CARL FRIEDRICH GAUSS: PRINCE OF MATHEMATICIANS

BY

Rand Lee Gray

Honors Thesis, ID 499

Dr. McCormick

June 2, 1971

Carl Friedrich Gauss: Prince of Mathematicians

Thesis: The life and the accomplishments of Carl Friedrich Gauss show that he should be ranked as one of the great mathematicians of all time, a prince of mathematicians.

I. Gauss' family life gives an idea of the meager conditions he had to overcome.
   A. Gauss' grandfather was Jürgen Goose.
   B. Gauss' father was Gebhard Dietrich Gauss.
   C. Gauss' mother was Dorothea Benze.
   D. Gauss' half-brother was George.
   E. Gauss displayed an early ability in mathematics at home.

II. Gauss' formal schooling prepared him for a life as professor and mathematician.
   A. Gauss' education was financed by Duke Carl Wilhelm Ferdinand of Brunswick.
   B. Gauss first attended St. Katherine's Volksschule (1784-1788)
   C. Gauss went to the gymnasium from 1788 to 1792.
   D. Gauss entered the Collegium Carolinum in 1792 and spent three years there.
   E. Gauss culminated his education when he attended the University of Göttingen (1795-1798).

III. Gauss made many discoveries and mathematical accomplishments while he still attended school.
   A. Gauss worked in the field of number theory.
      1. He worked with prime numbers.
      2. He worked with quadratic residues.
   B. Gauss spent time in creating non-Euclidean geometry.
   C. Gauss developed his method of least squares.
   D. Gauss discovered the inscriptability of a heptadecagon in a circle with Euclidean tools, which encouraged him to devote his life to mathematics.
   E. Gauss researched the area of eliptic and lemniscate functions.
   F. Gauss developed proofs for Lagrange's theorem and the fundamental theorem of algebra.

IV. After Gauss received his doctors degree he made a few new discoveries, but he also developed more fully those discoveries which he had already made as a student.
   A. Gauss published a formula for finding the date of Easter.
   B. Gauss published one of his greatest works, the <u>Disquisitiones Arithmeticae</u>.

C. In 1801 Gauss gave a proof of the fundamental theorem of arithmetic.
D. Gauss proved the law of quadratic reciprocity.
E. From 1801 to 1809 Gauss concentrated his time in the area of astronomy.
F. Gauss investigated imaginary numbers and helped to establish them for acceptibility by other mathematicians.
G. Gauss worked in the areas of infinite series and integration.
H. Gauss worked in geodesy and was commissioned to survey the kingdom of Hanover.
I. Gauss dealt with conformality mapping and other areas of surface theory.
J. Gauss in the years 1831 to 1840 spent his time in branches of science and physics more than in mathematics.
   1. Gauss worked in crystollography.
   2. Gauss investigated magnetism and electro-magnetism.
   3. Gauss worked with optics.
K. Gauss spent the last fifteen years of his life in keeping his mind active and keen.

V. Gauss was truly a prince of mathematicians, although much of his work went unnoticed.

CARL FRIEDRICH GAUSS:

PRINCE OF MATHEMATICIANS

Carl Friedrich Gauss was born at his home in Brunswick, Germany on April 30, 1777 to Gebhard and Dorothea Gauss. Of course "Gauss" is the Anglicized form of the name, indeed, Carl's grandfather spelled his name Jürgen Goose.

The Gauss family was rather poor, although Gebhard managed to provide his family with an adequate living. Gebhard had to care for his wife Dorothea and his son Johann George Heinrich, who was by his first marriage, and Carl. Gebhard was very strict, but a very good father and husband. On winter nights he would make Carl and George go to bed early in order to save on light and heat. In his attic room Carl would take a turnip, hollow it out, and roll a rough cotton wick for it. Using fat for fuel, he would study by dim light until cold and exhausted.[1]

Gebhard held many occupations during his lifetime, but none by which he could accumulate much wealth. He worked for the municipal waterworks, peddled meat from door to door, worked as a bricklayer, grew vegetables to sell at the market as a gardener, and served as an accountant in an insurance company.

Carl's mother, Dorothea, although unable to write and scarcely able to read, was kind and cheerful. She also helped in supporting the family as a servant in the home of a tannery proprietor.

[1] Guy Waldo Dunnington, Carl Friedrich Gauss: Titan of Science (New York, 1955), p. 15.

Carl showed a great ability in mathematics at a very early age. In fact before he turned four years of age he watched his father add up the payroll for the bricklaying crew at the company for which he worked. Carl noticed that he made a mistake in his calculations and called his father's attention to it. Upon checking the rows of figures Gebhard had found that little Carl had been right.[2]

Carl's half-brother, Johann George Heinrich, was not as studious as Carl. George left home while still quite young in order to learn a trade, then wandered about, finally returning in 1794. Eye trouble forced him to give up his trade, but his father would not permit him to just lie around the house. George decided to become a soldier, because he was unwilling to spend so much more time in preparing for another trade.[3]

When Carl reached the age of seven in 1784 he entered St. Katherine's Volksschule, which was directed by J. G. Buttner. Johann Martin Christian Bartels, an assistant to Buttner, was helpful in Carl's development. Carl's ability in mathematics was recognized by these two men when on one occasion, when Carl was ten years old, Buttner gave his class the tedious task of adding the integers from one to one hundred. Immediately Carl wrote 5050 on his slate and tossed it on the teacher's desk saying (in German of course), "There it is!"

[2]David Bergamini, Mathematics (Life Science Library) (New York, 1963), p. 150.

[3]Dunnington, Titan of Science, p. 8.

Carl was found to be the only student with the correct answer, once the other students had finished their laborious additions. Apparently Carl realized that 1 and 100, 2 and 99, 3 and 98, 4 and 97, etc. each add to 101, and therefore the fifty pairs together must add to 50 times 101 or 5050.[4]

Bartels was attracted to this exceptional student and they became close friends even though Carl was eight years younger than Bartels. By the time Carl was eleven, Bartels had taught him the binomial theorem and the theory of infinite series, two areas which later interested Carl very much and in which he made great strides. Bartels also helped him by telling persons of high rank about his ability, most importantly Eberhard August Wilhelm Zimmerman, who in turn discussed Carl's abilities with Duke Carl Wilhelm Ferdinand of Brunswick. Carl was granted a chance to entertain the Duke and the other court members with his calculating feats when he was still only fourteen. The Duke was so impressed that he granted him financial assistance for his schooling, including the purchase of the textbooks by the higher mathematicians in his university years.

Carl next entered the Katharineum Gymnasium in Brunswick, after his teachers at the Volksschule convinced Carl's father to let him attend a school for learning a profession instead of a school for learning a trade. Carl remained here from 1788 to 1792, excelling in ancient languages, mainly classical Latin but also Greek, and mathematics.

[4]Bergamini, p. 150.

Carl entered the Collegium Carolinum on February 18, 1792. He signed the register "Johann Friedrich Carl Gauss, of Brunswick," which is rather strange, for he never used the name "Johann", and on his writings is found "Carl Friedrich Gauss". He finished this part of his schooling in 1795, and decided to enter the University of Gottingen rather than the University at Helmstedt because of the former's superior library, but Carl was unsure as to whether to study languages or mathematics. Kastner was Gauss' professor of mathematics at Gottingen but since Carl already owned the publications of Kastner and had probably already mastered their content, Kastner's lectures were somewhat boring to him. As a result he buried himself in the library where he read from the works of Fermat, Euler, Lagrange, and Legendre. In reading the works of these great masters he found that he had already made some of the same discoveries, and in fact he found that he himself was a master.

Gauss made several discoveries while he was still a student at the University of Gottingen, in fact it has been said that he "spent about fifty years developing the inspirations that came to him before he was twenty-one, and he brought only a fraction of those ideas to maturity."[5]

One of the fields that Gauss investigated as a student was that of number theory, some of which appeared in his Analysis Residuorum, which appeared rewritten as part of the Disquisitiones Arithmeticae. His ideas in this field dealt with prime numbers and quadratic residues.

[5]Eric Temple Bell, The Development of Mathematics (New York, 1940), p. 227.

In 1791, at the age of 14, Gauss was the first to suggest in a purely empirical way, the asymptotic formula $\frac{x}{\log x}$ for $\phi(x)$, which is a function which determines all the prime numbers less than the given limit x. Still later in the years 1792 to 1793 he suggested another formula whose leading term is $\frac{x}{\log x}$. This formula is the integral, $\int_2^x \frac{dx}{\log x}$.[6]

These formulas are very closely related to the prime number theorem, which was discovered by Gauss. This theorem states that if A_n represents the number of prime numbers below n, then $\frac{A_n \log_e n}{n}$ approaches 1 as n becomes larger, that is $\frac{A_n}{n}$ is called the density of the prime numbers within the first n integers and is approximately given by $\frac{1}{\log_e n}$.[7]

In March of 1795, before Gauss was even eighteen, he discovered that one quadratic residue, -1, of prime numbers of the form 4n + 1 is a quadratic non-residue of prime numbers of the form 4n + 3, and he worked out a proof of this discovery. If a number divides the difference of the two numbers evenly then these two numbers are each residues of each other, if the difference is not evenly divisible, then they are non-residues. Another way of saying this, which may be more familiar, is that if the difference between the two numbers is evenly divisible by a third number, then the two numbers are congruent with respect to the third number. If they are not evenly divisible, then they are not congruent or incongruent.

[6]David Eugene Smith, A Source Book in Mathematics (New York, 1929), p. 127.

[7]Howard Eves, An Introduction to the History of Mathematics (New York, 1953), p. 140.

Gauss was not completely satisfied with this first proof, so, as was his character, he later produced six others in improving his discovery.

Gauss also considered the problem of the parallel axiom of Euclidean geometry while he was in school, in fact, in the year 1792 at the age of 15. First he tried to replace the axiom with a simpler one, but at this he failed. Next, he attempted to adopt an axiom contradicting Euclid's much like Saccheri had done; but, unlike Saccheri, Gauss decided that there could be other geometries as valid as Euclid's. Gauss had the ability and courage to create a non-Euclidean geometry, but he did not have the courage to face the mobs of critics who would have certainly declared him insane because of his discovery which did not agree with such a field as established as Euclidean geometry. Gauss decided to test his new geometry, to see if it was the geometry of the universe. He set up a triangle test by stationing observers on three mountain tops. He found that the sum of the angles of the triangle was within two seconds of 180 degrees, thus the measurements were so close that the difference could be attributed to errors of measurement. Hence, the experiment was indecisive. Gauss never published his findings about this new geometry because of the criticism it would stimulate and because he was unable to provide physical evidence of its validity, thus he did not gain popularity as a great mathematician from that discovery.

The method of least squares was used by Gauss in 1795. This method is a way of bringing observations into a calculation so that the unavoidable errors that creep into a calculation affect the results as little as possible, and so that the difference between the calculated value and the observational results is a minimum for any single case and for the entire set of observations. (He used this method in his astronomical work much later in his life.)

Gauss deduced the probability of error and from this he gave a full development of this method of least squares. To Gauss must go the credit for outlining the steps of the method for the investigation of the precision of results, for the method of correlatives for conditional observations, and for numerous practical applications. Gauss revised the method so that it would be reliable against all exceptions and he made it so easily useful that its utility could not be completely investigated or demonstrated until after 1820, when he published his "<u>Theoria</u> <u>combinationis</u> <u>observationum</u> <u>erroribus</u> <u>minimis</u> <u>obnoxiae</u>".

To the scientists Gauss' method of least squares (1) furnished common rules of procedure, (2) furnished common measures of precision, and (3) furnished common terminology.

Legendre rediscovered this method independently in 1806, and since Gauss did not publish his discovery at the time when he discovered it, there was a dispute as to the priority of the discoverer. It seems that Gauss could care less as to who discovered the method first, but Legendre on the other hand was quite bitter in defending his priority since Gauss publication appeared in 1809.

Probably one of the most important accomplishments that Gauss made while he was in school was that of the discovery of the possibility of inscribing heptadecagon, a regular seventeen sided polygon, in a circle with compass and straightedge.

This discovery was not so very important because of its contribution to scientific or mathematical knowledge, but because it influenced Carl to concentrate his mind and energies upon mathematics instead of philosophy and languages. This discovery was very satisfying to Gauss, because it had been undiscovered for so many centuries and had gone unnoticed by many great mathematicians, so pleasing in fact that he requested that it be inscribed on his tombstone (reminiscent of Archimedes tomb).

Gauss generalized his discovery by the statement that the construction of regular polygons with compass and straightedge alone is possible if and only if the polygon has n sides, where n is an integer of the form $2^s p_1 p_2 p_3 \ldots p_r$, $s \geq 0$ in which $p_1 p_2 p_3 \ldots p_r$ are r different primes each of the form of a power of two plus one.

From 1796 to 1799 were years that were very abundant in Gauss' accomplishments. Besides the discovery of the inscriptability of a heptadecagon, just mentioned, he discovered what is known as Lagrange's theorem and also discovered the connection between the elliptic quadrant and the arithmetico-geometric mean, and its connection with the power series, whose exponents are squares, and he did some work with elliptic and lemniscate functions.

In September of 1796 he investigated elliptic functions of the form $\int \frac{dx}{\sqrt{1-x^n}}$ and in January of the following year he studied lemniscate functions beginning with the inversion of $\int \frac{dx}{\sqrt{1-x^4}}$. These researches were interrupted by his revision of the *Analysis Residuorum* for publication as the *Disquisitiones Arithmeticae*, but he returned to them periodically in July, 1798 and again in May of 1799, when he had finished his doctoral thesis and had presented it to the Helmstedt faculty. During this month he discovered that the quotient of the periods of π and $\tilde{\omega}$ of the trigonometric and lemniscate functions is equal to the arithmetico-geometric mean of 1 and $\sqrt{2}$.[8]

Still later, December, 1799, he found the relation of the arithmetico-geometric mean $M(1, \sqrt{1-v^2})$ to the elliptic integral of the first order:

$$\frac{1}{M(1,\sqrt{1-v^2})} = \frac{2}{\pi} \int_0^2 \frac{d\phi}{\sqrt{1-v^2 \sin^2\phi}}.$$ [9]

Before the time of Niels Henrik Abel, nothing was publicly known about elliptic functions, for Gauss did not publish any of his research concerning them. All his discoveries lay untouched in his private papers. By examination of these papers after his death it has been found that in 1797 he had discovered the double periodicity of the lemniscatic function, and the general double periodic functions twenty-five years before Abel. These papers also contained formulas concerning the elliptic theta constants, which were rediscovered and brilliantly applied by Jacobi, but Gauss published nothing about elliptic functions, nor did he claim to have anticipated Abel or Jacobi, although their discoveries came in the 1820's. Even more amazing in realizing that Gauss was exceedingly ahead of his time is the fact that the elliptic modular functions which Gauss mastered in 1800 were only fully developed in recent times.

[8] Guy Waldo Dunnington, Carl Friedrich Gauss (Louisiana, 1937), p. 26.

[9] Guy Waldo Dunnington, Carl Friedrich Gauss (Louisiana, 1937), p. 26.

In 1799 Gauss received his doctor of philosophy degree from the University of Helmstedt <u>in absentia</u>. His dissertation probably contained one of the greatest accomplishments of his mathematical career: a proof of the "fundamental theorem of algebra", which can be stated as "every algebraic equation of degree n has exactly n roots." It seems that the term "fundamental theorem of algebra" was introduced by Gauss.

In all, Gauss gave four proofs of the theorem: (1) proof discovered in autumn 1797 and published as his dissertation in 1799 at Helmstedt, (2) an entirely algebraic proof given in 1816, (3) a third proof in 1816, and (4) a fourth published in 1850. Several attempts had been made to prove this theorem before Gauss' successful one, namely by: d'Alembert, 1746; Euler, 1749; Foncenex, 1759; Lagrange, 1772; and by Laplace, discovered in 1795 and published in 1812.

Gauss himself said that his first proof had a double purpose: (1) to show that previous proofs were "unsatisfactory and illusory" and (2) to give a newly constructed rigorous proof.[10]

Most of the great mathematicians of Gauss' time and before were more like engineers in their thinking. The formulas they discovered and proved were produced during a period of inspiration or intuition, but Gauss, rebelled against intuition in analysis. For Gauss, it was proof or nothing.[11]

[10]David Eugene Smith, <u>A Source Book in Mathematics</u> (New York, 1929), p. 293.

[11]Eric Temple Bell, <u>The Development of Mathematics</u> (New York, 1940), p. 293.

Gauss demanded of himself absolutely rigorous thinking, perhaps this is why he was rather wary of infinite quantities. Indeed, he stated, "I protest against the use of infinite magnitude... which is never permissable in mathematics."[12]

As an example of the rigorous limits he placed upon himself, there is a brief instance which occured when he was urged to complete for a prize offered by the French Academy in 1816 for a proof or disproof of Fermat's last theorem, whereupon he stated that guessing in arithmetic is undesirable and that he himself could manufacture any number of such conjectures which neither he nor anyone else could settle.[13] This critical acuteness which is so characteristic of Gauss he called rigor antiquus.

During 1800 Gauss continued his research on elliptic functions and in May he published his formula for determining the date of Easter. The reason for Gauss' interest in a way to determine the date of Easter was that he did not know the exact date of his birth. His mother could only remember that he was born on a Wednesday, eight days before Ascension Day; this started him on his search for a formula.

The results of his efforts are:

In the Julian Calendar let $m = 15$, $n = 6$

In the Gregorian Calendar from 1700 to 1799 let $m = 23$, $n = 3$; from 1800 to 1899 let $m = 23$, $n = 4$.

Divide the year by 19 and call the remainder "a".

[12] Morris Kline, Mathematics in Western Culture (New York), p.396.

[13] Bell, p. 295.

Divide the year by 4 and call the remainder "b"

Divide the year by 7 and call the remainder "c"

Divide (19a + m) by 30 and call the remainder "d"

Divide (2b + 4c + 6d + n) by 7 and call the remainder "e"

The result is that Easter Sunday is given by (22 + d + e) in March.[14]

For most of the time when Gauss was a student at Göttingen University, he was preparing his *Disquisitiones Arithmeticae* using another of his works, the *Analysis Residuorum*, as its basis. Gauss probably intended that his *Disquisitiones Arithmeticae* be his doctoral thesis, but its printing dragged on for such a long time that he turned to his proof of the fundamental theorem of algebra, which has already been mentioned.

Before the publication of his *Disquisitions Arithmeticae*, Gauss wanted Johann Heinrich Jakob Meyerhoff, a scholar excellent in classical Latin, to examine the Latin in his work and to make any corrections that were necessary. This action by Gauss is rather strange, for it is said that "if Cicero were to be able to understand the mathematics of Gauss' work he would have censured nothing."[15] Supposedly, Gauss wanted his Latin to be checked for the sake of professor Zimmermann. He wanted his first work to be perfect.

[14] Dunnington, *Titan of Science*, p. 70.

[15] *Ibid.*, p. 37.

The Duke of Brunswick furnished the funds for the publication of the *Disquisitiones Arithmeticae* and Gauss very gratefully dedicated the work to him when it was finally published on September 29, 1801. Gauss' appreciation for the Duke's assistance can be seen from Gauss' dedication of the work which follows:

> "I account it the highest joy, most gracious Prince, that you allow me to grace with your exalted name this work which I offer to you in fulfillment of the holy obligations of loyal love. For if your grace had not opened up for me the access to the sciences, if your unremitting benefactions had not encouraged my studies up to this day, I would never have been able to dedicate myself completely to the mathematical sciences to which I am inclined by nature. Indeed, the observations, some of which are presented in this volume ... the fact that I could undertake them, continue them for several years, and publish them, I owe only to your kindness which freed me from other cares, and permitted me to devote myself to this work. Your magnanimity has pushed aside all the obstacles which delayed publication." [16]

This work of Gauss has never been replaced nor ever been surpassed. Few people have mastered the work, partially because of the difficult terminology which Gauss used in his classification of numbers and their relations and partially because of the advanced concepts which the work considered, including Diophantine analysis and cyclotomy with its applications to transcendental functions.

Gauss also in the year 1801 deduced the fundamental theorem of arithmetic from Euclid's basic theorems on divisibility. This theorem states that a positive integer can be expressed as the product of prime numbers in only one way except that the arrangement of the factors may be different.

[16] Dunnington, *Titan of Science*, p. 37.

Another profitable accomplishment of Gauss during this year (1801), was his proof for the law of quadratic reciprocity, in fact he gave seven proofs within 17 years. Today this theorem is known as Legedre's law of quadratic reciprocity, because Legendre was the discoverer of the general theorem. Gauss said, "I discovered this theorem independently in 1795 at a time when I was totally ignorant of what had been achieved in higher arithmetic, and consequently had not the slightest aid from the literature of the subject. For a whole year this theorem tormented me and absorbed my greatest efforts until at last I obtained a proof."[17]

Beginning in 1801 Gauss concentrated more and more in the area of astronomy. Gauss liked teaching but it took up so much of his time that he thought his work in astronomy instead of in teaching might relieve him of this time consuming task and provide him with greater opportunity for research in pure science and mathematics.

At this time Nicolaus Fuss, the secretary of the St. Petersburg Academy of Sciences, was attempting to recruit German scholars for the Universities of Charkov and Kasan, as well as for other scientific positions, one of which was the position of director of the St. Petersburg observatory and astronomer to the Academy of Sciences. J. T. Pfaff called Fuss' attention to Gauss, and the position was offered to him.

[17]Smith, p. 113.

Gauss was tempted by the offer, but he did not want to leave his native Germany, and the Duke of Brunswick urged him to stay, so he rejected the offer.

Gauss' first major undertaking in the field of astronomy was the working out of a theory of the moon, but this work was interrupted by the discovery of Ceres by the astronomer, Piazzi. Piazzi was only able to make three sightings of this planetoid, because its orbit passes very close to the sun and Mercury, where it is difficult to see. Gauss was able to rediscover this planetoid since it was lost by computing its orbit from the few sightings in his possession. This rediscovery of Ceres by Gauss revealed to the world that he possessed a remarkable mathematical superiority over [illegible].

Gauss, later on in March of 1802, was called upon to calculate the orbit of a newly discovered planetoid found by Olbers, which Olbers later named Pallas.

Much of Gauss' time between 1801 and 1809 was devoted to his teaching, and much to his work in astronomy. These two areas of activity left him little time for anything else, but in 1809 Gauss had his "Theoria motus corporum coelestium in sectionibus conicis solem ambientium" published. This work introduced the principle of curvilinear triangulation, gave a complete system of formulas and processes for computing the movement of a body revolving in a conic section, explained a general method for determining the orbit of a planet or comet from three observed positions of the body, and concluded with a presentation of the method of least [illegible]um. This work is Gauss' major work in astronomy and probably one of the greatest works of astronomy in general. Indeed, Prince [illegible] sent Gauss a golden medal for this work, and another was sent by the Royal Society in London. Also, in recognition for his work in astronomy, Gauss was given the position of director of the Gottingen observatory in 1807, which he held until his death.

Several important events occured during this long period when he was so interested in astronomy, in Gauss' private life, one of which was his engagement to Johanna Maria Christine Ahr[illegible]holz on November 22, 1804. She was not an extremely beautiful girl and she lacked formal schooling, but she was very kind and possessed a very cheerful personality, and she was also very understanding. On October 9, 1805 they were married at St. Katharine's Church, and it was not long before they were blessed with a son, who arrived on August 21, 1806, and whom they named Joseph. Joseph was given a baby sister on February 29, 1808, which of course is leap year day. She was named Wilhelmine, in honor of Olber, who acted as godfather, but she was always called Minna by everyone in the family.

These two children brought much pleasure to Gauss' life, but tragedy struck just two months after Minna was born. In April, 1808, Gauss' father Gebhard Dietrich Gauss, was taken ill. His health became progressively worse, ending in his death at his home in Brunswick on April 14, 1808. Gauss was never very close to his father, but he had much respect for him and thought that he was a good provider, although he was very strict and domineering.

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Tragedy followed tragedy. On September 10, 1809 Gauss again became a father. The boy was named Ludwig, but called Louis by the family. The birth of Louis was very hard on Johanna and tragedy struck the Gauss family when she died on October 11, 1809 as a result of this birth. Tragedy came again when Louis suddenly died on March 1, 1810 when he was only five months old.

Gauss remarried soon after Johanna's death. There were two very understandable reasons for this. First he loved his first wife very much and when she died he was very lonely and his isolation was unbearable. The loneliness robbed him of his peace of mind, which he needed so much for his work. Second, his children needed a mother to love and care for them.

Gauss chose to marry Minna Waldeck whose full name was Friedrica Wilhelmine, who had been Johanna's best friend in Göttingen. Gauss and Minna were married on August 4, 1810 in St. John's Church in Göttingen. As early as 1818 Minna's health began to fail; she suffered from consumption, which grew worse day by day.

By Minna Gauss had three more children:
Peter Samuel Marius Eugenius (called Eugene), born July 29, 1811; Wilhelm August Carl Matthias (called Wilhelm), born October 23, 1813; and Henriette Wilhelmine Caroline Therese (called Therese), born June 9, 1816.

As Minna's health declined, she became practically a semi-invalid, and she later died on September 12, 1831, having been weakened by measles, which she had contracted in October of 1824.

With all these marriages, births, and deaths it seems that it would be very difficult to carry on teaching at the university, but then Gauss managed, and besides he continued his mathematical investigations. He turned his attention upon imaginary numbers.

Gauss was convinced that a satisfactory theory of complex numbers had not yet been devised. He created a geometrical representation on a two dimensional grid, much like the Cartesian coordinate system, which was very helpful in vector analysis in two dimensions, but he was not satisfied with a geometrical picture of what he thought dealt only with the question of number. He was convinced that only a "formal treatment" could produce a sound theory of complex numbers. By "formal treatment" Gauss meant the deduction of the properties of imaginary numbers from the accepted postulates of ordinary arithmetic. Gauss eliminated geometric intuition completely by defining the imaginary number, $a + bi$ as the number couple (a,b) such that:

$(a,b) = (c,d)$ means $a = c$, $b = d$

$(a,b) + (c,d)$ means $(a + c, b + d)$

$(a,b) \times (c,d)$ means $(ac - bd, ad + bc)$

by defining imaginary numbers in such a way, the mysterious "i" had vanished and was replaced by a "double algebra" of couples of real numbers using only the accepted laws of arithmetic and common algebra. Gauss was very influential in getting the complex numbers accepted as respectable portion of mathematics.

In 1819 he invented numbers to represent forces, velocities, and accelerations in more than two dimensions, and called them "hypercomplex numbers."

They were numbers of the form $a + bi + cj + dk$ in which the product of i,j, and k is 1.

In 1812 Gauss published his classic memoir on the hypergeometric series, in which, for the first time, the convergence of an infinite series was adequately investigated. Other mathematicians had stated tests for convergence, namely Leibniz and C. Maclaurin; but Gauss was the first to carry out a rigorous treatment. Gauss, in fact, by his work had opened the period for modern research upon infinite series.

Closely related to infinite series is integration, which Gauss studied briefly. In his letter to F. W. Bessel, Gauss stated that he had a proof for the following theorem: "The integral of a function round a closed contour is zero when the integrand does not become infinite within the region bounded by the contour."[18] This finding is very amazing, since it was found fourteen years before Cauchy published his memoir on complex integration. There is no reason to disbelieve what Gauss wrote to Bessel, but there is no evidence that he ever proved the theorem.

Gauss spent much of his time between 1818 and 1826 in making geodetic surveys in the neighborhood of Göttingen, for in 1818 he was commissioned to survey the Kingdom of Hanover. It was during this time that he invented the heliotrope, which was simply a device that used the reflected rays of the sun to make triangular geodetic surveys.

[18]Bell, p. 166.

The idea for this instrument came to him as he was making a survey in the field, when the reflection from a window interfered with his observation. Much of the time during the field work Gauss' health was affected by the summer heat, so he was grateful when cooler weather came. Gauss didn't mind the work so much, especially when it was cool, but after a while it did become tiresome and an accident in which a coach overturned and a box of instruments fell upon his leg helped to discourage him from continuing his geodetic work. The injury which he incurred was not very serious, but as was mentioned, it did have quite an effect upon his feelings about the survey.

In 1819 in a brief abstract that Gauss never had published, he wrote out the fundamental equations of mutations in space, which were essentially quarternions, or ordered quadruples. He did not do much in this area, but what he did produce indicated that he was adequately qualified to pursue this area of investigation if he would have taken the time to do so.

Gauss' memoir on conformal representation, which he wrote in 1822, was inspired by a prize problem of the Royal Society of Sciences in Copenhagen. He entitled this work the "General Solution of the Problem to so Represent the Parts of One Given Surface upon Another Given Surface That the Representation Shall be Similar," in its Smallest Parts, to the Surface Represented." This memoir won the prize and today may be considered the basis for the theory of conformal representation and as fundamental to the more modern theory of analytic functions of a complex variable.

In 1825 after he attacked the general problem of conformality mapping of either of two surfaces upon the other, where angles are preserved and distances (except in trivial cases) are distorted, he then applied his solution to geodesy.

Gauss made the first systematic study of quadratic differential forms in 1827 in his "Disquisitiones generales circa superficies curvas", in which the main theme in the curvature of surfaces begins with this work of Gauss. Two qualities of this work affected later developments in this area: the systematic use of curvilinear coordinates and the conception of a surface as a two-way extension.[19]

Gauss' work in surface theory was pioneer research for later mathematicians in two ways. First, Gauss used an infinite group, whereupon until this time only finite groups of transformations had been considered. Second, Gauss treated the theory of curved surfaces in such a manner that he paved the way for the theory to be easily extended to the general theory of n-dimensional space.[20]

In the years from 1830 to 1840 Gauss was primarily concerned with subjects that seem to be related more to physics and other branches of science than to mathematics, although they were connected with mathematics.

[19] Dunnington, Titan of Science, p. 165.

[20] Ibid.

In 1831 Gauss suddenly became interested in crystallography, probably as a hobby or diversion from his duties as a professor. Within only a few weeks he was able to fully master the subject as it was then known.

Another area of science which Gauss investigated was that of magnetism, especially terrestrial magnetism, and electromagnetism, two areas which he concentrated in the years from 1832 to 1836.

Gauss was interested in terrestrial magnetism as early as 1803, but he was too busy with his other pursuits at the time to devote himself to its study.

Later in 1829 he again became interested in magnetism due to the discovery of electromagnetism by Oerstedt and because of the encouragement of his friend Alexander von Humbolt.

In 1833 Gauss published a paper about the intensity of terrestrial magnetism entitled, "Intensitas vis magneticae terrestris ad mensuram absolutam revocata." In this work all the magnetic instruments were reduced to three basic units of measurements: mass, length, and time. With his studies in this area Gauss brought accuracy to this branch of physics.

Gauss confirmed Coulomb's law, very much according to his rigorous character, not accepting it as true until proven so.

Gauss postulated that every magnetic body must possess equal quantities of the two "magnetic fluids": north (positive) and south (negative). He was first to recognize that this was necessary in order to produce rational measurements of magnetic quantities.

Gauss also realized that magnetic forces depend upon temperature, although he did not have enough time to investigate this area of magnetism.

In January of 1833 Gauss proposed that a magnetic observatory be constructed and his proposal was immediately approved. The building was ready in the fall of 1833. It had double doors and windows in order to eliminate any air currents, and where iron was usually found in buildings there were copper replacements so that magnetic measurements and experiments in the building would not be affected. On account of this building and the great scientists who worked here, Göttingen became the center of magnetic research. Weber and Gauss even published a periodical to report all the research that was being done upon magnetism there.

In 1837 Gauss and Weber together invented a bifilar magnetometer, which was an instrument used to measure the magnetic force of direction.

Gauss considered all the necessary components of a general theory of terrestrial magnetism. He showed that the cause of terrestrial magnetism was in the interior of the earth and he was able to calculate the magnetic poles of the earth. In 1841 Captain Charles Wilkes found the magnetic south pole, which differed only slightly from Gauss' calculations and later Captain Ross found that the magnetic north pole differed from Gauss' calculations by only three degrees and thirty minutes.

Gauss made his first experiments with electromagnetism on October 22, 1832 and in January 1833 he discovered Kirchhoff's laws of branched circuits, which Kirchhoff did not discover until 1845. He also established the principle of maximum heat, which was also discovered by Kirchhoff later in 1848.

Gauss' main interest with electromagnetism was in the area of induction. He made his own apparatus and discovered what is known as "sympathetic vibrations". On January 23, 1835 he formulated the law of induction now known as the "Franz Neumann Law of Potential", although Neumann did not make the discovery until 1845.

Gauss and Weber developed an electromagnetic telegraph in November of 1833, but Gauss did not reveal this publicly until August 1834. By this invention they were the first scientists to use electrical current for communication, at least one year before Samuel F. B. Morse. The railroad industry became interested in this telegraph, but the cost was thought to be too high for its use. This discovery received little recognition and it was adopted by others as if it were their own.

Gauss received very little recognition for any of his contributions to the theory of magnetism. The little recognition he did receive was the use of his name in two terms in this area: "a gauss" is a unit of measurement of magnetism, and the term "degaussing" was a naval term which means taking the magnetism out of a ship as an antimine measure.

Another branch of science in which Gauss distinguished himself was that of optics, work on refraction of light and dioptic instruments. Gauss in a paper titled "Dioptic Researches" gave his findings on the path of rays of light through a system of refracting media. This work was the greatest advance of the century in the fundamental theory of dioptric instruments.[21]

In works before Gauss' work in this field the instruments had been considered as combinations of lenses in which both the incoming and outgoing rays were in air, but Gauss' work removed this limitation so his new theory applied not only to artificial dioptric instruments, but also to the natural dioptric instrument, the eye.

In the last years of Gauss' life, from 1847 to 1855, he was occupied with what he called geometria situs, today known as topology. In this field he investigated problems involving interwoven or knotted infinite or closed curves.

Also during these last years, he wanted to be sure that his mind would remain quick and keen so at the age of 62 he decided that he should learn a new language or science. He first tried to interest himself with botany and then with Sanscrit, but found them unsatisfactory. Finally he decided upon Russian and in two years he had mastered it by himself.

Gauss was very fond of music, especially of singing and when he heard a song which he liked he wrote it down, so listening to music was a favorite way to spend his time during his last years, up until his death in February 23, 1855.

[21] Oscar Faber (trans.), Ferraris' Dioptric Instruments (New York, 1920), p. XXIII.

Gauss was truly a great mathematician, at least in the eyes of his contemporaries. For instance, when Laplace was asked, "Who is the foremost mathematician in Germany?" he answered, "Pfaff". "But what of Gauss?", he was asked. "Oh", replied Laplace, "Gauss is the first mathematician in Europe."[22]

Bolyai also replied that Gauss would be the first mathematician in Europe when Gauss' mother asked him if her son would amount to anything.[23]

In November of 1838 the Royal Society of London conferred upon Gauss the Copley Medal, at the time the highest honor of the society.

Although Gauss' contemporaries knew about his greatness, many people who lived after him did not, mainly because he published almost none of his findings. Gauss' motto was "<u>pauca sed matura</u>" meaning "few but ripe", and he remained true to his motto as far as mathematical publications are concerned. Another reason for his lack of fame was because of his belief that a work should be a work of art. Because of this belief many works went unpublished because he lacked the time to perfect them. Still a third reason for publishing only a few of his works is that he was not willing to undergo the criticism that was sure to follow from the printing of some of his far-reaching results, as was the case with his discovery of non-Euclidean geometry. Reasons four and five for his lack of published material are his lack of time because of his duties as a professor, and the tragedies which were dealt to him and his family left him tired

[22] Bell, p. 234.

[23] Dunnington, <u>Titan of Science</u>, p. 30.

uneasy and unable to concentrate.

It has been said that Gauss was the last mathematician to know all that was possible to be known in mathematics. He has been ranked as one of the greatest mathematicians in history; the other two giants are Archimedes and Isaac Newton. He made contributions in almost every important field of pure mathematics, as well as in astronomy, optics, geodesy, electricity, and magnetism. For these reasons it seems only fitting that Gauss, who called mathematics the "queen of the sciences" and the theory of numbers the "queen of mathematics",[24] should be granted the title "prince of mathematicians".

[24] Dunnington, Titan of Science, p. 45.

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