Statistical Models for Corporate Bond Rates

An Honors Thesis (ID 499)

by

Michelle Ford

Thesis Director

John A. Beekman

John A. Beekman

Ball State University

Muncie, Indiana

May 1994

Expected date of graduation: May 1994

Table of Contents

Acknowledgments

I would like to thank Dr. Pierce at Ball State University for her constant support and encouragement. It was through her enthusiasm for the field of Statistics that I became excited about this topic. Her insight and teaching in this area allowed me to understand the background materials used for this project.

I would also like to thank Dr. Beekman at Ball State University for his support in the writing of this thesis. He was able to keep me on track during the busy times.

Introduction

The main purpose behind the rating of securities is to provide the investors with a quick and concise way of comparing the quality of various bonds to each other. The data presented in this paper was obtained for this purpose from the most recent publication of Moody's Bond Record.

The system employed in collecting and presenting the securities was originated in 1909 by John Moody. This particular system presents the individual investments in a form which is analogous to the graduations of the investment quality. Each graduation subgroup, displaying a broad range of the same characteristics, is indicated by a rating symbol. There are nine different symbols available:

Aaa Aa A   Baa Ba B   Caa Ca C

An investment carrying a rating of Aaa is considered to be the highest quality investment, which presents the smallest risk. The ratings continue down the scale from there, with an investment carrying a rating of C being the lowest quality with the highest risk factor. For a complete description of the definition of a particular bond rating refer to Appendix I which presents Moody's definition of each type of corporate rating.

Within each rating category from Aa to B Moody's also presents numerical modifiers, 1, 2, and 3. A subgroup rating of 1 indicates that the company ranking is at the highest end of that group; the modifier 2 indicates a mid-range ranking; and the modifier 3 indicates that the company falls in the lowest end of the category.

Once an investment has been rated, Moody's continues to monitor its progress and will change the rating accordingly if it is necessary to reflect a different risk factor. These changes are to be expected more frequently in the bonds that carry a lower initial rating than those considered as safer investments. The investor should, however, continue to monitor the rating of each bond no matter how stable it appears in the beginning. This is necessary if he is to respond properly to any changes.

Since Moody's ratings are only designed to reflect the overall investment quality an investor should be aware of the other characteristics involved in the specific bond being purchased. It is not a safe assumption to rely on the rating as a means of comparison between any two bonds of the same rating. Other factors, such as market price, money rates, and general economic trends should also be considered.

The rating reported by Moody's can also not be used merely as a measure of how the bond performed statistically in the past. In order to recognize the objective of an investor to predict future trends, Moody's takes all steps necessary to look at the potential "worst" return on the investment in the "visible" future.

The time series models which will be discussed and applied throughout this paper are only the surface of the theory behind the building of time series models. In order to not complicate the topic, only the basic models needed to adequately represent the trends involved in the data are being discussed. Interested readers are encouraged to refer to the detailed discussion of this topic presented in reference [4]. The time series models being discussed and the algorithms needed to analyze the data have been compiled from throughout this reference in a very brief manner.

Beginning Data Analysis

When the data being analyzed is in the form of a time ordered sequence it is most helpful to use a linear time series model to describe the data in a logical and predictable form. Often this identification of a suitable model is not immediate and can take many iterations. Each iteration can easily be broken down into three distinguishable stages: identification, estimation, and diagnostic checking. This process helps to identify a suitable model, estimate the parameters involved in the model, then test for adequacy of fit. If the final stage of the process does not reveal proof that the model is adequate, within pre-set boundaries, then the iterative process is continued. From here the original model will then be modified or discarded altogether until the estimated parameters meet the expectations.

Once a suitable model has been identified, the forecasting of future values is relatively simple. In order to forecast, the model is simply applied to the data. Consequently, in order to obtain reliable forecasts, which is usually the purpose behind building the model, the majority of the time required is in building the model.

It should also be noted that the adequacy of the forecasting procedure, developed from historical data, relies on the assumption that the future tends to behave in the same manner as the past. Once a model has been identified it is important to remember that, since it relies mainly on the behavior of past data, a model is seldom reliable for an extended period of time. It will be necessary to update the model to include the additional data available once the forecasting period has expired and the actual data can be obtained. The updated model can then be used to continue the short term forecasting.

The procedure developed in Box and Jenkins [4] will be the basic approach used. This will be the only method presented due to the straight forward algorithm it presents as well as ease in understanding and implementation. This method is currently being used by many researchers who are faced with the problem of forecasting business and economic problems [23].

Identification

The identification of a model deals mainly with the identification of prior beliefs about the series, and leads to a possible tentative model. The time series model in particular will reflect any dependence between the observation, therefore, the first concern in the identification of a tentative model lies in the autocorrelation coefficient:

$$r_k = \frac{\sum_{t=1}^{N-k} (y_t-\bar{y})(y_{t+k}-\bar{y})}{\sum_{t=1}^{N} (y_t-\bar{y})^2}, \quad k=1,2,,\ldots \tag{1}$$

where the "sample average" is:

$$\bar{y} = (1/N)\sum_{t=1}^{N} y_t$$

Once these autocorrelation coefficients have been calculated a simple line graph of their values will be used to test the fit of the data to the expected pattern. If they seem to oscillate between positive and negative values and die out rapidly then the correlation is significant and can be used to successfully identify a tentative model with which to begin the analysis. Appendix III, which present the various correlograms for different model types, can be used to match the pattern to the proper beginning model.

Corporate Bond Example

This section will present an in depth analysis of the identification and building of a time series model to fit the data found in the Aaa column of Appendix II. Although the other four categories, Average Corporation, Aa, A, and Baa will not be analyzed in a step by step fashion, the results obtained by performing the presented method will be summarized in a later section.

The corporate bond rates reported for all Aaa companies in the last five years have been plotted in Figure 1. The abscissa is a "time" axis, where time is assumed to be

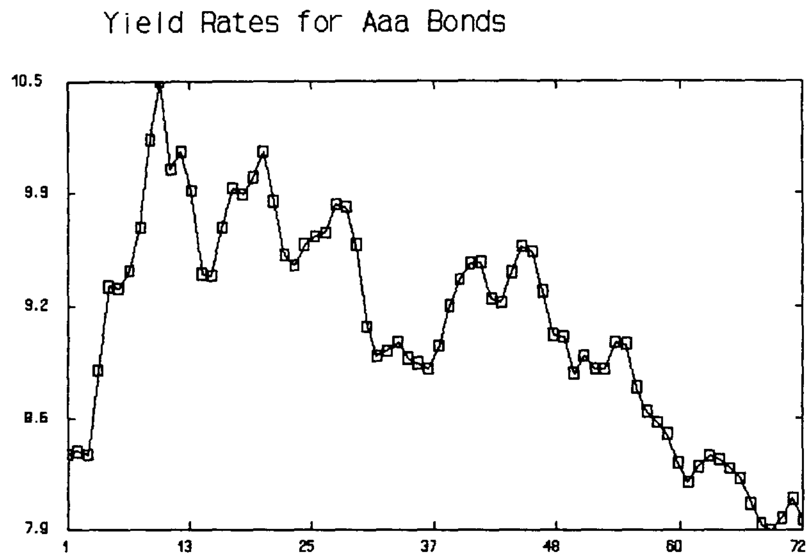


Figure 1- A discrete Time Series

measured in monthly intervals. The ordinate is the observation axis, where the data has been extracted from the second column of Appendix II. An observation at time t is denoted by y_t. These observations are connected to aid in the visualization of the pattern the data has followed over the past five years. Casual observation reveals that the observations do not appear to vary about a fixed point, but tend to follow a downward trend.

In order for the analysis to continue, the data must exhibit stationary characteristics. A series is said to be stationary if the following facts are true:

I. $E[y_t]$ and $Var[y_t] = E[y_t - E(y_t)]^2$ are constant functions of the index t.

II. $Cov[y_t, y_{t+k}] = E[y_t(y_{t+k})] - [E(y_t)]^2$ is some function of the lag k alone [23].

The first of these two requirements is obviously violated due to the decreasing nature of the data. As a result of this violation the autocorrelation coefficients can not be expected to provide any valuable information about the autocorrelation which may exist in this series. Figure 2 verifies the visual suspicion about the lack of stationarity through the use of the autocorrelation coefficient formula.

For the original series in Figure 1, with mean equal to 9.0957, the autocorrelation coefficient values are as follows:

$$r_0 = 1,$$

$$r_1 = \frac{\sum_{t=1}^{71} (y_t - \bar{y})(y_{t+1} - \bar{y})}{\sum_{t=1}^{72} (y_t - \bar{y})^2} = 0.920,$$

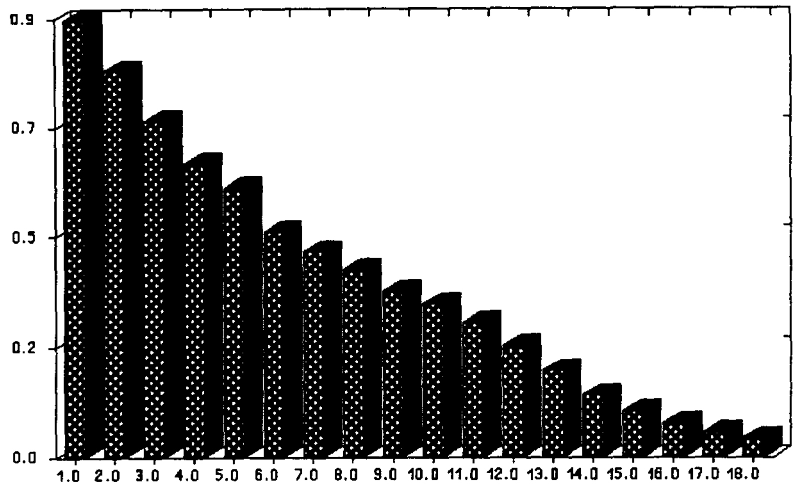


Figure 2- Autocorrelation Coefficients

$$r_2 = \frac{\sum_{t=1}^{70} (y_t-\overline{y})\,(y_{t+2}-\overline{y})}{\sum_{t=1}^{72} (y_t-\overline{y})^2} = 0.811, \ etc.$$

These persistently large values of r_k are an indication of the need for a transformation of the original series that will produce a stationary model that can then be analyzed. A reasonable transformation to consider is the forward difference. The amount of differencing needed to transform the data into a stationary model is the value of d for which the autocorrelation coefficients of the dth differenced series die out rapidly and present a recognizable pattern, within sampling fluctuations.

To illustrate the differencing algorithm, Figure 3 displays a graphical representation of the sample autocorrelation function of the original differenced series, $w_t = y_t - y_{t-1}$, t=1,2,...,N, followed by Table 1 which displays the actual autocorrelation coefficient values. The sample autocorrelation coefficients for the differenced series tended to die out quickly. It appears from this rapid dieing out that the original series was a realization of a stochastic process whose first difference is stationary.

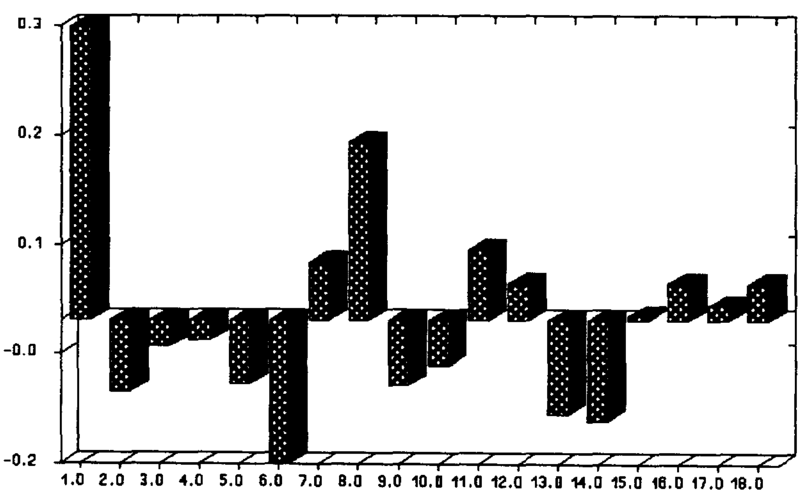


Figure 3 - First Differences

Once an order of differencing has been identified that transforms the series into a recognizable pattern the new series consisting of the differenced data can be represented by the proper time series model that best describes the behavior of the new data set.

Table 1 - Autocorrelation Coefficients for First Differences

Lag	Autocorrelation Coefficients	Lag	Autocorrelation Coefficients
1	0.344	10	(0.055)
2	(0.085)	11	0.083
3	(0.032)	12	0.044
4	(0.024)	13	(0.111)
5	(0.076)	14	(0.118)
6	(0.169)	15	0.005
7	0.068	16	0.044
8	0.208	17	0.017
9	(0.078)	18	0.043

Linear Time Series Model

First, the simple model will be considered:

$$Y_t = \mu + e_t \tag{2}$$

where

$$e_t \sim N(0, \sigma_e^2), \quad t=1,2,\ldots$$

$$E(e_t e_{t'}) = 0, \quad t \neq t'.$$

The types of stochastic time series models being discussed, in which successive values are highly dependent, can best be described as being generated from a series of independent "shocks" a_t [4]. These shocks are random observations from a fixed distribution assumed to be normal with a mean of zero and a specific standard deviation. A random series of these shock values within a small interval of the observations is sometimes referred to as white noise, "...'noise' because it is unsystematic, 'white' because it has a spectrum like that of white light" [15].

This model implies that the data varies about a fixed level and follows the laws of normality. Also, due to the independent nature of the individual random variables, it follows that the observations at time t are uncorrelated with past and future observations for all values of k:

$$\gamma_k = E(Y_t-\mu)(Y_{t+k}-\mu) = E(e_t e_{t+k}) = 0, \quad k \neq 0. \tag{3}$$

Moving Average Models

A logical progression from that of equation (3) is to allow the deviation of y_t from the mean to depend on the shock, e_{t-1}, which entered the system during the previous time period, as well as the current shock. The model then becomes:

$$Y_t = \mu + e_t - \theta_1 e_{t-1}$$

or equivalently

$$Y_t = Y_t - \mu = e_t - \theta_1 e_{t-1} \tag{4}$$

$$|\theta_1| < 1.$$

The type of model given by equation (4) is referred to as a "first-order moving average" (MA) process, also denoted MA(1). The number displayed in parentheses refers to the order of the model.

Subsequent MA models can be obtained by continuing the process of including additional shock values in the model. For example, the second-order moving average model is obtained by letting the deviation include not only the current shock and the previous shock, but also the shock from two time periods before, e_{t-2}. Thus the MA(2) model can be written:

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} \quad (5)$$

$$\theta_2 + \theta_1 < 1$$

$$\theta_2 - \theta_1 < 1 \quad (6)$$

$$-1 < \theta_2 < 1$$

Autoregressive Models

The other standard type of time series model often used is called the autoregressive model, AR. In this type of model the deviation at time t is not only a function of the shock at time t but also a function of the previous deviation as well. The AR(1) model can be written in the following form:

$$Y_t = \phi_1 Y_{t-1} + e_t, \quad |\phi_1| < 1 \quad (7)$$

$$E(t) = 0 \quad Var(e_t) = \sigma_e^2 \quad for\ all\ t \neq t'$$

$$E(e_t e_{t'}) = 0 \quad for\, t \neq t' \tag{8}$$

$$E(e_t Y_{t'}) = 0 \quad for\ t' < t$$

The order of an autoregressive model simply refers to the number of independent variables, or in other words the number of lags presented in the model. For example, the AR(2) model can be written as:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t, \tag{9}$$

where the following conditions must hold for the coefficients in order to ensure that the system remains stationary:

$$\phi_2 + \phi_1 < 1$$

$$\phi_2 - \phi_1 < 1 \tag{10}$$

$$-1 < \phi_2 < 1$$

Extension of the Basic Models

In order to reduce the number of parameters involved in a model it is often necessary to combine the MA and AR models together. This also helps to represent both the moving average processes and the autoregressive processes which are underlying in the data. Once the MA and AR models have been combined the following equation serves as a representation of an ARMA(1,1) model:

$$Y_t-\phi_1 Y_{t-1} = e_t-\theta_1 e_{t-1} \tag{11}$$

with the usual assumptions on each of the parameters and the error terms.

Another common variation on the normal representation of the MA and AR models is the use of differencing to obtain a stationary set of data. This type of model is referred to as an Autoregressive Integrated Moving Average model, or ARIMA(p,d,q) for short. This will be the type of model employed in the corporate bond rate example being explained.

The most basic of this type of model is the ARIMA(1,1,1). This implies that a first order MA model as well as a first order AR model is being used to represent the new data set that was obtained by taking the first difference of the original data set.

Corporate Bond Example

From the previous discussion of the pattern of the autocorrelation coefficients of the new data set it should now be clear that an ARIMA would be an appropriate model. From the large spike presented in the first lag, followed by significantly smaller negative spikes the conclusion can be drawn that an ARMA(1,1) model would have been appropriate if this had been the original data. It was, however, necessary to obtain the first differences of the data set first in order to maintain a stationary set. Therefore, the appropriate model to be employed is the ARIMA(1,1,1) in order to reflect the differencing algorithm performed initially.

Estimation of Parameters

The purpose of this stage in the model building process is to determine the "best" estimates of the unknown parameters of the given model. At this point it is convenient to refer to the ARIMA(p, d, q) process by the equation:

$$\phi(B)\, w_t = \theta(B)\, e_t \tag{12}$$

In general the calculation of the initial estimates is based on the first p + q + 1 autocovariances c_j[j = 0, 1, 2, 3,..., (p+q)]. Then proceeds in three stages:

1. The autoregressive parameters are estimated from the autocovariances c_{q-p+1}, ..., c_{q+1}, c_{q+2}, ..., c_{q+p}.
2. Using the estimates for the autoregressive parameters found in step one, the first q + 1 autocovariances c'_j (j = 0, 1, ..., q) of the derived series:

$$\acute{w}_t = w_t - \hat{\phi}_1 w_{t-1} - \ldots - \hat{\phi}_p w_{t-p} \tag{13}$$

   are calculated.
3. These new autocovariances are then used in an iterative calculation to complete the initial estimation of the moving average parameters and the residual variance.

In order to obtain the system of equations necessary to estimate the autoregressive parameters the following equation for the autocorrelations is obtained:

$$\rho_k = \phi_1\rho_{k-1} + \phi_2\rho_{k-2} + \ldots + \phi_p\rho_{k-p} \quad k \geq q+1. \tag{14}$$

The initial estimates of the parameters are then calculated by solving the p linear equations obtained from equation (14):

$$\begin{aligned} c_{p+1} &= \hat{\phi}_1 c_q + \hat{\phi}_2 c_{q-1} + \ldots + \hat{\phi}_p c_{q-p+1} \\ c_{q+2} &= \hat{\phi}_1 c_{q+1} + \hat{\phi}_2 c_q + \ldots + \hat{\phi}_p c_{q-p+2} \\ c_{q+p} &= \hat{\phi}_1 c_{q+p-1} + \hat{\phi}_2 c_{q+p-2} + \ldots + \hat{\phi}_p c_q . \end{aligned} \tag{15}$$

In order to proceed with the estimation of the parameters of the moving average process c'_j must first be defined as:

$$c'_j = \sum_{i=0}^{p} \phi_i^2 c_j + \sum_{i=1}^{p} (\phi_0\phi_i + \phi_1\phi_{i+1} + \ldots + \phi_{p-i}\phi_p)\, d_j \tag{16}$$

where

$$j = 0,\ 1,\ \ldots,\ q$$

$$d_j = c_{j+i} + c_{j-i}$$

$$\phi_0 = -1.$$

From here the linear convergence process can be employed by first identifying the autocovariance function of a MA(q) process as:

$$\gamma_0 = (1+\theta_1^2+...+\theta_q^2)\,\sigma_a^2 \qquad \textbf{(17)}$$

$$\gamma_k = (-\theta_k+\theta_1\theta_{k+1}+...+\theta_{q-k}\theta_q)\,\sigma_a^2 \quad k\geq 1$$

From equation (17) the parameters can then be estimated in reverse order as:

$$\sigma_a^2 = \frac{c_0}{1+\theta_1^2+...+\theta_q^2} \qquad \textbf{(18)}$$

$$\theta_j = -\left(\frac{c_j}{\sigma_a^2}-\theta_1\theta_{j+1}-\theta_2\theta_{j+2}-...-\theta_{q-j}\theta_q\right) \qquad \textbf{(19)}$$

In order to begin the estimation of the MA parameters it is necessary to first set each equal to zero. The values are then updated with the values from equation (19) as they are obtained.

By employing the principle of least squares the parameters can by updated until values are obtained which minimize:

$$S(\mu,\phi,\theta) = \sum\,[w_t-E(W_t|previous\ w's)]^2 \qquad \textbf{(20)}$$

Corporate Bond Example

The method described in the previous section can present a rather tedious problem when applied to a large data set. Therefore, it is advantageous at this point to introduce the use of a computer software to aid in this process of identifying the optimum parameter estimations. A common statistical software package called Minitab can be very useful for this reason. Table 2 presents the various estimations of the AR, MA, and constant terms. The table presents each of the estimations as they progressed toward a minimization of equation (17).

Table 2 - Estimation of the Parameters for Corporate Bonds With a Rating of Aaa

Iteration	SSE	AR Parameter	MA Parameter	Constant
0	3.49817	0.100	0.100	0.085
1	2.50164	0.248	(0.048)	0.024
2	2.43197	0.124	(0.198)	0.021
3	2.36801	0.006	(0.348)	0.016
4	2.30970	(0.104)	(0.498)	0.009
5	2.27006	(0.197)	(0.648)	(0.002)
6	2.26799	(0.172)	(0.646)	(0.006)
7	2.26799	(0.172)	(0.646)	(0.007)
8	2.26799	(0.171)	(0.646)	(0.007)
9	2.26799	(0.171)	(0.646)	(0.007)

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	(0.17130)	0.224	(0.770)
Ma 1	(0.64710)	0.173	(3.740)
Constant	(0.00563)	0.036	(0.180)

In summary the following is the model being proposed for Corporate Bonds carrying a rating of Aaa from Moody's:

$$Y_t = \mu - 0.1713(Y_{t-1} - \mu) + 0.6471e_{t-1} - 0.00563 + e_t$$

Diagnostic Checking

The model now having been identified and estimates of the parameters obtained, it is necessary to perform some type of diagnostic checking of the fitted model. This step in the identification of the proper model is useful in determining the adequacy of the chosen model, as well as obtaining suggestions as to any modifications that may be needed before the model can be used in forecasting future values.

In order to test the adequacy of the model, a new set of fitted values is obtained using the assumed model. In the case of an assumed ARMA model the following general equation can be applied for obtaining the fitted values:

$$\hat{y}_t = \hat{\mu} + \hat{\phi}(y_{t-1} - \hat{\mu}) - \hat{\theta} e_{t-1} \quad \textbf{(21)}$$

The test of adequacy that will be discussed deals with detecting "non-randomness" in the residuals of the model. The residuals are defined to be the difference between the observations and the corresponding fitted values:

$$\hat{e}_t = y_t - \hat{y}_t, \quad t = 2, \ldots, N \quad \textbf{(22)}$$

At this point it is important to realize that in order to assume the given model is correct the e_t's need to be uncorrelated at any lag with a mean of zero and a common variance. In addition the residual autocorrelations can be defined as:

$$r_k(\hat{e}) = \frac{\sum_{t=1}^{N-k} \hat{e}_t \hat{e}_{t+k}}{\sum_{t=1}^{N} \hat{e}_t^2}, \quad k = 1, 2, \ldots, K \tag{23}$$

A useful test for determining the adequacy of the chosen model is provided by comparing a Chi-squared distribution with (K-r) degrees of freedom, where r is the number of parameters being estimated, to the related Chi-squared estimate obtained from the residual autocorrelations:

$$\chi^2 = N \sum_{k=1}^{K} r_k^2(\hat{e}) . \tag{24}$$

Too large a value for X^2 can be viewed as evidence against model adequacy and the process of model identification and parameter estimation must begin again.

Corporate Bond Example

Chi-square statistics are automatically computed at lags of 12, 24, 36, and 48 when the model generator is employed through Minitab. The following hypothesis is used in determining if the values obtained at each of these lags is significant:

H_0: The specified ARIMA model is adequate for describing the data.

H_1: The specified ARIMA model is not adequate for describing the data.

Table 4 summarizes the findings of this test for the data being studied.

Table 4 - Chi-square Test for Fit

Lag	Degrees of Freedom	p-value
12	10	0.503844
24	22	0.918268
36	34	0.982271
48	46	0.943293

From the large p-values presented in Table 4 it can be concluded that the ARIMA(1,1,1) model being proposed is an adequate model for describing the yield rate for corporate bonds carrying a rating of Aaa from Moody's.

Forecasting Future Values

One of the major objectives of the formation of the time series model is to be able to forecast future values of the series. The future values should follow the same trends as the past data and should be a reasonable continuation of the series.

A word of caution is in order at this point. Since the forecasting of future values depends on the behavior of past data it is important that the lead time is not too large. The model should be updated periodically as new data becomes available in order to accurately depict any new and unexpected trends that may arise.

At this point the last value in the original data set will be referred to as z_t and the future values z_{N+k} $k = 1, 2, \ldots, q$ will be generated from subsequent iterations of the model:

$$\hat{z}_t(k) = z_t + \phi \frac{(1-\phi^k)}{(1-\phi)} (z_t - z_{t-1}) - \theta \frac{(1-\phi^k)}{(1-\phi)} e_t + e_{t-1} \qquad (25)$$

Corporate Bond Example

The model developed in a previous section can be applied to the job of forecasting future yield rates of Aaa rated bonds by subsequent iteration of the following model formula:

$$\hat{z}_t(k) = z_t - 0.1713\frac{(1-\phi^k)}{(1-\phi)}(z_t - z_{t-1}) + 0.6471\frac{(1-\phi^k)}{(1-\phi)} - 0.00563$$

Once again the Minitab software was utilized in performing this iterative process in order to maintain a high degree of accuracy in the calculations. The range of k was only allowed to span from one to twelve in order to maintain the integrity of the model in forecasting the behavior of the data. Table 3 contains a summary of the first twelve forecasts obtained from the model.

Table 3 - Forecast of Future Values of Aaa Rated Corporate Bonds

Lead	Forecast	Lower Bound	Upper Bound
1993			
January	7.89976	7.54202	8.25749
February	7.90697	7.26927	8.54468
March	7.89921	7.08962	8.70879
April	7.89400	6.94048	8.84752
May	7.88836	6.81035	8.96637
June	7.88280	6.69319	9.07240
July	7.87722	6.58563	9.16880
August	7.87164	6.48556	9.25772
September	7.86606	6.39153	9.34060
October	7.86049	6.30251	9.41846
November	7.85491	6.21774	9.49208
December	7.84933	6.13663	9.56204

Brief Presentation of Additional Types of Corporate Bonds

The following sections contain a brief presentation of the results for all five types of investments presented in Appendix II. The data will be presented in the same order as in the detailed description of the method in the previous sections to aid in the identification of equations used for each of the calculations involved. It is assumed at this point that a simple presentation of the data with a limited amount of explanation will be adequate for the reader to understand the outcome.

The results for each of the five investment ratings will be presented in tabular form along side each other to aid in the comparison between them. The charts will be presented in the following form:

Autocorrelation Coefficients

Lag	Average Corporate	Aaa	Aa	A	Baa
1	0.927	0.920	0.919	0.927	0.924
2	0.829	0.811	0.810	0.829	0.824
3	0.735	0.705	0.710	0.736	0.734
4	0.645	0.613	0.617	0.645	0.650
5	0.565	0.536	0.537	0.563	0.571
6	0.504	0.471	0.482	0.508	0.515
7	0.466	0.430	0.452	0.477	0.487
8	0.425	0.392	0.418	0.441	0.460
9	0.373	0.348	0.371	0.395	0.419
10	0.330	0.320	0.335	0.354	0.372
11	0.282	0.282	0.287	0.305	0.322
12	0.222	0.235	0.227	0.242	0.255
13	0.164	0.182	0.171	0.182	0.187
14	0.118	0.133	0.127	0.137	0.141
15	0.083	0.099	0.092	0.102	0.110
16	0.056	0.073	0.066	0.076	0.088
17	0.036	0.054	0.049	0.059	0.072
18	0.026	0.042	0.038	0.050	0.065

Difference Values for the Average of all Corporations

Avg Corp	First Difference	Second Difference	Third Difference	Fourth Difference
9.04				
9.03	(0.010000)			
8.99	(0.040000)	(0.050000)		
9.35	0.360001	0.320001	0.310000	
9.82	0.469999	0.830000	0.790000	0.780000
9.87	0.050000	0.520000	0.880000	0.840000
9.92	0.050000	0.100000	0.570000	0.930000
10.14	0.220000	0.270000	0.320001	0.790000
10.64	0.500000	0.720000	0.770000	0.820001
10.97	0.330000	0.830000	1.050000	1.100000
10.54	(0.430000)	(0.100000)	0.400000	0.620000
10.59	0.050000	(0.380000)	(0.050000)	0.450000
10.37	(0.220000)	(0.170000)	(0.600000)	(0.270000)
9.89	(0.480000)	(0.700000)	(0.650000)	(1.080000)
9.86	(0.030001)	(0.510000)	(0.730000)	(0.680000)
10.15	0.290000	0.259999	(0.220000)	(0.440001)
10.37	0.220000	0.510000	0.480000	0.000000
10.36	(0.010000)	0.210000	0.500000	0.469999
10.47	0.110001	0.100000	0.320001	0.610001
10.58	0.110000	0.220000	0.210000	0.430000
10.28	(0.300000)	(0.190001)	(0.080000)	(0.090000)
9.90	(0.380000)	(0.680000)	(0.570001)	(0.460000)
9.91	0.010000	(0.370000)	(0.670000)	(0.560000)
10.03	0.120000	0.130000	(0.250000)	(0.550000)
10.05	0.020000	0.140000	0.150001	(0.230000)
10.05	0.000000	0.020000	0.140000	0.150001
10.18	0.130000	0.130000	0.150001	0.270000
10.14	(0.040000)	0.090000	0.090000	0.110001
9.95	(0.190001)	(0.230000)	(0.100000)	(0.100000)
9.50	(0.450000)	(0.640000)	(0.680000)	(0.550000)
9.34	(0.160000)	(0.610000)	(0.800000)	(0.840000)
9.36	0.020000	(0.140000)	(0.590000)	(0.780001)
9.41	0.050000	0.070000	(0.090000)	(0.540000)
9.34	(0.070000)	(0.020000)	0.000000	(0.160000)
9.32	(0.020000)	(0.090000)	(0.040000)	(0.020000)
9.30	(0.020000)	(0.040000)	(0.110000)	(0.059999)
9.43	0.130000	0.110001	0.090000	0.020000
9.64	0.210000	0.340000	0.320001	0.300000
9.73	0.089999	0.299999	0.429999	0.410000
9.82	0.090000	0.179999	0.389999	0.520000
9.87	0.050000	0.140000	0.230000	0.440000
9.67	(0.200000)	(0.150000)	(0.059999)	0.030000
9.65	(0.020000)	(0.220000)	(0.170000)	(0.080000)

Difference Values for the Average of all Corporations (Continued)

Avg Corp	First Difference	Second Difference	Third Difference	Fourth Difference
9.84	0.190001	0.170000	(0.030000)	0.020000
10.02	0.180000	0.370001	0.350000	0.150001
10.03	0.009999	0.190000	0.380000	0.360000
9.85	(0.179999)	(0.170000)	0.010000	0.200001
9.63	(0.220000)	(0.400000)	(0.390000)	(0.210000)
9.62	(0.010000)	(0.230000)	(0.410000)	(0.400001)
9.36	(0.260000)	(0.270000)	(0.490001)	(0.670000)
9.43	0.070001	(0.190000)	(0.200000)	(0.420000)
9.33	(0.100000)	(0.030000)	(0.290000)	(0.300000)
9.32	(0.010000)	(0.110001)	(0.040000)	(0.300000)
9.45	0.130000	0.120000	0.020000	0.090000
9.42	(0.030000)	0.100000	0.090000	(0.010000)
9.16	(0.260000)	(0.290000)	(0.160000)	(0.170000)
9.03	(0.130000)	(0.390000)	(0.420000)	(0.290000)
8.99	(0.040000)	(0.170000)	(0.430000)	(0.460000)
8.93	(0.059999)	(0.099999)	(0.230000)	(0.490000)
8.75	(0.180000)	(0.240000)	(0.280000)	(0.410000)
8.64	(0.110000)	(0.290000)	(0.349999)	(0.389999)
8.75	0.110000	0.000000	(0.180000)	(0.240000)
8.81	0.060000	0.170000	0.060000	(0.120000)
8.77	(0.040000)	0.020000	0.130000	0.020000
8.71	(0.060000)	(0.100000)	(0.040000)	0.070000
8.63	(0.080000)	(0.140000)	(0.180000)	(0.120000)
8.44	(0.190001)	(0.270000)	(0.330001)	(0.370001)
8.29	(0.150000)	(0.340000)	(0.420000)	(0.480000)
8.26	(0.030000)	(0.179999)	(0.370000)	(0.450000)
8.41	0.150000	0.120000	(0.030000)	(0.220000)
8.51	0.100000	0.250000	0.220000	0.070001
8.35	(0.160000)	(0.059999)	0.090000	0.060000

Difference Values for Aaa Corporations

Aaa	First Difference	Second Difference	Third Difference	Fourth Difference
8.36				
8.38	0.020000			
8.36	(0.020000)	0.00000		
8.85	0.490001	0.470000	0.490001	
9.33	0.480000	0.970000	0.950000	0.970000
9.32	(0.010000)	0.469999	0.960000	0.940000
9.42	0.100000	0.090000	0.570000	1.060000
9.67	0.250000	0.350000	0.340000	0.820000
10.18	0.510000	0.760000	0.860001	0.850000
10.52	0.340000	0.850000	1.100000	1.200001
10.01	(0.510000)	(0.170000)	0.340000	0.590000
10.11	0.099999	(0.410001)	(0.070001)	0.440000
9.88	(0.230000)	(0.130000)	(0.640000)	(0.300000)
9.40	(0.480000)	(0.710000)	(0.610001)	(1.120001)
9.39	(0.009999)	(0.490000)	(0.719999)	(0.620000)
9.67	0.280000	0.270000	(0.210000)	(0.440000)
9.90	0.230000	0.509999	0.500000	0.020000
9.86	(0.040000)	0.190000	0.469999	0.460000
9.96	0.100000	0.060000	0.290000	0.570000
10.11	0.150000	0.250000	0.210000	0.440000
9.82	(0.290000)	(0.140000)	(0.040000)	(0.080000)
9.51	(0.309999)	(0.599999)	(0.450000)	(0.349999)
9.45	(0.060000)	(0.370000)	(0.660000)	(0.510000)
9.57	0.120000	0.059999	(0.250000)	(0.540000)
9.62	0.050000	0.170000	0.110000	(0.200000)
9.64	0.020000	0.070001	0.190001	0.130000
9.80	0.160000	0.180000	0.230000	0.350000
9.79	(0.010000)	0.150000	0.170000	0.220000
9.57	(0.220000)	(0.230000)	(0.070001)	(0.050000)
9.10	(0.469999)	(0.690000)	(0.700000)	(0.540000)
8.93	(0.170000)	(0.639999)	(0.860000)	(0.870000)
8.96	0.030000	(0.140000)	(0.610000)	(0.830000)
9.01	0.050000	0.080000	(0.090000)	(0.559999)
8.92	(0.090000)	(0.040000)	(0.010000)	(0.180000)
8.89	(0.030000)	(0.120000)	(0.070000)	(0.040000)
8.86	(0.030001)	(0.060000)	(0.150001)	(0.100000)
8.99	0.130000	0.099999	0.070000	(0.020000)
9.22	0.230000	0.360001	0.330000	0.300000
9.37	0.150000	0.380000	0.510000	0.480000
9.46	0.090000	0.240000	0.470000	0.600000
9.47	0.010000	0.100000	0.250000	0.480000
9.26	(0.210000)	(0.200000)	(0.110000)	0.040000
9.24	(0.020000)	(0.230000)	(0.220000)	(0.130000)

Difference Values for Aaa Corporations (Continued)

Aaa	First Difference	Second Difference	Third Difference	Fourth Difference
9.41	0.170000	0.150000	(0.060000)	(0.050000)
9.56	0.150001	0.320001	0.300000	0.090000
9.53	(0.030001)	0.120000	0.290000	0.270000
9.30	(0.230000)	(0.260000)	(0.110000)	0.060000
9.05	(0.250000)	(0.480000)	(0.510000)	(0.360000)
9.04	(0.010000)	(0.260000)	(0.490000)	(0.520000)
8.83	(0.210000)	(0.220000)	(0.470000)	(0.700000)
8.93	0.100000	(0.110000)	(0.120000)	(0.370000)
8.86	(0.070001)	0.030000	(0.180000)	(0.190001)
8.86	0.000000	(0.070001)	0.030000	(0.180000)
9.01	0.150001	0.150001	0.080000	0.180000
9.00	(0.010000)	0.140000	0.140000	0.070000
8.75	(0.250000)	(0.260000)	(0.110000)	(0.110000)
8.61	(0.140000)	(0.390000)	(0.400001)	(0.250000)
8.55	(0.059999)	(0.200000)	(0.450000)	(0.460000)
8.48	(0.070001)	(0.130000)	(0.270000)	(0.520000)
8.31	(0.169999)	(0.240000)	(0.299999)	(0.440000)
8.20	(0.110001)	(0.280000)	(0.350000)	(0.410000)
8.29	0.090000	(0.020000)	(0.190000)	(0.260000)
8.35	0.060000	0.150001	0.040000	(0.129999)
8.33	(0.020000)	0.040000	0.130000	0.020000
8.28	(0.050000)	(0.070001)	(0.010000)	0.080000
8.22	(0.059999)	(0.110000)	(0.130000)	(0.070000)
8.07	(0.150001)	(0.210000)	(0.260000)	(0.280001)
7.95	(0.120000)	(0.270000)	(0.330000)	(0.380000)
7.92	(0.030000)	(0.150000)	(0.300000)	(0.360000)
7.99	0.070000	0.040000	(0.080000)	(0.230000)
8.10	0.110001	0.180000	0.150001	0.030001
7.98	(0.120000)	(0.010000)	0.060000	0.030000

Difference Values for Aa Corporations

Aa	First Difference	Second Difference	Third Difference	Fourth Difference
8.86				
8.88	0.020000			
8.84	(0.040000)	(0.020000)		
9.15	0.309999	0.270000	0.290000	
9.59	0.440001	0.750000	0.710000	0.730000
9.65	0.059999	0.500000	0.809999	0.770000
9.64	(0.009999)	0.050000	0.490001	0.800000
9.86	0.219999	0.210000	0.270000	0.710000
10.35	0.490001	0.710000	0.700001	0.760000
10.74	0.389999	0.880000	1.099999	1.090000
10.27	(0.469999)	(0.080000)	0.410001	0.630000
10.33	0.059999	(0.410000)	(0.020000)	0.470000
10.09	(0.240000)	(0.180000)	(0.650000)	(0.260000)
9.60	(0.490000)	(0.730000)	(0.670000)	(1.139999)
9.59	(0.010000)	(0.500000)	(0.740000)	(0.680000)
9.86	0.270000	0.259999	(0.230000)	(0.470000)
10.10	0.240001	0.510000	0.500000	0.010000
10.13	0.030000	0.270000	0.540000	0.530000
10.26	0.130000	0.160000	0.400001	0.670000
10.37	0.110000	0.240000	0.270000	0.510000
10.06	(0.309999)	(0.200000)	(0.070000)	(0.040000)
9.71	(0.350000)	(0.660000)	(0.550000)	(0.420000)
9.72	0.010000	(0.340000)	(0.650000)	(0.540000)
9.81	0.090000	0.100000	(0.250000)	(0.559999)
9.81	0.000000	0.090000	0.100000	(0.250000)
9.83	0.020000	0.020000	0.110000	0.120000
9.98	0.150000	0.169999	0.169999	0.259999
9.94	(0.040000)	0.110000	0.129999	0.129999
9.75	(0.190000)	(0.230000)	(0.080000)	(0.060000)
9.29	(0.460000)	(0.650000)	(0.690000)	(0.540000)
9.14	(0.150000)	(0.610000)	(0.799999)	(0.839999)
9.14	0.000000	(0.150000)	(0.610000)	(0.799999)
9.23	0.089999	0.089999	(0.060000)	(0.520000)
9.19	(0.040000)	0.049999	0.049999	(0.100000)
9.14	(0.049999)	(0.089999)	0.000000	0.000000
9.11	(0.030001)	(0.080000)	(0.120000)	(0.030001)
9.27	0.160001	0.130000	0.080001	0.040001
9.44	0.169999	0.330000	0.299999	0.250000
9.51	0.070001	0.240000	0.400001	0.370000
9.64	0.130000	0.200001	0.370000	0.530001
9.70	0.059999	0.190000	0.260000	0.429999
9.49	(0.210000)	(0.150001)	(0.020000)	0.050000

Difference Values for Aa Corporations (Continued)

Aa	First Difference	Second Difference	Third Difference	Fourth Difference
9.47	(0.020000)	(0.230000)	(0.170000)	(0.040000)
9.63	0.160000	0.140000	(0.070000)	(0.010000)
9.77	0.140000	0.300000	0.280001	0.070001
9.77	0.000000	0.140000	0.300000	0.280001
9.59	(0.180000)	(0.180000)	(0.040000)	0.120000
9.39	(0.200000)	(0.380000)	(0.380000)	(0.240000)
9.37	(0.020000)	(0.220000)	(0.400001)	(0.400001)
9.16	(0.210000)	(0.230000)	(0.430000)	(0.610001)
9.21	0.050000	(0.160000)	(0.180000)	(0.380000)
9.12	(0.090000)	(0.040000)	(0.250000)	(0.270000)
9.15	0.030000	(0.060000)	(0.010000)	(0.220000)
9.28	0.130000	0.160000	0.070000	0.120000
9.25	(0.030000)	0.100000	0.130000	0.040000
8.99	(0.260000)	(0.290000)	(0.160000)	(0.130000)
8.86	(0.130000)	(0.390000)	(0.420000)	(0.290000)
8.83	(0.030000)	(0.160000)	(0.420000)	(0.450000)
8.78	(0.050000)	(0.080000)	(0.210000)	(0.470000)
8.61	(0.170000)	(0.220000)	(0.250000)	(0.380000)
8.51	(0.099999)	(0.270000)	(0.320000)	(0.349999)
8.67	0.160000	0.060000	(0.110000)	(0.160000)
8.73	0.059999	0.219999	0.120000	(0.050000)
8.69	(0.040000)	0.020000	0.179999	0.080000
8.63	(0.059999)	(0.099999)	(0.040000)	0.120000
8.56	(0.070000)	(0.129999)	(0.169999)	(0.110000)
8.37	(0.190001)	(0.260000)	(0.320000)	(0.360000)
8.21	(0.160000)	(0.350000)	(0.420000)	(0.480000)
8.17	(0.040000)	(0.200000)	(0.390000)	(0.460000)
8.32	0.150000	0.110000	(0.050000)	(0.240001)
8.40	0.080000	0.230000	0.190000	0.030000
8.24	(0.160000)	(0.080000)	0.070000	0.030000

Difference Values for A Corporations

A	First Difference	Second Difference	Third Difference	Fourth Difference
9.23				
9.20	(0.030000)			
9.13	(0.070000)	(0.099999)		
9.36	0.230000	0.160000	0.130000	
9.83	0.470000	0.700000	0.630000	0.600000
9.98	0.150000	0.620000	0.849999	0.780000
10.00	0.020000	0.170000	0.640000	0.870000
10.20	0.200000	0.220000	0.370000	0.840000
10.72	0.520000	0.720000	0.740001	0.890000
10.98	0.259999	0.780000	0.980000	1.000000
10.63	(0.349999)	(0.090000)	0.430000	0.630000
10.62	(0.010000)	(0.360000)	(0.100000)	0.420000
10.43	(0.190000)	(0.200000)	(0.549999)	(0.290000)
9.94	(0.490001)	(0.680000)	(0.690001)	(1.040000)
9.89	(0.049999)	(0.540000)	(0.730000)	(0.740000)
10.17	0.280000	0.230000	(0.260000)	(0.450000)
10.41	0.240000	0.520000	0.470000	(0.020000)
10.42	0.010000	0.250000	0.530000	0.480000
10.55	0.130000	0.140000	0.380000	0.660000
10.63	0.080000	0.210000	0.220000	0.460000
10.34	(0.290000)	(0.210000)	(0.080000)	(0.070000)
9.99	(0.350000)	(0.640000)	(0.560000)	(0.430000)
9.99	0.000000	(0.350000)	(0.640000)	(0.560000)
10.11	0.120000	0.120000	(0.230000)	(0.520000)
10.10	(0.009999)	0.110001	0.110001	(0.240000)
10.13	0.030000	0.020000	0.140000	0.140000
10.26	0.130000	0.160000	0.150001	0.270000
10.20	(0.060000)	0.070000	0.099999	0.090000
10.00	(0.200000)	(0.260000)	(0.130000)	(0.100000)
9.59	(0.410000)	(0.610000)	(0.670000)	(0.540000)
9.42	(0.170000)	(0.580000)	(0.780000)	(0.840000)
9.45	0.030000	(0.140000)	(0.550000)	(0.750000)
9.51	0.060000	0.090000	(0.080000)	(0.490000)
9.44	(0.070001)	(0.010000)	0.020000	(0.150001)
9.42	(0.020000)	(0.090000)	(0.030000)	0.000000
9.39	(0.030000)	(0.049999)	(0.120000)	(0.059999)
9.54	0.150000	0.120000	0.100000	0.030000
9.75	0.210000	0.360000	0.330000	0.310000
9.82	0.070000	0.280000	0.429999	0.400000
9.89	0.070001	0.140000	0.350000	0.500000
9.89	0.000000	0.070001	0.140000	0.350000
9.70	(0.190001)	(0.190001)	(0.120000)	(0.050000)
9.69	(0.010000)	(0.200001)	(0.200001)	(0.130000)

Difference Values for A Corporations (Continued)

A	First Difference	Second Difference	Third Difference	Fourth Difference
9.89	0.200001	0.190001	0.000000	0.000000
10.09	0.200000	0.400001	0.390000	0.200000
10.06	(0.030000)	0.170000	0.370001	0.360001
9.88	(0.180000)	(0.210000)	(0.010000)	0.190001
9.64	(0.240000)	(0.420000)	(0.450000)	(0.250000)
9.61	(0.030001)	(0.270000)	(0.450001)	(0.480000)
9.38	(0.230000)	(0.260000)	(0.500000)	(0.680000)
9.50	0.120000	(0.110000)	(0.140000)	(0.380000)
9.39	(0.110000)	0.010000	(0.219999)	(0.250000)
9.41	0.020000	(0.090000)	0.030000	(0.200000)
9.55	0.140000	0.160000	0.050000	0.170000
9.51	(0.040000)	0.100000	0.120000	0.010000
9.26	(0.250000)	(0.290000)	(0.150000)	(0.130000)
9.11	(0.150001)	(0.400001)	(0.440001)	(0.300000)
9.08	(0.030000)	(0.180000)	(0.430000)	(0.470000)
9.01	(0.070000)	(0.099999)	(0.250000)	(0.500000)
8.82	(0.190001)	(0.260000)	(0.290000)	(0.440001)
8.72	(0.099999)	(0.290000)	(0.360000)	(0.389999)
8.83	0.110000	0.010000	(0.180000)	(0.250000)
8.89	0.060000	0.170000	0.070001	(0.120000)
8.87	(0.020000)	0.040000	0.150000	0.050000
8.81	(0.059999)	(0.080000)	(0.020000)	0.090000
8.70	(0.110001)	(0.170000)	(0.190001)	(0.130000)
8.49	(0.210000)	(0.320001)	(0.380000)	(0.400001)
8.34	(0.150000)	(0.360000)	(0.470000)	(0.530000)
8.31	(0.030000)	(0.179999)	(0.389999)	(0.500000)
8.49	0.179999	0.150000	0.000000	(0.210000)
8.58	0.090000	0.270000	0.240000	0.090000
8.37	(0.210000)	(0.120000)	0.059999	0.030000

Difference Values for Baa Corporations

Baa	First Difference	Second Difference	Third Difference	Fourth Difference
9.72				
9.20	(0.520000)			
9.13	(0.070000)	(0.590000)		
9.36	0.230000	0.160000	(0.360001)	
9.83	0.470000	0.700000	0.630000	0.110000
9.98	0.150000	0.620000	0.849999	0.780000
10.00	0.020000	0.170000	0.640000	0.870000
10.20	0.200000	0.220000	0.370000	0.840000
10.72	0.520000	0.720000	0.740001	0.890000
10.98	0.259999	0.780000	0.980000	1.000000
10.63	(0.349999)	(0.090000)	0.430000	0.630000
10.62	(0.010000)	(0.360000)	(0.100000)	0.420000
10.43	(0.190000)	(0.200000)	(0.549999)	(0.290000)
9.94	(0.490001)	(0.680000)	(0.690001)	(1.040000)
9.89	(0.049999)	(0.540000)	(0.730000)	(0.740000)
10.17	0.280000	0.230000	(0.260000)	(0.450000)
10.41	0.240000	0.520000	0.470000	(0.020000)
10.42	0.010000	0.250000	0.530000	0.480000
10.55	0.130000	0.140000	0.380000	0.660000
10.63	0.080000	0.210000	0.220000	0.460000
10.34	(0.290000)	(0.210000)	(0.080000)	(0.070000)
9.99	(0.350000)	(0.640000)	(0.560000)	(0.430000)
9.99	0.000000	(0.350000)	(0.640000)	(0.560000)
10.11	0.120000	0.120000	(0.230000)	(0.520000)
10.10	(0.009999)	0.110001	0.110001	(0.240000)
10.13	0.030000	0.020000	0.140000	0.140000
10.26	0.130000	0.160000	0.150001	0.270000
10.20	(0.060000)	0.070000	0.099999	0.090000
10.00	(0.200000)	(0.260000)	(0.130000)	(0.100000)
9.59	(0.410000)	(0.610000)	(0.670000)	(0.540000)
9.42	(0.170000)	(0.580000)	(0.780000)	(0.840000)
9.45	0.030000	(0.140000)	(0.550000)	(0.750000)
9.51	0.060000	0.090000	(0.080000)	(0.490000)
9.44	(0.070001)	(0.010000)	0.020000	(0.150001)
9.42	(0.020000)	(0.090000)	(0.030000)	0.000000
9.39	(0.030000)	(0.049999)	(0.120000)	(0.059999)
9.54	0.150000	0.120000	0.100000	0.030000
9.75	0.210000	0.360000	0.330000	0.310000
9.82	0.070000	0.280000	0.429999	0.400000
9.89	0.070001	0.140000	0.350000	0.500000
9.89	0.000000	0.070001	0.140000	0.350000
9.70	(0.190001)	(0.190001)	(0.120000)	(0.050000)
9.69	(0.010000)	(0.200001)	(0.200001)	(0.130000)

Difference Values for Baa Corporations (Continued)

Baa	First Difference	Second Difference	Third Difference	Fourth Difference
9.89	0.200001	0.190001	0.000000	0.000000
10.09	0.200000	0.400001	0.390000	0.200000
10.06	(0.030000)	0.170000	0.370001	0.360001
9.88	(0.180000)	(0.210000)	(0.010000)	0.190001
9.64	(0.240000)	(0.420000)	(0.450000)	(0.250000)
9.61	(0.030001)	(0.270000)	(0.450001)	(0.480000)
9.38	(0.230000)	(0.260000)	(0.500000)	(0.680000)
9.50	0.120000	(0.110000)	(0.140000)	(0.380000)
9.39	(0.110000)	0.010000	(0.219999)	(0.250000)
9.41	0.020000	(0.090000)	0.030000	(0.200000)
9.55	0.140000	0.160000	0.050000	0.170000
9.51	(0.040000)	0.100000	0.120000	0.010000
9.26	(0.250000)	(0.290000)	(0.150000)	(0.130000)
9.11	(0.150001)	(0.400001)	(0.440001)	(0.300000)
9.08	(0.030000)	(0.180000)	(0.430000)	(0.470000)
9.01	(0.070000)	(0.099999)	(0.250000)	(0.500000)
8.82	(0.190001)	(0.260000)	(0.290000)	(0.440001)
8.72	(0.099999)	(0.290000)	(0.360000)	(0.389999)
8.83	0.110000	0.010000	(0.180000)	(0.250000)
8.89	0.060000	0.170000	0.070001	(0.120000)
8.87	(0.020000)	0.040000	0.150000	0.050000
8.81	(0.059999)	(0.080000)	(0.020000)	0.090000
8.70	(0.110001)	(0.170000)	(0.190001)	(0.130000)
8.49	(0.210000)	(0.320001)	(0.380000)	(0.400001)
8.34	(0.150000)	(0.360000)	(0.470000)	(0.530000)
8.31	(0.030000)	(0.179999)	(0.389999)	(0.500000)
8.49	0.179999	0.150000	0.000000	(0.210000)
8.58	0.090000	0.270000	0.240000	0.090000
8.37	(0.210000)	(0.120000)	0.059999	0.030000

Autocorrelation Coefficients for First Difference

Lag	Average Corporate	AAA	AA	A	Baa
1	0.360	0.344	0.338	0.400	0.373
2	-0.110	-0.089	-0.122	-0.121	-0.153
3	-0.047	-0.036	-0.075	-0.083	-0.164
4	-0.013	-0.024	-0.042	-0.037	-0.065
5	-0.135	-0.077	-0.155	-0.209	-0.197
6	-0.172	-0.172	-0.178	-0.177	-0.201
7	0.106	0.063	0.116	0.136	0.024
8	0.246	0.205	0.245	0.217	0.144
9	-0.034	-0.073	-0.020	-0.015	0.044
10	-0.047	-0.057	-0.018	-0.001	-0.005
11	0.077	0.085	0.066	0.103	0.122
12	-0.026	0.048	-0.057	-0.034	0.053
13	0.146	-0.111	-0.138	-0.160	-0.139
14	-0.103	-0.120	-0.073	-0.093	-0.138
15	-0.022	0.003	-0.020	-0.047	-0.090
16	0.013	0.044	0.008	-0.010	-0.015
17	0.014	0.016	0.056	0.039	0.008
18	0.047	0.041	0.073	0.097	0.069

Estimation of Parameters

Average Corporate

Iteration	SSE	AR Parameter	MA Parameter	Constant
0	3.28140	0.100	0.100	0.081
1	2.29191	0.250	(0.047)	0.023
2	2.21601	0.129	(0.197)	0.020
3	2.14466	0.013	(0.347)	0.016
4	2.07881	(0.092)	(0.497)	0.010
5	2.03396	(0.158)	(0.647)	(0.005)
6	2.02884	(0.066)	(0.585)	(0.011)
7	2.02851	(0.090)	(0.609)	(0.011)
8	2.02846	(0.080)	(0.600)	(0.011)
9	2.02845	(0.084)	(0.600)	(0.011)
10	2.02845	(0.083)	(0.603)	(0.011)
11	2.02845	(0.083)	(0.602)	(0.011)
12	2.02845	(0.083)	(0.602)	(0.011)

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	(0.08300)	0.222	(0.370)
Ma 1	(0.60230)	0.177	(3.400)
Constant	(0.01128)	(0.033)	(0.340)

Residuals:

SS = 2.02747
MS = 0.02982
DF = 68

Estimation of Parameters

Aaa

Iteration	SSE	AR Parameter	MA Parameter	Constant
0	3.49817	0.100	0.100	0.085
1	2.50164	0.248	(0.048)	0.024
2	2.43197	0.124	(0.198)	0.021
3	2.36801	0.006	(0.348)	0.016
4	2.30970	(0.104)	(0.498)	0.009
5	2.27006	(0.197)	(0.648)	(0.002)
6	2.26799	(0.172)	(0.646)	(0.006)
7	2.26799	(0.172)	(0.646)	(0.007)
8	2.26799	(0.171)	(0.646)	(0.007)
9	2.26799	(0.171)	(0.646)	(0.007)

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	(0.17130)	0.224	(0.770)
Ma 1	(0.64710)	0.173	(3.740)
Constant	(0.00563)	0.036	(0.180)

Residuals:

SS = 2.26438
MS = 0.0333
DF = 68

Estimation of Parameters

Aa

Iteration	SSE	AR Parameter	MA Parameter	Constant
0	3.25758	0.100	0.100	0.820
1	2.29765	0.243	(0.045)	0.210
2	2.23042	0.118	(0.195)	0.018
3	2.16914	(0.002)	(0.345)	0.014
4	2.11541	(0.110)	(0.495)	0.007
5	2.08824	(0.157)	(0.624)	(0.009)
6	2.08522	(0.088)	(0.561)	(0.010)
7	2.08511	(0.104)	(0.575)	(0.010)
8	2.08511	(0.099)	(0.571)	(0.010)
9	2.08510	(0.101)	(0.572)	(0.010)
10	2.08510	(0.100)	(0.572)	(0.010)
11	2.08510	(0.100)	(0.572)	(0.010)

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	(0.10030)	0.242	(0.410)
Ma 1	(0.57220)	0.199	(2.880)
Constant	(0.01005)	0.033	(0.310)

Residuals:

SS = 2.08349
MS = 0.3064
DF = 68

Estimation of Parameters

A

Iteration	SSE	AR Parameter	MA Parameter	Constant
0	3.13718	0.100	0.100	0.079
1	2.13671	0.250	(0.046)	0.027
2	2.04676	0.134	(0.196)	0.026
3	1.95960	0.025	(0.346)	0.023
4	1.87488	(0.070)	(0.496)	0.017
5	1.79714	(0.102)	(0.646)	(0.001)
6	1.78081	0.048	(0.557)	(0.011)
7	1.77990	0.001	(0.609)	(0.013)
8	1.77936	0.035	(0.576)	(0.013)
9	1.77921	0.014	(0.596)	(0.013)
10	1.77914	0.027	(0.583)	(0.013)
11	1.77912	0.019	(0.591)	(0.013)
12	1.77911	0.024	(0.586)	(0.013)
13	1.77911	0.021	(0.589)	(0.013)
14	1.77911	0.023	(0.587)	(0.013)
15	1.77910	0.022	(0.588)	(0.013)
16	1.77910	0.022	(0.588)	(0.013)

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.02200	0.204	0.110
Ma 1	(0.58790)	0.164	(3.590)
Constant	(0.01331)	0.030	(0.440)

Residuals:

SS = 1.77892
MS = 0.02616
DF = 68

Estimation of Parameters

Baa

Iteration	SSE	AR Parameter	MA Parameter	Constant
0	3.39143	0.100	0.100	0.073
1	2.36231	0.231	(0.032)	0.011
2	2.27789	0.113	(0.182)	0.088
3	2.19475	0.002	(0.332)	0.005
4	2.10861	(0.089)	(0.482)	(0.001)
5	2.00803	(0.073)	(0.632)	(0.019)
6	1.99176	0.069	(0.544)	(0.022)
7	1.99090	0.057	(0.568)	(0.024)
8	1.99081	0.068	(0.560)	(0.023)
9	1.99080	0.067	(0.562)	(0.023)
10	1.99080	0.067	(0.562)	(0.023)

Final Estimates of Parameters

Type	Estimate	St. Dev.	t-ratio
AR 1	0.06730	0.198	0.340
Ma 1	(0.56180)	0.163	(3.450)
Constant	(0.02348)	0.031	(0.760)

Residuals:

SS = 1.91342
MS = 0.02814
DF = 68

Diagnostic Checking

Average Corporate

Lag	Degrees of Freedom	p-value
12	10	0.381309
24	22	0.849042
36	34	0.967114
48	46	0.879002

Aaa

Lag	Degrees of Freedom	p-value
12	10	0.503844
24	22	0.918268
36	34	0.982271
48	46	0.943293

Aa

Lag	Degrees of Freedom	p-value
12	10	0.423074
24	22	0.918268
36	34	0.980642
48	46	0.933644

Diagnostic Checking

A

Lag	Degrees of Freedom	p-value
12	10	0.229260
24	22	0.583036
36	34	0.924807
48	46	0.809709

Baa

Lag	Degrees of Freedom	p-value
12	10	0.397721
24	22	0.711898
36	34	0.942602
48	46	0.849431

At this point it can be concluded from the large p-values that an ARIMA(1,1,1) model is appropriate for each of the bond ratings. The models used from this point on will be comprised of the final estimates of the parameters indicated in each of the individual estimation tables.

Forecast of Future Values

Average Corporation

Lead	Forecast	Lower Bound	Upper Bound
1993			
January	8.25644	7.91794	8.59495
February	8.25293	7.63723	8.86863
March	8.24194	7.44899	9.03490
April	8.23158	7.29366	9.16950
May	8.22116	7.15791	9.28441
June	8.21075	7.03545	9.38604
July	8.20033	6.92278	9.47789
August	8.18992	6.81771	9.56213
September	8.17951	6.71876	9.64025
October	8.16909	6.62488	9.71331
November	8.15868	6.53529	9.78207
December	8.14827	6.44938	9.84716

Aaa

Lead	Forecast	Lower Bound	Upper Bound
1993			
January	7.89976	7.54202	8.25749
February	7.90697	7.26927	8.54468
March	7.89921	7.08962	8.70879
April	7.89400	6.94048	8.84752
May	7.88836	6.81035	8.96637
June	7.88280	6.69319	9.07240
July	7.87722	6.58563	9.16880
August	7.87164	6.48556	9.25772
September	7.86606	6.39153	9.34060
October	7.86049	6.30251	9.41846
November	7.85491	6.21774	9.49208
December	7.84933	6.13663	9.56204

Forecast of Future Values

Aa

Lead	Forecast	Lower Bound	Upper Bound
1993			
January	8.15993	7.81677	8.50308
February	8.15791	7.54729	8.76853
March	8.14806	7.36587	8.93025
April	9.13900	7.21575	9.06225
May	8.12986	7.08450	9.17522
June	8.12073	6.96608	9.27537
July	8.11159	6.85716	9.36603
August	8.10246	6.75560	9.44932
September	8.09333	6.66000	9.52666
October	8.08420	6.56932	9.59908
November	8.07506	6.48281	9.66732
December	8.06593	6.39989	9.73197

A

Lead	Forecast	Lower Bound	Upper Bound
1993			
January	8.23898	7.92190	8.55606
February	8.22278	7.62185	8.82372
March	8.20912	7.41787	9.00360
April	8.19551	7.25152	9.13949
May	8.18190	7.10665	9.25714
June	8.16828	6.97615	9.36042
July	8.15467	6.85612	9.45323
August	8.14106	6.74418	9.53795
September	8.12745	6.63872	9.61619
October	8.11384	6.53861	9.68908
November	8.10023	6.44300	9.75746
December	8.08662	6.35127	9.82197

Forecast of Future Values

Baa

Lead	Forecast	Lower Bound	Upper Bound
1993			
January	8.22651	7.89766	8.55535
February	8.19336	7.56475	8.82196
March	8.16764	7.33261	9.00267
April	8.14242	7.14221	9.14264
May	8.11724	6.97546	9.25902
June	8.09206	6.82443	9.35969
July	8.06688	6.68481	9.44895
August	8.04170	6.55396	9.52944
September	8.01652	6.43014	9.60290
October	7.99134	6.31211	9.67057
November	7.96616	6.19894	9.73338
December	7.94098	6.08995	9.79200

Appendix I

Key to Moody's Corporate Ratings*

Aaa

Bonds which are rated Aaa are judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged." Interest payments are protected by a large or by an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes ar can be visualized are most unlikely to impair the fundamentally strong position of such issues.

Aa

Bonds which are rated Aa are judged to be of high quality by all standards. Together with the Aaa group they comprise what are generally known as high grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present which make the long-term risk appear somewhat larger than the Aaa securities.

A

Bonds which are rated A Possess many favorable investment attributes and are to be considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present which suggest a susceptibility to impairment some time in the future.

Baa

Bonds which are rated Baa are considered as medium-grade obligations, (i.e., they are neither highly protected not poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be

Key to Moody's Corporate Ratings (Continued)

characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and in fact have speculative characteristics as well.

Ba

Bonds which are rated Ba are judged to have speculative elements; their future cannot be considered as well-assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.

B

Bonds which are rated B generally lack characteristics of the desirable investment. Assurance of interest and principal payments or of maintenance of other terms of the contract over any long period of time may be small.

Caa

Bonds which are rated Caa are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.

Ca

Bonds which are rated Ca represent obligations which are speculative in a high degree. Such issues are often in default or have other marked shortcomings.

Key to Moody's Corporate Ratings (Continued)

C

Bonds which are rated C are the lowest rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.

* The above table is a complete quote of the key presented in the most recent publication of Moody's Bond Record.

Appendix II

Moody's Corporate Bond Yield Averages

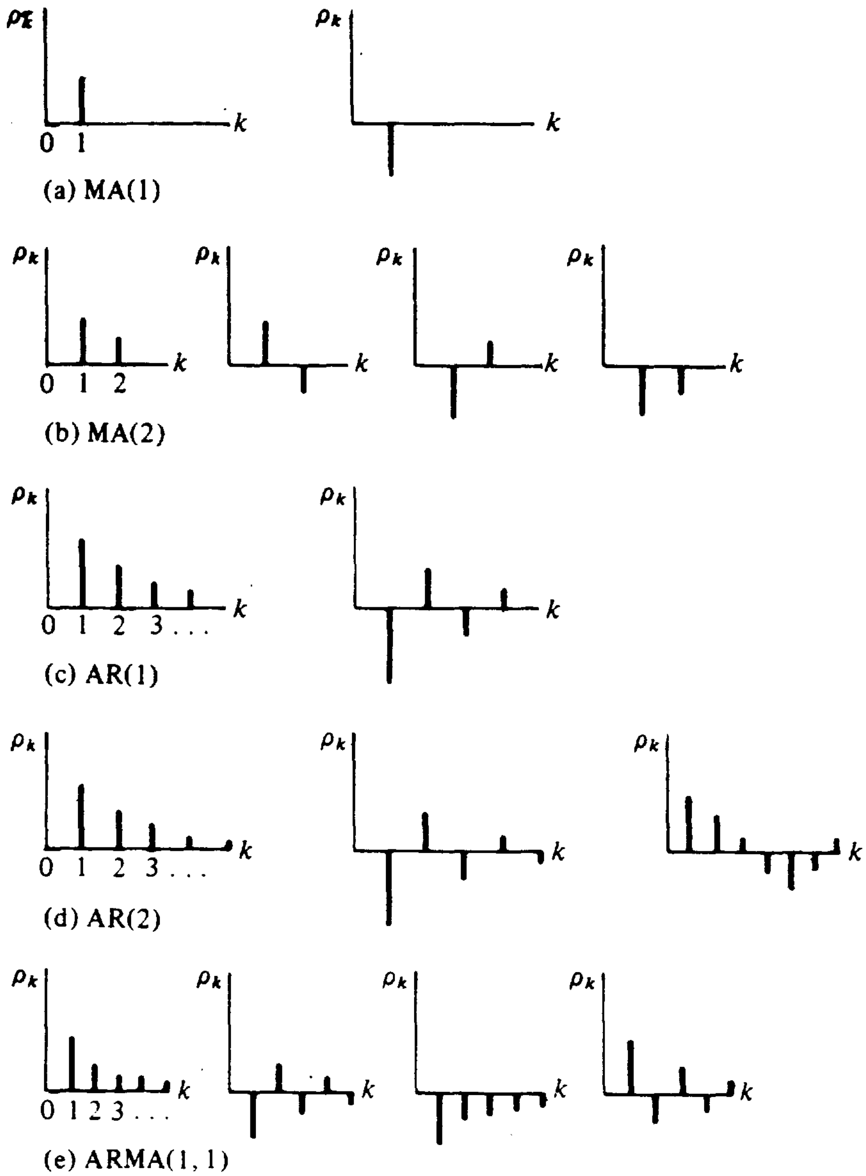
Date	Average Corporate	Aaa	Aa	A	Baa
1987					
January	9.04	8.36	8.86	9.23	9.72
February	9.03	8.38	8.88	9.20	9.65
March	8.99	8.36	8.84	9.13	9.61
April	9.35	8.85	9.15	9.36	10.04
May	9.82	9.33	9.59	9.83	10.51
June	9.87	9.32	9.65	9.98	10.52
July	9.92	9.42	9.64	10.00	10.61
August	10.14	9.67	9.86	10.20	10.80
September	10.64	10.18	10.35	10.72	11.31
October	10.97	10.52	10.74	10.98	11.62
November	10.54	10.01	10.27	10.63	11.23
December	10.59	10.11	10.33	10.62	11.29
1988					
January	10.37	9.88	10.09	10.43	11.07
February	9.89	9.40	9.60	9.94	10.62
March	9.86	9.39	9.59	9.89	10.57
April	10.15	9.67	9.86	10.17	10.90
May	11.37	9.90	10.10	10.41	11.04
June	10.36	9.86	10.13	10.42	11.00
July	10.47	9.96	10.26	10.55	11.11
August	10.58	10.11	10.37	10.63	11.21
September	10.28	9.82	10.06	10.34	10.90
October	9.90	9.51	9.71	9.99	10.41
November	9.91	9.45	9.72	9.99	10.48
December	10.03	9.57	9.81	10.11	10.65
1989					
January	10.05	9.62	9.81	10.10	10.65
February	10.05	9.64	9.83	10.13	10.61
March	10.18	9.80	9.98	10.26	10.67
April	10.14	9.79	9.94	10.20	10.61
May	9.95	9.57	9.75	10.00	10.46
June	9.50	9.10	9.29	9.59	10.03
July	9.34	8.93	9.14	9.42	9.87
August	9.36	8.98	9.14	9.45	9.88
September	9.41	9.01	9.23	9.51	9.91
October	9.34	8.92	9.19	9.44	9.81
November	9.32	8.89	9.14	9.42	9.81
December	9.30	8.86	9.11	9.36	9.82

Moody's Corporate Bond Yield Averages (Continued)

Date	Average Corporate	Aaa	Aa	A	Baa
1990					
January	9.43	8.99	9.27	9.54	9.94
February	9.64	9.22	9.44	9.75	10.14
March	9.73	9.37	9.51	9.82	10.21
April	9.82	9.46	9.64	9.89	10.30
May	9.87	9.47	9.70	9.89	10.41
June	9.67	9.26	9.49	9.70	10.22
July	9.65	9.24	9.47	9.69	10.20
August	9.84	9.41	9.63	9.89	10.41
September	10.02	9.56	9.77	10.09	10.64
October	10.03	9.53	9.77	10.06	10.74
November	9.85	9.30	9.59	9.88	10.62
December	9.63	9.05	9.39	9.64	10.43
1991					
January	9.62	9.04	9.37	9.61	10.45
February	9.36	8.83	9.16	9.38	10.07
March	9.43	8.93	9.21	9.50	10.09
April	9.33	8.86	9.12	9.39	9.94
May	9.32	8.86	9.15	9.41	9.86
June	9.45	9.01	9.28	9.55	9.96
July	9.42	9.00	9.25	9.51	9.89
August	9.16	8.75	8.99	9.26	9.65
September	9.03	8.61	8.86	9.11	9.51
October	8.99	8.55	8.83	9.08	9.49
November	8.93	8.48	8.78	9.01	9.45
December	8.75	8.31	8.61	8.82	9.26
1992					
January	8.64	8.20	8.51	8.72	9.13
February	8.75	8.29	8.67	8.83	9.23
March	8.81	8.35	8.73	8.89	9.25
April	8.77	8.33	8.69	8.87	9.21
May	8.71	8.28	8.63	8.81	9.13
June	8.63	8.22	8.56	8.70	9.05
July	8.44	8.07	8.37	8.49	8.84
August	8.29	7.95	8.21	9.34	8.65
September	8.26	7.92	8.17	8.31	8.62
October	8.41	7.99	8.32	8.49	8.84
November	8.51	8.10	8.40	8.58	8.96
December	8.35	7.98	8.24	8.37	8.81

Appendix III

Common Correlograms*



*The above correlograms have been extracted from citation [18].

References

[1] Anderson, O. D. *Analyzing Time Series*. New York: North-Holland Publishing Co., 1980.

[2] Anderson, Oliver, D. B.Sc., M.A., M.Sc., A.F.I.M.A., M.I.S., F.S.S. *Time Series Analysis and Forecasting: The Box-Jenkins Approach*. Reading, Mass.: Butterworth Inc., 1975.

[3] Bowerman, Bruce, L, and O'Connell, Richard, T. *Forecasting and Time Series: An Applied Approach*. Belmont, California: Duxbury Press, 1993.

[4] Box, G. E. P., and Jenkins, G. W. *Time Series Analysis Forecasting and Control*. San Francisco, Calif.: Holden-Day, 1970.

[5] Box, G. E. P., and Pierce, D. A. "Distribution of Residual Autocorrelations in Autoregressive-integrated Moving Average Time Series Models," *Journal of the American Statistical Association*, LXV (1970), 1509-26.

[6] Butler, William, F., Kavesh, Robert, A., and Platt, Robert, B. *Methods and Techniques of Business Forecasting*. Englewood Cliffs, New Jersey: Prentice-Hall Inc., 1966.

[7] Chambers, J. C., Mullick, S. K., and Smith, D. D. "How to choose the Right Forecasting Technique," *Harvard Business Review*, July-August, 1971.

[8] Chatfield, C. *The Analysis of Time Series: an Introduction*. New York: Chapman and Hall, 1989.

[9] Graupe, Daniel. *Time Series Analysis, Identification and adaptive filtering*. Malabar, Florida: Robert E. Krieger Publishing Co., 1984.

[10] Greville, T. N. E. "Discussion of 'Six Term Formulas for Routine Actuarial Interpolation,'" RAIA, XXXIV (1945), 35-39.

[11] Harvey, Andrew, C. *Forecasting, Structural Time Series Models and the Kalman Filter.* New York: Cambridge University Press, 1989.

[12] Holt, Charles, A., and Shore, Robert, W. *Bayesian Analysis in Economic Theory and Time Series Analysis.* New York: North-Holland Publishing Co., 1980.

[13] Jenkins, G. M., and Watts, D. G. *Spectral Analysis and Its Applications.* San Francisco, Calif.: Holden-Day, 1968.

[14] Jenkins, W. A., and Lew, E. A. "A New Mortality Basis for Annuities," TSA, I (1949), 369-466.

[15] Kendall, Maurice, G., M.A., Sc.D., F.B.A. *Time-Series.* London: Charles Griffin and Company Limited, 1976.

[16] Leuthold, R. M., MacCormick, A. J. A., Schmitz, A., and Watts, D. G. "Forecasting Daily Hog Prices and Quantities: a Study of Alternative Forecasting Techniques," *Journal of the American Statistical Association*, LXV (1970), 90-107.

[17] Macaulay, Frederick, R. *Some Theoretical Problems Suggested by the Movements of interest Rates, Bond Yields and Stock Prices in the United States Since 1856.* New York: National Bureau of Economic Research, 1938.

[18] Miller, Robert, B., and Wichern, Dean, W. *Intermediate Business Statistics: Analysis of Variance, Regression, and Time Series.* New York: Holt, Rinehart, and Winston, 1977.

[19] Mirer, Thad, W. *Economic Statistics and Econometrics.* New York: Macmillan Publishing Co., 1988.

[20] Nelson, C. R. "The Prediction Performance of the FRB MIT PENN Model of the U.S. Economy," *American Economic Review*, LXII (1972), 902-17.

[21] Parker, B. B., and Segura, E. L. "How to Get a Better Forecast," *Harvard Business Review*, March-April, 1971.

[22] Ray, Christina, I. *The Bond Market: Trading and Risk Management*. Homewood, Illinois: Business One Irwin, 1993.

[23] Theil, H. *Applied Economic Forecasting*. Amsterdam: North-Holland Publishing Co., 1966.