EXPONENTS IN EQUATIONS Name___________________________

Equations appear in this grid horizontally, vertically, and diagonally. Insert +, -, x, ÷, =, parenthesis, and * to form as many equations as you possible can. A * is used to show exponents. That is, $8^2 = 64$ is shown in the puzzle as 8 * 2 = 64. Be sure that each of your equations involves exponents.

8	2	64	1	2	3	7	1
6	4	3	1	27	1	4	0
5	2	32	1	27	1	4	0
9	1	2	8	25	27	8	31
4	3	64	6	2	3	36	6
7	16	4	8	2	1	49	16
1	18	36	7	2	2	3	27
64	9	4	8	32	1	25	1

THE CONQUISTADORS Name________________________________

In 1531, Francisco Pizarro set sail from Panama with only 180 men in three small ships. His destination was Peru where he planned to conquer the Incas, a highly developed nation stretching over 2000 miles of coast. The Andes mountains were dotted with elaborate cities and fortresses the Incas had built, and their highways and water system rivaled those of the Romans.

Pizarro marched his conquistadors across thousands of miles of rugged terrain and deserts until they finally reached the Inca city of Cajamarca, high in the Andes. There, Inca King Atahualpa, with an army of 40,000 men, was awaiting the Spaniards.

To learn how Pizarro defeated the Incas, work the problems below.

FIRST, change the word expressions to symbol statements and locate each answer in the table.
SECOND, place the letter by the answer in the blank next to the problem.
THIRD, match the numbers by the picture with the corresponding letters.

1.______ 5 more than 2

2.______ 3 les than 8

3.______ greater by 7 than x

4.______ 5 more than z

5.______ 5 less than y

6.______ y less than 5

7.______ 3 times x plus 1

8.______ sum of x, y, and 2x

9.______ z more than y

10.______ one part of 9 is 3, the other part is___

11.______ z greater than x - y

12.______ x less than x - y

13.______ five more than z y

14.______ one part of 9 is x, the other part is___

15.______ 5 less than z y

16.______ z less than x - y

17.______ greater than x by y

18.______ x less than y

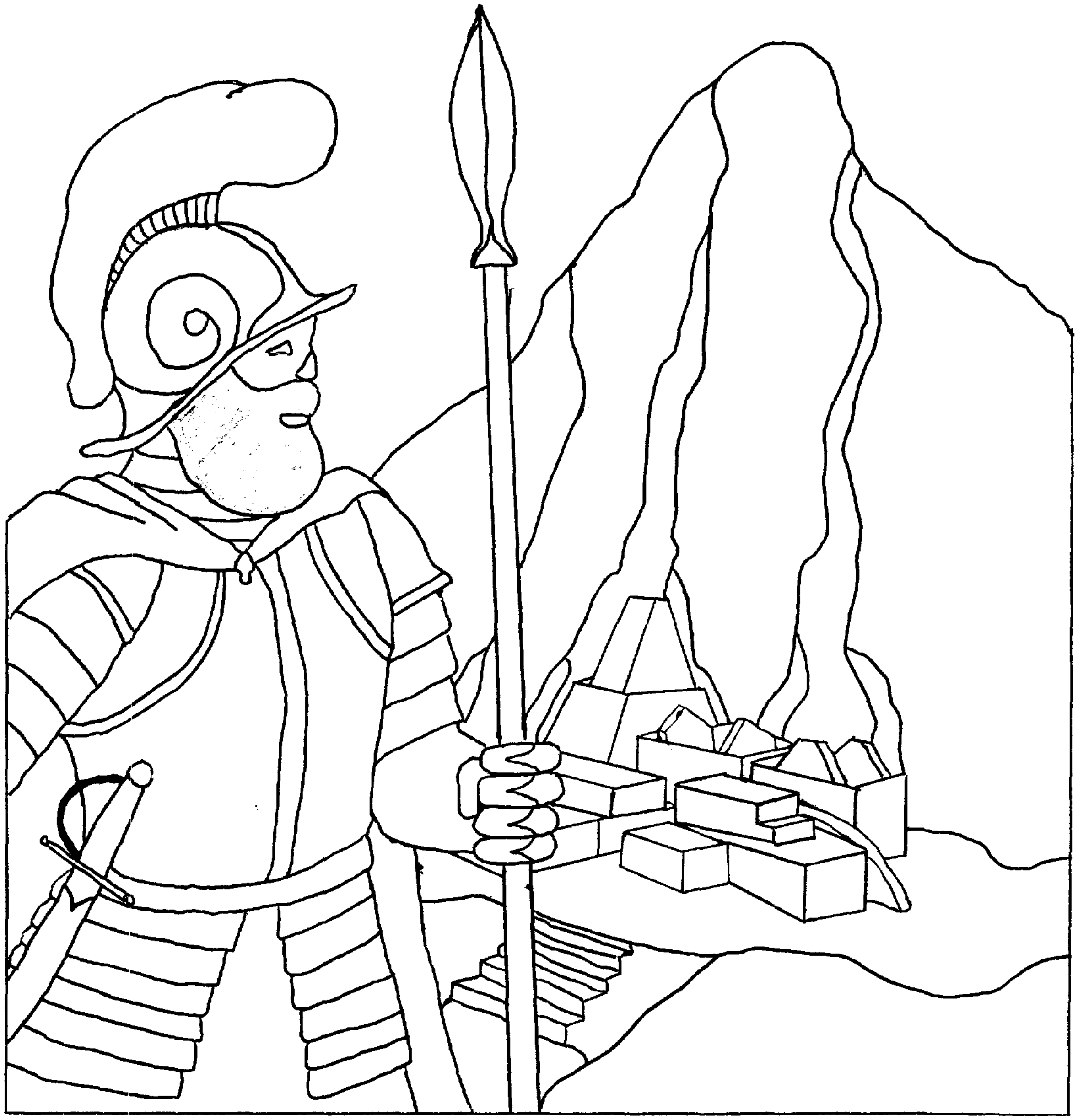
19.______ one part of x is 9, the other part is ___

20.______ y less than x

21.______ x - y less than z

22.______ x - y less than x

A = 5 - y	G = 3x + 1	N = 3x + y	T = z + y - 5
B = z + y + 5	H = x + y	O = -y	U = x - y
C = x + 7	I = 5	P = y - 5	V = y
D = x - y - z	K = 9 - x	R = y - x	W = z + 5
E = x - 9	L = 6	S = z + y	Z = x - y + z
F = z - x + y	M = 7		



5-2-11-6-18-18-12 2-8-22-2-15-19-16 15-17-19 14-2-8-7 15-12 17-2-9 3-6-1-5 6-8-16 3-6-5-15-20-18-19-16 17-2-1 2-8 6-8 6-1-13-20-9-17. 9-2-8-3-19 6-10-10 5-12-4-19-18 18-19-9-15-19-16 2-8 15-17-19 14-2-8-7, 5-2-11-6-18-18-12 4-12-8 3-12-8-15-18-12-10 12-21 15-17-19 2-8-3-6 8-6-15-2-12-8.

THE ISLES OF THE BLESSED   Name______________________

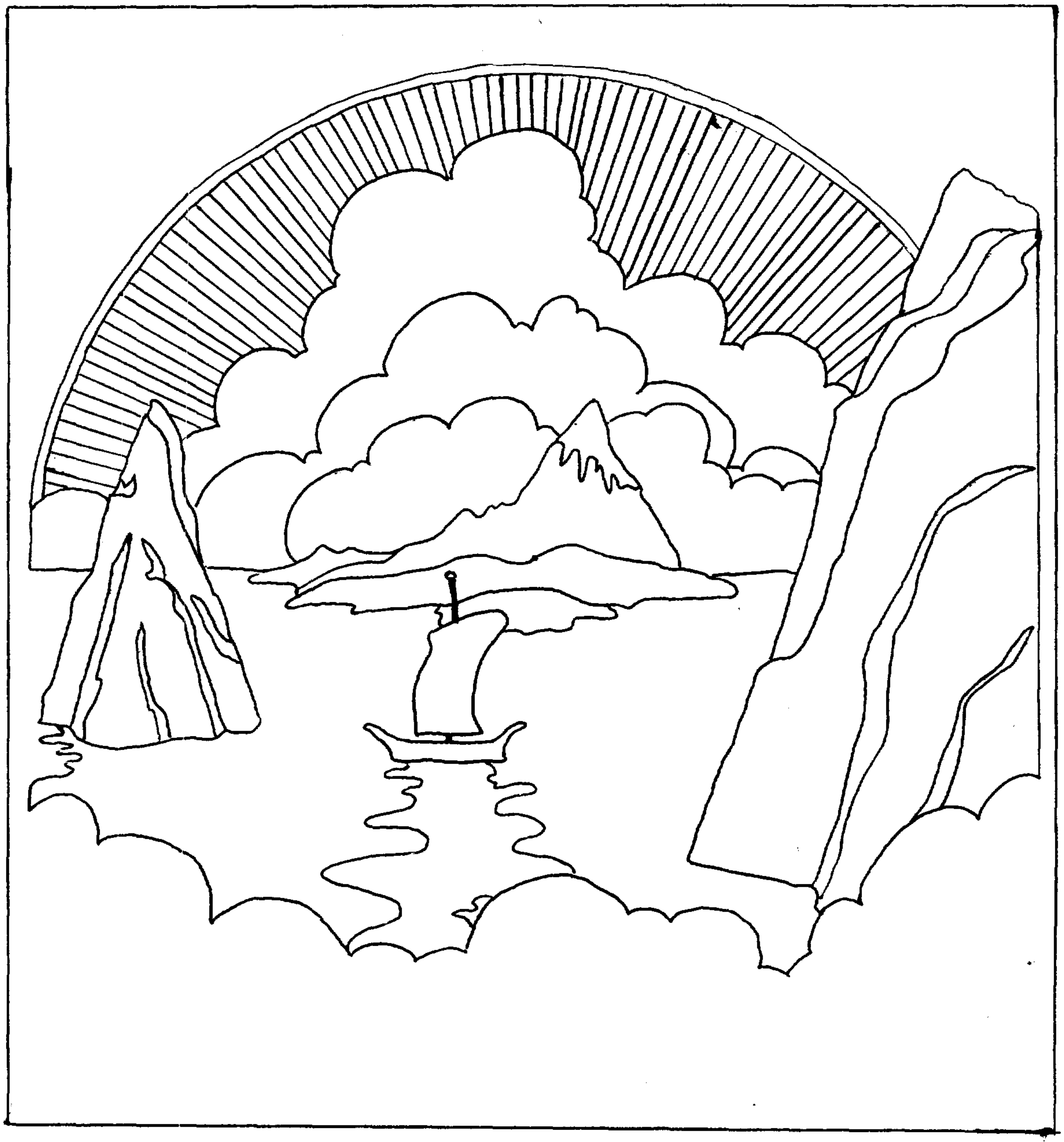
During the Stone Age, there was a tribe living on the Canary Islands that built wooden huts, half sunk into the ground and covered with earthen roofs. The wealthy carved their houses from stone near the rocky cliffs. Tribal leaders distinguished themselves by wearing long hair while tribe members shaved their heads.

The Islanders had some unusual laws for criminal punishment. If a murderer entered the home of his victim through the front door, he faced no penalty. But if the murderer was an assassin, regardless of how he entered the building, he was severely punished. To learn what the punishment was, work the equations below.

FIRST, solve the equations for the unknowns and locate the value of each unknown in the table at the bottom of the page.
SECOND, place the letter by the answer in the blank next to the problem.
THIRD, match the numbers by the picture with the corresponding letters.

1.______	$a + 1 = 2$	11.______	$-4b = 24$
2.______	$r + 3 = 8$	12.______	$8 - 12z = 92$
3.______	$2 + m = -1$	13.______	$-7h = -63$
4.______	$7 + 2y = 3$	14.______	$3m + 7 = 5$
5.______	$3 + 3b = 0$	15.______	$2 - 5a = 3$
6.______	$3x - 5 = -5$	16.______	$15t = -60$
7.______	$1.5a + 4 = -3.5$	17.______	$2 + 3c = 1.1$
8.______	$3m - 7 = -5$	18.______	$4 - z = 2$
9.______	$8 + 4y = -2$	19.______	$4 + m = -5$
10.______	$2a = 8$		

A = -1	E = -1/5	M = -7	R = -5/2	V = -2/3
B = -6	H = -3	N = 2/3	S = -4	W = -3/10
C = 0	I = 2	O = 5	T = -9	X = -2
D = 9	L = -5	P = 1	U = 4	



3-15 17-5-16 5-7-7-2-17-15-13 19-2 7-18-14-15,

11-10-19 19-3-15 1-15-9-16-2-8 3-15 7-2-14-15-13

12-2-16-19 17-5-16 15-4-15-6-10-19-15-13.

SWORD EXCALIBUR Name________________________

One of the famous legends of King Arthur, who ruled England during the sixth century, is the story of how he became king. When he was 15 his father died and Arthur was elected king, amidst strong opposition. Many proud knights, anxious for power, felt themselves better fit to rule than Arthur.

On the steps of the church appeared a stone with a sword in it. Anxious to unify the country under a popular king, the Bishop declared that the man who could remove the sword from the stone would be King of England. Many knights tried but even the strongest were unable to budge the sword. Arthur effortlessly freed the sword from the stone. He became king, unopposed, for it was thought that ability to remove the sword was an endorsement from heaven. King Arthur really did live in England. To learn more about him, work the problems below.

FIRST, solve the equations for the unknowns and locate the value for each unknown in the table.
SECOND, place the letter by the answer in the blank next to the problem.
THIRD, match the numbers by the picture with the corresponding letters.

1.______ $s - 4 = 7$

2.______ $2x = x + 5$

3.______ $3y = -2y + 10$

4.______ $t + 4 = 7$

5.______ $-a = 2a + 6$

6.______ $2b + 3 = -9$

7.______ $r - 4 = -7$

8.______ $-4h = 16$

9.______ $c^3 = 3c + c^3$

10.______ $-a - 3 + 4a = 7 + 5a$

11.______ $3t^2 - t + t^2 = 4t^2 + 9$

12.______ $3(x - 5) = x + 5$

13.______ $a + 4 = -7$

14.______ $r - 4 = 3r - 16 - r$

15.______ $3z - 5 = 4z + 5$

16.______ $-7(y - 4) = -y - 26$

17.______ $-3y = -12$

18.______ $2(3h - 2) - 2h = 2(h - 3)$

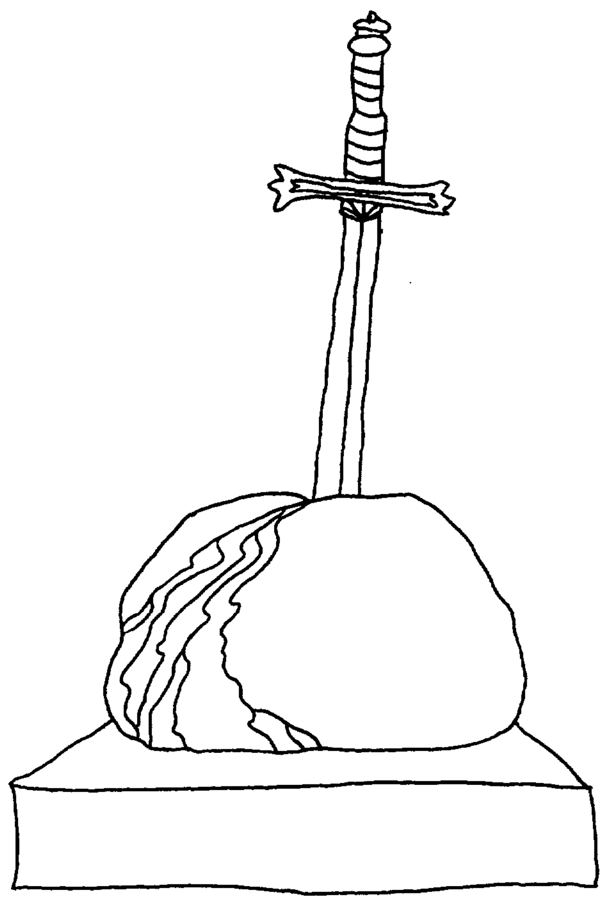
19.______ $12v - 6 = 3(v + 4) + 54$

20.______ $5w + 4 - 12w + 49 = 4$

21.______ $7x + 28 = -14x - 119$

22.______ $d(d - 1) = (d + 1)(d - 1)$

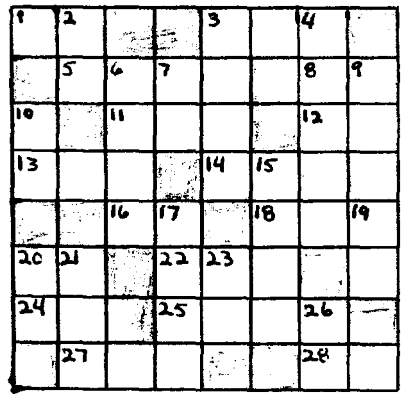
A = 0	F = 2	L = 11	R = -11	V = 4
B = -9	G = -10	N = 12	S = -6	W = -4
C = -5	H = 8	O = -2	T = -3	X = -7
D = 5	I = 3	P = 7	U = 10	Y = 9
E = -1	J = 1			



9-13-7-19-12-13 8-9-6 9 20-13-4-14-10-18 5-3 9 7-13-4-11-18 5-3 11-13-4-7-5-14-6 10-9-1-1-18-2 6-4-1-12-13-18-6. 19-18 2-18-3-18-9-7-18-2 7-19-18 6-9-21-5-14 4-14-17-9-2-18-13-6 4-14 7-8-18-1-17-18 11-9-7-7-1-18-6 11-18-3-5-13-18 19-4-6 2-18-9-7-19 9-7 7-19-18 19-9-14-2-6 5-3 19-4-6 14-18-20-19-18-8.

(1)

CROSS NUMBER PUZZLE Name________________



Across

1. $x + 7 = 22$
3. $m - 6 = 296$
5. $w - 34 = 4295$
8. $a + 15 = 49$
11. $\frac{x}{2} = 86$
12. $52 = n - 9$
13. $b + 9 = 874$
14. $3x = 39$
16. $5x = 105$
18. $\frac{w}{4} = 51$
20. $2m - 19 = 39$
22. $3a + 11 = 341$
24. $5y + 9 = 69$
25. $n - 50 = 1992$
27. $3x - 21 = 1479$
28. $9b + 24 = 150$

Down

2. $b + 5 = 59$
3. $x - 12 = 3909$
4. $m + 6 = 242$
6. $\frac{a}{8} = 394$
7. $n + 5 = 32$
9. $3x = 123$
10. $2y = 36$
15. $b - 56 = 3148$
17. $2a + 9 = 2249$
19. $4x - 9 = 175$
20. $3m + 4 = 67$
21. $\frac{n}{25} = 37$
23. $9y - 47 = 43$
26. $6m + 11 = 137$

(*)

ALGEBRA LINGO

Circle all of the algebra words you can find in the letters below. They may be written upward, downward, forward, backward, or on a left or right diagonal.

S	T	K	E	P	G	R	A	P	H	N	C	Y	A	L	I	M	E	F
O	D	G	M	W	R	Y	P	A	O	Y	T	X	S	N	C	E	W	R
Q	U	T	V	B	W	S	A	R	R	I	P	F	I	O	D	Q	U	E
U	N	A	T	D	P	V	I	A	I	M	N	E	S	E	R	A	G	P
A	Y	B	L	E	R	G	C	B	Z	J	O	T	R	S	U	N	N	R
D	G	C	P	H	I	E	D	O	O	B	C	L	E	B	A	T	W	V
R	U	M	X	N	S	L	A	L	N	I	P	U	R	R	O	Q	U	N
A	O	T	E	M	V	S	B	A	T	Y	E	K	W	I	C	L	O	E
N	R	A	H	O	A	G	T	A	A	P	O	S	Z	B	N	E	A	M
T	O	M	V	E	R	X	R	E	L	A	T	I	O	N	C	L	P	J
M	A	I	D	G	I	D	S	L	B	N	O	K	P	I	W	T	Y	T
S	H	N	T	C	A	X	I	S	S	A	O	V	M	A	G	D	W	I
P	N	O	M	U	B	R	V	N	H	B	R	C	S	M	I	L	D	F
O	V	I	Q	D	L	S	M	D	A	E	R	O	K	O	A	L	S	K
D	J	T	F	H	E	O	G	B	P	T	Q	O	W	D	O	E	I	R
U	T	A	Y	Z	M	X	S	N	C	V	E	R	T	I	C	A	L	B
V	G	U	H	F	J	C	L	D	K	S	L	D	A	Y	T	U	A	R
I	W	Q	O	P	I	E	O	S	V	B	W	I	M	X	H	C	I	N
L	S	E	A	S	P	Z	P	O	X	I	C	N	U	V	Y	B	M	T
N	R	R	S	M	E	W	E	Q	U	A	H	A	S	J	D	K	O	F
L	G	A	W	N	E	M	L	A	N	O	I	T	A	R	T	C	N	A
P	A	E	O	S	I	D	U	F	Y	G	H	E	R	J	E	K	I	W
L	Z	N	K	W	C	J	E	H	V	R	G	S	B	T	F	N	B	Y
D	M	I	U	S	Z	I	W	P	T	A	L	G	X	M	V	D	S	J
K	P	L	A	E	X	H	S	F	B	M	T	R	I	O	W	P	X	A

ALGECABULARY

Circle the algebra words hidden in this array of letters. They may be written horizontally, vertically, or diagonally. Also, some of the words may be in reverse order.

Q	U	R	O	S	I	V	I	D	J	S	E	Q	U	I	V	A	L	E	N	T	T	X
V	A	R	I	A	T	I	O	N	M	R	E	U	J	B	L	X	I	U	T	J	P	I
V	S	B	I	N	A	R	Y	E	O	C	X	A	O	M	I	I	N	M	U	C	A	R
D	S	T	S	U	B	S	E	T	N	A	P	D	T	I	W	O	E	V	E	C	R	T
G	O	M	R	O	C	E	A	T	O	E	O	R	S	T	E	M	A	T	R	A	A	A
J	C	N	A	T	L	E	N	U	M	G	R	A	L	I	A	G	R	E	E	R	B	M
P	I	R	F	U	O	U	N	C	I	T	I	O	N	O	R	H	C	A	L	T	O	D
Y	A	F	O	P	S	E	T	N	A	I	I	D	E	N	T	I	T	Y	R	E	L	I
T	T	R	N	T	E	E	R	E	L	N	V	G	A	L	P	J	N	I	Q	S	A	S
H	I	R	A	Y	D	U	N	A	V	L	N	T	H	R	I	C	G	T	Y	I	U	C
A	V	S	J	B	M	O	U	E	L	A	T	A	O	R	O	O	T	N	E	A	O	R
G	J	E	Q	U	I	A	R	X	R	T	L	C	I	O	N	M	N	S	J	N	S	I
O	T	N	T	N	E	S	L	P	F	R	A	U	S	M	E	P	C	T	I	L	C	M
R	Y	E	U	L	E	O	H	O	R	L	A	W	E	Y	D	L	R	N	A	N	O	I
E	F	U	N	C	T	I	O	N	D	N	M	T	H	E	D	E	U	I	M	E	A	N
A	A	M	D	L	N	G	A	E	I	J	R	C	A	L	O	X	G	L	M	A	M	A
N	C	I	S	A	D	U	R	N	A	I	R	R	A	T	I	O	N	A	C	E	U	N
S	T	J	L	E	S	H	E	T	C	O	O	R	D	I	N	A	T	E	D	N	L	T
M	O	A	O	P	E	N	D	E	S	R	O	R	E	L	A	T	I	O	N	B	T	E
P	R	O	P	O	R	T	I	O	N	A	L	I	G	C	D	W	I	R	M	T	I	R
A	T	N	E	L	J	E	S	L	D	E	T	E	R	M	I	N	A	N	T	S	P	E
L	A	M	G	R	A	P	H	R	A	S	N	D	E	S	H	L	A	W	N	E	L	M
C	S	A	D	N	J	A	M	O	D	A	H	I	E	M	P	T	Y	S	E	T	E	I

GEOMETRY

MIRROR SYMMETRICS

One objective of mathematics instruction is to determine an awareness in the student of mathematics in situations frequently taken for granted. Symmetry exists in many areas with which the student is quite familiar. These areas can be used to strengthen the understanding of the concept.

Instructions

Each student will need an activity sheet with the printed alphabet and regular polygons. Each will also need a mirror.

The students will use the mirror to find the line of symmetry in the figures by placing it perpendicular to the paper on the figures and finding the placement needed to make the half-figure showing and the reflection the same as the original figure.

Encourage the students to be flexible and open in their thinking, thus helping them to understand that they might need to view the letters and shapes from "non-standard" vantage points. When placing the mirror on the shape or letter, care must be taken that the plane of the mirror is perpendicular to the plane of the paper on which the shape or letter is printed.

(MIRROR SYMMETRICS)

1. How many lines of symmetry does an equilateral triangle have?

______

2. How many does an isosceles triangle have? ______ Where is it?

____

3. How about a scalene triangle? ______

4. Make a general statement about the lines of symmetry for triangles.

____

____

5. How many lines of symmetry do the following quadrilaterals have?

a) Square ______

b) Rectangle ______

c) Rhombus ______

d) Parallelogram ______

e) Isosceles Trapezoid ______

f) Trapezoid ______

6. Make a general statement about the lines of symmetry for quadrilaterals.

____

____

7. Considering a regular pentagon, a regular hexagon, and other regular polygons, make a statement about lines of symmetry of these regular figures.

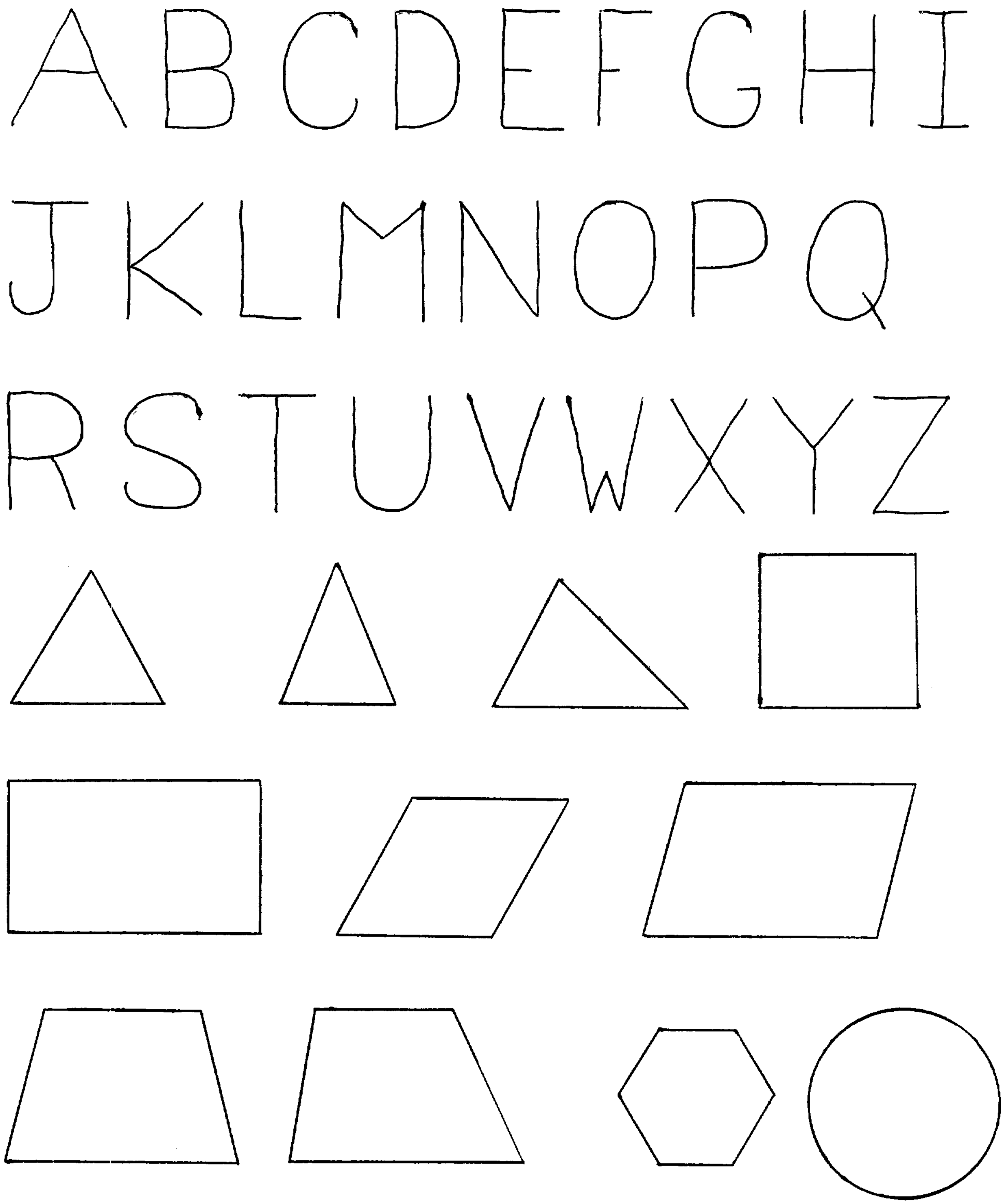
____

____

8. How many lines of symmetry does a circle have? ______

MIRROR SYMMETRICS Name________________________

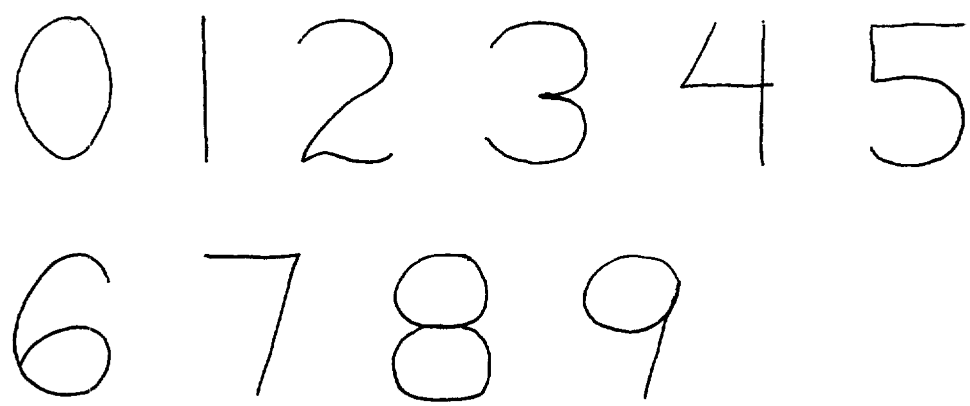
Symmetry lines are found in shapes and letters as well as in many other things. By placing your mirror perpendicular to the plane of the paper, find the lines of symmetry in the following letters or shapes.



(MIRROR SYMMETRICS)

What word is this? Put your mirror on the dotted line to find out. Make up some "half words" of your own.

Identify and describe lines of symmetry for the digits 0 through 9. Can you combine digits to form numerals which have vertical lines of symmetry? Is there any multi-digit numeral that has a horizontal line of symmetry?



(5)

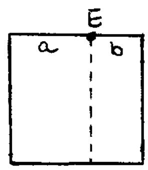
FOLDING THE SQUARE OF SUMS

This activity provides a geometric demonstration for $(a + b)^2 = a^2 + 2ab + b^2$.

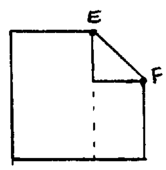
Instructions

Each student will need one piece of square paper.

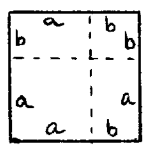
1. Label a point E on one edge. Fold this edge over on itself to form a vertical crease parallel to the adjacent edge. Label the longer dimesion "a" and the shorter dimension "b."



2. Fold the upper right-hand corner over onto the crease to locate point F. Folding this way, point F will be the same distance from the corner as point E.

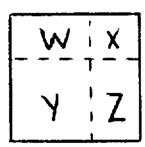


3. Fold a horizontal crease through F and all outside dimensions.



4. Note that two square regions and two rectangular regions are formed.

area of square Y: a^2
area of square X: b^2
area of rectangle W: ab
area of rectangle Z: ab



5. Show that these areas together must equal $(a + b)^2$.

Analysis

The original square paper measures $a + b$ on each edge and hence has an area of $(a + b)^2$. The four smaller parts combine to give an area of $a^2 + 2ab + b^2$. But together their areas must equal that of the original square. Therefore,

$$(a + b)^2 = a^2 + 2ab + b^2$$

(5)

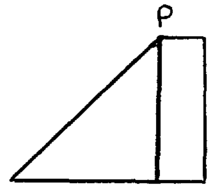
FOLDING THE DIFFERENCE OF SQUARES

This activity provides a geometric demonstration for $(a \quad b)(a - b) \quad a^2 - b^2$.

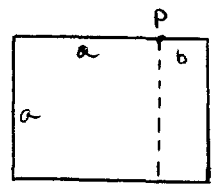
Instructions

Each student will need one rectangular sheet of paper.

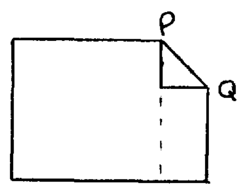
1. Fold the left edge down onto the bottom edge to locate point P.



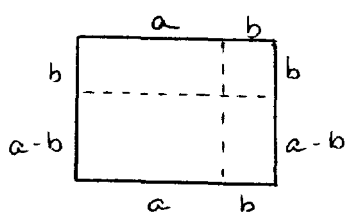
2. Fold vertically through point P. This forms a square on the left. Label the longer dimension "a" and the shorter dimension "b."



3. Fold the upper right-hand corner over on the crease to locate point Q.



4. Fold a horizontal crease through Q and label all outside dimensions.



5. Note the three rectangular and one square region formed.

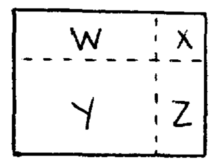
area of square X: b^2

area of rectangle W: ab

area of rectangle Y: $a(a - b) = a^2 - ab$

area of rectangle Z: $b(a - b) = ab - b^2$

6. Show that the areas of Y and Z together equal both $(a + b)(a - b)$ and $a^2 - b^2$.



Analysis 1

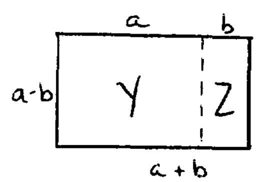
This rectangle has an area of $(a + b)(a - b)$. But its area is also the sum of the areas of Y and Z.

area $Y = a(a - b) = a^2 - ab$

area $Z = b(a - b) = ab - b^2$

sum of areas $\quad a^2 - b^2$

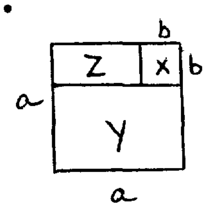
Hence $(a + b)(a - b) = a^2 - b^2$.



Analysis 2

Arrange pieces X, Y, and Z to form a square with area a^2. Removing piece X gives an area of $a^2 - b^2$. But pieces Y and Z can be rearranged to form a rectangle with area $(a + b)(a - b)$.

Hence $a^2 - b^2 = (a + b)(a - b)$.

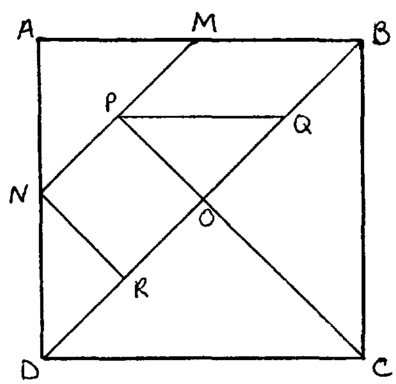


TANGRAMS

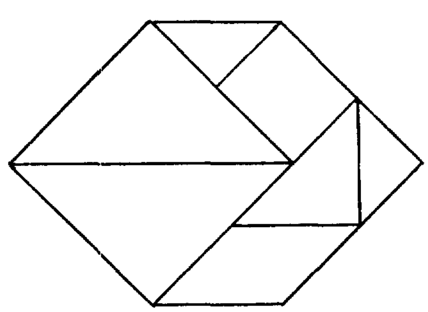
One of the oldest known puzzles is the ancient Chinese puzzle game of tangrams. Having amused and challenged people for thousands of years, it is certain to capture the interest of many students as well.

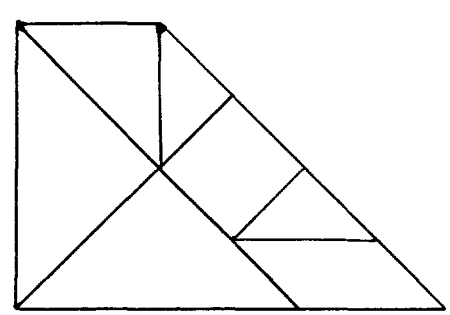
Instructions

Use a large square-shaped piece of card board or construction paper, and complete the figure shown. To draw this figure, first locate points M and N, the midpoints of sides AB and AD. Draw MN. Then draw diagonal BD and part of the other diagonal of the square, shown as PC. Finally, draw PQ parallel to AB and NR parallel to PC.



Next instruct the students to cut the figure along the lines drawn. This should give them seven separate pieces, consisting of five triangles, one square, and one parallelogram. A tangram puzzle now consists of arranging these seven pieces in the form of a given figure. Let the students try to use the seven pieces to form a triangle or a quadrilateral. Shown are two possible figures that can be formed from the seven pieces.





Extensions

A convex polygon is one with all its diagonals contained in the polygon and its interior. There are 13 possible tangrams altogether that are in the form of convex polygons, two of which are shown in the preceding figure. Of these 13 tangrams, one is a triangle, six are quadrilaterals, two are pentagons, and four are hexagons. Challenge your students to find all 13, and arrange these in a bulletin board display.

(5)

EXPERIMENTS WITH GEOMETRIC MODELS

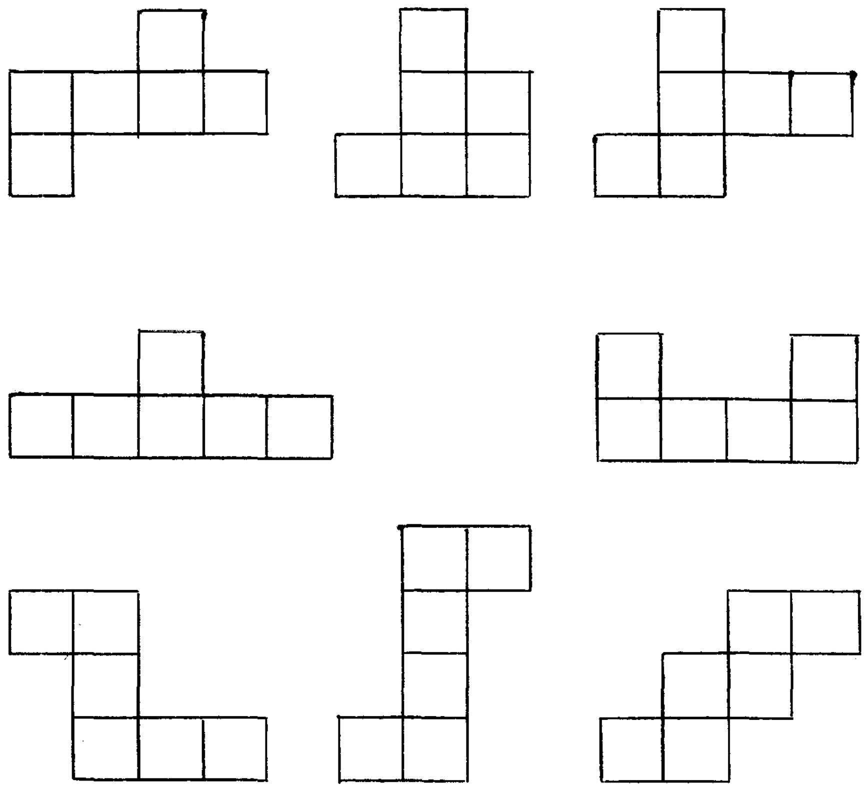
Few topics are better suited for laboratory experiments than geometric models. They seldom require more than a few sheets of heavy paper, a ruler, a pair of scissors, and some tape or glue.

Instructions

The experiments that follow are very appropriate for investigation by each individual student. But as the teacher, you need to become actively involved with students as they work through these experiments, offering helpful hints and suggestions where needed. It is also wise to have extra questions ready to challenge the better students to further exploration.

CAN YOU SPOT THE CUBES? Name____________________

Many different patterns can be used to form models of cubes. Each pattern must have six squares for faces arranged so that, when assembled, no faces overlap.



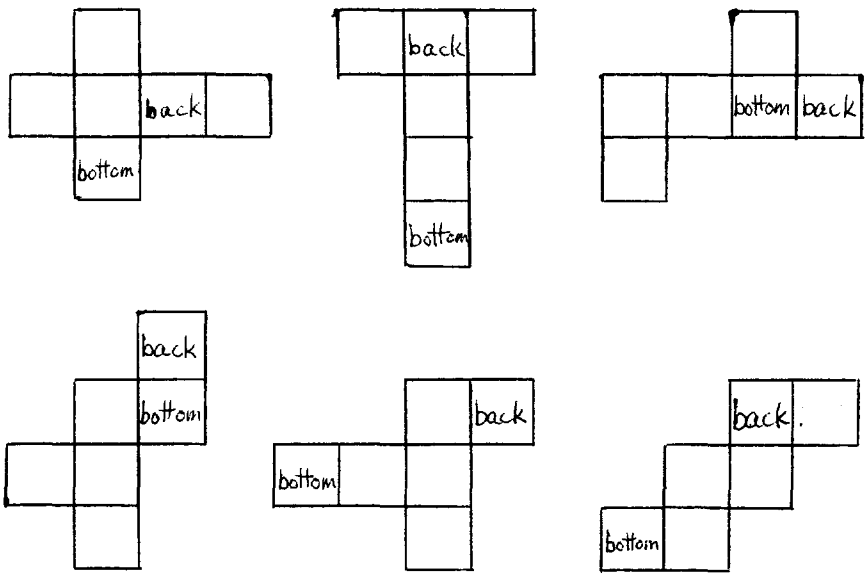
1. Study the patterns given here and circle those that you think can be used for a cube. Be careful!

2. If you want to check your answers, copy the figures onto construction paper, cut them out, and try to assemble them.

3. Copy those that work on to a sheet of graph paper. Then draw as many more as you can find. Remember, count only those that are different patterns, not those that are different positions of the same pattern.

WHICH FACE IS WHERE? Name________________________

All the patterns shown can be used to form a model of a cube. Assume that each cube has been assembled and properly positioned with the words on the outside and with the bottom face down and the back face in the rear.

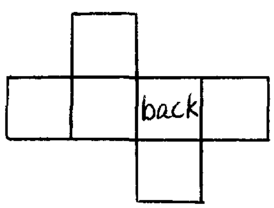


1. Mark the location of the remaining four faces on each pattern using this code:

   F: front
   L: left
   R: right
   T: top

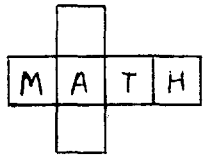
2. If you want to check your answers, you could copy each pattern onto construction paper, cut them out, and assemble them.

3. Copy this pattern on a sheet of paper several times. Then see if you can find the four different ways you can label all the remaining faces so that the cube can still be properly assembled and positioned.

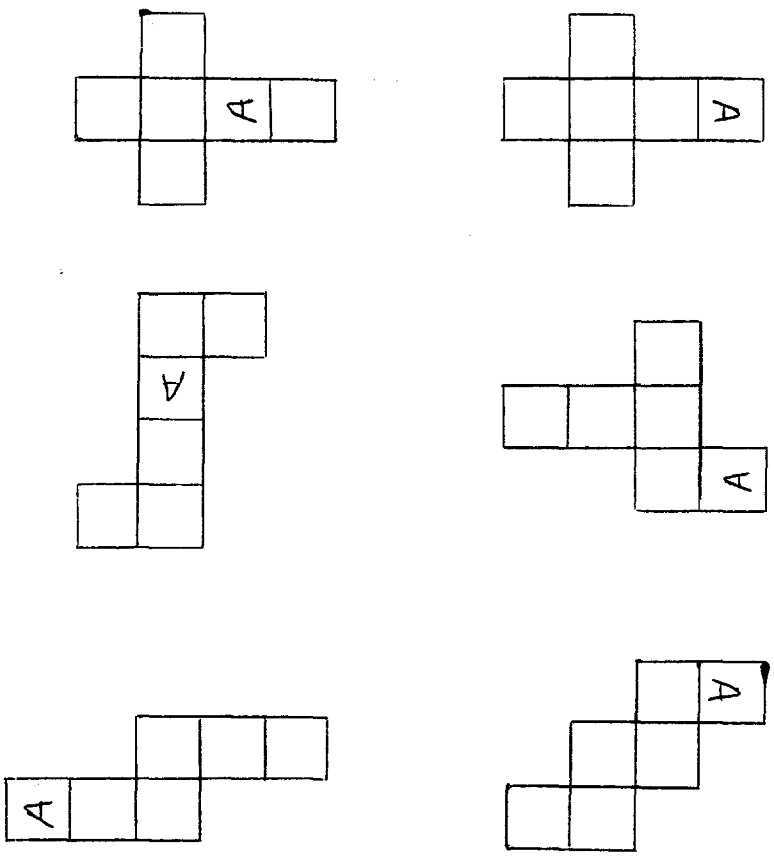


CAN YOU SPELL "MATH" ?   Name_______________________

If the pattern below were assembled to form a cube, it would spell MATH around four of its faces.



Place the letters M, T, and H on the patterns below so that they too will spell MATH when assembled the same way.



FOLDING TRIANGLES

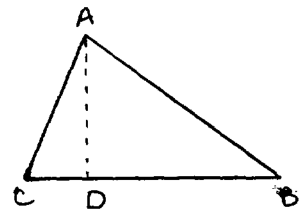
Many geometric properties can be effectively and dynamically illustrated through paper-folding activities. Three are described below.

I. SUM OF THE ANGLES IS 180°

Instructions

Each student will need a triangular piece of paper. It would add interest if the triangles were not all identical.

1. Begin with a triangular region ABC.
2. Fold vertex C onto side BC so that the crease passes through vertex A.
3. The crease AD is the altitude through vertex A.
4. Fold vertex A onto point D.
   Fold vertex C onto point D.
   Fold vertex B onto point D.
5. Show that the sum of the measures of the angles A, B, and C is 180°.



Analysis

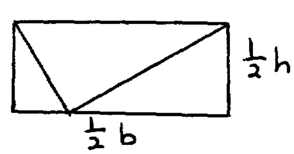
The three angles have been folded onto a straight line segment. Hence, the sum of their measures must be 180.

II. AREA OF A TRIANGLE

Instructions

Each student will need a triangular piece of paper. The one used in the previous experiment may be used here, also.

1. Note that twice the area of the rectangle is the area of the triangle.



2. The length of the rectangle is half the base of the triangle. The width of the rectangle is half the height of the triangle. Show that the area of the triangle is $\frac{1}{2}bh$.

Analysis

$$\text{Area of triangle} = 2(\text{area of rectangle}) = 2(l \times w) = 2(\tfrac{1}{2}b \times \tfrac{1}{2}h) = 2(\tfrac{1}{4}bh) = \tfrac{1}{2}bh$$

III. ANGLE BISECTORS, ALTITUDES, AND MEDIANS

Instructions

Each student will need three triangular pieces of paper.

1. With one triangle, fold the three angle bisectors. An angle bisector can be formed by folding one side of the angle on top of the other side and creasing. The crease through the vertex is the angle bisector.

2. Fold the three altitudes using another triangle. An altitude from a vertex can be formed by folding the opposite side upon itself such that the crease passes through the given vertex.

3. With the third triangle, fold the three medians. To fold a median to a given side, first bisect the side. This can be done by folding it upon itself such that the endpoints coincide. Then fold a crease through this midpoint and the opposite vertex.

Analysis

The angle bisectors of a triangle are concurrent, as are the altitudes and medians. The common point for the angle bisectors and medians will always lie within the triangle. However, the triangle must be acute for the altitudes to meet within it.

(5)

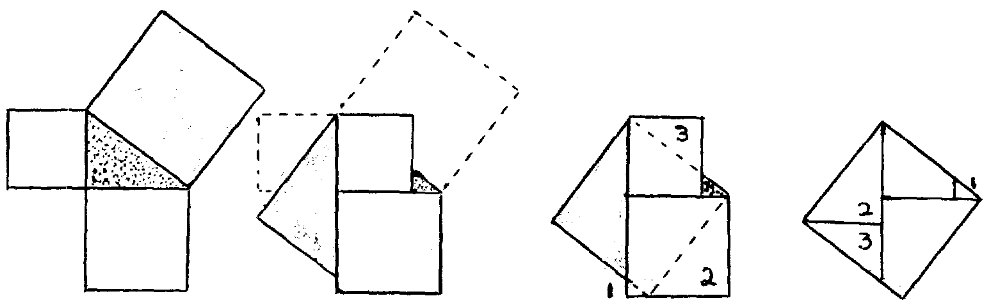
PYTHAGOREAN CONSTRUCTION

This aid is best suited for review of the Pythagorean property. It is simple to make and use by the teacher and dynamic and visual to the viewer. Although it is not a proof, it does leave in the eyes of the observer a convincing result for any right triangle.

Instructions

Cut a right triangle of any size from red paper. Then cut a large square of green paper to fit the hypotenuse and two smaller squares of white paper to fit the two legs. Tape the squares onto the sides of the triangle as shown here.

I. The object is to show a simple method of cutting the squares on the legs so that the pieces can be easily rearranged to fit the square on the hypotenuse.



Step 1: Fold the square on the smaller leg over the triangle. Fold the square on the hypotenuse under the triangle.

Step 2: Cut off the three triangular pieces that extend beyond the large square as shown.

Step 3: Relocate these pieces as shown to completely cover the square on the hypotenuse.

By using the colors suggested, the white areas (that of the squares on the two legs) will completely cover the green area (that of the square on the hypotenuse), thereby illustrating the theorem.

II. Repeat with several triangles of other sizes. Then let the students construct some of their own to illustrate the same property. Follow the same steps; only the sizes of the pieces cut and rearranged will differ.

EULER'S FORMULA

Leonard Euler, Switzerland's most famous mathematician, lived from 1707 to 1783. (His name is pronounced like the word "oiler.") Among Euler's many contributions were the beginning of topology, with his careful analysis of the famous Koenigsberg bridge problem, and his intriguing study of polyhedrons. Although known to Descartes more than 100 years earlier, this simple relationship among the number of vertices, faces, and edges of a polyhedron was independently discovered by Euler and now bears his name.

EULER'S FORMULA	If V is the number of vertices, E the number of edges, and F the number of faces of a simple polyhedron, then $V + F = E + 2$.

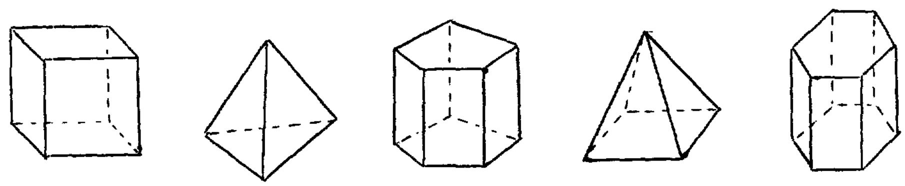
The experiment described on the following student activity sheet is designed for the individual student to follow in exploring these numbers, V, F, and E, for various polyhedrons. Hopefully, it will lead him to discover and support Euler's formula.

(5)

COUNTING VERTICES, FACES, AND EDGES OF POLYHEDRONS

Name ____________________

A polyhedron is a figure in space that has flat faces bounded by polygons. The prisms and pyramids below are examples of polyhedrons.



1. Count the number of vertices, faces, and edges for each prism and pyramid shown. Then record the numbers in the table.

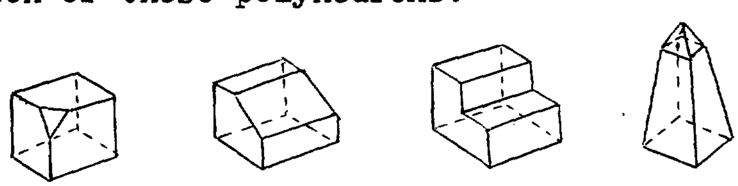
Polyhedron	Number of		
	Vertices	Faces	Edges
cube			
tetrahedron			
pentagonal prism			
square pyramid			
hexagonal prism			

2. Study the values in the table. For each of these polyhedrons, is the number of edges E greater than the number of vertices V and greater than the number of faces F? ______ ______

   Is E always less than the sum of V and F? ______

   Can you discover a relationship among the number of edges E and the number of vertices V and faces F? Express this relationship in symbols.

3. Support your formula from step 2 using the V, F, and E from each of these polyhedrons.



(2)

GEOCABULARY

R	E	A	N	I	L	L	O	C	P	B	H	Q	U	O	P	R	I	S	I	O	N	T
G	D	E	A	E	X	A	C	O	N	K	Y	A	N	O	I	T	C	E	L	F	E	R
M	G	Y	E	H	B	R	A	N	N	C	P	A	C	U	S	D	X	E	C	A	F	B
L	E	B	I	L	E	N	P	E	R	H	O	M	B	U	S	A	Q	U	I	L	G	F
S	W	I	E	A	V	C	N	H	D	A	T	R	A	A	L	E	E	T	U	T	A	V
C	A	C	R	T	O	L	Y	R	A	L	J	H	E	X	O	T	M	E	D	I	A	N
S	E	O	N	Y	W	A	S	E	C	A	N	T	I	I	P	A	R	T	L	T	C	L
O	T	N	N	O	V	E	T	C	U	R	U	S	V	O	E	F	T	A	F	U	I	L
H	Y	D	N	T	E	S	E	O	R	C	S	D	O	M	U	G	L	R	A	D	S	A
O	J	I	N	H	A	K	O	N	T	A	E	P	O	S	T	U	L	A	T	E	S	C
I	R	T	U	E	R	A	M	C	N	H	G	U	A	B	A	G	M	Y	U	R	B	I
S	O	J	N	S	J	M	U	E	E	S	T	D	J	T	N	E	M	G	E	S	I	T
T	C	O	S	I	N	E	H	N	U	T	S	B	U	B	G	H	W	U	H	H	H	R
C	H	N	A	S	E	S	L	T	R	E	H	S	L	I	E	Z	A	B	Y	E	T	E
A	R	A	V	S	I	L	L	R	G	A	Y	M	L	I	N	E	R	I	P	J	T	V
R	N	L	O	C	U	S	J	M	I	N	L	P	X	K	T	T	I	H	R	E	I	H
R	I	Q	A	I	Z	R	T	C	O	R	O	L	L	A	R	Y	Z	A	R	H	E	C
I	N	S	C	R	I	B	E	D	C	G	T	O	D	E	R	F	E	V	F	R	O	E
D	V	K	M	R	D	N	X	N	Y	P	E	T	O	I	M	O	N	E	D	O	R	L
E	E	E	E	Y	I	E	A	I	S	O	S	C	E	L	E	S	I	C	L	E	E	H
A	R	W	T	Q	U	F	H	L	E	I	I	D	O	F	R	U	S	T	R	U	M	N
T	S	U	P	O	T	T	C	I	B	A	S	A	H	W	N	Y	W	O	O	N	K	T
V	E	R	T	E	X	N	T	T	D	E	S	F	H	A	G	R	U	R	A	L	X	O

(5)

More than 60 words, phrases, and names associated with geometry are hidden in this array. They are spelled horizontally, vertically, diagonally, and both forward and backward. How many can you find?

R	T	L	U	H	F	A	C	E	I	L	C	N	A	I	D	E	M	E	Y	P
A	O	D	O	O	E	T	I	N	T	E	R	C	E	P	T	A	E	R	A	O
L	F	T	O	G	W	E	A	I	Q	G	J	E	B	D	L	O	R	E	Z	L
I	S	R	C	V	I	L	X	S	R	S	N	A	W	T	R	X	O	H	S	Y
M	P	I	E	E	Z	C	R	A	P	I	K	M	I	O	L	O	E	P	N	G
I	I	A	M	C	S	R	D	X	L	V	P	T	G	R	A	P	H	S	O	O
S	N	N	E	A	Q	I	U	I	A	E	U	L	P	I	U	T	T	C	I	N
E	D	G	R	P	U	C	B	S	N	D	N	A	M	G	Q	T	N	D	T	D
Q	U	L	T	S	A	T	N	R	E	G	R	G	E	I	E	R	A	E	C	F
U	C	E	X	N	R	N	E	O	A	A	O	I	T	N	D	A	E	F	E	Z
I	T	M	E	O	E	Q	G	Q	L	L	R	N	S	H	C	P	R	I	S	M
L	I	P	C	Y	R	M	D	L	O	C	U	S	O	L	G	E	O	N	C	A
A	O	O	L	R	O	P	E	N	T	E	U	C	L	I	D	Z	G	I	I	E
T	N	L	S	T	O	L	S	L	O	P	E	R	I	I	M	O	A	T	N	B
E	A	K	I	E	T	J	X	I	P	K	V	I	D	D	H	I	H	I	O	T
R	U	N	W	M	T	V	O	L	U	M	E	B	J	A	N	D	T	O	C	D
A	A	R	G	O	I	Y	M	I	U	O	O	E	I	R	X	E	Y	N	E	F
L	E	C	F	E	S	T	R	V	K	I	L	C	Z	O	B	A	P	G	N	D
D	G	Q	U	G	N	S	H	H	E	X	A	G	O	N	Y	A	R	R	A	Y
P	O	I	N	T	C	T	T	J	P	A	N	H	W	B	I	E	S	G	E	C
A	R	B	M	D	E	D	U	C	T	I	O	N	T	H	E	O	R	E	M	P

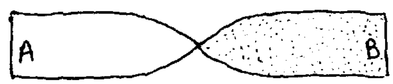
MISCELLANEOUS ENRICHMENT ACTIVITIES

MOBIUS STRIP

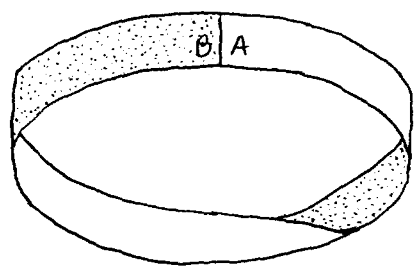
Discovered by the German mathematician August Mobius, the Mobius strip is a fascinating item that lends itself very well to a worthwhile enrichment topic that can be presented in the form of a laboratory exercise.

Instructions

Have each student begin with a strip of paper about 20 inches long and 4 inches wide for ease of handling. Mark one end A and the back of the opposite end B. Turn over one end of the paper so as to form a half-twist.



Now, join the ends so as to form a figure known as the Mobius strip.



The Mobius strip is a one-sided figure. Start at A and draw a line down the middle of the strip. You will ultimately reach B without having to cross an edge, even though B was on the opposite side of the strip after making the initial half-twist.. Now cut the figure down the middle. Instead of two figures as expected, you will end up with one band!

Extensions

1. Cut the newly formed figure down the middle again to see what type of figure is obtained, but first ask the class to predict the outcome. (It will be two two-surfaced bands, linked within each other, impossible to separate.)

2. Form a new Mobius strip, but this time cut along a line that is approximately one-third of the way across the band. (This will give two strips intertwined; one will be a new mobius band, the other a two-surfaced figure.)

3. Form a band with two half-twists. Cut it down the center and discover the resulting figure. (Two two-surfaced bands linked together.)

(MOBIUS STRIP)

4. Repeat the preceding extension, but begin with a band that has three half-twists. (A twisted band that is tied in a knot.)

(5)

NUMBER PATTERNS

The following two activities provide the student with opportunities to discover number patterns.

Instructions

Each student should receive a dittoed activity sheet. Although this is designed to be an individual activity, some student discussion and comparison of ideas could help them discover the patterns more quickly.

Answers

COUNTING SQUARES

Number of divisions	1	2	3	4	5
Number of small squares	1	4	9	16	25

Number of divisions	1	2	3	4	5
Number of all squares	1	5	14	30	55

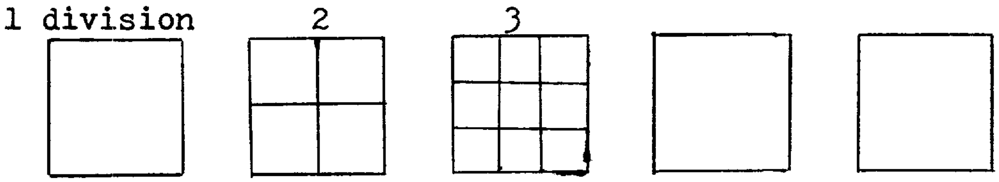
COUNTING TRIANGLES

Number of folds	0	1	2	3	4	5
Number of triangles	1	3	6	10	15	21

(5)

COUNTING SQUARES Name________________________

Repeatedly divide a square into smaller and smaller squares. Count the number of the smallest squares formed after each successive division.



Number of divisions	1	2	3	4	5
Number of small squares					

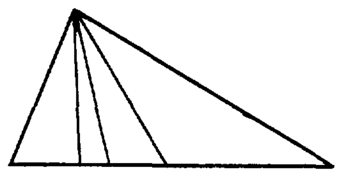
Repeat the process, only this time count all squares of all sizes.

Number of divisions	1	2	3	4	5
Number of all squares					

(5)

COUNTING TRIANGLES   Name____________________

Repeatedly fold a triangle through one of its vertices. Count the total number of triangles formed after each fold.



3 folds

Number of folds	0	1	2	3	4	5
Number of triangles	1					

Can you discover the pattern?

(5)

MAGIC SQUARES

Magic squares are square arrays of numbers. They date back to ancient times when people believed they held strange mystic powers because of their special properties.

Instructions

Each student will need a dittoed activity sheet which will contain full directions.

PROPERTIES OF MAGIC SQUARES Name______________________

Add the numbers in each row, column, and diagonal in this magic square. What property do you discover?

103	19	79
43	67	91
55	115	31

Sums of rows: _______, _______, and _______

Sums of columns: _______, _______, and _______

Sums of diagonals: _______ and _______

A square array of numbers is a magic square if the sums of the numbers in each row, column, and diagonal are the same. Find these sums for each of these magic squares.

7	2	16	9
12	13	3	6
1	8	10	15
14	11	5	4

9	2	25	18	11
3	21	19	12	10
22	20	13	6	4
16	14	7	5	23
15	8	1	24	17

2/3	1/12	1/2
1/4	5/12	7/12
1/3	3/4	1/6

(PROPERTIES OF MAGIC SQUARES)

Study these square arrays. Then complete them so that they become magic squares.

16	2	12
	18	

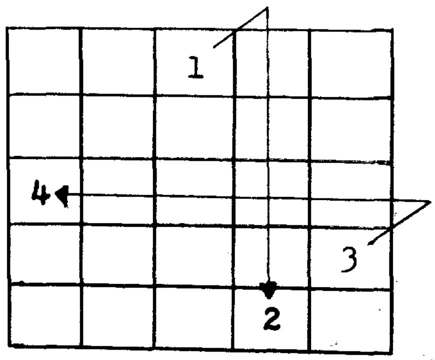
8		14	3
15	2	9	4
	11		13

7/6		
	11/12	
1		2/3

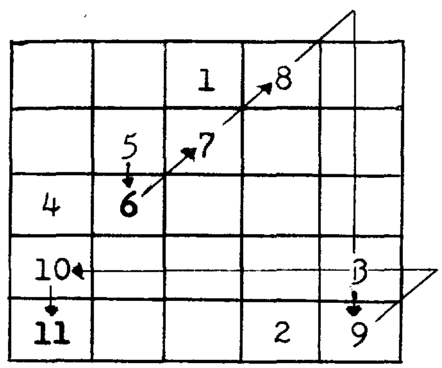
CONSTRUCTING ODD-ORDER MAGIC SQUARES   Name____________________

Here is a method that can be used to construct odd-order magic squares, those with an odd number of rows and columns.

Step 1: Write the first number, 1, in the middle of the top row. Fill in successive counting numbers moving diagonally upward and to the right. When the next position is off the square, enter the number at the opposite end of the next row or column. Note how this has been done for the entries 2 and 4.



Step 2: Continue working diagonally upward and to the right. When you come to a position already occupied, enter the number in the spaced immediately below the last one filled. Note how this has been followed for the numbers 6 and 11.



Step 3: The space that follows the upper right-hand corner is the lower left-hand corner. Since this space is occupied by 11, the 16 is placed below 15, following the rule from step 2.

		1	8	**15**
	5	7	14	**16**
4	6	13		
10	12			3
11			2	9

Step 4: Continue the process until the 5-by-5 magic square is finished. A 5-by-5 magic square has 5 x 5, or 25, entries. Since 1 was the first number, 25 is the last.

Step 5: Check to see that the completed array is indeed a magic square by adding the numbers in each row, column, and diagonal.

Use the same method to complete these magic squares.

Start this magic square with a 1.

	1	
3		
		2

Start this 5 x 5 magic square with an 8.

		8		
				10
			9	

Start this 7 x 7 magic square with a 1.

			1			

Check each array to be sure that is is a magic square. Then see if you can construct your own 9-by-9 magic square.

PUZZLES AND LOGIC

1. How can you cook an egg for exactly 15 minutes, if all you have is a 7-minute hourglass and an 11-minute hourglass?

   (Start both hourglasses simultaneously. At the moment that the 7-minute hourglass is empty, the 11-minute one has 4 minutes left at the top of the glass. Start timing the egg at this point; it will then take 4 minutes to complete. Then turn the 11-minute hourglass over and let it run its full 11 minutes; 4 11 15.)

2. How can a 24-gallon can of water be divided evenly among three men with unmarked cans whose capacities are 5, 11, and 13 gallons?

   (Fill the 13-gallon can; pour out 5 gallons into the 5-gallon can, leaving 8 gallons. Pour this into the 11-gallon can. Repeat the procedure to obtain another 8 gallons in the 13-gallon can, leaving the last 8 gallons in the 24-gallon can.)

3. Nine coins are in a bag. They all look alike, but one is counterfeit. It weighs less than the others. Use a balance scale and find the fake coin in exactly two weighings.

   (Divide the coins into three sets of three each and weigh two sets. If they balance, the counterfeit coin is in the remaining set of three. Weigh two of these against each other; if they balance, the remaining coin is fake. If they do not balance, the fake coin is the one that shows to be lighter on the balance scale. In the original weighing, if the two sets of three coins do not balance, the fake coin is one of these that weigh less. In that case proceed as just described with the three coins.)

4. There are 12 coins, of which one is counterfeit, weighing less than the others. Use a balance scale and find the fake coin in exactly three weighings.

   (Divide the coins into three sets of four each. In the first weighing, balance a set of four against another set of four. This will determine which set of four coins contains the counterfeit one. In the second weighing, balance two against two to determine which set of two coins contains the counterfeit. In the final weighing, balance one against one of these final coins to see which is lighter. Note that a more difficult problem consists of determining the counterfeit coin in exactly three weighings where you are not told whether the bad coin is lighter or heavier than the rest.)

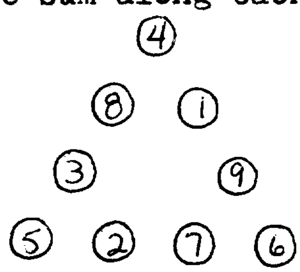
5. It is decided that whenever a certain club meets, everyone must shake hands with everyone else. If only two people meet, then only one hadshake is necessary. If three people meet, then three handshakes are necessary.
(a) How many handshakes are necessary if four people meet?
(b) How many handshakes are necessary if five people meet?

((a) 6   (b) 10)

6. A man went into a store and purchased a pair of shoes worth $5, paying with a $20 bill for his purchase. The merchants, unable to make change, asked the grocer next door to make change. He then gave the customer his shoes and the $15 change. After the customer had left, the grocer discovered that the $20 bill was counterfeit and demanded that the shoestore owner make good for it. The shoestore owner did so, and turned the counterfeit bill over to the FBI. How much did he lose by this transaction?

($15 and a pair of shoes.)

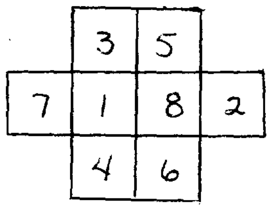
7. Use the digits 1, 2, 3, 4, 5, 6, 7, 8, and 9. Place exactly one of these in each position in the figure so that the sum along each side of the "magic" triangle is 20. As an extension, use the same digits and construct a "magic" triangle with the smallest and the largest possible sum along each side.



8. Use the digits 2, 3, 4, 5, 6, 7, 8, 9, and 10. Place exactly one of these in each position in the figure so that the sum for each row, column, and diagonal is 18.

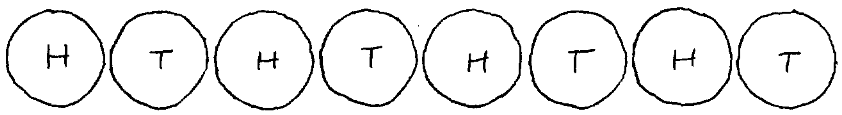
7	2	9
8	6	4
3	10	5

9. Arrange the numerals 1 through 8 in the figure so that no two consecutive integers touch at a side or on a corner.

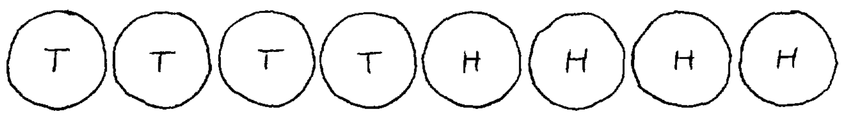


(In puzzles 10 through 14 it is suggested that the student attempt these problems by a trial-and-error process, using physical objects to experiment.)

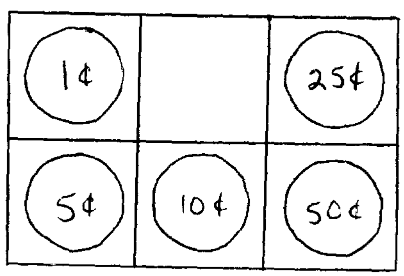
10. Arrange eight coins as in this figure:



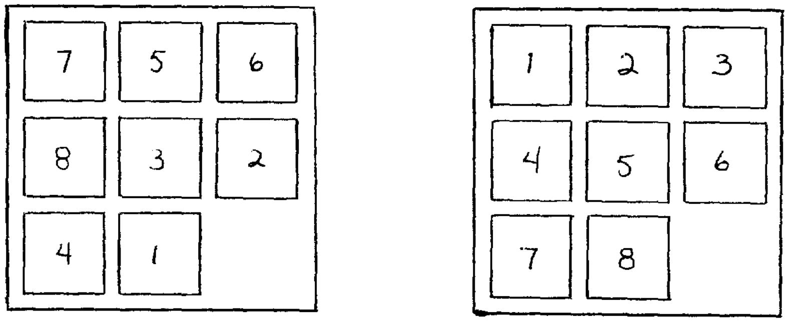
Moving two adjacent coins at a time, try to obtain the following arrangement:



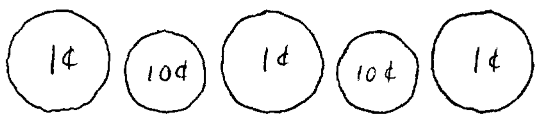
11. Interchange the penny and the nickel in the figure by sliding the coins from square to adjacent square, one at a time:



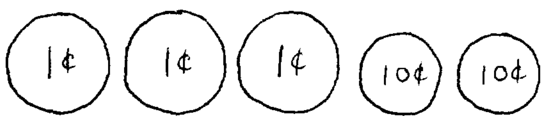
12. Draw a square, number eight pieces of paper, and arrange them as shown in the figure on the left. By sliding only one piece at a time into an open square, arrange the pieces as shown in the figure on the right:



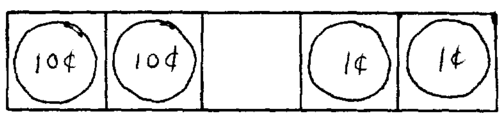
13. Arrange five coins as in this figure:



Try to obtain the following arrangement by moving two adjacent coins at a time, but each pair of coins moved must consist of a penny and a dime and must not be interchanged during the move:



14. Place two pennies and two dimes on a set of five squares as in the figure:



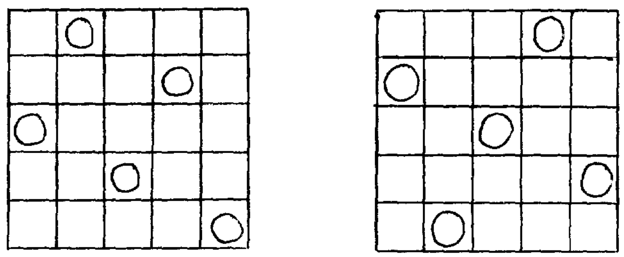
The object of the game is to interchange the positions of the pennies and the dimes. The rules are that dimes may be moved only to the right, and pennies may be moved only to the left. Coins may be jumped (without removal), but only one square at a time, as in checkers. The interchange can be made in a minimum of 8 moves. As an extension consider a set of seven squares, and use three pennies and three dimes. Follow the same rules and attempt to make the interchange in 15 moves.

15. Arrange three piles of toothpicks, chips, or other similar objects so as to have 6 items in pile A, 7 items in pile B, and 11 items in pile C. In exactly three moves you are to attempt to obtain 8 items in each pile. The rules for movement are that you may only move to a pile as many items as are already there, and all items moved must come from a single other pile.

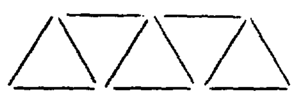
(In the first step move 7 items from pile C to pile B. In the second step move 6 items from pile B to pile A. In step three move 4 items from pile A to pile C.)

16. Place five pennies on five squares so that no two pennies are in the same row, column, or along a diagonal:

(Many solutions are possible; here are two:)



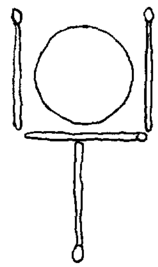
17. Eleven toothpicks are arranged as shown to give five triangles:



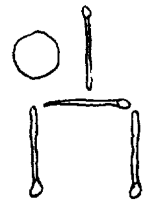
(a) Remove one toothpick to show four triangles.
(b) Remove two toothpicks to show four triangles.
(c) Remove two toothpicks to show three triangles.
(d) Remove three toothpicks to show three triangles.

(a) (b) (c) (d)

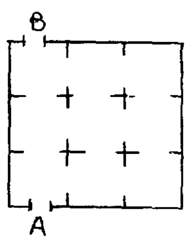
18. A coin is in a "cup" formed by four matchsticks. Try to get the coin out of the cup by moving <u>only</u> two matchsticks and with these forming a congruent "cup" but in a new position.

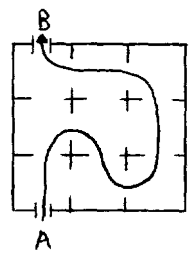


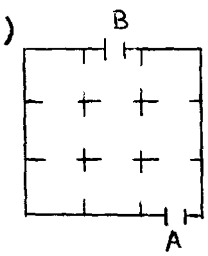
(Slide the horizontal matchstick one-half its length to the right. Then move the upper left stick to the lower right position shown.)



19. Here are floor plans for two houses. In each case try to get from door A to door B by walking through each room once and only once.

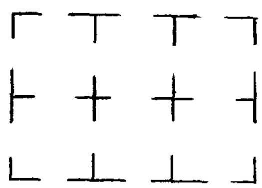
(a) 

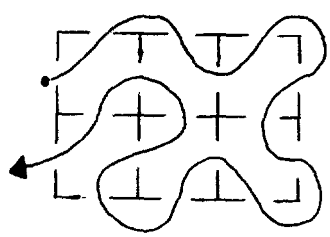


(b) 

Impossible

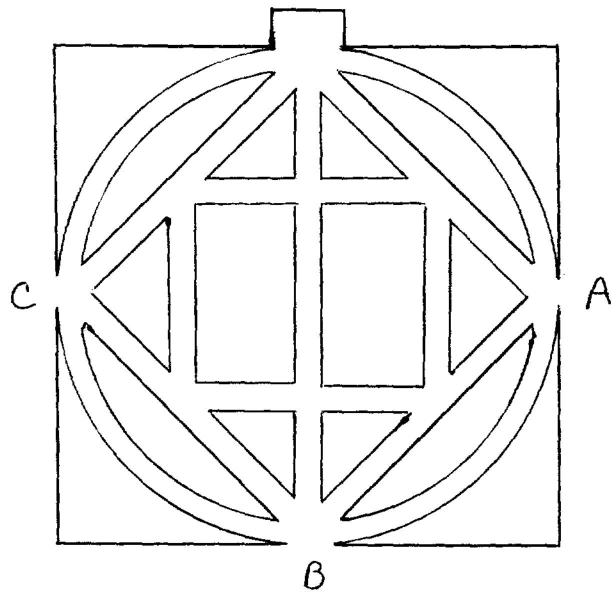
20. Can you walk through every door in this house exactly once, entering through the first and leaving through the last?



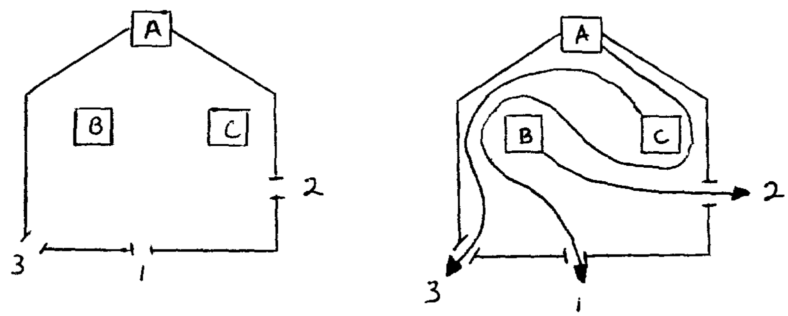


21. Try to walk through the maze. Start at the top, walk through each path exactly once, and come out at B. Then try to walk through each path once and only once and come out at A or C.

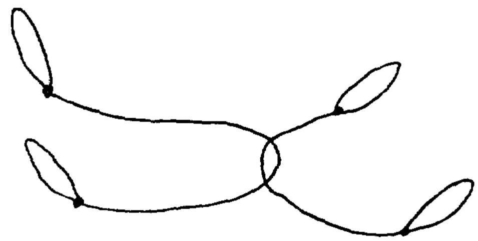
(The first path can be traced in several ways; the second is not possible.)



22. Three houses are located in the figure at A, B, and C. There are also three gates, numbered 1, 2, and 3. Draw a path from house A to gate 1, from house B to gate 2, and from house C to gate 3 so that no path crosses another.

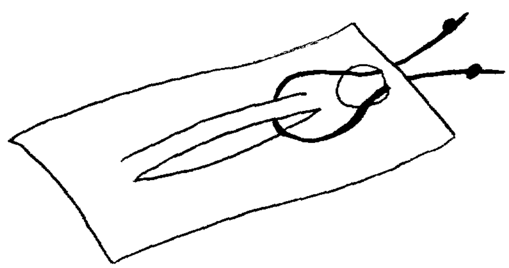


23. Loosely, but securely, tie together the hands of two people, looping one string around the other as shown. (Each string should be about 1 yard long.) Now get them apart without untying or cutting the string. It can be done!



(Pass a loop formed at the center of your string between the wrist and loop of your partner and up over his/her hand. Then pull and the strings will be separated.)

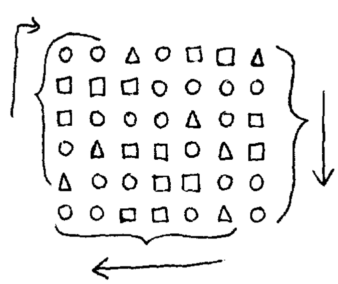
24. Cut a tag like this one. Pass a string through the hole, under the narrow center strip, out again, and back through the hole. Knot the ends. The trick is to get the string off the tag without cutting the string, tearing the tag, or passing the knotted ends of the string back through the hole.



(Make the center strip narrower than the punched hole. Then bend the strip through the hole, without tearing, and pull one knotted end of the string.)

25. Find the pattern in this array.

○	○	△	○	□	□	△
□	□	□	○	○	○	○
□	○	○	○	△	○	□
○	△	□	□	○	△	□
△	○	○	□	□	○	○
○	○	□	□	○	△	○



(Spiraling into the center, the pattern is triangle, circle, square, square, circle, circle.)

26. Each of the following will form a single familiar mathematical word when rearranged. How many can you decipher?

(a) CAN IT FOR (b) LUMTY LIP (c) MAD LICE
(d) CART TUBS (e) ME RUN B (f) I SON VIDI

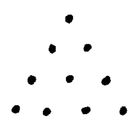
((a) FRACTION (b) MULTIPLY (c) DECIMAL
(d) SUBTRACT (e) NUMBER (f) DIVISION)

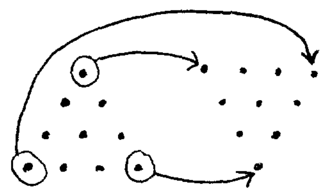
27. Can you unscramble the names of these famous mathematicians?

(a) A SCALP (b) SEED CARTS (c) UL DICE
(d) SOAP THY RAG (e) NOT NEW (f) MAIDS CHEER

((a) PASCAL (b) DESCARTES (c) EUCLID
(d) PYTHAGORAS (e) NEWTON (f) ARCHIMEDES)

28. Move just three dots to form an arrow pointing down instead of up.





29. Form four equilateral triangles with just six toothpicks.

(Use three of the toothpicks to form one equilateral triangle. Then place one end of each of the remaining three at each of the vertices of the triangle with the other ends all meeting at a point above the center of the triangle, forming a tetrahedron.)

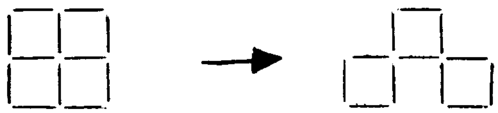
30. How many pennies can you arrange such that each penny touches every other penny?

(Five--three in a triangle formation with one above and one below.)

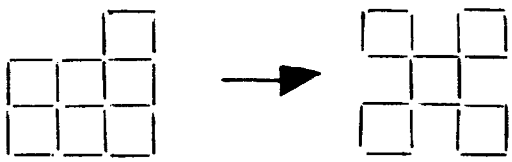
31. To mount a picture, two thumbtacks are needed in any two corners. What is the least number of tacks needed to mount four pictures?

(Three)

32. Rearrange three toothpicks to form a figure that consists of three squares of the same size.



33. Rearrange three toothpicks to form a figure that consists of five squares of the same size.



34. The digits 0 through 8 have been classified with the letters A, B, and C. How would you classify the digit 9?

A	1	4	7	
B	2	5		
C	0	3	6	8

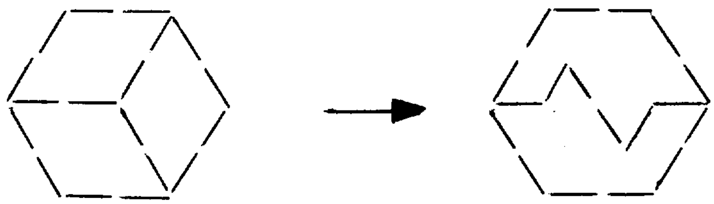
(C--formed with curves only.)

35. Here the letters A through H have been classified with the digits 1, 3, and 5. Discover the secret and classify the rest of the letters in the alphabet.

1	A	E	F	H
3	C			
5	B	D	G	

(1	I	K	L	M	N	T	V	W	X	Y	Z
3	O	S									
5	J	P	Q	R	U						)

36. These 18 sticks form a regular hexagon divided into three congruent regions. Move just four sticks to divide it into only two congruent regions.

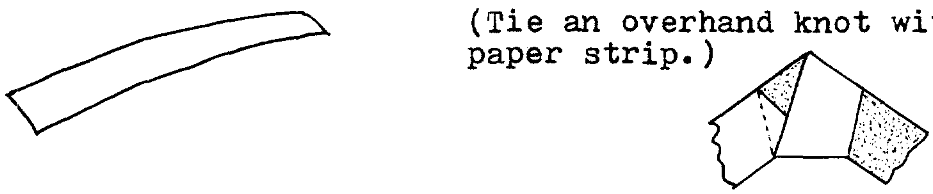


37. A solid has this for both its top and front view. Draw its side view.

(Other answers are possible.)

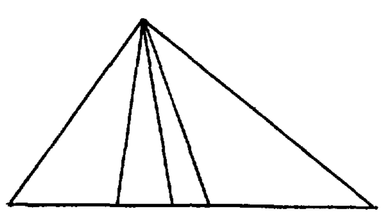
38. Take a single strip of paper and fold it into a regular pentagon.

(Tie an overhand knot with the paper strip.)



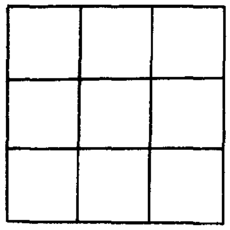
39. Without lifting your pencil from the paper, try to draw four connected lines that pass through all nine points. Remember, the lines must be straight.

40. How many triangles are in this figure?



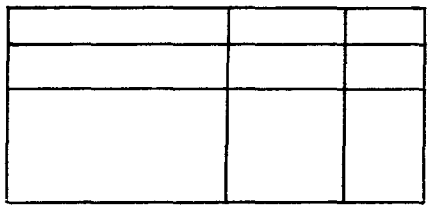
(10)

41. How many squares are in this figure?



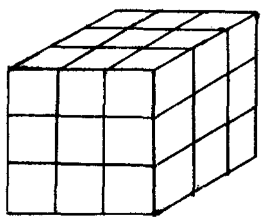
(14)

42. How many rectangles are in this figure?



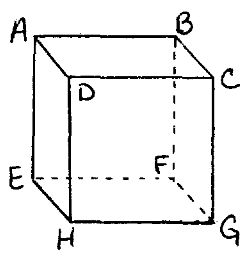
(36)

43. Count all the 1 x 1 x 1, 2 x 2 x 2, and 3 x 3 x 3 cubes in this figure.



(36)

44. List all the possible paths along the edges of this cube that pass through all eight vertices just once in going from vertex A to vertex G.

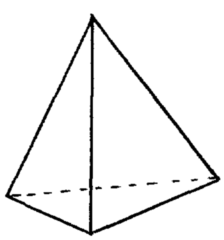


(ABCDHEFG ABFEHDCG

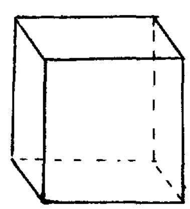
ADCBFEHG ADHEFBCG

AEHDCBFG AEFBCDHG)

45. A tetrahedron and a cube are to be painted but no two adjacent faces on either can be colored the same. What is the least number of colors needed for each?



(4)



(3)

46. Starting with the A at the top of the array, move down to the left or right one letter at a time to spell out ALGEBRA. How many different paths are possible? Do they all spell ALGEBRA?

```
      A
     L L
    G G G
   E E E E
  B B B B B
 R R R R R R
A A A A A A A
```

(All 64 possible paths spell ALGEBRA.)

47. Starting with any A on top, move down to the left or right, one letter at a time. How many different ways can you spell out the word ALGEBRA?

```
A A A A A A A
 L L L L L L
  G G G G G
   E E E E
    B B B
     R R
      A
```

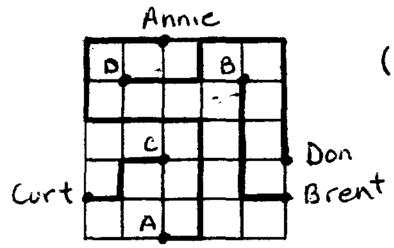
(64)

48. Start with the top, left A and move down, or to the right, or diagonally down and to the right, one letter at a time. How many ways can you spell out the word ALGEBRA?

```
A L G E B R A
L L G E B R A
G G G E B R A
E E E E B R A
B B B B B R A
R R R R R R A
A A A A A A A
```

(127)

49. This is a street map of a certain town. Annie lives at A, Brent lives at B, Curt lives at C, and Don lives at D. How can each person get home without running into another person's path? (They must walk along the streets. No shortcuts!)



(Answers may vary.)

50. Using just a 3-liter pitcher and a 5-liter pitcher, how can you measure out 4 liters of water?

(Fill the large pitcher. Pour water from it to fill the small pitcher. Empty the small pitcher. Pour the 2 liters from the large pitcher into it. Fill the large pitcher again, and pour water from it into the small pitcher until the small one is full. The large pitcher now has 4 liters.)

51. Amos, Christopher, and Geraldine work in the circus. They are the ringmaster, lion tamer, and elephant trainer, not necessarily in that order. 1) Geraldine has red hair; 2) Amos has curly hair; 3) the ringmaster is taller than Amos; and 4) the lion tamer is bald. Who is the elephant trainer?

(Amos)

52. Using quarters, dimes, nickels, and pennies, how many different combinations of coins have a sum of exactly 27¢? What are they?

(13 different combinations: 2 dimes, 7 pennies----1 d, 3 n, 2 p----1 d, 2 n, 7 p----1 d, 1 n, 12 p----1 d, 17 p----5 n, 2 p----4 n, 7 p----3 n, 12 p----2 n, 17 p----1 n, 22 p----27 p----1 q, 2 p----2 d, 1 n, 2 p)

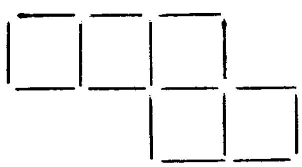
53. Paula and Peter Piper are picking pickled peppers. They are putting the pickled peppers in a basket. The number of peppers they pick doubles every minute. In one hour the basket is full. When was the basket half full?

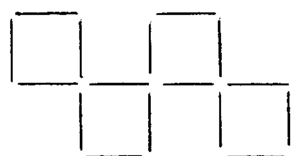
(The 59th minute.)

54. There are 3 red hats and 2 black hats in a drawer. Sheri, Brad, and Paula line up in a single file, and a hat is placed on each one's head. They are asked to figure out what color hat they are wearing. Paula, who could look ahead and see Brad and Sheri, says, "I don't know." Brad, who could see only Sheri, says, "I don't know." But Sheri, who could see nobody, says, "I know." What color is Sheri's hat and how does she know?

(Sheri's hat is red. If Brad and Sheri both had black hats on, then Paula would know she had a red one, but she doesn't know her color, so Brad and Sheri must either have 2 reds or a black and a red. If Brad could see a black hat on Sheri's head, then he would know his was red, but he doesn't know his color. Therefore, he must see a red hat on Sheri, so she knows hers is red.)

55. Here are five squares made from sticks. It is possible to make four squares by moving just two of the sticks. Can you do it?

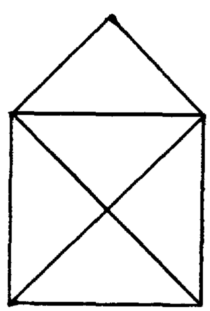


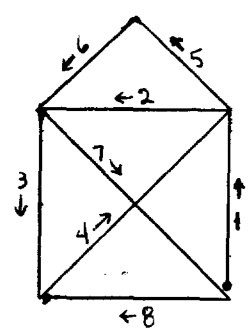


56. Without using paper, how much is twenty-eight thousand six hundred and twenty-five subtracted from twenty-six thousand twenty-six hundred and twenty-six?

(one)

57. Make this figure without lifting your pencil from the paper and without going over the same line twice.





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(*) original material