Lesson Plans for Implementing the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics*

An Honors Thesis (HONRS 499)

by

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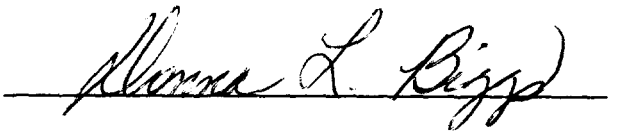

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ABSTRACT

Purpose of Thesis

This project is a collection of lesson plans which are classroom ready--perfect for the beginning mathematics teacher. In the beginning of a mathematics teaching career it is important to be prepared, at this time more than ever, due to the guidelines set out by the National Council of Teachers of Mathematics.

In their book *Curriculum and Evaluation Standards for School Mathematics*, the National Council outlines a vision for mathematics instruction for the twenty-first century. However, at this time materials to implement these standards are few and far between.

This project begins with a brief overview of the Standards and the vision outlined in them. The lesson plans follow this section. They are divided into specific categories, although many lessons would fit quite well into other categories.

ACKNOWLEDGMENTS

I would like to thank my family for all of their support during my school career. They have kept me going through many difficult times. I would especially like to thank "my kids"-Amanda, Ashley, Aaron, Andrew, Taylor, and Dustin-one of the reasons I have realized that education is so important and that the world needs teachers who are willing to take the time to recognize and reward talent.

I would like to thank Mrs. Donna Biggs without whom this project would have never been possible. She has opened up her classroom and her vast knowledge of mathematics and mathematics teaching so graciously. I can never thank her enough for serving as the advisor to this project.

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Overview of the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics*

As the beginning of the twenty-first century grows closer, schools and teachers are taking stock of what and how they are teaching. Questions such as, "Will this prepare our students for the developing technologies of the next century?" and "How can we better address the changes our students will face once they enter the work force?" are commonly heard in teacher's meetings and curriculum planning sessions. Several national teaching organizations are joining in this effort to make changes to an educational system that is failing to meet the needs of all students.

The National Council of Teachers of Mathematics (NCTM) has published three major documents to help guide mathematics teachers through the sea of change. The NCTM began its efforts in 1989 by publishing the *Curriculum and Evaluation Standards for School Mathematics*. Two years later, the *Professional Standards for Teaching Mathematics* (1991) was added to the series. In May of 1995 a third document, the *Assessment Standards for School Mathematics*, was added to the series. In addition to these three books the NCTM has published a series of addenda books which begin to address the need for materials better suited to the teaching methods discussed in the *Standards* series than are currently available from most textbook series.

The *Standards* are not meant to be a list of objectives or a curriculum plan which all schools should adopt. "The *Standards* document is meant to facilitate grass-roots change in mathematics curriculum and instruction, not to impose reform from above." (Schoen, 1989)

The *Standards* series will undoubtedly lead to changes in the way mathematics is taught as more and more school systems embrace the vision set forth by the NCTM. As beginning teachers join the staffs of these school systems

they may feel a lot of anxiety about these changes. In most cases, the teaching methods and the subject focus called for in the *Standards* are unlike anything a beginning teacher has ever experienced. This project can help a beginning teacher understand what types of teaching and assessment methods should be used for real change to begin to take place in mathematics education.

Mathematics education has been under fire for many years due to low success rates by a majority of students. Documents such as *School Mathematics: Options for the 1990's* (Romberg and Stewart, 1985) and *Workforce 2000* (Johnston,1987) show that students are not getting what they need mathematically. However, "graduation rates and career choices depend heavily on the mathematics skills students possess upon entering the university (Waits et al., 1988)." While mathematics education has been reformed many times in the past century (i.e., the "new math" and "back-to-basics" movements), these attempts have always met with failure due to several factors. One factor which lies at the heart of the problem was alleviated during the writing of the *Standards*. "Lessons from the past have taught us that we must involve teachers in all steps of curriculum development and implementation and that we must build a broad base of support among parents, business partners, and school administrators (Frye, 1989)." Teachers now have a voice in the mathematics education they are supposed to facilitate for students.

At the heart of the *Standards* are the realization that in the twenty-first century there will be new societal goals. These are a need for (1) mathematically literate workers, (2) lifelong opportunity, (3) opportunity for all, and (4) an informed electorate. In order to attain these goals their must be a shift in the focus of what is taught. Students graduating from high schools today learn in a "computation-dominated curriculum more suitable for the nineteenth century. (National Council of Supervisors of Mathematics, 1989)" These new societal

goals can only be met through teaching students how to use these computational skills in realistic situations. In order to accomplish this the NCTM has called for a focus on problem solving, mathematical communication, reasoning, and connections within mathematics and to other curriculum areas.

In order to become a mathematical problem solver students must learn to assess a situation, apply appropriate techniques, and evaluate their decision for reasonableness.

> Solving a problem is a little like building a building in that many different jobs must be done successfully to obtain the desired results. Unlike in the construction of a building, though, you as problem solver must perform all the roles. Furthermore, you must continuously monitor your present role and the progress being made, and make decisions concerning when to switch hats and assume a new role.
>
> (Jensen, 1987)

Students should be given chances to not only solve problems in many different contexts but also to create problems based on things which interest them. According to the National Council of Supervisors of Mathematics, "Solving word problems in texts is one form of problem solving, but students also should be faced with nontext problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. Students should see alternate solutions to problems; they should experience problems with more than a single solution. (1989)"

So how should a beginning teacher, or for that matter any teacher who is looking to change their methods in order to address the *Standards*, go about implementing the *Standards*.

> After a study of the document, you should assess your current beliefs about how students learn

> mathematics, your familiarity with the new content and areas of emphasis, your instructional strategies, your assessment procedures, and your use of technology. By comparing them to those described in the *Standards*, you would determine what you need to learn and what you need to begin to change in a systematic and incremental way. For example, you might want to focus on connecting mathematics and other disciplines through applications or to initiate interviews and conferences as a way to assess students' understanding and progress. (Frye, 1989)

Since a beginning teacher has little to base this assessment on concerning their own teaching experiences they should reflect on their own educational experiences and decide on one or more small pieces of these experiences which they would like to do differently for their own students. For instance, questions concerning assigning homework are often a central focus of a beginning teacher's classroom plan. However, if this teacher takes time to reflect back on their own mathematics education they might recall times when an instructor assigned a pattern of problems such as all the evens or just multiples of three. It is obvious that little or no thought went into this assignment so why should the student put thought and effort into completing the assignment. (Marquis, 1989) The beginning teacher, in fact all teachers, should give careful consideration to the problems they assign to students. Through this process the students can see that the teacher values what they are doing and has given thought to the types of activities the students are to do.

If a beginning teacher had several technology classes or classes in which they were able to use technology they might consider assisting other teachers in their building in implementing the use of calculators and computers in their classrooms. In many instances, a beginning teacher entering a school may be the only person who has worked with the vast array of graphing calculators and computer software.

It is also important for beginning teachers to get involved in their school and community.

> ...a teacher can do some things [to implement curriculum change] outside [their] classroom. [The teacher] can -
> - get involved in school and district curriculum committees;
> - begin educating administrators, school boards, and parents;
> - invite observers into your classroom to see what [they] are doing. Students engaged in doing mathematics and talking, writing, and even arguing about mathematical ideas can not help but impress the greatest skeptic. (Mumme et al., 1989)

The *Standards* call for both mathematics students and teachers to become empowered. The most basic element of empowerment is involvement.

Now is the most exciting time in the history of mathematics education to become a mathematics teacher. However, with the excitement comes much anxiety about being able to successfully meet the challenge set forth in the *Standards* series. The lessons which follow will help a beginning teacher, or any teacher who is beginning to implement the *Standards*, to successfully reach the new goals and help form a mathematically literate society.

Title: What did you think?

Grade Level: 5 - 12

Time Required: 3 - 5 minutes

Objectives:
1. Students will react to a presentation/demonstration providing teacher with immediate feedback.
2. Students will ask questions about areas which were not made clear during the presentation.

Notes to the Teacher:

If you are looking for a quick way to begin using writing in the classroom, this is the perfect activity. Not only will students feel that their opinion matters which will encourage their mathematical empowerment; you will also get immediate feedback on the understandings and/or misunderstandings of your students.

It is quite important that these cards not be graded. If students feel that a negative opinion or an obvious misunderstanding they would like to express will lead to a lower grade they will not feel free to express themselves.

Once students have become fully acquainted with this activity you might want to adapt the activity to a simple suggestion (or question) box which can be kept on a table in the classroom which students can drop notes in about questions/concerns whenever they arise.

Procedure:

At the end of the lesson or class period have students write two or three sentences on what they thought of the lesson, what they do not understand or what they found especially confusing. You may find it necessary to phrase specific questions to the students when using this activity for the first few times. Some questions may be:

- Were the examples used during the lesson clear and understandable? Which example made the most or the least sense? Why?
- What did you like most about the lesson? Why? What did you like least? Why?
- What do you think was the objective of today's lesson?
- What is the biggest question you have about the skill/method you learned today?

You should not have students put their names on the cards. This will allow students to feel free to express themselves honestly. It is also important that you address the questions/concerns on the cards during class time.

Title: Grade Tracking Sheet

Grade Level: 5 - 12

Objectives:
1. Students will develop responsibility for tracking their progress throughout the course of the year.
2. Students will use basic computational skills such as addition, finding averages, finding percents, etc.

Materials Needed:
___ copy of grade tracking sheet for each student.

Notes to the Teacher:

The grade tracking sheet can serve many purposes. First of all, students will be taking responsibility for keeping track of their assignments, daily grades, and course grade.

Second, this can serve as an important checking device between the teacher's grade book and the student's idea of their grade. When a discrepancy occurs between the grade the teacher and student have reached it will be simple to go back and find where the discrepancy arose. Students should be encouraged to keep all homework, quizzes, tests, and projects with their grade sheets in case a discrepancy does arise.

Finally, the grade tracking sheet can serve as a communication device between parents, teachers, and students. By letting parents know you are using the grade sheet, they will have a way of keeping a handle on how their child is doing in class. Some parents may wish for you to initial the sheet periodically or you may wish to have parents do the same in order to maintain communication.

In order for students to see value in this activity it is a good idea to announce the total points possible, have students figure their grade, and collect the grade sheets. Award points for accuracy in record keeping (suggested 3 - 5 points).

Procedure:

Pass out grade tracking form. Explain procedure necessary for keeping tracking form.

Make extra copies of tracking sheet so as students fill up a page they can get another copy. Keep these extra copies in a prominent location.

Grade Tracking Form

Name: ______________________________ Class: ____________________

Date	Description of Assignment	Your Points	Points Possible

Title: Using the mathematics notebook

Grade Level: 5 - 12

Objectives:
1. Students will develop skills of organization in order to better their understanding of mathematics.
2. Students will manage their own records of progress for the course.

Materials Needed:
___ notebook description
___ three-hole punch (to remain available all year)

Notes to the Teacher:

After implementing the mathematics notebook in their classroom, a teacher may be surprised to see how many uses it will have.

The obvious reason for using mathematics notebooks is to increase students' organizational skills and aid them in preparing for exams. Mathematics notebooks will help to eliminate the, "I can't remember" response used all too frequently by students when teachers try to involve them in discussion. A quick look back in the notebook should lead to answers, evidence to support conjectures, etc.

But supporting claims and preparing for exams are far from the only uses for the mathematics notebook. The mathematics notebook can also serve as a starting point for portfolio assessment. Students will have all materials at hand to find items they and you feel demonstrate their growth in the course.

The mathematics notebook can also be used during parent-teacher conferences as a starting point for discussion. It can also help you understand where a student's difficulties began.

The benefits to the student of keeping their notebook in order could also be had by the teacher who used a notebook of their own. Since you will probably need to teach students some simple organizational skills and demonstrate how the mathematics notebook should look, you should consider using your notebook to hold lesson plans, evaluations, assessment activities, and your mathematics journal.

Procedure:

Hand out the notebook description during the first week of classes or of the grading period. Allow students time to read through the sheet and ask questions.

As the grading period progresses set an example for students by placing items in your own notebook.

Assessing Mathematics Notebooks:

To help instill that this is a valuable assignment you should consider collecting the notebooks periodically and awarding points for completeness, organization, neatness, etc. How often you pick up the notebook and how thoroughly you go through the notebook when grading it will depend on the amount of time you have available. A rotation schedule of picking up one group of students' notebooks every week instead of trying to look at the entire class's notebooks at once. Just be sure you have picked up everyone's notebook the same number of times each grading period.

A second option is to not pick up the notebooks, but make homework or note quizzes and allow students to use only their notebooks on the quiz.

A third assessment method is to simply "browse" the notebooks while students are working on do now problems, activities, or checking homework in groups. This method involves a very brief check and therefore should be used once students have developed their own sense of the value of the notebook.

Mathematics Notebook Description

This year you will be required to keep a mathematics notebook. This notebook will serve several purposes throughout the year. You will use your notebook when reviewing for quizzes and tests and when preparing your portfolio. I will use your notebook to assess your progress during the year.

Your notebook will serve as an organizational tool for class notes, handouts, assignments, and journal entries. To help you get started i would like you to divide the notebook into the following sections:

- Course Information
- Class Notes and Lecture Handouts
- Assignments, Quizzes, and Tests
- Mathematics Journal

The section "Course Information" should contain this sheet, class calendar and announcements, and your grade tracking form. You should use this section to keep track of assignments and due dates.

Since I will collect the notebooks to check its contents and to check your grade sheet it is important you keep these items up to date. Your notebook grade will count towards your class participation grade.

Title: Summarizing Notes for Exams

Grade Level: 7 - 12

Time Required: 15 to 20 minutes

Objectives:
1. Students will organize class notes in order to prepare for exams.

Materials Needed:
___ Copies of note card and exam guidelines

Notes to the Teacher:

While in college I found an organized method to prepare for exams. I created a note card like those allowed by some instructors on exams. In some cases, I was able to use this card on the exam. However, in some cases where this was not permitted I still found the process invaluable. For that reason, I believe high school students should be allowed to use note cards on exams in advanced mathematics classes.

The sign-off portion of the guidelines sheet is provided so students take responsibility for reading the guidelines and abiding by them.

One of the guidelines for note cards is that they must be turned in at the beginning of the class period of the day before the exam. This guideline is used to encourage students not to wait until the last minute to prepare for exams. Cards are picked up at the beginning of the period, checked to be sure they have been started, and are returned at the end of the period that same day so that students may use them while studying. When first implementing the use of note cards you may want to use this short check of cards to decide on concepts which may need to be reinforced before the exam. You can check over cards while students go over homework in groups or work on a short set of review problems.

Procedure:

Pass out note card and exam guidelines to students. Allow students time to read handout and ask any questions. Have students sign bottom of form, detach, and return.

Adapted From:

Clopton, Edwin L., "Summary Cards for Tests." *Mathematics Teacher* 85: (February 1992). 110.

Note Card Guidelines

We will be covering quite a bit of information this semester. When test time comes around, organizing all of this information may seem overwhelming. In order to help you organize this information and prepare for exams you will be permitted to use a note card while taking chapter and unit tests and other designated exams. Note cards will not be allowed during quizzes unless announced otherwise.

Note cards must meet the following guidelines. If a note card does not meet the all of the guidelines you will not be permitted to use the note card.

1. Note cards can be no larger than a 4" x 6" index card (a surface area of 243 cm^2).
2. Note cards must be handwritten in your own handwriting.
3. Note cards must have your name and the date of the exam printed in the top right corner of one side of the card.
4. Note cards may have any information you feel is necessary or useful for the exam including definitions and worked examples.
5. Note cards must be handed in at the beginning of the period the day before the exam. They will be returned at the end of that period so that you may use them to prepare for the exam.
6. Note cards will be handed in with the exam.

Tips on preparing your note card:

- After each class period go through your notes and mark what you feel are the most important ideas and examples from the lesson. This will serve as a starting point for preparing your note card.
- You may discuss the contents of your note card with fellow classmates. It is vital, though, that you concentrate on the information YOU need to know.
- Do not try to include examples of every type of problem, definition, formula, etc. Look for problems which illustrate more than one concept at a time.
- Keep all note cards to use when preparing for the midterm and final exams.

Read the above form. Ask any questions about any material on the form you do not understand.

Read the following statement. Signing below indicates that you have read this entire sheet and agree to abide by its guidelines.

I agree to follow the guidelines for using note cards on exams. I understand that the use of notecards is a privelege, not a right which means that failing to comply with a guideline will result in my not being permitted to use a note card on the exam in question.

Signed: _____ Date: _____

Title: Cooking up Problems--a problem writing activity

Grade Level: 5 - 8

Time Required: 10 - 15 minutes

Materials Needed:

___ Large cooking pot
___ Index card for each student
___ For effect you may wish to wear an apron and/or chef's hat
___ several object about which students can write problems

Notes to the Teacher:

In this activity students and teacher get to have a little fun while writing (oftentimes) silly word problems based on a few simple everyday objects. This is a good activity to use as a do-now or focus type of class starter. After the teacher has presented students with the ingredients and the students have begun writing, the teacher can take care of attendance/management aspects as well as have a little time to write in their own journal.

This lesson lends itself to several different methods of assessment. You may choose one or more of those described here or use a technique of your own.

1) Evaluate the problems themselves using a rubric such as the one below. This assessment technique is most suitable when first using this lesson to provide students feedback on their problem creation technique. You might consider giving bonus points for creativity.

A problem receives

- an A if all items in the pot are used and an appropriate solution and correct answer are provided on the back of the card.
- a B if most items are used or if the solution given is not appropriate or accurate.
- a C if fewer than half of the items are used or the solution is inappropriate or the answer given is not based on the work provided.
- a D if little effort was put forth in creating the problem or no solution is provided.
- an F if the student makes no attempt to create a problem.

2) Choose two or three of the problems and put them on a quiz or test. Be sure to choose problems which address all ability levels of your students.

3) Use problems as lesson starters if a problem fits well into a particular area of the curriculum.

Procedure:
Place five to seven items in the cooking pot and place lid on top. Put pot on table in front of room. Inform students they will be cooking up problems using all of the items pulled out of the pot. Stress that students may use other items in the problems they write, but they must use all of the items pulled out of the pot.

If you are teaching methods of problem solving you may want to specify the type of problem students should write. However, the results will probably be better if students are allowed to write any type of problem they wish.

Once all items have been pulled out of the pot, pass them around the room. Students should write rough drafts of their problems and work out solutions in their journals. Students should write their name and problem on one side of the card and the solution on the other side.

Adapted From:
Fisher, Julie. "Sharing Teaching Ideas: What's in the Pot?" *Mathematics Teacher.* 86 (March 1993): 214-215.

Title: Die Hard: With a Vengeance--Adventure into Problem Solving

Grade Level: 6 - 10

Time Required: 25 - 30 minutes

Objectives:
1. Students will work in cooperative groups to solve a famous mathematical problem.
2. Students will attempt to generalize a solution method to similar problems.

Materials Needed:
___ Video clip from Die Hard 3: With a Vengeance
___ Tubs of water and 3 and 5 unit containers with no graduation marks on or in them.

Procedure and Teacher Discussion:
In the movie, Die Hard with a Vengeance a terrorist by the name of Simon (Jeremy Irons) puts New York police detective John McClane (Bruce Willis) and his civilian partner Zeus (Samuel L. Jackson) through several "mental challenges." If they accomplish their tasks the bomb, which Simon says he has placed in a school, will not explode.

One of the tasks John and Zeus must accomplish is the famous water jars problem. The problem is to use a three-gallon (or unit) and five gallon (or unit) jug to get exactly four gallons of water. (Neither jug has any type of measurement marks) While the entire movie is not appropriate, the clip can be used as an appropriate attention getter to introduce the problem to the class. It is important that you view the clip prior to class, editing any parts which would be deemed inappropriate (some of the language is quite strong) by your school administrators.

Show the portion of the clip where the problem is posed then set students to work. Students should work in small groups in which one student keeps a careful record of the "moves" made. Moves recorded should include any time a jug is filled or emptied and where it is filled from or emptied to.

Once all groups have found the solution show the rest of the clip. Discuss whether or not Jackson and Willis actually solved the problem or if it just sounds and looks as if they did.

Solution:
Fill the three gallon jug from water source. Empty contents of three gallons into five gallon container. Fill the three gallon container from water source again. There are now three gallons in each jug. Fill the five gallon jug exactly to the top using two of the three gallons in the three gallon container. This leaves exactly one gallon in the three gallon container. Empty the five gallon container back into

water source. At this point there should be one gallon in the three gallon container and nothing in the five gallon container. Place the one gallon from the three gallon container into the five gallon container. Fill the three gallon container and add this to the one gallon in the five gallon container. This gives exactly four gallons.

A step by step summary is shown by the chart below.

3 gallon container	5 gallon container
3	0
0	3 (from 3-gal container)
3 (from source)	3
1(dump above to 5-gal container)	5 (from 3-gal container)
1	0 (dump out above)
0 (dump above to 5-gal container)	1 (from 3-gal container)
3 (from source)	1
0 (dump above to 5-gal container)	4 (from 3-gal container)

Assessment:

Students should write in their journals about whether or not the solution their group found was the most efficient. What should they have done differently if there solution was not the most efficient? What difficulties, if any, did they have in solving the problem.

Note:

You may consider having students look for other sets of numbers which will work and examining patterns. Some combinations that work are 8 and 3 gallons to yield 7 gallons, and 4 and 9 gallons to yield six gallons. Have students look for patterns in both the combinations of whole numbers which will lead to solutions and the actual pattern of "dumps" in the solutions.

Depending on the ability level of your students you might want to consider looking for an algebraic solution of the problem which is described in this lesson plan.

Resources:

Kane, William H. "Don't Give Up!" *Mathematics Teacher.* 86 (February 1993): 110-112.

Title: Is it always 34?

Grade Level: 9 - 12

Time Required: one or two class periods

Objectives:
1. Students will analyze a mathematical trick to see how much of a trick it really is.
2. Students will write and simplify algebraic expressions.

Materials Needed:
___ Transparencies made from each of the five transparency masters
___ Overhead projector and markers.

Notes to the Teacher:
To many mathematicians, much of the beauty of mathematics lies in knowing that there is almost always an explanation for the seemingly unexplainable. Mathematics can account for or be used to analyze almost all phenomenon, from the number of spikes on a pine cone to the "knots" formed by DNA in human genes.
Most students are intrigued by mathematical "tricks" in which you start with some number, perform a series of operations on this number, and then get a result which the "magician" predicted before beginning the trick.
This lesson is intended for college-intending students looking for an interesting challenge. However, any student up for a good bit of problem solving would be motivated to work on this problem.
As an assignment for this activity (or assessment method) have students come up with a "trick" on their own and prove using algebra that it will always work.

Procedure:
Display transparency 1. Tell students you are going to write a number on a piece of paper which will be the sum of the numbers a student is about to choose. Write 34 on the piece of paper. Choose a player. Have that person choose a number in the first row and then cross off all numbers below the number they choose. The should continue in this manner for all four rows. have the player total the numbers. They should get 34. With some great flourish reveal that your prediction was correct.

Repeat this procedure with the 5 by 5 grid. The total here is 65.

Pose the question to students - Does this always happen? Is the sum of a 4 x 4 grid always 34? Is the sum of a 5 x 5 grid always 65? Why?

One way to approach this problem is to rename every entry in the array in terms of multiples of the dimension of the array. For example the 4x4 array could be rewritten as:

4(1)-3	4(1)-2	4(1)-1	4(1)
4(2)-3	4(2)-2	4(2)-1	4(2)
4(3)-3	4(3)-2	4(3)-1	4(3)
4(4)-3	4(4)-2	4(4)-1	4(4)

Students should discover that every game sum will have the form:

$$4(1) + 4(2) + 4(3) + 4(4) - 3 - 2 - 1$$

Have students use this same process on the 5x5 array. The number resulting from this array will always be in the form

$$5(1) + 5(2) + 5(3) + 5(4) + 5(5) - 4 - 3 - 2 - 1$$
$$= 5(1 + 2 + 3 + 4 + 5) - (1 + 2 + 3 + 4)$$

At this point have students work in pairs to find a method (formula) for summing the first 4, 5, 6, *n* counting numbers. Allow students time to discover:
The sum of the first n natural numbers is given by $\frac{n(n+1)}{2}$.

From here demonstrate that the 5x5 array will always have a game sum of

$$5(1 + 2 + 3 + 4 + 5) - (1 + 2 + 3 + 4)$$
$$= 5\left(\frac{5(6)}{2}\right)\left(\frac{4(5)}{2}\right) = 75 - 10 = 65$$

Allow students time to work out the game sums for several larger games. Finally have them generalize this result to an $n \times n$ array:

$$n(1 + 2 + 3 + 4 + \ldots + n) - (1 + 2 + 3 + \ldots + (n - 1))$$

$$= n\left(\frac{n(n+1)}{2}\right) - \left(\frac{n(n-1)}{2}\right)$$

Have students "simplify this expression" and verify their answer with a partner. The correct result should be $\frac{n^3 + n}{2}$

Adapted From:

Roche, James W. "Sharing Teaching Ideas: Blackstone's mathmagic." *Mathematics Teacher* 86 (December 1993): 733 - 734.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Title: So Many Numbers--How do I Keep Track?

Grade Level: 4 - 7

Time Required: 50 minutes

Objectives:
1. Students will respond to the notion that mathematics is difficult because there are so many numbers involved.
2. Students will identify uses of large numbers in society.
3. Students will use estimation skills to describe large distances.
4. Students will use large numbers in both written and numerical form to describe a situation.

Materials Needed:

___ Family Circus Transparency
___ Family Circus Journal Page
___ Overhead Projector
___ *Apollo 13* Videocassette
___ Worksheet 1
___ "NASA Media Guide"

Notes to the Teacher:
Most textbook exercises which cover naming and using large numbers do so in a drill and practice format with little or no real world application in problem solving and or communication situations. This lesson takes out the drill and practice nature of this valuable activity toward the development of number sense by students.
Many students are interested in astronautics and space and almost all students are interested in movies. During this lesson you show a portion of the 1995 film Apollo 13 starring Tom Hanks. While it is important to view this section of the film before class to edit any parts your community members and administration would find inappropriate, this clip provides an excellent opportunity to assess students sense of large distances and their abilities to communicate using large numbers. If your students have access to the Internet you may wish to have them find information on the Apollo 13 in order to make their story more accurate. You may also wish to have students go to the library to use reference materials.

Procedure:

Display Family Circus Transparency on overhead projector. Discuss with students the misconception Billy has about the way numbers are formed. Hand out journal page and have students write their responses to your discussion and the cartoon.

Tell students about the *Apollo 13* mission to the moon. Show portion of video tape where flight controller is talking to mission controller about how the astronauts do not have enough supplies to make it home.

After viewing the segment draw the same picture seen on the clip on the chalkboard or overhead projector (see figure 1). Discuss with students how far they think the moon is from the Earth and how far they think the astronauts were from the Earth when the accident occurred based on the flight controller's drawing. Students will probably give estimates of varying size. Record all estimates on the chalkboard or overhead projector. Discuss different ways of writing these estimates in word and numerical form.

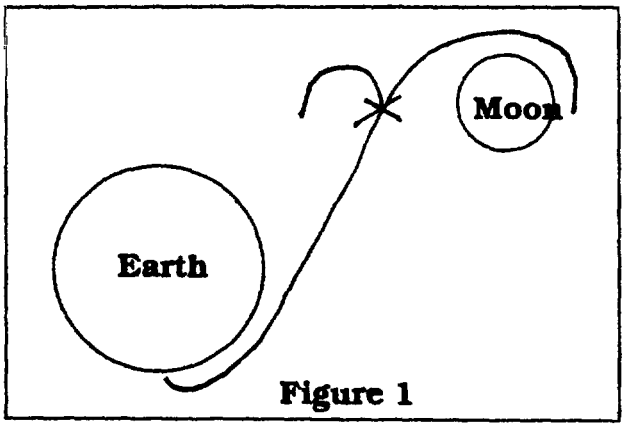


Figure 1

Pass out worksheet 1. You may have students work on this in "research pairs." Have students explore various reference materials such as encyclopedias, world wide web and gopher sites, or books on the space program.

References:

Cain, Barbara, ed. *Cartoon Corner: Alphabet Soup.* Mathematics Teaching in the Middle School. 1:(5). 383.

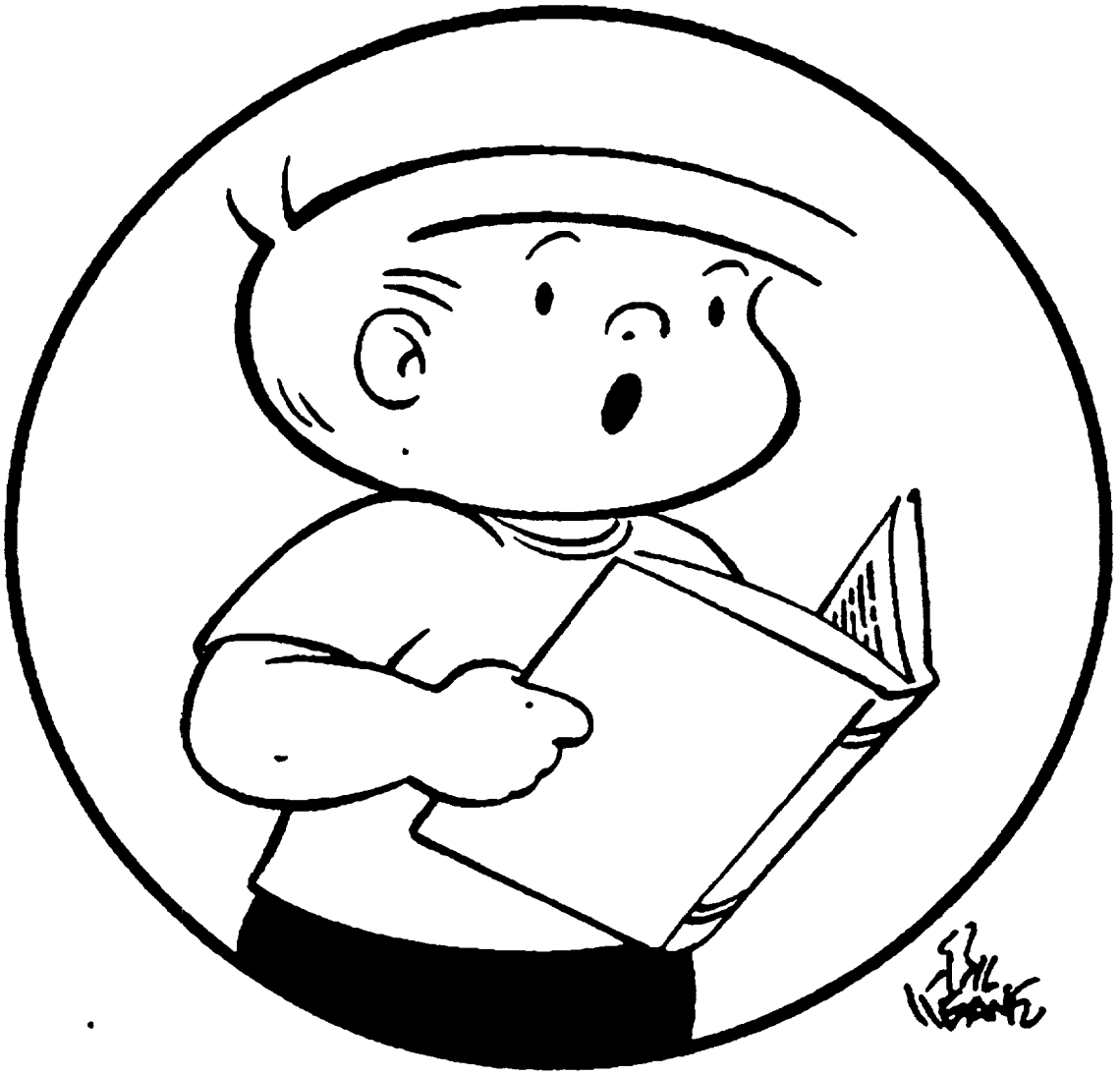
Howard, Ron, dir. *Apollo 13.* With Tom Hanks. MCA Universal Home Video. 1995.

"Space Exploration." Microsoft Encarta. 1994.

"Moon." Microsoft Encarta. 1994.

"The Planets: Earth." Microsoft Encarta. 1994.

The Family Circus *by Bil Keane*

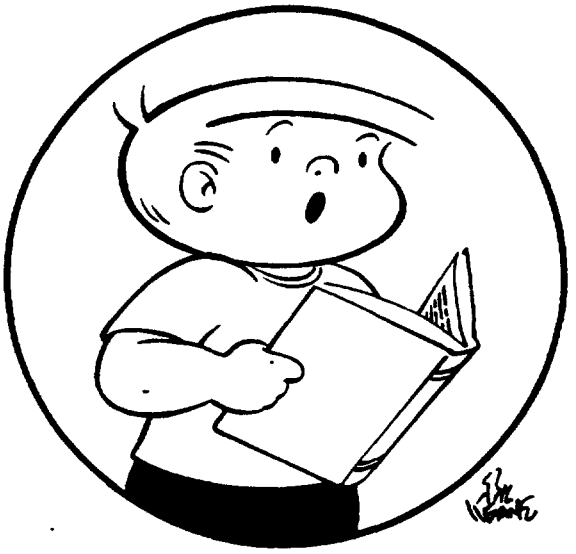


"Reading is easier than math. There are only 26 letters, but millions of numbers."

So Many Numbers--How do I keep track of them all?

Use the space below to react to our discussion and your ideas about the cartoon. Add this page to your journal.

The Family Circus *by Bil Keane*

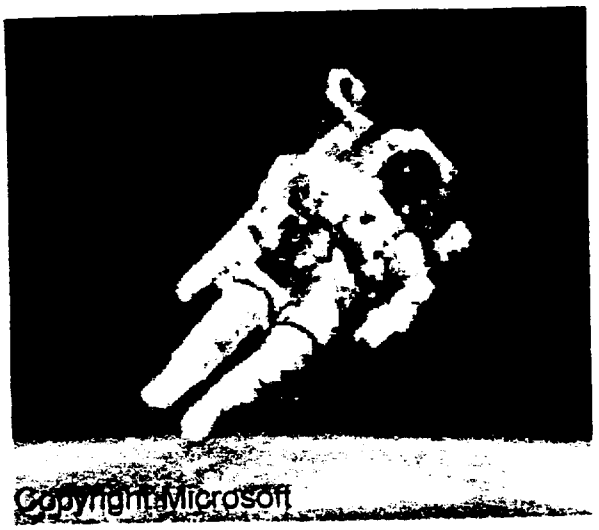


"Reading is easier than math. There are only 26 letters, but millions of numbers."

So Many Numbers

Worksheet 1

Name: _____ Date: _____ Class: _____



The astronauts on the Apollo 13 mission faced several dangers during their journey. The most obvious was how far they were from home. As the Apollo 13 astronauts and mission controllers worked to get the astronauts home safely the world could only watch and wait. Imagine you were assigned the job of writing a newspaper article describing the situation faced by the astronauts. Use the information in the Media Guide and other classroom resources to write your article. You should include several numerical descriptions; some in standard form and others in written form. REMEMBER: We know the story has a happy ending, but at the time you would have been writing your article the fate of the astronauts was far from certain.

NASA Media Guide

Mission: Apollo 13
Crew: Jim Lovell
Fred Haise
Jack Swigert
Flight director: Gene Kranz

The three astronauts are using the Lunar Module as a lifeboat because the main craft, the Odyssey, was damaged in an explosion. The cause of the explosion is still unclear. Astronaut Ken Mattingly is working at mission control to come up with a plan to start up the spacecraft systems on a very limited power supply in order to land the craft.

THE MOON

DISTANCE FROM EARTH	
Shortest	356,399 km (221,456 mi)
Greatest	406,699 km (252,711 mi)
Mean	384,403 km (238,857 mi)

DIAMETER
3476 km (2160 mi)

AGE
4.6 billion years

TEMPERATURE AT EQUATORIAL MARIA
127° C (260° F)*
-173° C (-280° F)**

CIRCUMFERENCE
10,927 km (6790 mi)

ROTATION PERIOD ABOUT AXIS
27 days, 7 hours, 43 minutes

REVOLUTION PERIOD AROUND THE EARTH
27 days, 7 hours, 43 minutes

LENGTH OF DAY AND NIGHT
15 earth-days each

SURFACE GRAVITY
1/6 that of the earth

ESCAPE VELOCITY
2.4 km (1.5 mi) per second

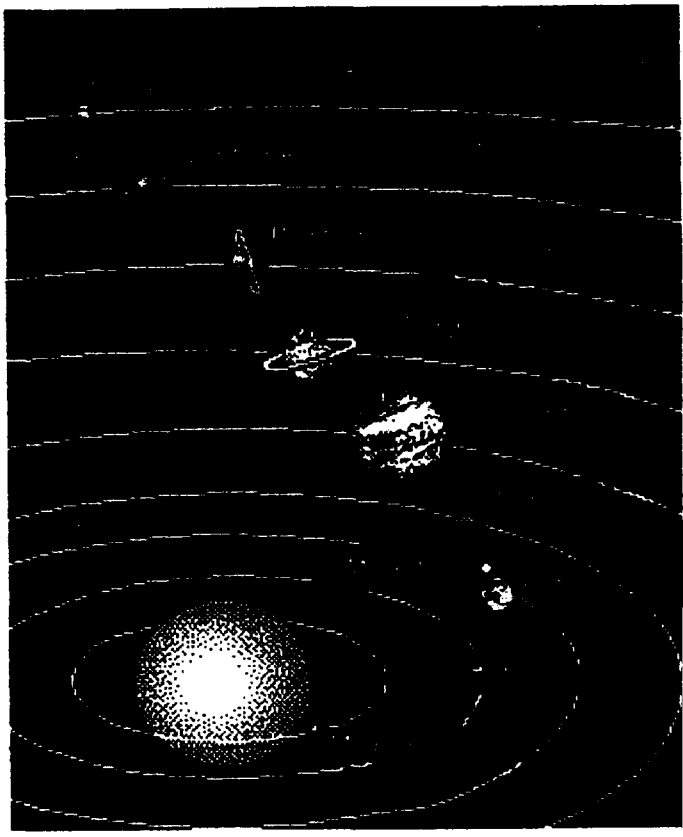
MASS
1/81 that of the earth

VOLUME
1/50th that of the earth

ATMOSPHERE
Little or none

*Sun at zenith ** Lunar night

Microsoft Table



Title: Can you carry one million dollars?

Grade Level: 5 - 7

Time Required: 50 minutes (longer may be desired)

Objectives:
1. Students will use problem solving skills and estimation to answer the question in the title.
2. Students will write a written statement of their findings.

Materials Needed:
___ Large Suitcase
___ Overhead Projector
___ Extra! Extra! transparency
___ Paper money or real $1 bills (a few for each group)

Procedure:

Place empty suitcase in visible area at the front of the room. Put Extra! Extra! transparency on overhead. Have a student read the story on the transparency aloud.

Pose the following set of questions to the students-

> *Can $1 million in $1 bills fit into a suitcase? If so, how much would the suitcase weigh? Could you lift the suitcase? If $1 million in $1 bills could not fit in the suitcase, then what amount could fit?*
>
> (McReynolds, 1995)

Carry the suitcase around the room to allow students the chance to get an idea of size. You may also want to hand out paper money or real $1 bills to groups as they begin working on the problem.

As students work on the problem you may want to provide them with some or all of the following information (McReynolds, 1995):

- *About five $1 bills will fit on a sheet of notebook paper.*
- *The highest denomination currently being printed is a $100 bill.*
- *Paper currency, no matter the denomination, is strapped, or wrapped, in stacks of 100 bills*
- *These straps are then packaged into what are called bricks. Each brick is ten straps long by ten straps high. A brick of $1 bills would be worth $10,000. A brick of $5 bills would be worth $50,000. A brick of $10 bills would be worth*

$100,000, and so on, up to a brick of $100 bills being worth $1 million.

Adapted From:

McReynolds, Kimberly. *Readers Write: What's a Brick Worth?* Mathematics Teaching in the Middle School. 1(7). 514, 553.

Additional Resources:

Exploring the Size of a Million Dollars. Developing Number Sense in the Middle Grades. Addenda Series Grades 5-8. National Council of Teachers of Mathematics. 1991. 22.

Extra! Extra!
Vacation Robbers captured!

AP-Northshores, FL-One of American National Bank's problems was solved early this morning when police apprehended the so-called "Vacation Robbers." An anonymous tip led police to a motel in Bayside where the couple had rented a room.

When police took the couple into custody, though, a disturbing question was left unanswered--Where was the money? The robbers placed $1 million in $1 bills into a suitcase and fled from the bank in the getaway car. However, the money was not found in the motel room and the alleged robbers have given no hints as to its whereabouts.

Title: Teaching Addition with Base Ten Blocks

Grade Level: 4 - 8

Time Required: At least two class periods

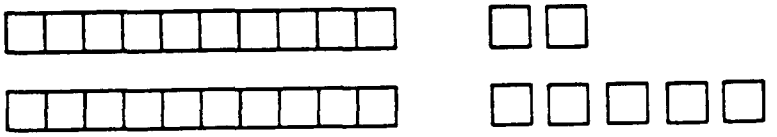
Objectives:
1. Students will model addition problems using base ten blocks.
2. Students will write and solve problems which use addition.

Materials Needed:
___ Base 10 blocks for each student or group of students.
___ Overhead Base 10 Pieces
___ Addition with Base 10 Blocks Worksheet

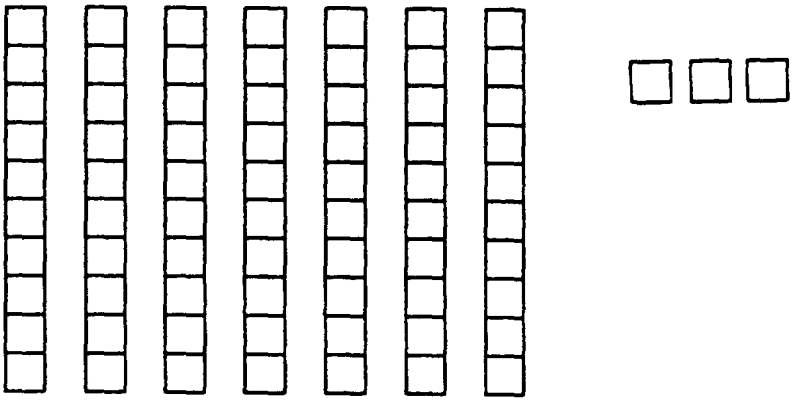
Notes to the Teacher:
The use of manipulatives is highly recommended by the National Council of Teachers of Mathematics. Base 10 blocks are one of the many manipulatives which can be used to model basic addition, subtraction, multiplication, and division. Base 10 blocks are an extremely versatile investment because they can also be used for other topics such as rounding and approximation of square roots where other manipulatives are limited to use in just one or two topic areas. If your school does not have Base 10 blocks you can make a simple set out of paper using the blackline master at the end of this lesson. For the purposes of this lesson students will need at least 10 - one hundreds, 25 - tens, and 30 - ones.
When students are working on the worksheet you may wish to have them use the worksheet only as a guide and do work on their own paper. When working problems have them draw the Base 10 pieces which would model the method of solving that particular problem.

Procedure:
Begin by demonstrating how to model each of the following using Base 10 blocks:

27



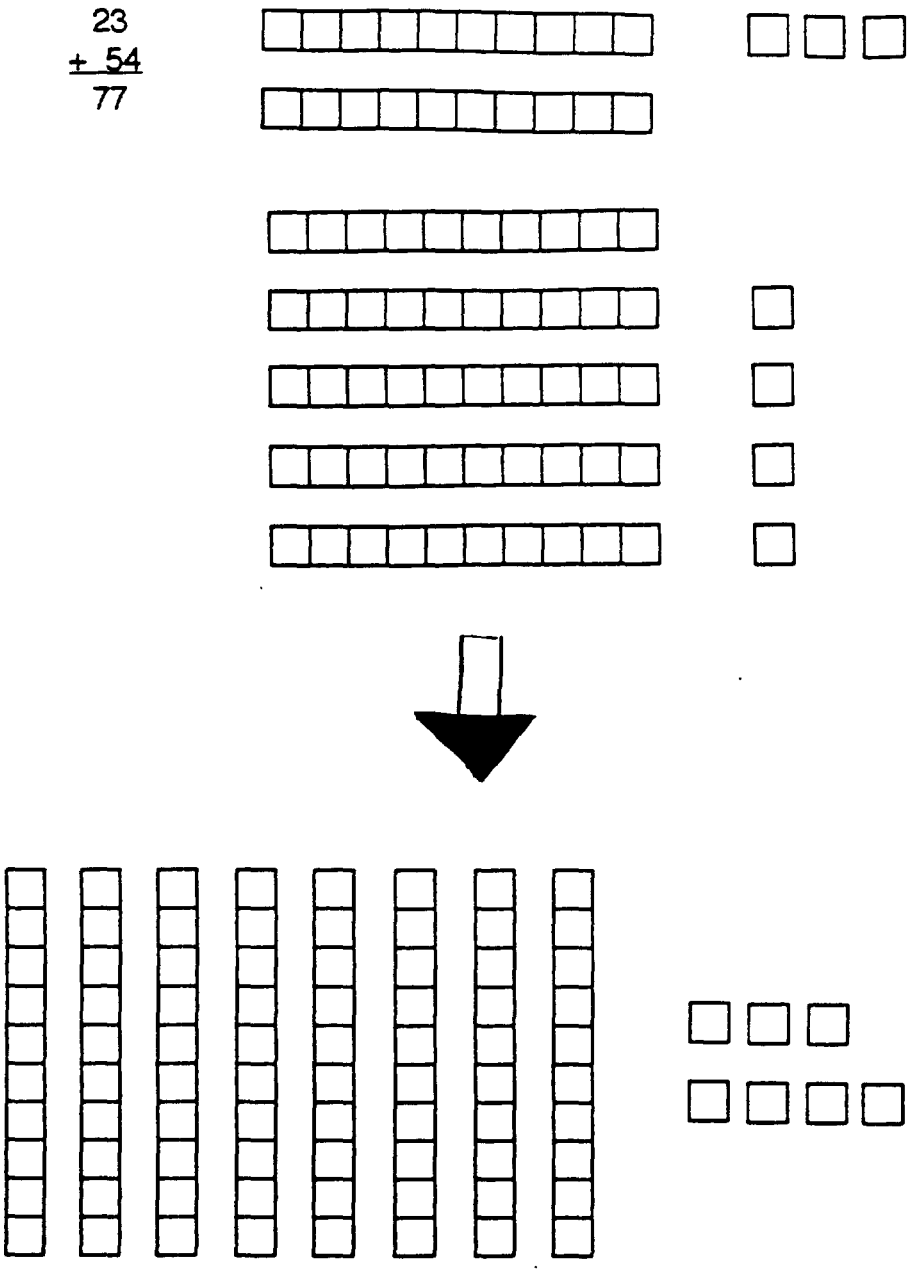
73



Allow students time to model numbers of their own. Challenge them to find any numbers that can not be modeled.

Model addition of two numbers without renaming.

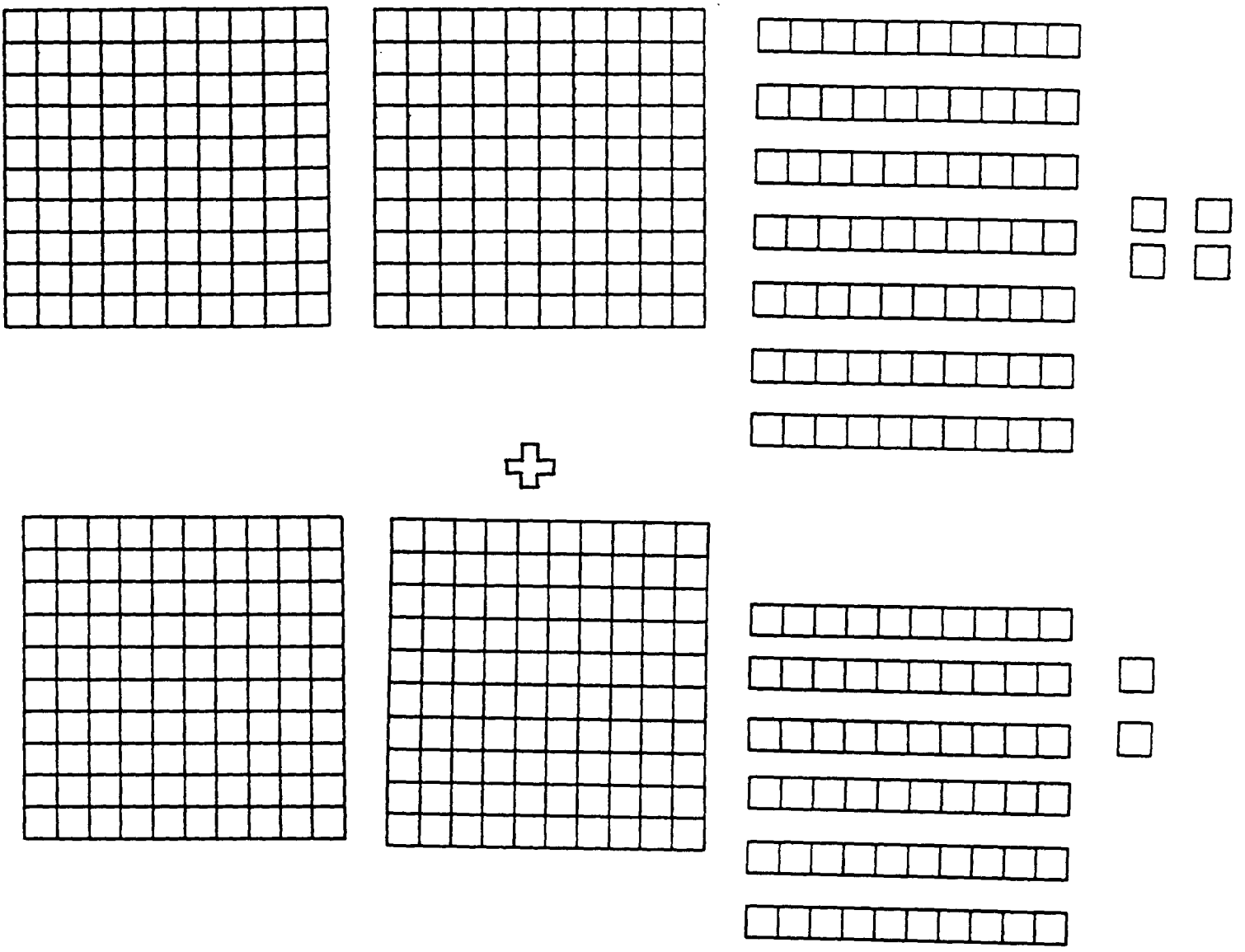
$$\begin{array}{r} 23 \\ +\ 54 \\ \hline 77 \end{array}$$

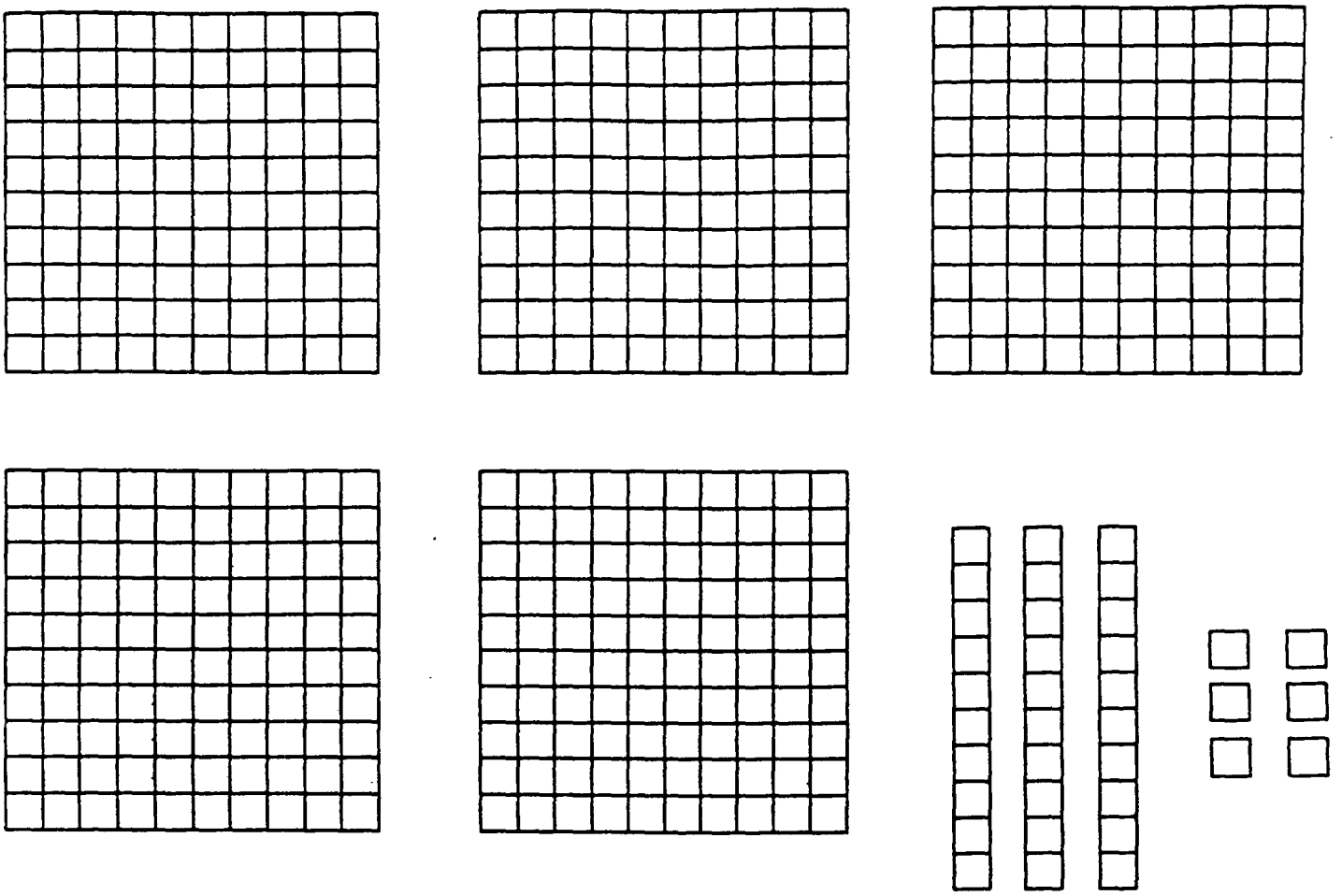


Have students create two or three problems where no renaming is involved and record the problem and the modeled answer in their journals. Have volunteers model a couple of the class's problems on the overhead projector.

Now model addition of numbers with one renaming.

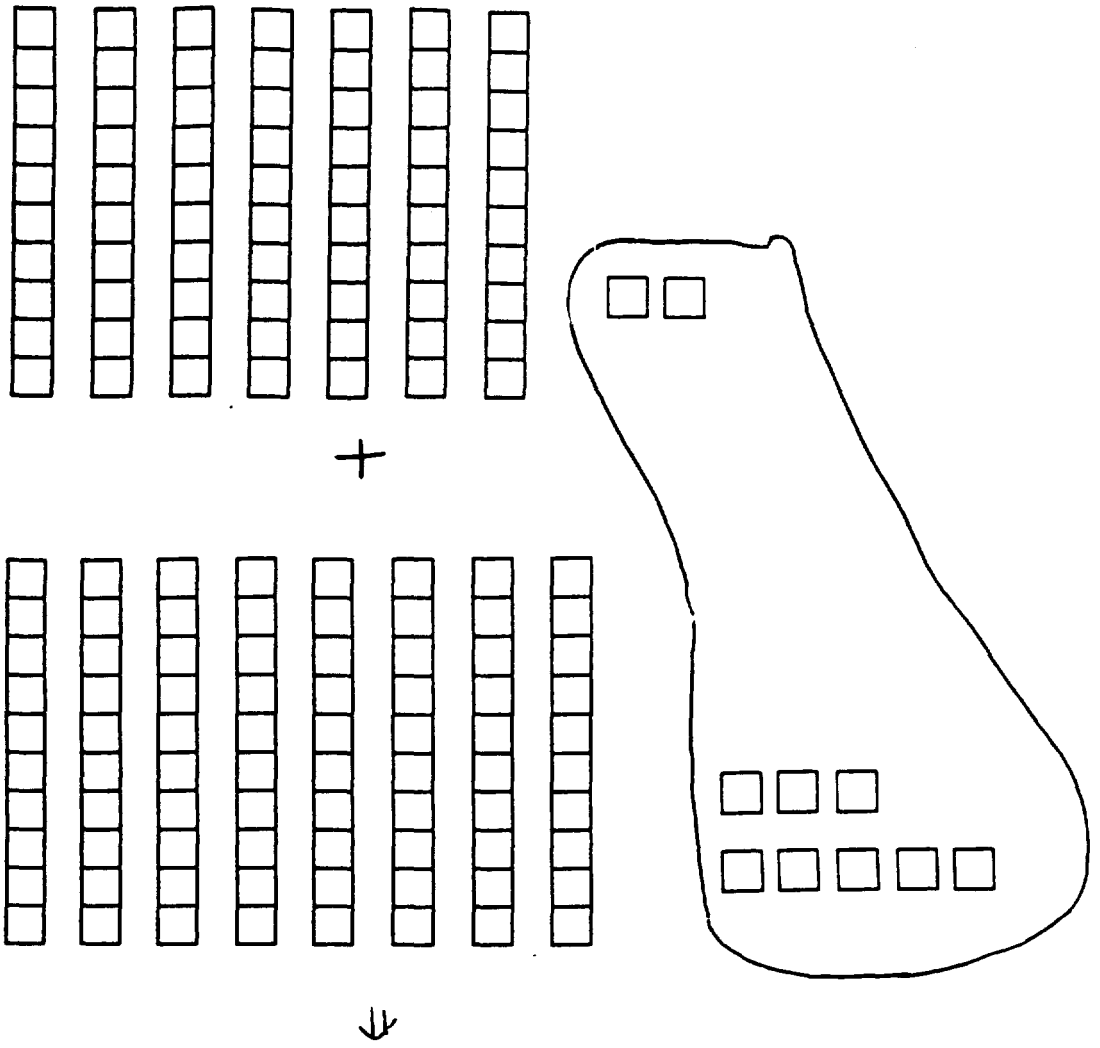
$$\begin{array}{r} 274 \\ +\ 262 \\ \hline 536 \end{array}$$

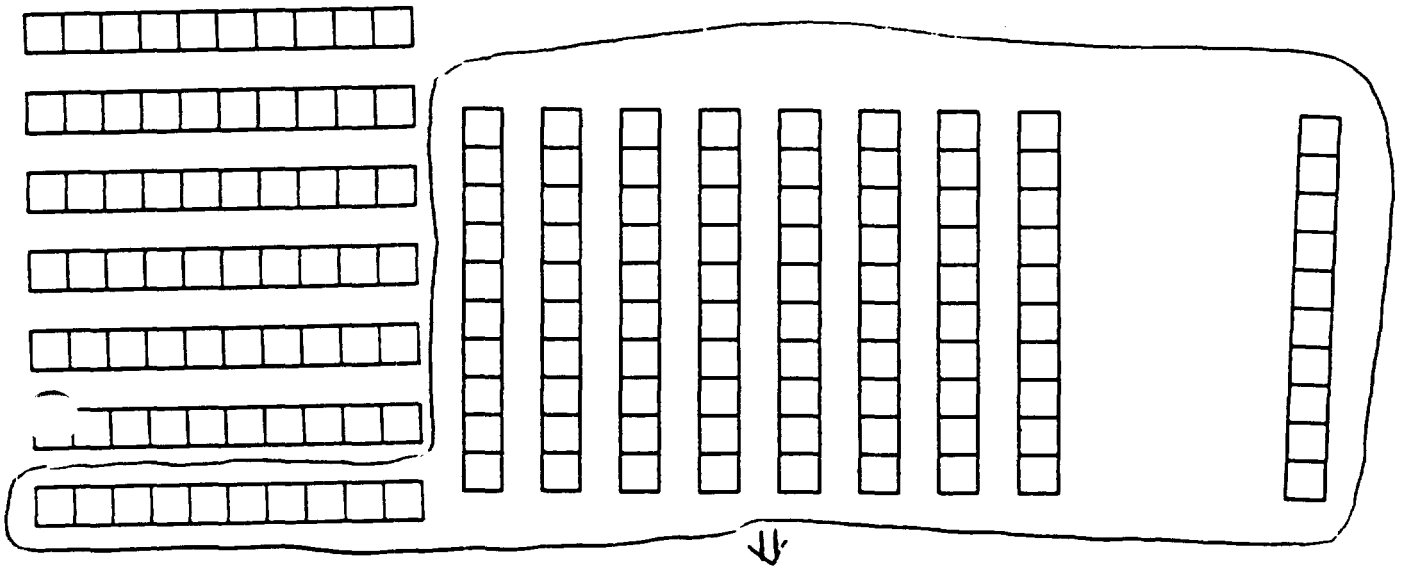


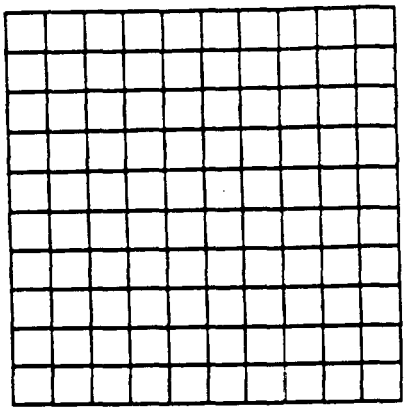


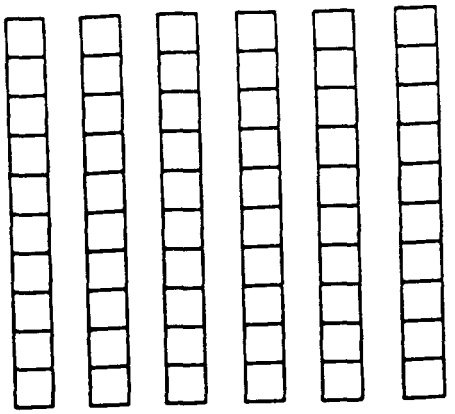
Finally model addition with two or more renamings such as

$$\begin{array}{r} 72 \\ +\ 88 \\ \hline \end{array}$$









Brainstorm with students times when you need to know how to add and keywords that tell you your job as a mathematician is to add two numbers together. You should record these on the overhead projector and students should record them on the worksheet. Allow groups time to work on the worksheet in class.

Addition with Base 10 Blocks

Name: _____Date: _____ Class: _____

Write down the words or phrases that let tell you your job as a mathematician is to add two numbers together:

Work on the following problems. Be sure to show all work in the space provided. Keep this page in your journal so that you can refer to it while working on classwork for the rest of the semester.

1. 23 + 46

2. 98 + 86

3. 147 + 292

4. 3826 + 720

5. 429 + 399

6. 872 + 721

Write three addition problems of your own and solve them below.

1.

2.

3.

Write two word problems that are solved by addition. Include solutions to your problems.

Addition with Base 10 Blocks

Name: Answer Key Date: _____ Class: _____

Write down the words or phrases that let tell you your job as a mathematician is to add two numbers together:

Answers will vary

Work on the following problems. Be sure to show all work in the space provided. Keep this page in your journal so that you can refer to it while working on classwork for the rest of the semester.

1. $\begin{array}{r} 23 \\ +\ 46 \\ \hline 69 \end{array}$

2. $\begin{array}{r} 98 \\ +\ 86 \\ \hline 184 \end{array}$

3. $\begin{array}{r} 147 \\ +\ 292 \\ \hline 439 \end{array}$

4. $\begin{array}{r} 3826 \\ +\ 720 \\ \hline 4546 \end{array}$

5. $\begin{array}{r} 429 \\ +\ 399 \\ \hline 828 \end{array}$

6. $\begin{array}{r} 872 \\ +\ 721 \\ \hline 1593 \end{array}$

Write three addition problems of your own and solve them below.

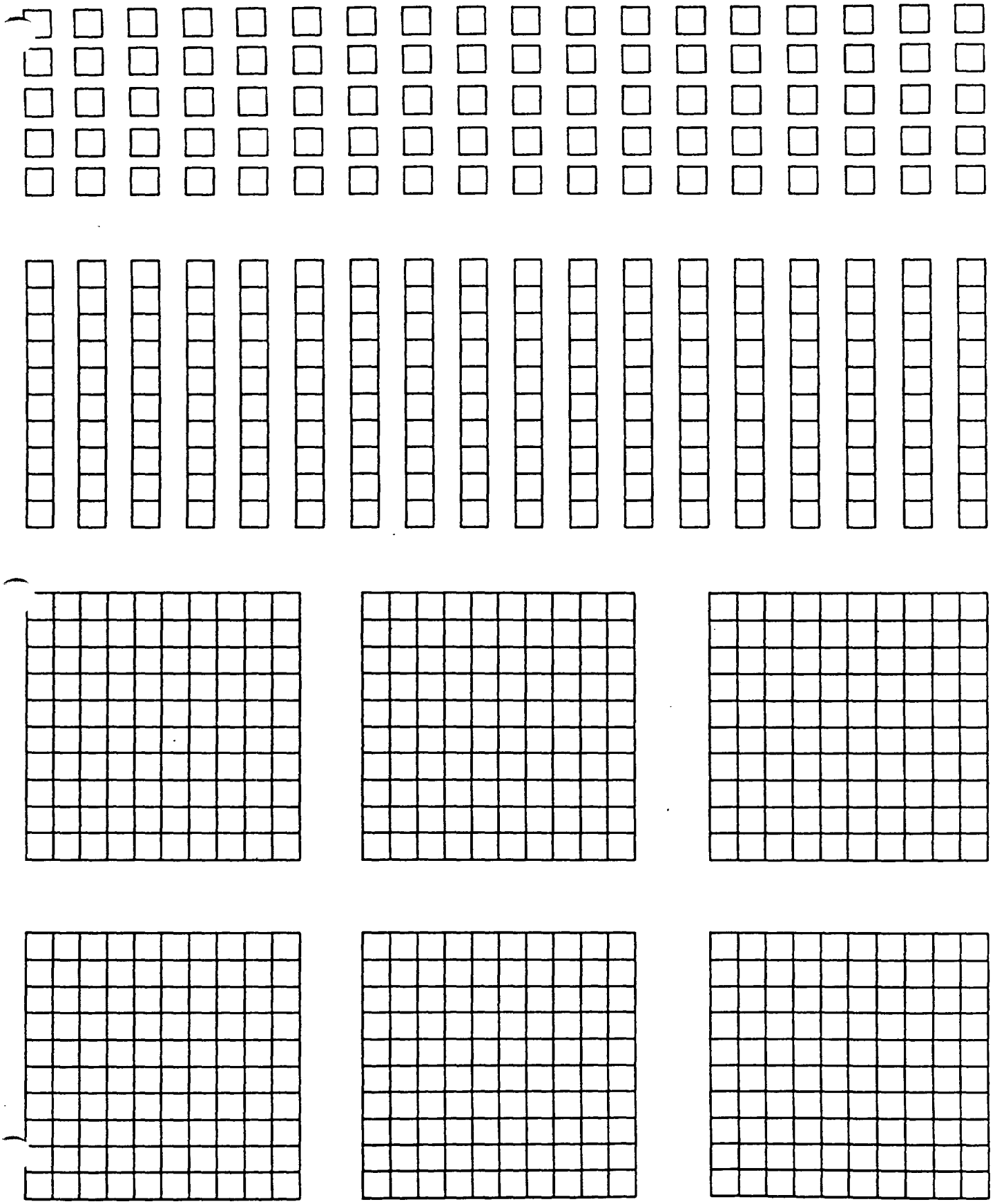
1. 2. 3.

Answers will vary

Write two word problems that are solved by addition. Include solutions to your problems.

Answers will vary

Base Ten Block Pieces



Title: A Year of Operations

Grade Level: 5 - 12

Time Required: one class period to several days

Objectives:
1. Students will demonstrate proficiency in mathematical operations while engaged in a problem solving activity.

Procedure:
Pose the following problem to students. Use only the digits in the current year and any mathematical operations necessary to write statements equivalent to each of the numbers from 1 to 100.

For example: 1996

$6 - \sqrt{9} - \sqrt{9} + 1 = 1$

or

$\frac{6!}{9} + (6 - 1) = 85$

or

$1 + \frac{9}{\sqrt{9}} + 6 = 10$

Students should be encouraged to find as many possible representations of each number and keep accurate records of each solution.

Adapted From:
Greenberg, Claire. *Readers Write: 1995-A Year of Operations*. Mathematics Teaching in the Middle School. 1(6): 434.

Additional Sources:
Dyson, Myrna. *Readers Write: More on 1995*. Mathematics Teaching in the Middle School. 1(6): 434.

Yurecko, Rae. *Readers Write: 1989*. Mathematics Teaching in the Middle School. 1(4): 272.

Title: Using funny 1's to teach addition of fractions

Grade Level: 5 - 9

Time Required: 55 minutes

Objectives:
1. Students will recognize the need for "commonness" when adding and subtracting fractions.
2. Students will try a non-traditional approach to adding and subtracting fractions.

Materials Needed:
___ Overhead projector, pens, and transparencies or chalkboard and chalk
___ Funny 1's and Fractions worksheet
___ several apples, oranges, nickels, quarters, pens, pencils, etc.

Notes to the Teacher:

This lesson in its present form is written to be used after students have been introduced to the traditional method of addition of fractions through finding a common denominator. The process of finding a common denominator when adding fractions can be very difficult for students to grasp and can even be detrimental to them in more advanced mathematics courses. The method proposed can help students begin to make a connection between the concepts of fractions and rational expressions usually studied during first year algebra. In first year algebra a typical exercise might be to simplify:

$$\frac{x+1}{x+6} + \frac{x-4}{x-2}$$

The easiest way to simplify this expression is by multiplying each fractional expression by

$$\frac{(x+6)(x-2)}{(x+6)(x-2)}$$

which gives

$$\frac{(x+1)(x-2)+(x-4)(x+6)}{(x+6)(x-2)} = \frac{(x^2-x-2)+(x^2+2x-24)}{(x^2+4x-12)}$$

$$= \frac{2x^2+x-26}{x^2+4x-12}.$$

In this example the solution is found by multiplying by 1 written in a "funny" form. By using the lesson plan described here students will be introduced to this concept in order to perform basic fraction addition.

Procedure:

Begin by giving students several examples such as adding 2 apples and 3 apples gives five apples; 3 nickels and 7 nickels gives 10 nickels; two pencils and

six pencils gives eight pencils, and so on. Then extend this to ask if you have two apples and three oranges then what do you have? four nickels and five quarters? three pencils and six pens? Students should see that it is necessary to convert to some sort of common unit like fruit, coins, and writing instruments.

Ask students how they think this relates to addition and subtraction of fractions. Put up an example such as :

$$\begin{array}{r} \frac{3}{8} \\ + \frac{1}{6} \end{array}$$

Students should recognize that these fractions are not common. Students may or may not be familiar with the "criss-cross" method of getting common denominators in which you think "8 goes into 24 three times so multiply three times three to get nine."

Instead of going through this see if there is a "funny 1" you can multiply by to obtain like fractions. For example

$$\begin{array}{r} \frac{3}{8} \cdot \frac{3}{3} = \frac{9}{24} \\ + \frac{1}{6} \cdot \frac{4}{4} = \frac{4}{24} \\ \hline \frac{13}{24} \end{array}$$

Students having difficulty with this can use the LCM type approach in which you begin multiplying by 2/2, 3/3, 4/4... on each fraction until the common fractions. For example

$$\frac{3}{8} = \frac{6}{16} = \frac{9}{24} = \frac{12}{32} = \frac{15}{40} \ldots$$

$$\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24} = \frac{5}{30} = \frac{6}{36} \ldots$$

Work a few more examples. Have students work in groups on the worksheet.

Adapted From:

Crossfield, Don. "The Power of 1." *Mathematics Teacher* 86: (October 1993). 549 - 552.

Funny 1's and Fractions Worksheet

Name: _____ Period: ____ Date: _____

Indicate the funny 1 you should multiply the fraction on the left by to obtain the fraction on the right.

1. $\frac{2}{5} \times \frac{\square}{\square} = \frac{6}{15}$

2. $\frac{7}{12} \times \frac{\square}{\square} = \frac{28}{48}$

3. $\frac{3}{4} \times \frac{\square}{\square} = \frac{18}{24}$

4. $\frac{5}{9} \times \frac{\square}{\square} = \frac{10}{18}$

Indicate what fraction on the left was multiplied by the funny 1 shown to obtain the fraction on the right.

5. $\frac{\square}{\square} \times \frac{5}{5} = \frac{25}{30}$

6. $\frac{\square}{\square} \times \frac{7}{7} = \frac{49}{70}$

7. $\frac{\square}{\square} \times \frac{3}{3} = \frac{21}{33}$

8. $\frac{\square}{\square} \times \frac{6}{6} = \frac{36}{42}$

Add each of the following fractions by first getting common fractions by multiplying by a "funny 1".

9. $\begin{array}{r} 1 \\ 2 \\ 3 \\ +\ 4 \end{array}$

10. $\begin{array}{r} 3 \\ 7 \\ 5 \\ +\ 6 \end{array}$

11. $\begin{array}{r} 2 \\ 3 \\ 4 \\ +\ 9 \end{array}$

12. $\begin{array}{r} 3 \\ 5 \\ 4 \\ +\ 7 \end{array}$

13. $\begin{array}{r} 6 \\ 11 \\ 3 \\ +\ 1 \end{array}$

14. $\begin{array}{r} 4 \\ 5 \\ 2 \\ +\ 7 \end{array}$

On the back of this sheet make up two problems which could be solved by adding fractions and solve them using the funny 1's approach.

Funny 1's and Fractions Worksheet

Name: ANSWER KEY Period: ____ Date: _____

Indicate the funny 1 you should multiply the fraction on the left by to obtain the fraction on the right.

1. $\frac{2}{5} \times \frac{\boxed{3}}{\boxed{3}} = \frac{6}{15}$

2. $\frac{7}{12} \times \frac{\boxed{4}}{\boxed{4}} = \frac{28}{48}$

3. $\frac{3}{4} \times \frac{\boxed{6}}{\boxed{6}} = \frac{18}{24}$

4. $\frac{5}{9} \times \frac{\boxed{2}}{\boxed{2}} = \frac{10}{18}$

Indicate what fraction on the left was multiplied by the funny 1 shown to obtain the fraction on the right.

5. $\frac{\boxed{5}}{\boxed{6}} \times \frac{5}{5} = \frac{25}{30}$

6. $\frac{\boxed{7}}{\boxed{10}} \times \frac{7}{7} = \frac{49}{70}$

7. $\frac{\boxed{7}}{\boxed{11}} \times \frac{3}{3} = \frac{21}{33}$

8. $\frac{\boxed{6}}{\boxed{7}} \times \frac{6}{6} = \frac{36}{42}$

Add each of the following fractions by first getting common fractions by multiplying by a "funny 1".

9. $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$
$+ \frac{3}{4} \times \frac{1}{1} = \frac{3}{4}$
$\frac{5}{4}$

10. $\frac{3}{7} \times \frac{6}{6} = \frac{18}{42}$
$+ \frac{5}{6} \times \frac{7}{7} = \frac{35}{42}$
$\frac{53}{42}$

11. $\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$
$+ \frac{4}{9} \times \frac{1}{1} = \frac{4}{9}$
$\frac{10}{9}$

12. $\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$
$+ \frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$
$\frac{41}{35}$

13. $\frac{6}{11} \times \frac{1}{1} = \frac{6}{11}$
$+ \frac{3}{1} \times \frac{11}{11} = \frac{33}{11}$
$\frac{39}{11}$

14. $\frac{4}{5} \times \frac{7}{7} = \frac{28}{35}$
$+ \frac{2}{7} \times \frac{5}{5} = \frac{10}{35}$
$\frac{38}{35}$

On the back of this sheet make up two problems which could be solved by adding fractions and solve them using the funny 1's approach.

ANSWERS WILL VARY

Title: GORP--a cooperative assessment activity for fractions

Grade Level: 5 - 8 General Mathematics/ Pre-Algebra

Time Required: 1 class period

Objectives:
1. Students will work cooperatively to solve a real-world problem with 75% accuracy.
2. Students will compare fractions to determine which is largest and smallest.
3. Students will determine which measuring cups (1/4, 1/3, 1/2, and 1 cup) should be used to obtain "non-standard" amounts of ingredients.
4. Students will assess their own learning and performance.

Materials Needed: (Amounts are for a class of 24 divided into 6 groups of 4; adjust amounts as needed to fit your class)
___ 6 copies of GORP assessment page
___ 6 small tape recorders and blank tapes
___ 6 sets standard measuring cups (1/4, 1/3, 1/2, and 1 cup)
___ Bowls containing ingredients (Ingredients in this activity are M & M candies, peanuts, raisins, small pretzels, Crispix Cereal; other ingredients might be dried fruits, Reese's pieces candies, chocolate or candy coated chips, etc.)
___ 6 large zip-loc plastic bags
___ Small bags or cups for students to take a share of their recipe
___ copies of GORP post-assessment for each student

Notes to the Teacher:
In the National Council of Teachers of Mathematics' *Assessment Standards for School Mathematics* the Council urges a shift in evaluation, "toward evidence from several sources judged by teachers" and "away from a single test judged externally." The type of activity described in this lesson fits naturally into this shift in assessment. This activity not only assesses a students knowledge of fractions but also their knowledge of how to use fractions in a "real" situation.
Learning to assess an activity such as this one objectively may take quite a bit of time and practice for the beginning teacher. Assessment should begin with observing students during execution of the lesson. During this time watch for students who take a leadership role in each group and students who may try to let the rest of the group carry the load. If this is happening it may be necessary to remind students that the cassette tapes will be used later to measure individual contributions. A key to the GORP activity sheet is included at the end of this lesson. Each question is marked with a suggested number of points and information on assigning partial credit. It is left up to the individual teacher how they would like to include observations made from the cassette tapes and information from the post-assessment sheet in the grade for students. Teachers

may wish to use these two pieces to assign an individual portion of a grade (40% of the total points) and the GORP activity sheet to assign a group grade (60% of the total points).
An extension of this activity is the classic water jars problem (See: Die Hard: With A Vengeance--Adventure into Problem Solving).

Procedure:
Set up stations for each group prior to class if possible. Give each group a copy of GORP assessment page. The group number should be written on both the assessment page and the cassette tape label. The cassette recorder allows you the freedom to be "in two places at once" by floating around the room during the activity but being allowed to listen in on the work of each group after the fact to record leadership and cooperative problem solving.

Clarify that the goal of this activity is to make a batch of GORP as efficiently and accurately as possible. For example, if you can add an ingredient you should do it using the smallest number of "dumps" possible. For example, 1 cup of an ingredient can be added in several ways (Have students list these for you)

1 - 1 cup
2 - 1/2 cup
1 -1/2 cup and 2 - 1/4 cup
3 - 1/3 cup
4 - 1/4 cup

Of course 1 - 1 cup dump is the most efficient.
Have groups turn on tape recorders and begin. Groups should fill in assessment sheet as they work.

Assessment Options:
Since this is a cooperative activity you may want to give two marks for the activity- an individual grade and a cooperative (or group) grade.

DO NOT EAT ANYTHING UNTIL TOLD TO DO SO!!

GORP Assessment

Group Members: _____ Group Number: _____
_____ Date: _____
_____ Class Period: _____
_____

GORP
1/2 cup M&M candies
5/6 cup peanuts
2/3 cup raisins
7/12 cup small pretzels
3/4 cup Crispix cereal
Combine all ingredients in large ziploc bag. Seal tightly and shake to mix ingredients.

You are to work together to make the recipe given on the recipe card above. As your group works please remember that you are to add each ingredient as efficiently as possible. This means you should add using as few "dumps" as possible. Record the dumps you make below and then answer the questions that follow.

Add 1/2 c. M & M candies. Record the dump(s) made.

Add 5/6 c. peanuts. Record the dump(s) made.

Add 2/3 c. raisins. Record the dump(s) made.

Add 7/12 c. small pretzels. Record the dump(s) made.

Add 3/4 c. Crispix cereal. Record the dump(s) made.

Combine all ingredients and mix.

Without measuring, how many cups of GORP did you yield?

Which ingredient did you add in the largest amount?

Which ingredient did you add in the smallest amount?

How could you add 1/12 c. Reese's pieces candies most efficiently. Record the dump(s) you would make here.

Divide the mixture into four equal parts so that each member receives the same amount of GORP. Record your procedure here.

DO NOT EAT ANYTHING UNTIL TOLD TO DO SO!!

GORP Assessment

Group Members: Answer Key _____ Group Number: _____

_____ Date: _____

_____ Class Period: _____

_____

GORP
1/2 cup M&M candies
5/6 cup peanuts
2/3 cup raisins
7/12 cup small pretzels
3/4 cup Crispix cereal
Combine all ingredients in large ziploc bag. Seal tightly and shake to mix ingredients.

You are to work together to make the recipe given on the recipe card above. As your group works please remember that you are to add each ingredient as efficiently as possible. This means you should add using as few "dumps" as possible. Record the dumps you make below and then answer the questions that follow.

Add 1/2 c. M & M candies. Record the dump(s) made.

1/2 cup

Add 5/6 c. peanuts. Record the dump(s) made.

1/3 cup and 1/2 cup

Add 2/3 c. raisins. Record the dump(s) made.

1/3 cup and 1/3 cup

Add 7/12 c. small pretzels. Record the dump(s) made.

1/4 cup and 1/3 cup

Add 3/4 c. Crispix cereal. Record the dump(s) made.

1/2 cup and 1/4 cup

Combine all ingredients and mix.

Without measuring, how many cups of GORP did you yield?

$3\frac{1}{3}$ cups

Which ingredient did you add in the largest amount?

peanuts

Which ingredient did you add in the smallest amount?

M & M's

How could you add 1/12 c. Reese's pieces candies most efficiently. Record the dump(s) you would make here.

Fill 1/3 cup with Reese's pieces. Carefully fill the 1/4 cup from this 1/3 cup. You are left with 1/12 c. in the 1/3 c. measuring cup.

Divide the mixture into four equal parts so that each member receives the same amount of GORP. Record your procedure here.

Each person should receive $3\frac{1}{3} \div 4$ or $\frac{5}{6}$ cup of the mixture. Therefore each person receives a dump of both the 1/3 and 1/2 c. measuring cups.

GORP Post-Assessment Sheet

Name: _____ Group Number: _____

Rate each of the following statements by placing an x in the area on the line which matches your feeling toward the statement.

1. Our group worked together to complete the activity.

Strongly Agree 5 —— 4 —— 3 ——2 ——1 Strongly Disagree

2. Each member of my group contributed equally to the task.

All members contributed equally 5 —— 4 —— 3 ——2 ——1 One member dominated or no members contributed.

3. The tasks were completed efficiently.

Very Efficiently 5 —— 4 —— 3 ——2 ——1 Task not completed

4. I contributed ideas and suggestions to my group.

Many contributions 5 —— 4 —— 3 ——2 ——1 I did not contribute

5. My groups considered all ideas fairly before making a decision.

All ideas considered 5 —— 4 —— 3 ——2 ——1 Only one person's ideas considered

6. I offered support and encouragement to my fellow group members.

Support offered 5 —— 4 —— 3 ——2 ——1 I did not offer support

7. This activity accurately assessed my knowledge of fractions.

Accurate assessment 5 —— 4 —— 3 ——2 ——1 Did not assess

8. I enjoyed this activity.

Enjoyed 5 —— 4 —— 3 ——2 ——1 I hope we never do this again!

9. This activity was worthwhile.

Worthwhile 5 —— 4 —— 3 ——2 ——1 Not worthwhile

10. Please write a short paragraph describing which part(s) of this activity you thought was (were) most important and why you think this?

Title: Three-Dimensional Description

Grade Level: 5 - 8

Time Required: 30 - 55 minutes

Objectives:
1. Students will build a three dimensional object using interlocking pieces.
2. Students will verbally explain how they built an object clearly enough for another student to create the same object.

Materials Needed:
___ several interlocking shapes for each student (triangles and squares should be available at first. If this activity is used more than once you may wish to use additional shapes).
___ small cassette recorders and blank cassette tapes for each group of students
___ activity evaluation sheet

Notes to the Teacher:

"Mathematics as communication" and "Mathematics as problem solving" are two of the unifying goals which can be found through the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards for School Mathematics*. In this lesson a problem is given to students and theft hey must communicate their solution to another student verbally.

Evaluation activities such as this may seem difficult since to truly evaluate each student's success would involve watching and listening to them directly. However, by using the tape recorder allows the teacher to observe the activity during class and then react to it later. As you listen to a cassette of the students have their activity evaluation with you. A grade assigned for this activity could be assigned using the following rubric:

Willingness to try	3 pts
Use of correct mathematical language	5 pts
Figures are the same at the end of activity	2 pts

Procedure:

Students should be placed in homogeneous groups of two or three. If groups of two are used then one students builds and describes while the other one builds what is described. if groups of three are used then two students will build what is described.

Set up a barrier between the students. have one student build a three dimensional object. After they have built their object they should turn on the tape recorder. The student should then describe to the other student(s) how they built

the object. The other student(s) should attempt to build the object being described to them.

Once the student has an object assembled the two students should compare their objects. They should discuss what went well and areas which were incorrect or of particular difficulty. If you use portfolio assessment it would be a good idea to take a picture of the students holding their objects standing side-by-side. Copies of these pictures and the activity evaluation sheet could be placed in the portfolio.

Students should trade jobs and repeat the building and description processes.

Once every student has had the chance to be the describer have students fill out the evaluation sheet.

Extensions:

This activity probably will not be extremely challenging at first, however you can make this activity more challenging by adding some requirements (or taking away requirements). For example, the first time students do the activity you may want to specify that students use four squares and eight triangles. This requirement limits the types of closed objects which could be created.

Other requirements to consider are:

1. Build an object with a given volume or surface area.
2. Build an object using shapes other than triangles and squares.

You may also want to have students use cubes or Cuisenairre rods instead of interlocking pieces to build three dimensional objects.

Assessment:

Activities such as this are often considered fun by students, but they are not valued by students because they are not assessed formally.

Of course, you want to assess the activity informally at first. However, if this type of activity is used frequently it lends itself nicely to performance assessment.

One method to use is to schedule time during an exam for students to complete a similar activity such as giving students an assembled object and have them describe how they would have built the object directly to you. It is important that during this time you remain objective and uninterfering. Allow students to work without providing encouragement or without correcting the student if they begin to make mistakes.