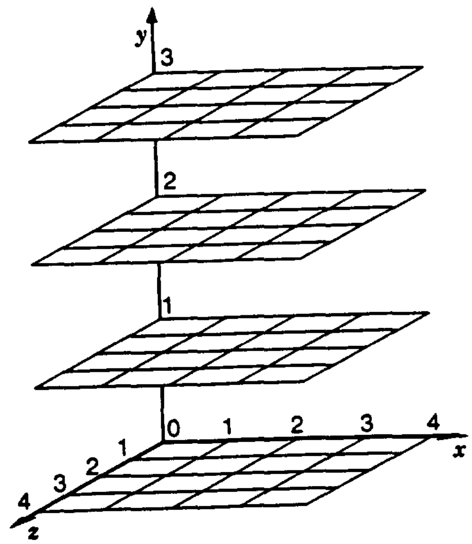
Allow students to play in groups of two or three. You may want to organize a short single-elimination tournament.

After students have played the game a few times introduce them to the three dimensional graph paper. Demonstrate how to plot points in space. Have students plot the following points: A(0,0,0), B(4,0,0), C(0,0,4), D(0,3,0), E(4,3,0), and F(0,3,4). Have students connect these points to form a rectangular prism.

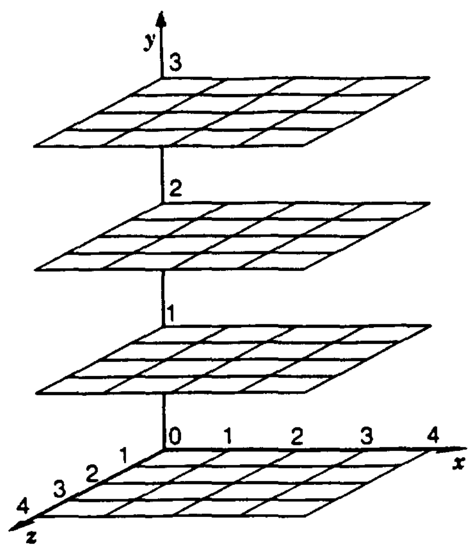


Students should set aside this page and take out a clean sheet of 3-D graph paper and three different color crayons.

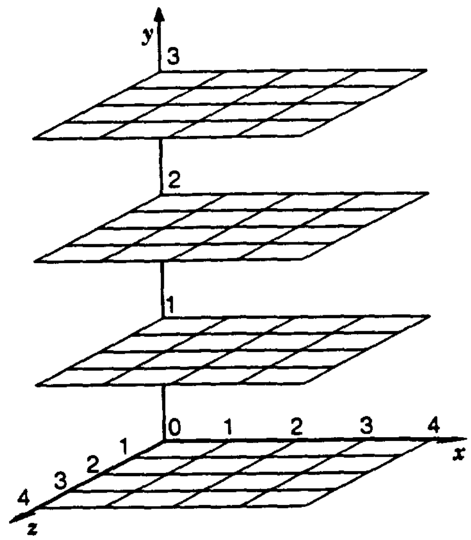
Explain that every set of four noncoplanar points determines a tetrahedron, or triangular pyramid. Beginning with one color crayon, plot the tetrahedron determined by A(0,0,0), D(0,3,0), E(4,3,0), and F(0,3,4).



Have students change colors and form the tetrahedron determined by A(0,0,0), B(,0,0), C(0,0,4), and F(0,3,4) on the same set of axes.



Change colors again. Plot A(0,0,0), B(4,0,0), F (0,3,4), and E(4,3,0).

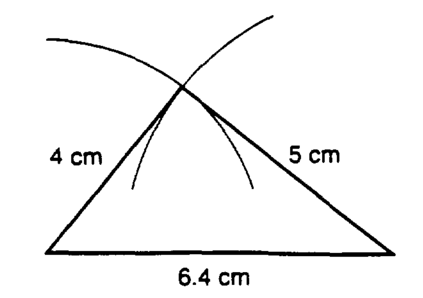


This figure shows how a triangular prism can be subdivided into three tetrahedrons.

Next distribute centimeter graph paper to students. They will use this to construct the three tetrahedrons. Students will use these models to discover that any two of these tetrahedrons, although not congruent, to each other, have the same volume.

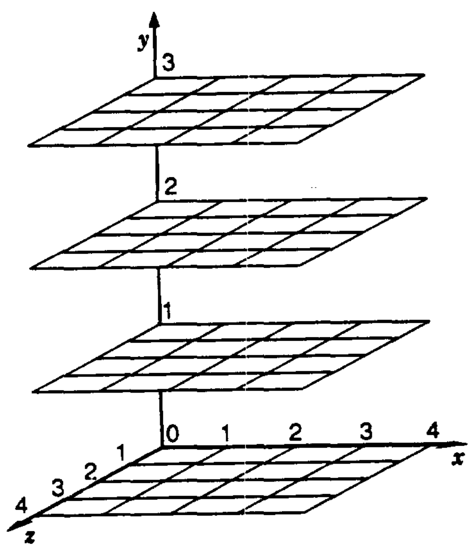
Using the centimeter graph paper, compass, straightedge, side-side-side triangle construction method, and three dimensional distance formula:

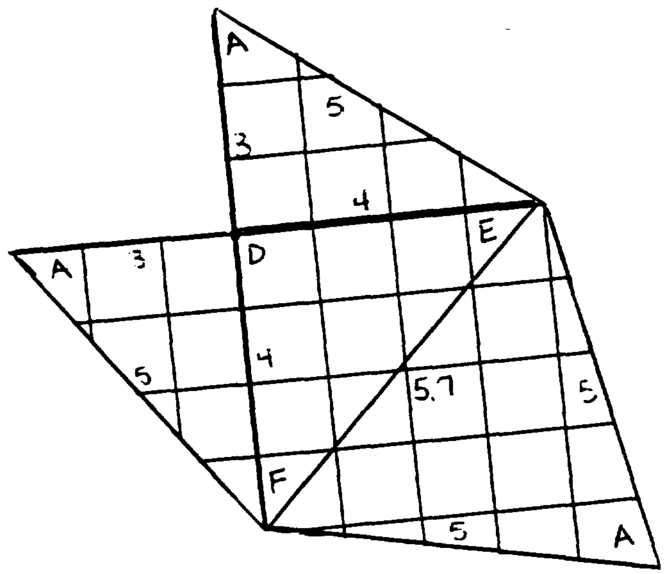
$$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2} .$$

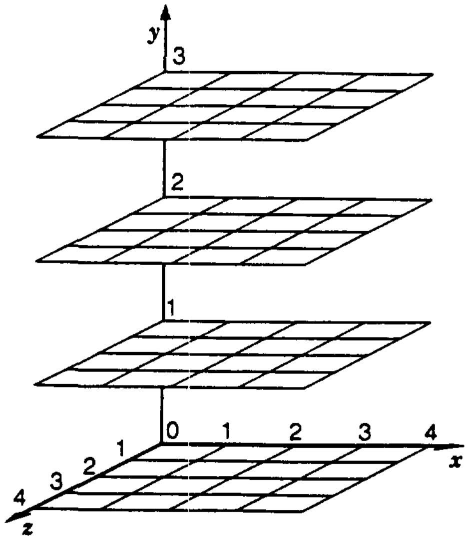


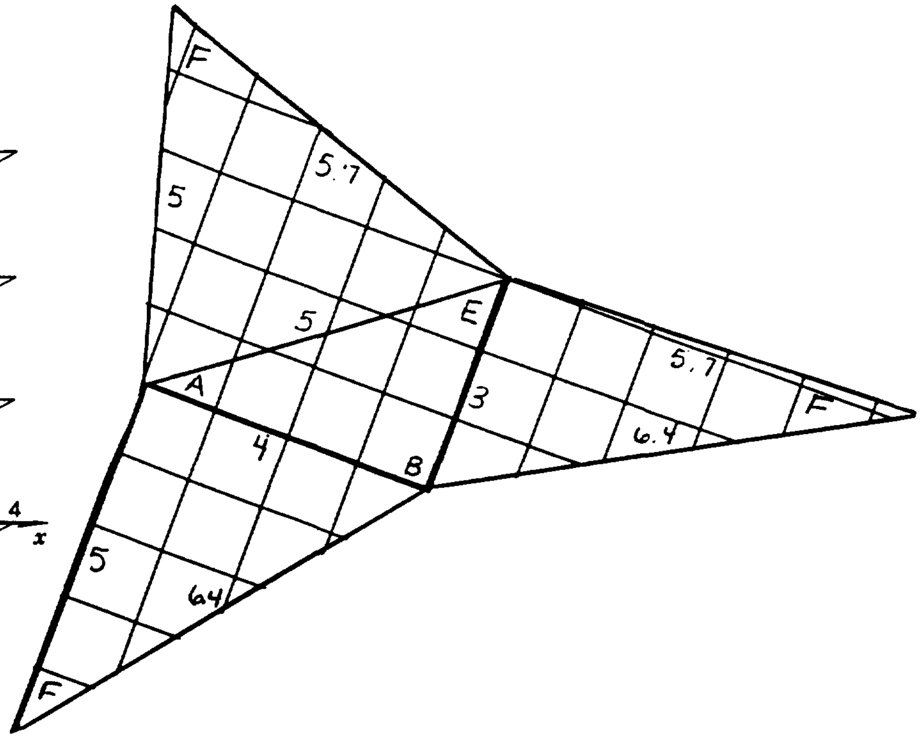
Side-side-side construction of one triangle

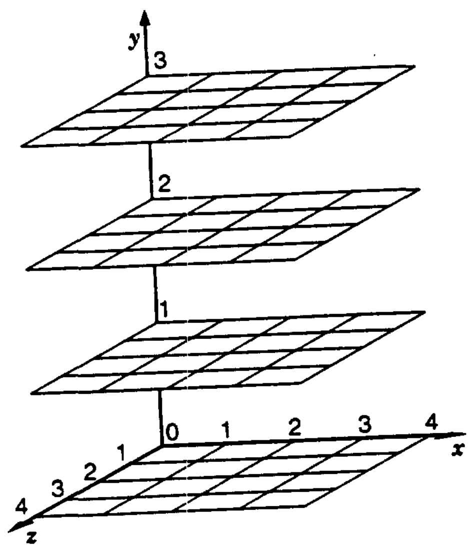
Once students have drawn the flat model of their tetrahedrons they should color each one a different color, which corresponds to the colors on the original graph; label all vertices, cut each one out, and tape it together.

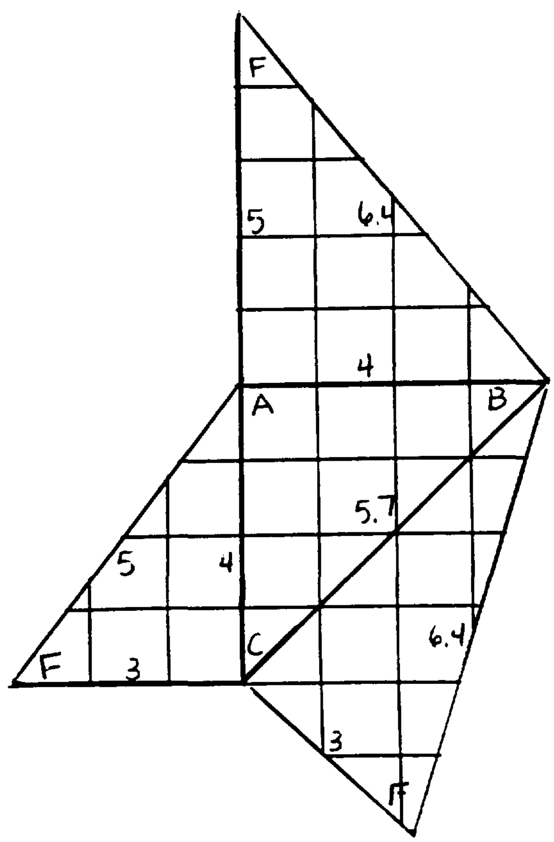




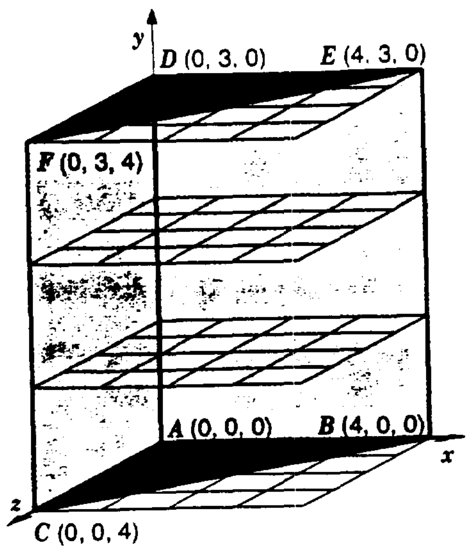

y
3
2
1
0 1 2 3 4
x
1 2 3 4
z

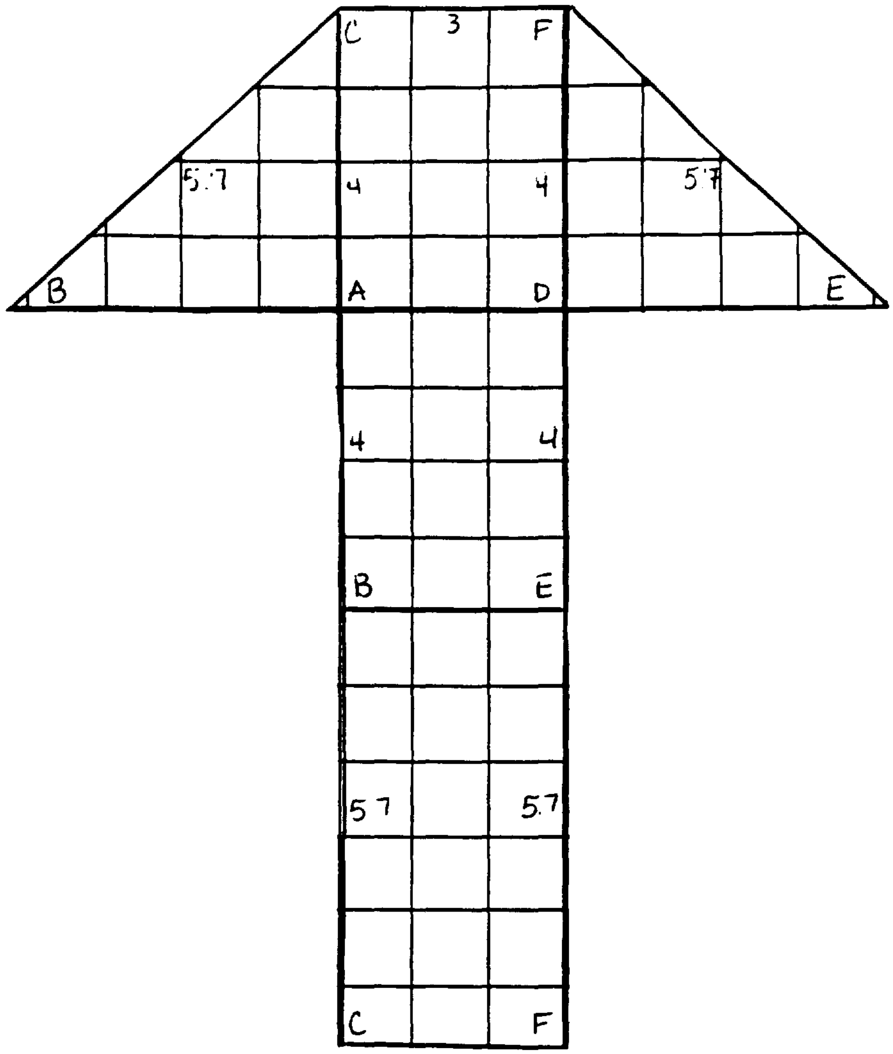

F
5.7
5
5
E
A
3
5.7
F
4
B
6.4
5
6.4
F


y
3
2
1
0 1 2 3 4
x
1 2 3 4
z


F
5
6.4
4
A
B
5.7
5
4
C
6.4
F
3
3
F

Finally, the students should construct the triangular prism from the first drawing. Once assembled, the triangular prism serves as a box to hold the three tetrahedrons.





Now it is time to make some discoveries about volume.
Students should begin by finding two tetrahedrons which have a common face; for example, the blue tetrahedron (FABE) and the green tetrahedron (ABCF) share face FAB. Have students place the two tetrahedrons they have found on the desk, common face down. Ask students what they observe. They should notice that the altitudes are the same. So these tetrahedrons have congruent bases and altitudes. Therefore the volumes must be the same. Repeat this procedure with one of the tetrahedrons you already have and the tetrahedron you have not used yet. Once the same relationship is established between these two then by transitivity all three tetrahedrons must have equal volumes.

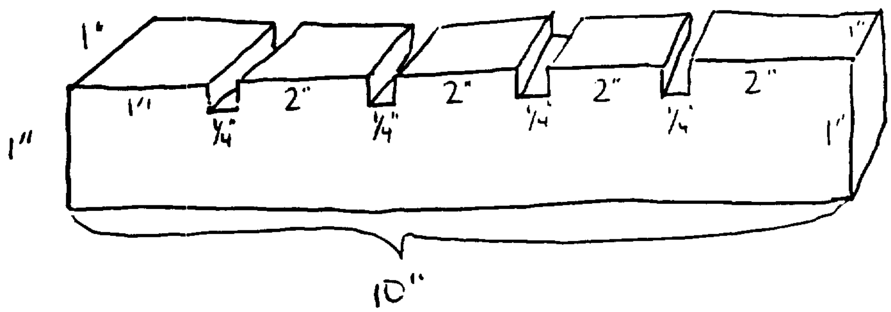
Adapted From:

Marino, George. *Graphing a Solid: A classroom activity*. Mathematics Teacher. 86(9): 734-737.

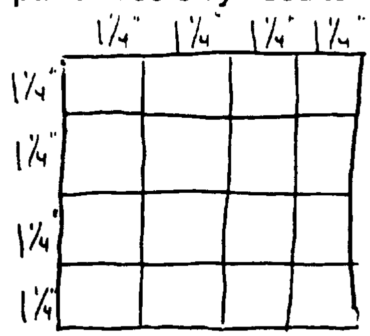
Constructing a Three Dimensional Tic-Tac-Toe Board

Materials Needed:
4 - 10" by 1" by 1" pieces of wood
4 - 5" by 5" clear plastic sheets-1/4" thick
Paint for legs of stand
1/4" wide colored tape or marker/paint which can be used to mark clear sheets (shelves) with tic tac toe grid
20 - playing pieces in each of 3 colors or styles

Procedure:
1. Notch (as shown below) each of the pieces of wood with 1/3" - 3/8" deep notches that are just wide enough to hold shelves.



2. Paint the pieces of wood (legs) as desired and allow sufficient time to dry.
3. While legs are drying, mark each of the four shelves as shown below using the 1/4" tape, marker, or paint. You only need to mark one side of each shelf.



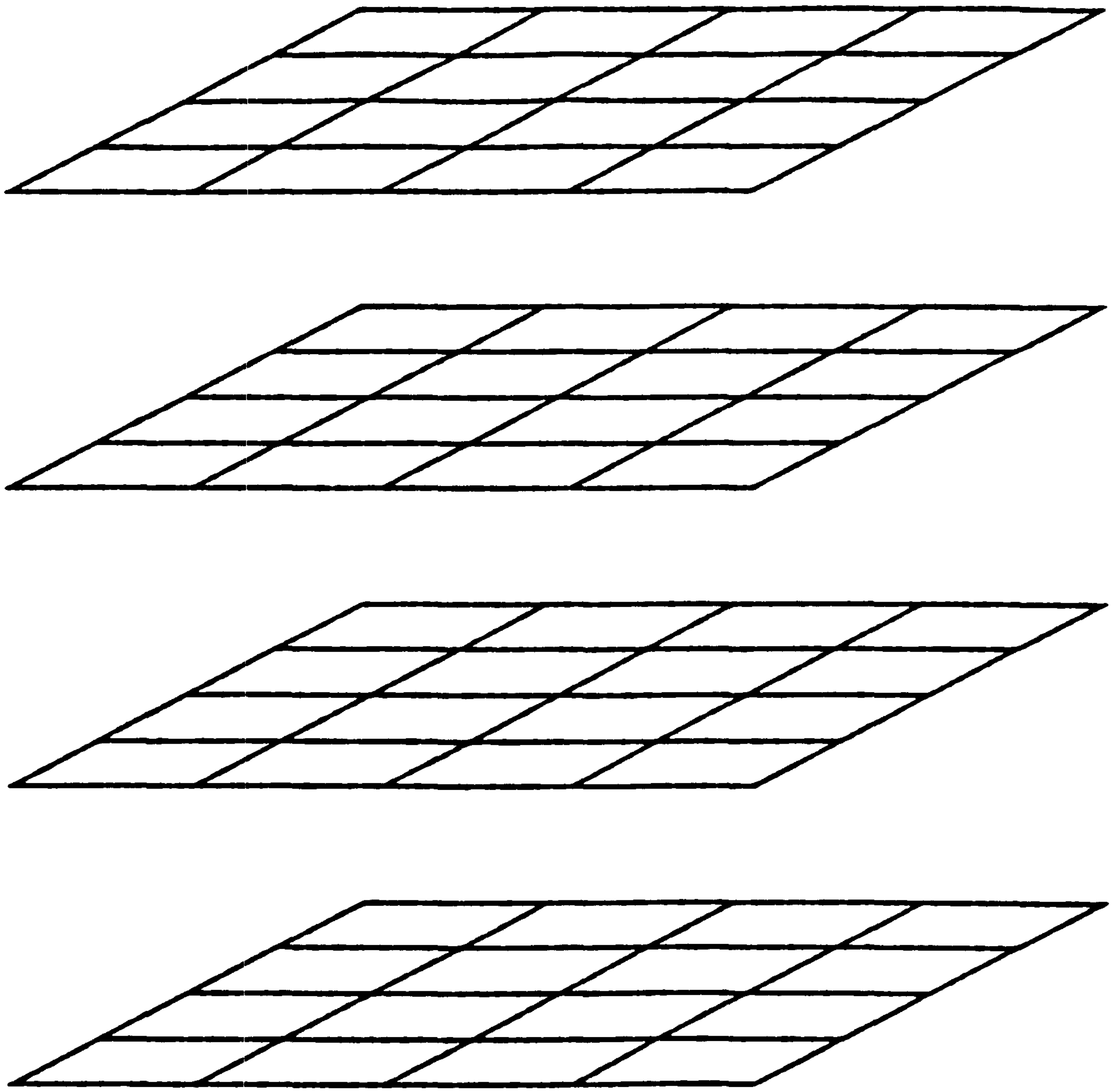
4. Once legs (and shelves if you painted them) are dry, place two legs on a level surface about 2-1/2" apart. Stand a shelf in each set of grooves with the gridded side facing toward the top of the legs. Place other two legs on top of the four shelves.

5. Place the stand upright and adjust shelves so that there is even overhang on all sides.

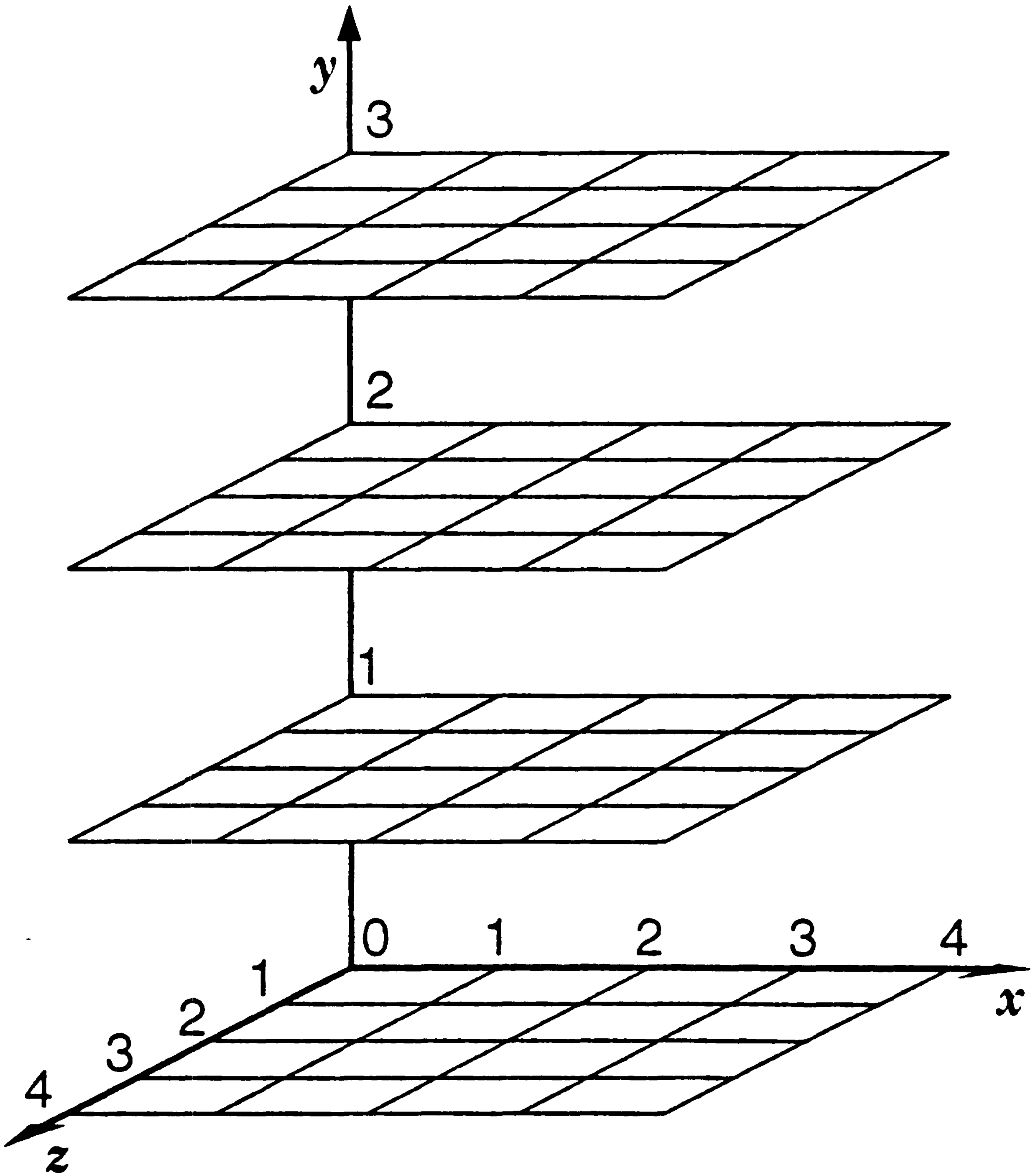
6. Play a couple of games using the playing pieces.

Centimeter Graph Paper

Three Dimensional Tic-Tac-Toe Board



Three Dimensional Graph Paper



Title: Absolutely Valuable Exploration

Grade Level: 9 - 12

Time Required: 50 minutes

Objectives:
1. Students will solve absolute value equations using a graphing calculator.
2. Students will draw an appropriate graph for an absolute value given the number of solutions the equation should have.
3. Students will identify conditions for which an absolute value equation will have extraneous roots.

Materials Needed:
___ Graphing calculator which will graph absolute values for each student.
___ Worksheet

Procedure:
Begin class with a discussion of what the equals sign in equations such as

$$x - 2 = 3$$
$$x^2 + 8 = 8$$
$$| x - 3 | = 4$$

means. This idea is fundamental to how you solve absolute value equations on the graphing calculator.

Have students solve the equation $| x + 5 | = 6$. By algebraic means, students should come to the solutions $x = -11$ and $x = 1$. To see this idea graphically have students graph $y_1 =| x + 5 |$ and $y_2 = 6$. Since these two functions must be equal for the equation to be satisfied students should look for places where the two graphs coincide.

Students should examine some of the following equations and record their observations.
$| 2x - 3 | = | 3x + 7 |$
$| 2x - 7 | = 9$
$| 3x + 4 | = | x - 4|$

At this point pass out worksheets and allow students to work in cooperative groups on the problems, providing explanation and direction when necessary.

Adapted From:
Horak, Virginia M. *Investigating Absolute-Value Equations with the Graphing Calculator.* Mathematics Teacher. 87(1): 9 - 11.

<u>Investigating Absolute Values--A Graphing Calculator Activity</u>

Name:____________________________ Date: _______________ Class: __________

1. Solve $| x + 3 | = 4x - 3$ algebraically.

$x =$ ______ and $x =$ ______

2. Graph $y_1 = | x + 3 |$ and $y_2 = 4x - 3$ on your graphing calculator. Sketch your graph below and record any observations.

Graph

Observations

____
____
____
____
____
____

3. Graph $y_1 = x + 3$ and $y_2 = 4x - 3$ on your graphing calculator. Sketch your graph below and record any observations.

Graph

Observations

____
____
____
____
____
____

4. Graph $y_1 = x + 3$ and $y_2 = -4x + 3$ on your graphing calculator. Record your graph and any observations below.

Graph

Observations

____
____
____
____
____

5. Why do you believe the extraneous root resulted in the algebraic solution?

6. Solve $| x - 4| = | x + 2|$ algebraically.

7. Graph $y_1 = | x - 4|$ and $y_2 = | x + 2|$. Sketch the graph and record any observations.

Graph

Observations

____
____
____
____
____
____

8. Graph $y_1 = x - 4$ and $y_2 = x + 2$. Sketch the graph and record any observations.

Graph

Observations

____
____
____
____
____
____

9. Graph $y_1 = x - 4$ and $y_2 = -x -2$. Sketch the graph and record any observations.

Graph

Observations

____
____
____
____
____
____

11. Are there any general conditions under which the algebraic solution of an equation of the form $| m_1x + a | = | m_2x + b |$ would result in an extraneous root?

Test your conjecture on at least three equations. Record those tests here and on the back of this page if necessary.

12. Draw the graph of an absolute value equation which has absolute values on each side for which there would be no solution.

13. Write the corresponding equation.

14. Draw the graph of an absolute value equation which has absolute values on each side for which the solution set is infinite.

15. Write the corresponding equation.

Title: A graphical approach to radical equations

Grade Level: 11 - 12

Time Required: 2-fifty minute class periods or 1-ninety to one-hundred minute class period.

Objectives:
1. Students will approach the solution of a problem generally solved algebraically, from a geometric standpoint.
2. Students will account for extraneous solutions which are often a result in the algebraic solutions of algebraic equations.

Materials Needed:
___ Graphing calculators or computer graphing package
___ Worksheet 1
___ Worksheet 2

Notes to the Teacher:
Too often, algebraic manipulations are taught to students without any attempt being made to connect these topics to their geometric (graphical) representations. However, connecting these two topics can provide students with not only another method to use when solving radical equations but when solving almost any type of equation.
When using a graphing calculator, a typical difficulty is knowing how to set the range or window. Examples 1, 2, and 3 in the first part of the procedure are accurately viewed with a range of xMin = -10; xMax = 10; xScl = 1; yMin = -10; yMax = 10; and yScl = 1. Examples 4 and 5 can be accurately viewed when the xMin is changed to -5, the xMax to 40 and the xScl to 5. The graphs produced with these settings are shown next to each example in the procedure section. The answer key for worksheet 1 also contains the graphs of each equation and a convenient range/window for viewing that equation. If extra practice in graphing is desired assign students to choose three or four of these problems to verify using the graphing calculator.
Students may find worksheet 2 challenging. However, the main goal of the teacher should be to challenge students to think for themselves, use their reasoning skills, and support the conjectures they make. For this reason, the teacher should not use worksheet 2 as a discussion guide. Students should work together to complete the activities, without more than subtle guidance from the teacher.

Procedure:

Begin by introducing the algebraic method for solving radical equations with examples such as

* 1. $\sqrt{x+2} = x$

$x + 2 = x^2$

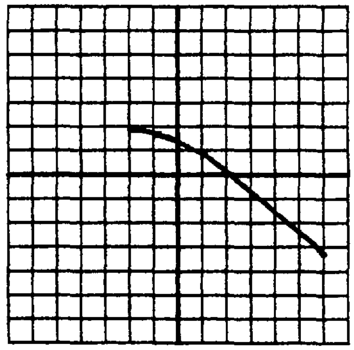
$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x = 2$ or -1

-1 is extraneous

Sketch



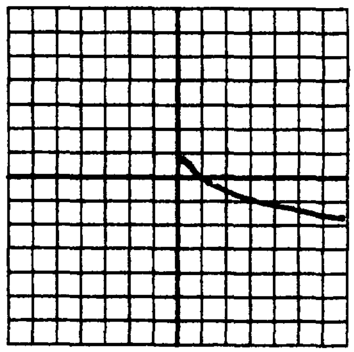
2. $\sqrt{x+3} = \sqrt{5x-1}$

$x + 3 = 5x - 1$

$4 = 4x$

$1 = x$

Sketch



* 3. $\sqrt{3a-2} - \sqrt{2a-3} = 1$

$\sqrt{3a-2} = 1 + \sqrt{2a+3}$

$3a - 2 = 1 + 2\sqrt{2a-3} + 2a - 3$

$3a - 2 = 2a - 2 + 2\sqrt{2a-3}$

$a = 2\sqrt{2a-3}$

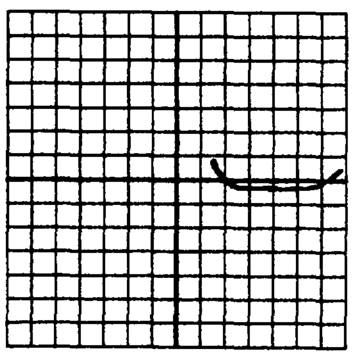
$a^2 = 4(2a-3)$

$a^2 - 8a + 12 = 0$

$(a-6)(a-2) = 0$

$a = 6$ or 2

Sketch



Suggested Range

x Min = -1

x Max = 10

x Scl = 1

y Min = -1

y Max = 3

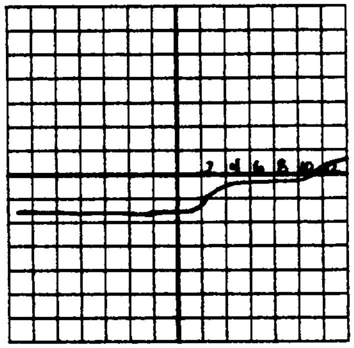
y Scl = 1

4. $\sqrt[3]{x-3} = 2$

$x - 3 = 8$

$x = 11$

Sketch



Suggested Range
xMin = -15
xMax = 15
xScl = 5
yMin = -10
yMax = 10
yScl = 2

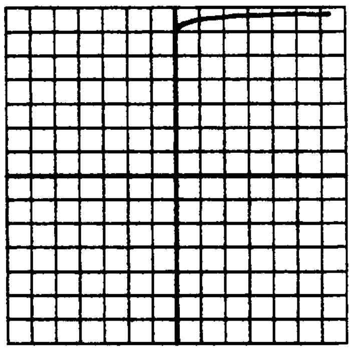
5. $\sqrt{x} + 6 = 0$

$(\sqrt{x})^2 = (-6)^2$

$x = 36$

36 is extraneous

Sketch



Allow students a chance to practice some of this type of problem (Worksheet 1). Students may or may not finish all of worksheet 1 before you begin the next part of the lesson.

Next, demonstrate how to graph radical equations on a graphing calculator. For example, to graph $\sqrt{x+2} = x$, input y1 $= \sqrt{x+2} - x$. Students can see that -1 is not a solution of the equation but x = 2 is. Demonstrate as many equations as necessary until students understand the technique. Pass out worksheet 2 and allow students time to work in groups.

Adapted from:
Naraine, Bishnu. *An Alternative Approach to Solving Radical Equations.* Mathematics Teacher. 86(3): 204-205.

Worksheet 1--Solving Radical Equations

Name: ______________________________ Date: ______________________

Solve each of the following equations remembering to show all work. Write all solutions in the blanks provided. **CIRCLE** any extraneous roots. If an equation has no real solution, say so.

1. $\sqrt{4x-3} = 5$ 1. ____________

2. $3\sqrt{t} - 5 = 13$ 2. ____________

3. $\sqrt{2x^2 - 7} = 5$ 3. ____________

4. $\sqrt[3]{2d} + 5 = 3$ 4. ____________

5. $\sqrt{2n+3} = n$ 5. ____________

6. $\sqrt{t-2} + t = 4$ 6. ____________

7. $5 + \sqrt{a + 7} = a$

7. __________

8. $\sqrt{2x + 5} - 1 = x$

8. __________

9. $\sqrt{5y - 1} - \sqrt{7y + 9} = 2$

9. __________

10. $\sqrt{x} + \sqrt{3} = \sqrt{x + 3}$

10. __________

11. $\sqrt{2n - 5} - \sqrt{3n + 4} = 2$

11. __________

12. $\sqrt{x + 7} + \sqrt{x} = 7$

12. __________

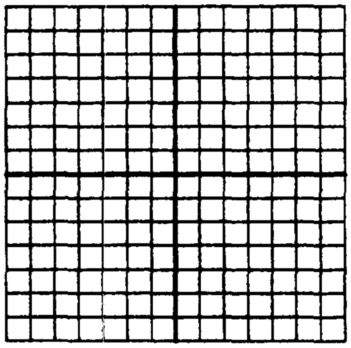
Worksheet 2--An Alternative Approach to Solving Radical Equations

Name: ______________________________ Date: ____________________

Reevaluate problems 3, 4, 6, 7, 10, 11, and 12 from worksheet 1 using the graphing calculator. Sketch the graph below each problem. Write the solutions you found using the graphing calculator in the blank provided.

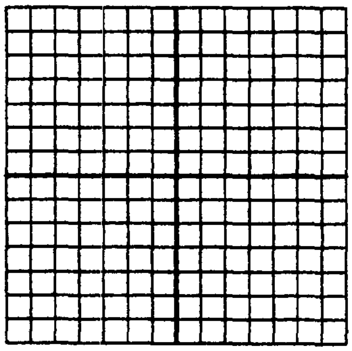
3. $\sqrt{2x^2 - 7} = 5$ 3. ______________

Sketch



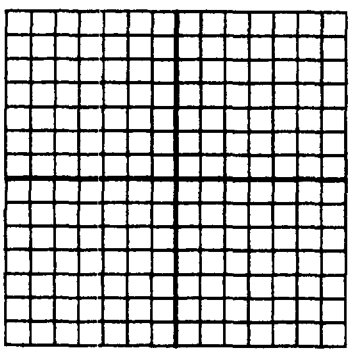
4. $\sqrt[3]{2d} + 5 = 3$ 4. ______________

Sketch



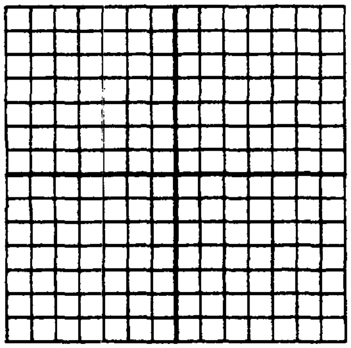
6. $\sqrt{t - 2} + t = 4$ 6. ______________

Sketch



7. $5 + \sqrt{a + 7} = a$

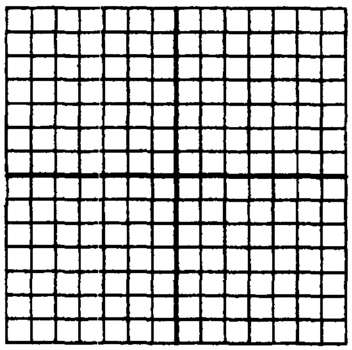
Sketch



7. ____________

10. $\sqrt{x} + \sqrt{3} = \sqrt{x + 3}$

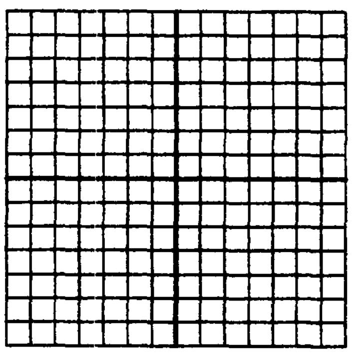
Sketch



10. ____________

11. $\sqrt{2n - 5} - \sqrt{3n + 4} = 2$

Sketch

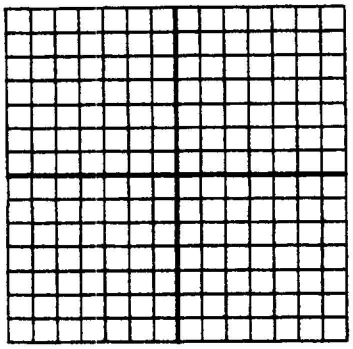


11. ____________

12. $\sqrt{x+7} + \sqrt{x} = 7$

12. ____________

Sketch



Questions

1. After squaring both sides of $\sqrt{2n+3} = n$ (Exercise 5), you should have gotten $2n + 3 = n^2$. There are three other equations like $\sqrt{2n+3} = n$ which would yield $2n + 3 = n^2$ when both sides are squared. List these below.

(1) $\sqrt{2n+3} = n$
(2) ________________
(3) ________________
(4) ________________

2. Do you notice anything about these four equations? Can you group these four equations into two groups of two equations which have something in common?

Group 1	Group 2
____________________	____________________
____________________	____________________

3. Why did you group the equations in this manner?

____
____
____
____

4. Substitute both values you got in the algebraic solution of exercise 5 into all four equations from above. If a true statement results put a "+" in the box; if a false statement results put a "-" in the box.

Equation	n value ___	n value ___
$\sqrt{2n+3} = n$		

5. Compare the results from your grouped equations. Are there any similarities within each group of equations? If so, explain. If not, return to question 2, look for a different way to group the equations, and repeat.

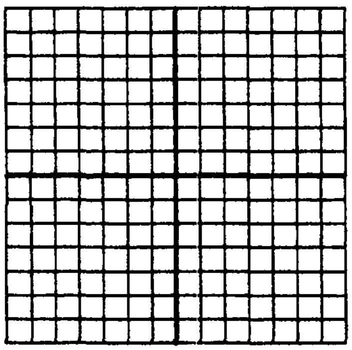
6. Why do you believe these results occurred? (i.e., What do you do in the algebraic solution of radical equations which might cause an extraneous root to result?)

7. Graph the four equations on the graphing calculator and record the graphs below.

Group 1

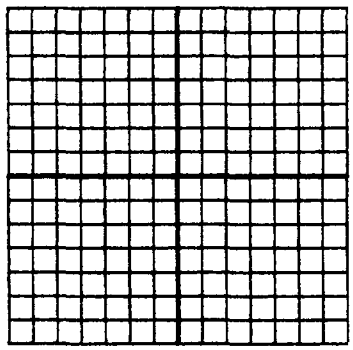
Equation ______________

Sketch



Equation ______________

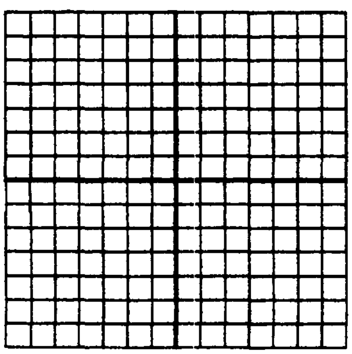
Sketch



Group 2

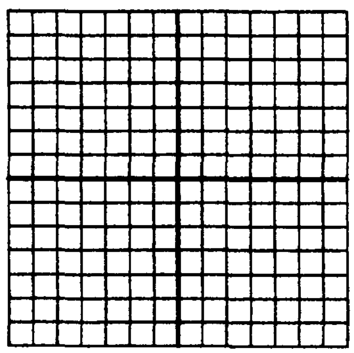
Equation ______________

Sketch



Equation ______________

Sketch



8. Do the graphs confirm your reasoning from question 6? How do they or do they not confirm this?

Worksheet 1--Solving Radical Equations

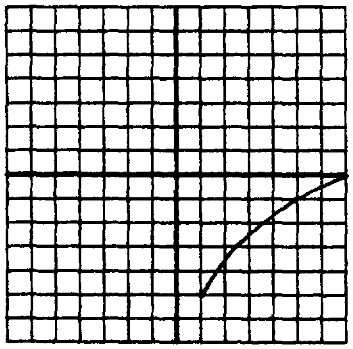
Name: Answer Key Date: ____________

Solve each of the following equations remembering to show all work. Write all solutions in the blanks provided. **CIRCLE** any extraneous roots. If an equation has no real solution, say so.

1. $\sqrt{4x-3} = 5$

$4x-3=25$
$4x=28$
$x=7$

Sketch



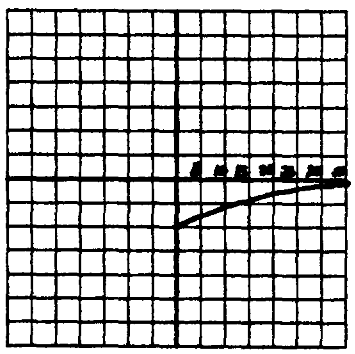
1. $x=7$

$y_1 = \sqrt{4x-3} - 5$

2. $3\sqrt{t} - 5 = 13$

$3\sqrt{t} = 18$
$\sqrt{t} = 6$
$t=36$

Sketch



2. $t=36$

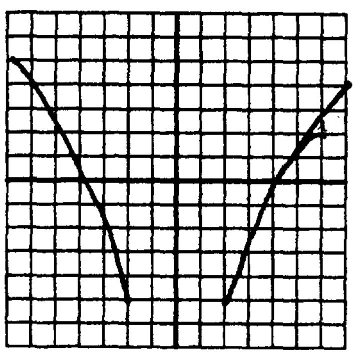
$y_1 = 3\sqrt{t} - 18$

Range
XMin = -50
XMax = 50
XScl = 5
yMin = -50
y Max = 50
y Scl = 5

3. $\sqrt{2x^2-7} = 5$

$2x^2-7=25$
$2x^2=32$
$x^2=16$
$x=\pm 4$

Sketch



3. $x=4, -4$

$y_1 = \sqrt{2x^2-7} - 5$

Range
x Min = -10
x Max = 10
x Scl = 1
y Min = -10
y Max = 10
y Scl = 1

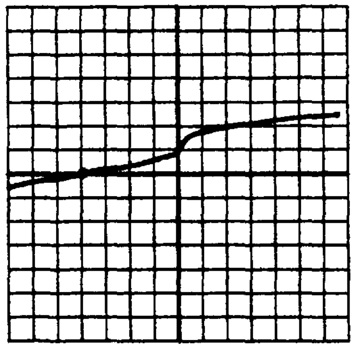
4. $\sqrt[3]{2d} + 5 = 3$

$\sqrt[3]{2d} = -2$

$2d = -8$

$d = -4$

Sketch



4. $d = -4$

$y_1 = (2d)^\wedge(1/3) + 2$

5. $\sqrt{2n+3} = n$

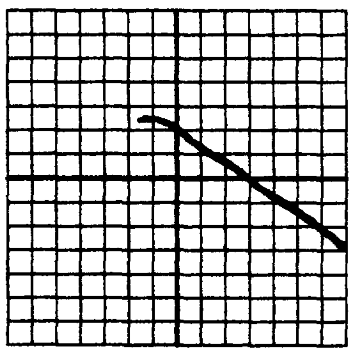
$2n + 3 = n^2$

$0 = n^2 - 2n - 3$

$0 = (n-3)(n+1)$

$n = 3 \text{ or } -1$

Sketch



5. $n = 3, \textcircled{-1}$

$y_1 = \sqrt{2n+3} - n$

6. $\sqrt{t-2} + t = 4$

$(\sqrt{t-2})^2 = (4-t)^2$

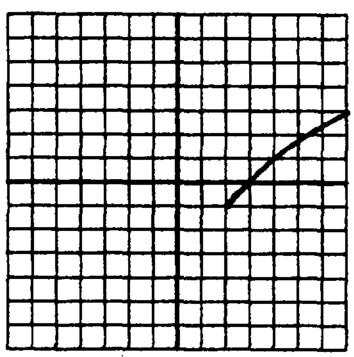
$t - 2 = 16 - 8t + t^2$

$0 = t^2 - 9t + 18$

$0 = (t-6)(t-3)$

$t = 6, 3$

Sketch



6. $t = \textcircled{6}, 3$

$y_1 = \sqrt{t-2} + t - 4$