

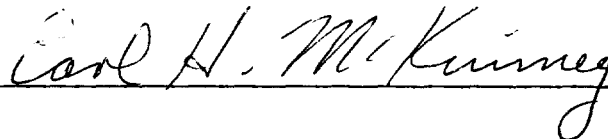
Linear Programming and the Simplex Method

An Honors Thesis (ID 499)

By

Terri A. Tevis

Thesis Director

A handwritten signature in cursive script, reading "Earl H. McKinney", is written over a solid horizontal line.

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ABSTRACT

This thesis discusses the basic problems of solving a linear programming problem. A definition of the linear programming problem is stated.

Basic linear algebra methods are necessary to solve a linear programming (LP) problem. A short synopsis of the necessary methods used are presented, including examples.

The simplex algorithm is introduced and an example is used to demonstrate its applicability. A computer program is presented which utilizes the basic simplex algorithm and the extended tableau form.

The problem of degeneracy as it affects the simplex method is explained. Research of more efficient programs are always in progress. The power of the simplex method is evident in the business applications area.

The linear programming problem may be defined as an optimization problem where the greatest possible numerical value (the maximum) or the least possible numerical value (the minimum) of a mathematical function of independent variables is found. There may be constraints upon the variables in the form of equalities and/or inequalities. Therefore, the solution to the linear programming (LP) problem is the set of variables which gives an optimal value to the objective function and at the same time does not violate the constraints imposed upon the variables.

The general form of a LP problem is a linear function called the objective function and a list of restrictions upon the variables of this function. Together, these equations form a system of linear equations, and it is for this system that a solution is sought.

The matrix of co-efficients of a system of  $m$  equations in  $n - 1$  variables augmented with the constant vector forms an  $m \times n$  matrix. The underlying principles are explained briefly:

$$3x + 5y = 8$$

$$4x + y = 5$$

Example 1a

Example 1a is a system of linear equations in two variables which will form a  $2 \times 3$  augmented matrix, as shown in Example 1b.

$$\begin{bmatrix} 3 & 5 & : & 8 \\ 4 & 1 & : & 5 \end{bmatrix}$$

## Example 1b

A solution set for this system of equations is the ordered pair (x,y) which solve both equations (i.e., (1,1)). To find the solution to a system of linear equations, elementary matrix row operations may be used. These are:

- 1.) Interchange any two rows
- 2.) Multiply any row of the matrix by any non-zero constant
- 3.) To any row may be added a non-zero constant multiple of any row in the matrix.

The method that will be used to solve systems of equations is called Gauss-Jordan elimination. The algorithm is summarized as follows.

- 1.) Choose a pivot element.
- 2.) Change the pivot element to the value 1 by performing the appropriate row operation on the pivot row.
- 3.) Now, perform subsequent operations such that all of the elements in the pivot row and pivot column are changed to zero except the pivot element.
- 4.) Perform this algorithm until the identity matrix is produced and the values of the variables can be read off from the right hand column.

The steps taken to solve the system given in Example 1b will now be demonstrated to explain the Gauss-Jordan method.

$$\left[ \begin{array}{cc|c} 3 & 5 & 8 \\ 4 & 1 & 5 \end{array} \right]$$

Choose 1 as the pivot element since it is already equal to 1. Now, change the elements in the pivot column to zero. To change the 5 in the pivot column to zero we must add -5 times the second row to the first row as shown.

$$\left[ \begin{array}{cc|c} -17 & 0 & -17 \\ 4 & 1 & 5 \end{array} \right] \quad (-5)R_2 + R_1$$

Next, the 4 in the pivot row is changed to zero by adding 4/17 times the first row to the second row. The resulting matrix is:

$$\left[ \begin{array}{cc|c} -17 & 0 & -17 \\ 0 & 1 & 1 \end{array} \right] \quad (4/17)R_1 + R_2$$

All of the elements in the pivotal row and column have now been changed to zero, except the pivot element itself.

Now, choose a new pivot element and change its value to 1.

$$\left[ \begin{array}{cc|c} -17 & 0 & -17 \\ 0 & 1 & 1 \end{array} \right] \quad (-1/17)R_1$$

The result is:

$$\begin{bmatrix} 1 & 0 & | & 1 \\ 0 & 1 & | & 1 \end{bmatrix}$$

x   y

The procedure is complete because the 2 x 2 identity matrix has been produced. The values for x and y can be read from the right hand column. As stated earlier,  $x = 1$  and  $y = 1$ . To prove the solution is correct, simply substitute the values for x and y into the original equations.

The solution to a system of inequalities is analogous to the solution of a system of equalities. This is important because constraints placed upon variables are often in the form of inequalities.

Inequalities are changed to equalities by adding the constant S which takes up the slack between the right and left hand sides.

$$Ax + By < C$$

is equivalent to

$$Ax + By + S = C$$

$$\text{where } S \geq 0$$

S is called a slack variable and is used in less-than type inequalities. For an inequality of the greater-than type, the variable T is used to represent the surplus and is subtracted from the left-hand side.

$$Ax + By > C$$

is equivalent to

$$Ax + By - T = C$$

where  $T \geq 0$

T is called a surplus variable.

One method of solving linear programming problems was developed by George Dantzig. It is an iterative process name the Simplex Method which makes use of Gauss-Jordan elimination techniques discussed previously. The Simplex method is very simple and may be used in a variety of LP problems. Since it is iterative in nature, it may be easily adapted to computers. A FORTRAN program will be developed to demonstrate the suitability of this method to the computer.

The goal of the Simplex method is to obtain a feasible solution to the objective function based upon the constraints placed upon the variables. An optimal solution will be defined as the solution set which satisfies all the constraints of a problem and optimizes the objective function. Each iteration of the simplex method approaches the optimum solution of the objective function which describes the problem. When each iteration is complete, the results are checked, and if the solution is not optimal the process is repeated using the data obtained from the previous iteration.

Applications of the simplex method to business related problems are quite common. The objective function may concern maximizing



certain functions such as profit, margin, returns, utilization, capacities, occupancy, and time. On the other hand, such functions as cost, scrap, distance, weight and time may wish to be minimized. The Simplex method can handle these objectives easily on the condition that the problem must be able to be described by mathematical notation, and all constraints upon the variables involved be in the form of inequalities with non-negative variables.

Basic Requisites<sup>1</sup> for the Simplex method approach are:

- 1.) A well-defined objective must be clearly stated and expressed by a function.
- 2.) All data entered in the description of the problem must be expressed in quantitative form.
- 3.) An alternative choice among factors must be feasible, such as the possibility of choosing between machine work, machines and processes, space or machines, space and time, and all conceivable and possible discrimination among the productive factors.
- 4.) Only linear functions expressing constraints and restrictions will be used. That is, the direct proportional change in output, when a variation of input is verified, must be absolutely present. (The reason for the name "linear" programming lies in this general requirement.)
- 5.) The problem must be completely described by mathematical notation.

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<sup>1</sup> Introduction to Linear Programming Processes by di Roccoferrera; Cincinnati, South-Western Pub. Co., 1967, pp. 424-425.

All equations being linear are of the first degree. No variables may have a power greater than 1.

In the Simplex method, the basic maximal or minimal solution is found when no improvements are possible upon the solution at hand. This solution is called the optimum solution.

One nice result of the Simplex method is that the maximum and the minimum of a problem are found at the same time. This is so because the maximum result of any linear function is equal to the negative of the minimum of that linear function. Or restated, "The maximum of the negative objective function equals the minimum of the positive objective function." Therefore, the Simplex method results in:

- 1.) A solution to a maximum LP.
- 2.) A solution to its minimum dual.
- 3.) Their common value.

To demonstrate the iterative procedure of the Simplex method, a LP problem will be given and the steps taken to reach the optimal solution will be shown in Tableau Form. The linear programming problem will be given as an objective function and the constraints upon the variables.

Let the objective function be

$$Z = 10x_1 + 12x_2 + 14x_3$$

subject to the following conditions:

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$x_3 \geq 0$$

$$2x_1 + 2x_2 + x_3 \leq 9$$

$$2x_1 + 3x_2 + x_3 \leq 12$$

$$x_1 + x_2 + 5x_3 \leq 15$$

Maximize the objective function  $Z$  subject to the stated restrictions.

The next step in solving the LP problem is to derive a system of equations using slack variables (since the constraints are all of the less-than-or-equal-to type) to form equalities from the inequalities.

Doing this gives:

$$2x_1 + 2x_2 + x_3 + S_1 + 0S_2 + 0S_3 = 9$$

$$2x_1 + 3x_2 + x_3 + 0S_1 + S_2 + 0S_3 = 12$$

$$x_1 + x_2 + 5x_3 + 0S_1 + 0S_2 + S_3 = 15$$

The simplex tableau may now be set up and steps involved in the simplex method demonstrated.

The tableau consists of rows made up of basic and non-basic variables. These correspond to the original variables and the slack variables given in the problem. The tableau will appear as in Example 2a.

	non basic			basic			
	$x_1$	$x_2$	$x_3$	$S_1$	$S_2$	$S_3$	
	2	2	1	1	0	0	9
	2	3	1	0	1	0	12
	1	1	5	0	0	1	15
Z	-10	-12	-14	0	0	0	0

Example 2a

The first three columns are labeled non-basic, which are the variables which are set equal to zero. The basic variables are the variables at a given stage which are in the basis. Row Z is the negative of the objective function. With the tableau prepared, the Simplex Algorithm will now be explained.

The object of the algorithm is to drive out the negative coefficients in the objective function, Z. This is done through a series of Gauss-Jordan elimination steps. But, first a pivot must be chosen.

STEP 1 - To choose a pivot element, scan the objective row (row Z) and choose the most negative value. The column it is in will be the pivotal column.

STEP 2 - Form a ratio of the constraint constants of the right hand side column to the elements in the pivotal column. The element which produces the smallest ratio will be chosen as the pivot element.

STEP 3 - Convert the pivot element to a 1 by dividing the pivot row by the pivot element.

STEP 4 - Convert all remaining values in the pivotal column to zero using Gauss-Jordan elimination and the pivot element.

STEP 5 - Repeat these steps until there are no negative values left in the objective row. Never pivot on a negative value or a value in the objective row.

Example 2b will show the effects of the Simplex method upon the problem given.

2	2	1	1	0	0	9
2	3	1	0	1	0	12
1	1	⑤	0	0	1	15
-10	-12	-14	0	0	0	0

Example 2b

5 is chosen as the first pivotal element by STEPS 1 and 2. STEP 3 is performed by dividing by the pivotal element 5. Then Gauss-Jordan elimination was used to reduce the column values to zero (STEP 4). The results can be seen in the following example.

9/5	9/5	0	1	0	-1/5	6
9/5	14/5	0	0	1	-1/5	9
1/5	1/5	1	0	0	1/5	3
-36/5	-46/5	0	0	0	14/5	42

The objective row is again scanned for negative values. Since there are still negative values, the process repeated, beginning with STEP 1. The resulting matrix is shown below.

	9/14	0	0	1	-9/14	-1/14	3/14
	9/14	1	0	0	5/14	-1/14	45/14
	1/14	0	1	0	-1/14	3/14	33/14
Z	-9/7	0	0	0	23/7	15/7	501/7

Since there is still a negative value in row Z, the process is repeated, again!

	1	0	0	14/9	-1	-1/9	1/3
	0	1	0	-1	1	0	3
	0	0	1	-1/9	0	2/9	7/3
Z	0	0	0	2	2	2	72

Since, there are no negative values in the objective row, the process is complete. Now, all that is left is to read the values of the variables from the tableau. To do this, look at the columns corresponding to the variables  $x_1$ ,  $x_2$ , and  $x_3$ . If the column is in standard form, i.e.

a single element equals 1 and the rest of the elements are equal to zero, and there are no other columns in the basic variable area equal to this column, then assign the variable of the column the value in the final column of the tableau which appears in the same row as the value 1. If any column is not in standard form, then the assigned value of the corresponding variable is zero. If two or more of the columns are in standard form and are equal, then assign one variable the value from the corresponding constant column and assign the value zero to the other variables of the columns equal to the columns in question. This case will most probably occur with the condition of degeneracy.

Applying this to the previous example, the value for  $x_1$  is found to be equal to  $1/3$ ,  $x_2 = 3$  and  $x_3 = 7/3$ . Therefore, the objective function can be written as

$$10(1/3) + 12(3) + 14(7/3) = 72$$

Thus, the maximum value of the objective function is equal to 72 with the variable values  $(1/3, 3, 7/3)$  in accordance with the imposed constraints.

The solution to this LP problem has been verified through the use of a computer program which will be presented shortly. The output of the program is shown below.

AFTER 4 ITERATIONS THE OPTIMAL OBJECTIVE VALUE IS 72.000

1.000	0.000	0.000	1.556	-1.000	-0.111	0.333
0.000	1.000	0.000	-1.000	1.000	0.000	3.000
0.000	0.000	1.000	-0.111	-0.000	0.222	2.333
0.000	0.000	0.000	2.000	2.000	2.000	72.000

The concept of duality was touched upon with the mention of minimization of a linear programming problem. The example presented dealt only with constraints of the less-than-or-equal-to type. This type of LP will be called the Primal problem. Therefore, the minimization problem related to this Primal problem is called the Dual of the LP problem. Its constraints are in the form greater-than-or-equal-to and seeks the minimum value of the objective function as can be determined by the given constraints upon the variables. Every LP problem has a Dual. The Fundamental Theorem of Linear Programming (John von Neumann, 1947) relates the Primal and its Dual as follows:

"If a Linear Program and its Dual LP both have a feasible point, then they both have a best feasible point and the values of both programs are equal. If either program has no feasible point, then the other one has no best feasible point."

Therefore, the maximum value of the Primal problem will be equal to the minimum value of its dual.

To set up the Dual of the example in 2a:

The objective function is

$$9x_1' + 12x_2' + 15x_3' = Z'$$

subject to:

$$x_1', x_2', x_3' \geq 0$$

$$2x_1' + 2x_2' + 1x_3' \geq 10$$

$$2x_1' + 3x_2' + 1x_3' \geq 12$$

$$1x_1' + 1x_2' + 5x_3' \geq 14$$



To solve the minimization LP problem with "greater-than" constraints one can solve the dual problem instead. Then, from the Z row in the tableau in example 2c, the values of the variables in the problem may be read. They appear in the columns corresponding to the slack variables. As coefficients of the objective function, they are (2, 2, 2) with a minimum functional value of 72.

Applications of Duality may be used in situations such as production maximization and cost-minimization. This allows any LP problem to be viewed from two different aspects and if possible to choose the simplest LP problem to solve.

Through working out the solution to a LP problem it is easy to see the repetitive steps required to reach the optimal solution. If the number of variables were great, just think of the number of calculations that would be necessary to reach the desired value, and also the possibility for error in hand-calculation! Because of the iterative nature of the Simplex method, its adaptability to a computer is evident. Programmers have spent countless hours developing and coding Simplex algorithms. The programs range from very complex algorithms which include basic and non-basic variables, artificial variables, condensed tableaux and other aspects of the Simplex method, to very basic calculation of a simple maximization problem.

The program that will be used to demonstrate the simplex algorithm will be developed, now. If the reader desires to see examples of more

elaborate Simplex programs, suggested sources are Linear Programming with FORTRAN by C.S. Wolfe, Applied Linear Programming by J.R. Frazer, and Linear Programming for Decision Making by Anderson, Sweeney and Williams.

The program being discussed uses three subroutines developed by Carvel S. Wolfe, author of Linear Programming with FORTRAN, which perform the steps necessary to choose a pivotal element and execute Gaussian elimination upon the matrix. (A couple of minor changes have been made to the subroutines to adapt them to the uses discussed in this paper.) The complete program and output may be found in the Appendix.

The input to the FORTRAN program is:

M - the number of rows in the tableau

N - the number of columns in the tableau

TABLE - The coefficients of the constraints, the slack variables and the constants. The last row of TABLE is the negative of the objective function.

The maximum dimensions of TABLE have been set to 10 x 10 for demonstration purposes, only. In reality, the dimensions of the tableau may have to be much larger.

The first subroutine is named COL. COL chooses the pivotal column by scanning the objective row of the tableau and seeking the element with the greatest negative value. Parameters passed from the main routine to COL include TABLE(Y), M(M) and N(N). IQ becomes the number of the column that the pivotal element will be located based upon a search of the

objective row and the location of the most negative value in that row. If no negative number is found in the objective row, IQ is assigned the value -1. The value of IP, along with the other parameters are returned to the main routine. If the value of  $IQ \leq 0$ , the optimal solution has been found and the procedure is complete. Otherwise, the program continues.

The next subroutine to be summoned is ROW. ROW determines the row from which the pivotal element will be chosen by calculating the ratios of the constant values with the corresponding element in the pivotal column. The same parameters are passed to ROW as were passed in COL, with the addition of IP. IP will become the number of the row which contains the smallest ratio value. If a suitable pivot element is not found, the value of IP returned is -1, else the row number is returned to the main routine. If  $IP \leq 0$ , this may signal degeneracy, or that the variables become infinite and the solution Z is unbounded.

The subroutine, PIVEX, performs the Gauss-Jordan elimination upon TABLE, which is named Y in the subroutine. All known values are passed to PIVEX, including IP and IQ, which are the indices of the pivot element. A is set equal to the pivot element. The statement  $Y(IP,J) = Y(IP,J)/A$  with J from 1 to N, changes the pivot element to 1 and divides the entire pivotal row by the value of the pivot element. Next, the elimination process takes place, changing the elements in the pivot column to zeros and performing the necessary operation upon each row.

This is carried out by the statement  $Y(I,J) = Y(I,J) - B*Y(IP,J)$  with I from 1 to M and J from I to N. When the process is complete the new table is passed to the main routine. The entire program is repeated until no new pivotal element can be found.

The subroutine WTAB simply outputs the entire tableau. II is a counter which is increased by one for each iteration of the entire algorithm. This number is also output at the end of the program to give the number of complete times the Simplex method was applied successfully to the LP problem.

The complete program for the Simplex method may be found in the Appendix, along with the output for the following LP problem.

A manufacturer produces three types of tables, each of which must go to a cutter, assembler, and finisher. The man-hours required to completely produce the Standard Table is Cutter - 1.8 hours, Assembler - 1.6 hours, Finisher - 1.0 hours. The time required to produce the Deluxe model is Cutter - 3.6 hours, Assembler - 1.2 hours, and Finisher - 1.0 hours. The Bargain model needs Cutter - 0.9 hours, Assembler - 1.2 hours, and Finisher - 2.0 hours. The total man-hours available for cutting the furniture is 162 per day. There is also available 120 man-hours a day for Assembling, and 130 man-hours a day for Finishing.

The profit made on the Standard model is \$16.00, profit on the Deluxe model is \$14.00, and for the Bargain model the profit is \$18.00.

Find the amount of production of each model which will maximize profit.

The objective function of this problem will be

$$Z = 16.00x_1 + 14.00x_2 + 18.00x_3.$$

The input to the computer will be

$$M = 4 \text{ and } N = 7, \text{ since we will add surplus variables.}$$

The constraints on the variables are

$$1.8x_1 + 3.6x_2 + 0.9x_3 \leq 162$$

$$1.6x_1 + 1.2x_2 + 1.2x_3 \leq 120$$

$$1.0x_1 + 1.0x_2 + 2.0x_3 \leq 130$$

The coefficients of the variables and the constant terms, along with the basis and the constants will be inputted to the program.

The tableau will appear as:

1.800	3.600	0.900	1.000	0.000	0.000	162.000
1.600	1.200	1.200	0.000	1.000	0.000	120.000
1.000	1.000	2.000	0.000	0.000	1.000	130.000
-16.000	-14.000	-18.000	0.000	0.000	0.000	0.000

After the first application of the Simplex Algorithm, the tableau will be:

1.350	3.150	0.000	1.000	0.000	-0.450	103.500
1.000	0.000	0.000	0.000	1.000	-0.600	42.000
0.500	0.500	1.000	0.000	0.000	0.500	65.000
-7.000	-5.000	0.000	0.000	0.000	9.000	1170.000

It is possible to note that 2.0 was chosen as the first pivot element by the program. This may be verified by hand calculation of the ratios.

$$162/9 = 180; \quad 120/1.2 = 100; \quad 130/2.0 = \underline{65}$$

The next iteration produces:

0.000	2.340	0.000	1.000	-1.350	0.360	46.800
1.000	0.600	0.000	0.000	1.000	-0.600	42.000
0.000	0.200	1.000	0.000	-0.500	0.800	44.000
0.000	-0.800	0.000	0.000	7.000	4.800	1464.000

And the final tableau appears as:

0.000	1.000	0.000	0.427	-0.577	0.154	20.000
1.000	0.000	0.000	-0.256	1.346	-0.692	30.000
0.000	0.000	1.000	-0.085	-0.385	0.769	40.000
0.000	0.000	0.000	0.342	6.538	4.923	1480.000

The optimum value of the function is 1480.00. To read from the table the quantity of each model to be produced simply find the value 1.00 in the column and read horizontally from the constant column. According to the table, 30 tables of the Standard model should be produced, 20 tables of the Deluxe type and 40 tables of the Bargain brand should be made. If this production schedule is met a maximum profit of

$$16.00x_1 + 14.00x_2 + 18.00x_3 = 1480.00 \quad \text{will be made.}$$

Verify this by substituting in the appropriate values of the variables.

$$\$16.00(30) + \$14.00(20) + \$18.00(40) = \$1480.00$$

From this example it is easy to see the value of this method to the business world. A problem with many variables and constraints would become unwieldy to calculate by hand! The use of computers and sophisticated

programs yields much valuable information necessary for businesses to make important decisions quickly.

Degeneracy is another problem faced by many linear programming problems. An LP problem is said to be degenerate when an optimum solution cannot be found. The conditions which point to degeneracy are:

- 1.) If any of the rows of the tableau are linearly dependent, that is, one row is a multiple of another row, then the problem is said to be degenerate.
- 2.) Also, when comparing ratios to determine a pivot element, if two or more of the ratios are equal the problem is degenerate.

Degeneracy may result in no feasible solution, perhaps negative values.

A phenomena known as cycling may occur as a result of degeneracy. This means that the basic solutions of the problem will begin to repeat themselves, never converging to the optimum solution. This does not occur too often, but it is quite possible.

When choosing the pivot element by the ratio method and the ratios are equal, then the next iteration will obviously produce a zero in the basic variable column. Mathematicians have varying views upon the problem of degeneracy. In dealing with degeneracy with the Simplex method, if equal ratios do occur, the simplest method is to simply choose the upper row as the pivot row and assign the value zero to the corresponding basic variable(s) in the row(s) not chosen.

In conclusion, there are still many problems and complexities not

touched upon in this paper which add to the intricacies of the Simplex method as a method of solving LP problems. From the topics discussed, it is possible to grasp the power of the Simplex method. It may easily be applied to many business situations, or any situation which may be expressed in the mathematical form specified.

Its simplicity evolves from the fact that nothing more than basic mathematic calculations of addition, multiplication, subtraction and division are required in order to obtain a fairly accurate solution to a problem. The ease with which this method may be adapted to the use of today's modern digital computers, allowing fast and accurate solutions to massive linear programming problems has been demonstrated. The Simplex method may not be the only method available to solve LP problems, but from an applications point of view it simply is the easiest and most efficient of the methods available. Research continues to find even more efficient methods of the Simplex method. The revised Simplex method is one of these, which utilizes a condensed tableau and therefore, less storage space is required in a computer's memory. This allows even larger problems to be solved faster and more efficiently. Linear Programming is certainly a dynamic aspect of mathematics, especially in relation to its use with today's computers and technology.



APPENDIX

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Subroutines COL, PIVEX, & ROW developed by Carvel S. Wolfe; Linear Programming with FORTRAN; Glenview, IL, Scott, Foresman, 1973.

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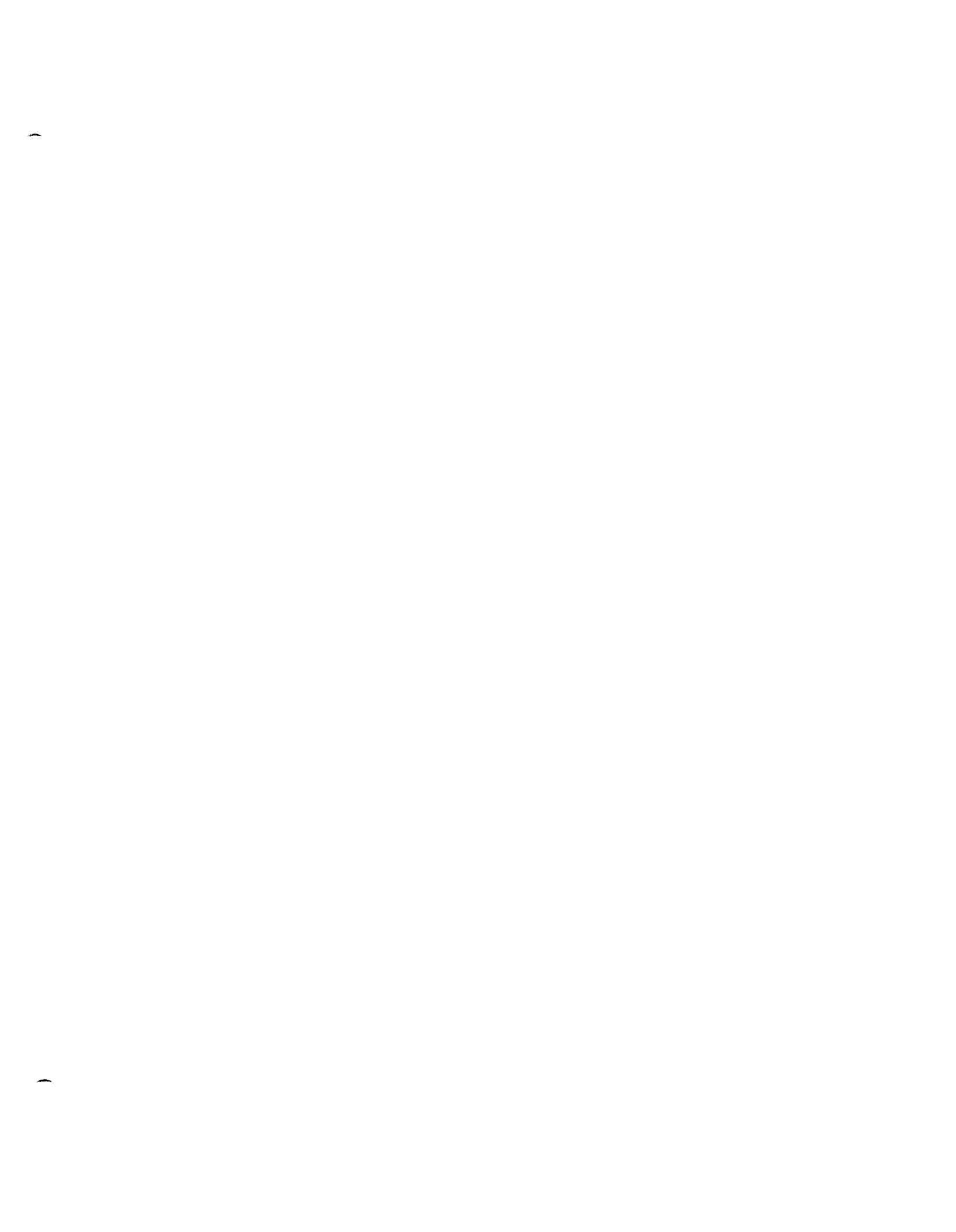
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0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000

APPENDIX 1 - OPERATIONS OF OPTIMAL CONTROL FOR 1991-92

0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000
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0.000	0.000	0.000	0.000	0.000	0.000	0.000

FAD OF EXECUTION

ORGANIZATION OF THE ALGORITHM FOR THE OPTIMAL CONTROL

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