

**Students' Misconceptions
In Middle School Mathematics**

An Honors Thesis (HONRS 499)

by

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Abstract

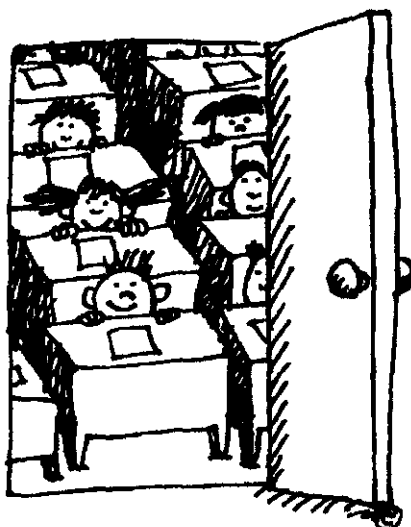
Middle school is a significant period in students' mathematical development, a period in which they crystallize their understanding of mathematical concepts and procedures and make the unconscious decision as to whether they will be successful in math. The project is intended to be an instructional resource providing teachers of middle school mathematics with information about common mathematical misconceptions held by students. The data for this research is taken from actual student work on the applied skills sections of the 2002 6th and 8th grade ISTEP+ Mathematics Assessments. The misconceptions are categorized according to specific standards as defined by NCTM, and corresponding Indiana Academic Standards are identified.

Acknowledgements

- I would especially like to thank Dr. Elizabeth Bremigan for helping me to develop this idea and advising me throughout the process. Thank you for being always available to help me along the way, and offering me guidance and sharing your experience.
- I owe a great thanks also to Donna Biggs and Marilyn DeWeese for your willingness to allow me to use the test data and your copy machine and also for making room for me to come in to the school frequently to work on my research.

STUDENTS' MISCONCEPTIONS IN MIDDLE SCHOOL MATHEMATICS

By Lindsay M. Keazer



common mathematical misconceptions exhibited by students, so that educators can work to restructure students' conceptions of specific mathematical content areas and avoid these misconceptions from occurring or continuing in their students.

The ISTEP+ Mathematics Assessment consists of two parts: a basic skills section and an applied skills section. The multiple choice questions from the basic skills section are not released after the test is scored, but copies of the actual student work from the open-ended questions in the applied skills section are returned to their specific schools. These open-ended questions are what I used in my analysis. Because my data comes from one specific school, the results are in some ways singularly representative of this school. Perhaps a misconception existing in a certain mathematical concept area at one school is not present at another. However, these results have the potential to be of great use to math educators at other schools, because the fact that a misconception exists within some students at one school indicates that there is the possibility of that misconception being created elsewhere. Thus, the results of this study can be used as a tool to inform educators of mathematical concepts that should be handled in the classroom with careful forethought, and to provide them with an awareness of potential misconceptions in order to avoid the creation of these in their classroom.

At points in my research, I found that the data did not provide clear conclusive evidence to support a specific mathematical misconception, but only suggested that a misconception could exist. In my results, I present all that I have found from the data, both the clear conclusive misconceptions, as well as the hints of trouble spots that exist. Some misconceptions were widespread among a large portion of the population, while in

others they represent a smaller percentage of the students under analysis. I tried to stress the importance of those misconceptions that are prevalent in my results.

Group Under Study

The data analyzed represents 75 middle school students from a small Indiana public school. Of these students, 39 were enrolled in 6th grade and 36 were enrolled in 8th grade in the fall of 2002 when the ISTEP+ test was administered to them. They were assessed with standardized tests based on the Indiana academic standards for the previous grade level. Hence, students in grade 6 are given tests based upon the academic standards for grade 5. The ISTEP+ tests given to 6th and 8th graders in 2002 are the first tests that assess the Indiana Academic Standards adopted in 2000.

Data Analysis

The data for my research consisted of student responses to open-ended questions taken from the fall 2002 6th and 8th grade ISTEP+ assessments (IDOE). Initially, I reviewed all the questions on the tests in order to select those that I wanted to analyze. Any assessment question in which the students were not required to elaborate on their procedures and/or thought processes was dropped, as it did not reveal explicit information about existing student misconceptions. I was left then with 19 questions to analyze. Studying each question and the variety of responses existing, I created a unique rubric to use in developing a coding system to analyze each question. My coding system indicated both the strategies used and the errors made, in order to identify misconceptions and categorize students' strategies. Then, by further study of the results of my analysis, I reached conclusions about the misconceptions that clearly existed and provided suggestions of misconceptions that may exist.

One example of a rubric I created to analyze an item that asked students to identify a triangle as right, acute, or obtuse and explain their reasoning, was:

Correct Answer: Obtuse	Correct Explanation	Labeled individual angles	wrong: acute or right	no rational given	because "how it looks"	said acute & obtuse	because "2 acute angles"	"sum of angles < 90"
		X	X					
			X	X				
		X	X		X			
		X				X		
X				X				
			X	X				
X			X	X				
			X				X	
		X	X		X			
X			X	X				X
X				X				
		X				X		
			X				X	
X			X		X			
X	X			X				
			X	X				
X			X					
X	X				X			
X	X							
		X	X	X				X
X	X							
X			X	X				
X	X							
X			X				X	
X	X				X			
X	X							

After coding all the open-response items in rubrics like the one above, I returned to these rubrics to perform an analysis of each item.

Of the 19 open-response items analyzed, six items are not included in this final analysis, either because the majority of students' responses to those items demonstrated a solid understanding of the concepts under assessment, or the responses led to no conclusive evidence pointing towards a misconception. The following 13 items address areas of middle school students' mathematical misconceptions, whether simply hints of them or explicit occurrences were present. The misconceptions under study have been categorized by specific standards. The five content standards as defined by NCTM are **Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability**, and one process standard, **Problem Solving**, is also included. The Indiana Academic Standard(s) corresponding to each task has been identified before the item analysis (IDOE).

The following issues addressed as misconceptions are not an exhaustive list. They are, however, misconceptions gathered from student work on the 2002 ISTEP+ exams, and can be used to inform math educators of common misconceptions that are easy to miss as a teacher, yet vital to students' understanding. When recognized, these misconceptions can be addressed and replaced with a complete conceptual understanding.

Number and Operations

TASK (Grade 6): Look at the following number sentence. $227 \times 56 = 1,025$

- On the lines below, use estimation to explain why the number sentence is incorrect.
- Now solve 227×56 . Write your answer on the line below.

Indiana Academic Standards:

- 5.2.6 Use estimation to decide whether answers are reasonable in addition, subtraction, multiplication, and division problems.
- 5.7.6 Know and apply appropriate methods for estimating results of rational-number computations.

DISCUSSION: In the first part of this problem, it is clearly stated that an estimate is to be used, and 46% of the students did this successfully. However, one third of the students responding solved this part of the task using exact computation, which was intended to be done only in the second part of the problem. Students may not realize that estimation is an actual skill they are expected to exhibit, and view it as simply a “shortcut” to getting an answer. They also may see an exact solution as a better answer because estimation can lead to multiple solutions, while the exact multiplication provides only one clear-cut answer. Those misconceptions may lead students to provide the exact solution in place of an estimated answer.

TASK (Grade 6): Sarah has \$20.00. She wants to buy a shirt for \$12.99, and a hat for \$4.55, both including tax. She also wants to buy socks for \$2.99, including tax.

- On the lines below, explain how Sarah could estimate whether she has enough money for the socks if she buys the shirt and hat.
- Exactly how much money does Sarah have left to spend? Write your answer on the line below.

DISCUSSION: Although the success rate on this problem was a little over half of the students, this question proved to be a fascinating one all because of the words *including tax*. 30% of the students in this particular school interpreted this as “make sure you include tax in your calculations.” Even though no percent of taxation was given, they calculated some sort of tax for each item, creating a very complicated solution process. Another misconception within the problem existed in the second part, when students are asked to account for how much money is remaining. Because they determined in the previous answer that Sarah didn’t have enough money to buy all the things she wanted, another 30% of students gave answers to the second part such as *no money left*, *\$0.00*, or gave a negative amount of money left.

Show All Work

$$\begin{array}{r} 20 \overline{) 13.00} \\ \underline{22} \\ 55 \\ \underline{5} \\ 478 \end{array}$$
$$\begin{array}{r} 20 \overline{) 13.00} \\ \underline{22} \\ 55 \\ \underline{5} \\ 478 \end{array}$$
$$\begin{array}{r} 13.00 \\ + 0.65 \\ \hline 13.65 \end{array}$$
$$\begin{array}{r} 11.1 \\ 13.61 \\ 4.77 \\ + 3.24 \\ \hline 21.62 \end{array}$$
$$\begin{array}{r} 20 \overline{) 2.99} \\ \underline{24} \\ 99 \\ \underline{80} \\ 125 \\ \underline{120} \\ 324 \end{array}$$

Answer \$ -81.62

This student response shows a student who tried to calculate a 5% sales tax onto each item, and then answered for the amount of money Sarah would be in debt if she had bought everything she wanted.

The misconceptions in this problem appear to stem from a misinterpretation of the language. Perhaps students are accustomed to seeing price tags of items in real life before the tax has been computed and added, and *including tax* was not familiar terminology to them.

Algebra

TASK (Grade 6): Find the value of $12x - 7$ when $x = 4$.

Indiana Academic Standard:

5.3.2 Write simple algebraic expressions in one or two variables and evaluate them by substitution.

DISCUSSION: Only 13% of students successfully answered this problem, while 23% chose to leave it blank, for reasons unknown. Of those students who responded unsuccessfully, one significant misconception stood out. In fifth grade, the grade level that this test covers, students have just learned the concept of substitution for variables. However, many students viewed the variable x next to a coefficient as placeholder for a missing number.

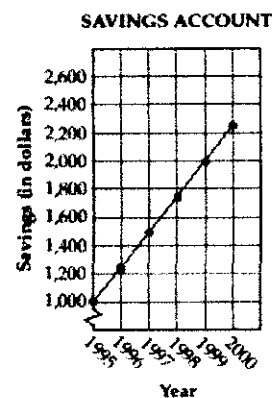
40% of the students that answered this question displayed in their work something to the effect of $12x - 7 = 124 - 7$, where x is simply a placeholder where the number 4 goes, rather than a multiplication problem. The idea of using variables to represent

unknown numbers is still new to students; it is much different from their elementary experiences where a blank spot or empty box indicates the need to fill in a missing number. Students' only previous experience using variables (or letters) in math may be as labels of units, such as ft. or in., in which the label is connected to the number it proceeds, not multiplied by it. Perhaps getting beyond these previous experiences is the only obstacle to constructing a new understanding of the variable and coefficient.

$$x=4 \quad \begin{array}{r} 12x = 124 \\ - 7 \\ \hline 117 \end{array}$$

TASK (Grade 8): Ms. Thompson opened a savings account in 1995. The graph shows the amount of money in Ms. Thompson's savings account since she opened it.

The slope of the graph is 250 and the y-intercept is 1,000. On the lines below explain what these two numbers represent in the graph.



Indiana Academic Standard:

7.3.6 Define slope as vertical change per unit of horizontal change and recognize that a straight line has a constant slope or rate of change.

DISCUSSION: 17% of the students chose not to answer this item, for reasons unknown. However, of the students who answered this question, 36% gave a correct explanation of the slope of the graph, and 50% gave a correct explanation of the y-intercept. Other

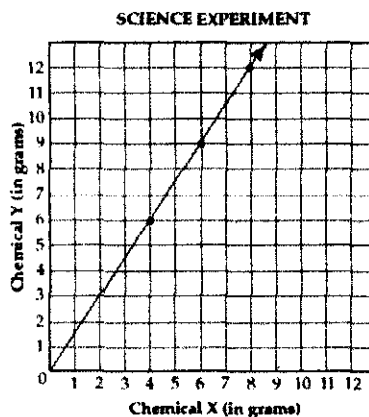
student responses included: simply giving the formula for how to calculate slope, repeating the information that was given, and some unclear responses such as: “These numbers represent how much Ms. Thompson’s savings is increasing and at what year it makes this increase.”

Though students may know how to *use* slope and y-intercept in finding the function representing the line, *understanding* what those numbers represent is equally important. There is a tendency to define the concepts of slope and y-intercept in terms of the graph only, and not in terms of the interpretation in the data. Textbooks can contribute to the limited understanding of these concepts. For instance, one commonly used pre-algebra textbook defines slope as *rise over run*, or *the ratio of the change in y to the corresponding change in x*, and the y-intercept as *the y-coordinate of the point where the graph crosses the y-axis*, but says nothing about what these two concepts actually mean when interpreting data (Leschensky, W., et. al. 400–406).

TASK (Grade 8): In Ms. Porter’s science class, students were given the table below describing the amounts of two chemicals, chemical X and chemical Y, in an experiment.

SCIENCE EXPERIMENT	
Chemical X (in grams)	Chemical Y (in grams)
4	6
6	9
8	12

The students were asked to graph the information in the table and then write an equation describing the graph. Larry’s graph and equation are shown below.



Larry’s equation: $y = \frac{2}{3}x$

- Did Larry graph the information in the table correctly? On the lines below, explain your answer.
- Did Larry write a correct equation? On the lines below, explain your answer.

Indiana Academic Standards:

7.3.6 Define slope as vertical change per unit of horizontal change and recognize that a straight line has a constant slope or rate of change.

7.3.7 Find the slope of a line from its graph

DISCUSSION: An example of an exemplary response to the first part of this task, as written in the *ISTEP+ Teacher’s Scoring Guide* is as follows: “Yes, Larry graphed the

information correctly. The pairs of numbers in the table are the same as the coordinates of the points in the graph” (IDOE 12). Almost the entire student group explained with no difficulty that this was correct because the points or “dots” from the chart all matched those of the graph.

However, in the second part of this question a much wider variety of answers were given. When asked if Larry’s equation was correct, 31% of these students answered correctly, saying that Larry’s equation was not correct, and most of these students even provided the correct equation to the graph: $x = \frac{2}{3}y$, which was not required to get full points for the problem. Many other students misinterpreted what Larry’s equation represented. Some actual student responses were as follows:

- *Yes, Larry’s equation was correct. It’s correct because 4/6, 6/9, and 8/12 can all be simplified to 2/3.*
- *Larry is correct because 4 is 2/3 of 6, 6 is 2/3 of 9, and 8 is 2/3 of 12.*
- *Yes, Larry divides each level of chemical X and chemical Y. Every time the answer was 2/3. For every part of chemical Y there is 2/3 part chemical X.*

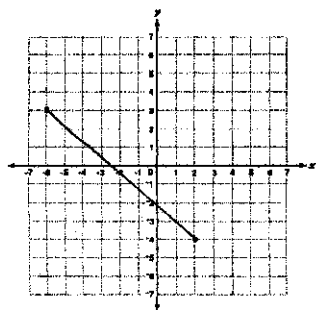
Student responses of this type display an incomplete understanding of functions and slope. The function $y = \frac{2}{3}x$ was frequently misread as meaning “chemical x is 2/3 of chemical y,” a statement which is a correct representation of the data, but not a correct interpretation of the function, and therefore many students failed to recognize that the slope of Larry’s function was incorrect. Teaching students how to read and interpret an equation is a skill easily overlooked, yet essential to a total understanding.

TASK (Grade 8): On the grid below, graph the points (-6, 3) and (2, -4). Then draw the line that passes through the two points.

Indiana Academic Standard:

7.3.8 Draw the graph of a line given the slope and one point on the line, or two points on the line.

DISCUSSION: 36% of the students answering this question were able to successfully plot the points above. However, 70% of those students drew the line segment between the two points, rather than “the line that passes through the two points” as was asked for.



The *ISTEP+ Teacher’s Scoring Guide* does not clarify whether this is an acceptable solution or not, it only states that the full score of two points is given if the student correctly graphs the two points and draws the line passing *through* the points, and a score of one point is given if the two points are correctly plotted but the line is not correctly drawn (IDOE 16). Perhaps this misconception stems from students not yet understanding the difference between a line and a line segment. In the elementary math levels,

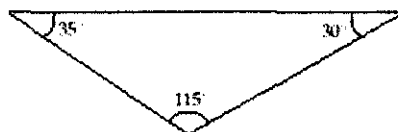
sometimes the term *line* is used when referring to both. The Indiana standards for

geometry do not state explicitly at what age the distinction between lines and line segments should be learned.

44% of the students used a correct strategy for plotting the points, but made an error reversing the x and y-coordinates, and one student made an error reversing the +,- signs. The misconceptions causing these procedural errors address aspects that are integral to student's understanding of the process of plotting points.

Geometry

TASK (Grade 6): Look at the triangle below.



- Identify the triangle as right, acute, or obtuse. Write your answer on the line below.
- On the lines below, explain why you identified the triangle as right, acute, or obtuse.
-

Indiana Academic Standard:

5.4.2 Identify, describe, draw, and classify triangles as equilateral, isosceles, scalene, right, acute, obtuse, and equiangular.

DISCUSSION: From this question it is evident that it is not enough for students to simply be able to classify a triangle, but they must also be able to explain how the triangle is identifiable as such. 44% of students correctly answered that the triangle pictured was obtuse, but less than half of those students were able to explain why they knew it to be an obtuse triangle. Many students labeled every individual angle of the triangle, and some went on to say that it was an obtuse and acute triangle, because both types of angles were present. Others labeled it acute because there were two acute angles in it. If that were the requirements for an acute triangle, however, obtuse triangles would not exist.

Some sample student-responses follow:

Answer 2 acute, 1 obtuse

I put two acute because an acute angle is smaller than a right angle. I also put one obtuse angle because an obtuse angle is bigger than a right angle. That is how I got my angle!

[I put two acute because an acute angle is smaller than a right angle. I also put one obtuse angle because an obtuse angle is bigger than a right angle. That is how I got my angle!]

Answer acute

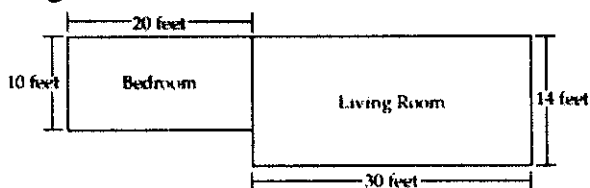
I identified the triangle as a acute angle because it has 2 acute angles in it

[I identified the triangle as a acute angle because it has 2 acute angles in it.]

Perhaps the misconception here stems from students having already mastered the skill of identifying *angles* of the triangle as acute or obtuse, but not yet understanding how to use that previous skill in the more complex task of identifying *triangles*. It can be confusing to students that all the angles of an acute triangle are acute, and yet not all the angles in an obtuse triangle are obtuse (in fact only one angle of an obtuse triangle is obtuse and the other two angles are acute.) Having students experiment with drawing triangles and attempting to draw a triangle with more than one obtuse angle could eliminate this misconception.

Measurement

TASK (Grade 6): Janie wants to put a new carpet in one of the bedrooms and the living room of her house. A diagram of the rooms is shown below.



What is the area, in square feet, to be carpeted?

$$\text{Area of rectangle} = \text{length} \times \text{width}$$

Indiana Academic Standards:

5.5.3 Use formulas for the areas of rectangles and triangles to find the area of complex shapes by dividing them into basic shapes.

DISCUSSION: 40% of the students who answered this problem took a correct approach and found the area of each room separately, and added the two areas. Only one student chose to leave the task blank, for reasons unknown. Of those that attempted other strategies, there were three primary methods tried by these 6th graders. Some of these students found the perimeter of each room and then added those together. Others simply added the four measurements together: $10+14+20+30$. The most common of the unsuccessful strategies was to combine the measurements of the two unique rectangles into one rectangle with only one length, one width. This was either $14' \times 50'$ or $24' \times 50'$, and students then found the area of this rectangle to answer the task. However, since the two different rooms have different widths and heights, simply combining the measurements creates a new room greater than the sum of the two.

Two examples of student work are:

Show All Work

$$\begin{array}{r} 20 \\ + 30 \\ \hline 50 \end{array} \quad \begin{array}{r} 10 \\ + 14 \\ \hline 24 \end{array} \quad \begin{array}{r} \times 50 \\ 00 \\ 1000 \\ \hline 1000 \end{array}$$

Answer 1000 square feet

$$\begin{array}{r} 50 \\ \times 14 \\ \hline 200 \\ 500 \\ \hline 700 \end{array} \quad \begin{array}{r} \times 30 \\ 50 \\ \hline 1500 \end{array}$$

Answer 700 square feet

These students understood that the formula for calculating the area of a rectangle was needed for this problem, yet the difficulty arose when they were not familiar with using this formula in finding the area of two rectangles comprising a more complex shape. Perhaps this misconception could be eliminated by increasing student understanding of the concept of area, beyond simply recognizing that $\text{area} = \text{length} \cdot \text{width}$. Understanding what area represents would cause students to see that it is summative, and that by adding the lengths and widths of the two rectangles you would be calculating the area for a rectangle much larger than the sum of the two areas.

Data Analysis and Probability

TASK (Grade 6): The numbers below show how many tickets nine students sold for the talent show.

12, 7, 25, 5, 17, 10, 8, 15, 20

On the lines below, explain the steps necessary to find the MEDIAN of the number of tickets sold.

Indiana Academic Standard:

5.6.2 Find the mean, median, mode, and range of a set of data and describe what each does, and does not, tell about the data set.

DISCUSSION: The *ISTEP+ Teacher's Scoring Guide* provides an exemplary answer to this item: *Put the numbers in order from smallest to largest, and the median is the number in the middle* (IDOE 41). Only one student successfully completed this task, and 31% chose to give no response, for reasons unknown. 30% of the students who answered this question had the concept of *median* confused with the concept of *mean*, adding the nine numbers and dividing the sum by nine. Two students confused it with *mode*.

Of all the measures of center of a data set, the mean, or the average, is the most commonly taught and used. Perhaps the student misconception displayed here is the result of a well-developed understanding of mean, without much awareness of median. However, the other measures, such as median and mode, have equally valuable places in data analysis. Perhaps students need to recognize that there are instances in which finding the median is much more useful than finding the mean of a set of data. By comparing and contrasting the different measures of center and finding cases in which each measure would be more effective than the others, students will learn to differentiate between them.

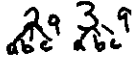
TASK (Grade 8): Elizabeth is a chef who specializes in cheesecakes. She makes 3 flavors of cheesecakes and customers have a choice of 3 sauces and 3 fruit toppings. How many different combinations of flavor, sauce, and topping are possible?

Indiana Academic Standard:

7.6.7 Find the number of possible arrangements of several objects using a tree diagram.

DISCUSSION: 67% of the students were able to successfully complete this task. There are three primary ways of approaching this problem, all of which were used by students: using a tree diagram, listing all possible combinations, and multiplication. The Indiana standards suggest the use of a tree diagram strategy, an approach attempted by 22% of the students. The following student was able to use this strategy very successfully.

Show All Work



$$(9+9+9) \cdot 3$$

$$27$$

This student only used a tree diagram to find that there were 9 combinations for one flavor, and then he/she added 9 three times to find the total, using a combination of two strategies.

Answer 27 combinations

Of those who did not complete this task successfully, most tried to create an exhaustive list of all the varieties of cheesecakes. Some cleverly gave variables to represent the different choices and then began to list all the combinations of these variables, but never was anyone successful at listing and counting all twenty-seven combinations. The student work below illustrates one such attempt at listing all the possible combinations of cheesecakes.

Show All Work

	Sauces	Fruits	
	A	1	$A+X+1, A+Y+2, A+Z+3$
Fruits	B	2	$B+1+X, B+2+Y, B+3+Z = 9$
X	C	3	$C+1+X, C+2+Y, C+3+Z$
Y			
Z			
	$A+X+2$	$B+X+2$	$C+X+2$
	$A+X+3$	$B+X+3$	$C+X+3$
		$\times 2 = 12$	
			<u>21</u>

Answer 21 combinations

This particular student came up with only 21 total combinations, by what seems to be a listing strategy. However, what he/she did not realize was that there is a pattern for determining total combinations—multiplying the number of options for each choice together to count the total number of outcomes—and the list failed to account for all combinations. Perhaps the students who chose this strategy did so because he/she could not recount the formula for finding the total combinations of a group of choices. By studying the groupings of combinations in a collection of objects, students can learn to recognize patterns that exist and understand how to determine combinations with a more reliable method than listing.

Problem Solving

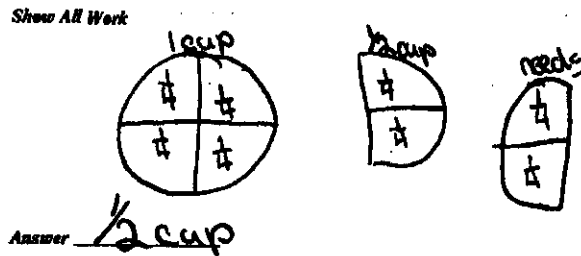
TASK (Grade 6): Jason needs $\frac{1}{4}$ cup of plant food for each of his 8 plants. He has a bag containing $1\frac{1}{2}$ cups of plant food. He has determined that he will not have enough plant food. On the lines below, describe one approach that Jason may have used to determine that he does not have enough plant food.

Indiana Academic Standard:

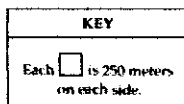
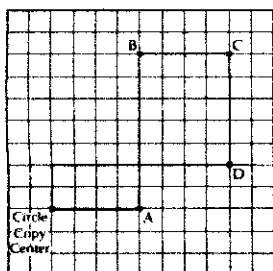
5.7.3 Apply strategies and results from simpler problems to solve more complex problems.

DISCUSSION: This question falls into the NCTM Process Standard of *Problem Solving*, implying that it is a task for which the solution process is not obvious beforehand. Certain students struggle with these kinds of problems perhaps because they have been taught to look for a formula for every problem. A handful (about 13%) of the students took this problem and simply added $1\frac{1}{2} + 4$, perhaps for the lack of a known formula to apply to this task.

A little over half of the students were able to successfully complete this task. 13% of the students were able to do so and display a conceptual understanding of the process even without multiplying or dividing fractions, simply by drawing pictures and diagrams or employing a counting strategy. These responses show that mathematical concepts must be taught with understanding so that students can produce their own strategies when a formula is not available. An example of one student's diagram follows that illustrates this skill.



TASK (8th grade): The map below shows Tom's daily pickup and delivery route for Circle Copy Center.



Tom drives from Circle Copy Center to each of the 4 stops (A,B,C,D), then back to Circle Copy Center along the route shown in the map. What is the distance per week in KILOMETERS, that Tom drives on his route if he drives his route twice a day and works 5 days a week?

Indiana Academic Standard:

7.7.4 Apply strategies and results from simpler problems to solve more complex problems.

DISCUSSION: This answer has several parts: Students must find the distance of the entire route, then account for the number of times (10) that Tom drives the route per week, and then they must convert meters to kilometers, not necessarily in that order. Students have the least success rate with problems that integrate more steps, perhaps because there are more opportunities for mistakes.

31% of students solved this task successfully, while 69% did not. Of the students whose solution was incorrect, only 40% had recognizable errors: calculation errors, forgetting to multiply the route by 10 trips, and finding the area inside of the route rather than the perimeter. The remaining students used some sort of strategy that was not recognizable and did not appear to be moving towards the desired solution.

In a complicated problem of this sort, it is integral that students learn how to organize the information given and integrate multiple strategies from more familiar, simple problems together into an organized progression. It is easy for students to become overwhelmed by a lengthy task such as this, but by developing the skills to break a task into a sequence of simple steps, they can approach it comfortably and be able to think clearly about the problem at hand.

TASK (Grade 8): Quinn is using 1-square-foot tiles to cover her living room floor. The living room is 11 feet by 19 feet. **ESTIMATE** the number of tiles she will need to cover the living room floor.

Indiana Academic Standard:

7.7.8 Select and apply appropriate methods for estimating results of rational-number computations.

DISCUSSION: The task above specifies, in bold letters, that estimation is the intended strategy for students to use. However, 57% of students, such as the one whose work is pictured at right, gave an answer of 209, the exact solution to the problem. The *ISTEP+ Teacher's Scoring Guide* says that any answer between the range of 190 and 220 is acceptable, so in this instance the exact answer is still acceptable for the full points (IDOE 63). However, in previous ISTEP+ assessment items, such as the Fall 2001 6th grade assessment (Session 4–Item 5), the exact answer was not considered to be an exemplary response, and only one point was awarded instead of two (IDOE 57). As mentioned in a previous task when discussing estimation, perhaps students view the estimated answer as subordinate in merit to the exact answer, and therefore choose to calculate the exact answer since they have the mathematical capabilities to do so.

Handwritten student work showing a multiplication problem: $11 \times 19 = 209$. The work is written vertically, with 11 on the top line, 19 on the second line, and 209 on the bottom line. There are small marks above the 19 and below the 209, possibly indicating carries or alignment.

Closing Remarks

In performing an analysis of these students' responses to open-ended ISTEP + tasks, I have learned much about the mathematical misconceptions existing among middle school students. Each school does not have the time nor resources to perform such an extensive analysis on their test data every year, so I hope that I have reached my objective and provided information that can be used as an instructional resource for other math educators. Hopefully it will be a tool used by others to recognize the mathematical misconceptions addressed above, in order to restructure students' understanding as well as help prevent those misconceptions from occurring or continuing in their students.

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