AN EXAMINATION OF THE COPENHAGEN INTERPRETATION OF QUANTUM MECHANICS

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STATEMENT OF PURPOSE

Since its origin over sixty years ago quantum mechanics has developed a consistent mathematical structure that is expressible in the form of a set of basic postulates. From these postulates, the remainder of quantum mechanical theory can be derived. It is in this manner that the subject is generally presented. (For example, see Introduction to Quantum Mechanics by Chalmers W. Sherwin).

This approach, however, does not clearly indicate the great revision in physical concepts that was necessitated by the development of quantum theory. According to present views the basic concepts of classical physics, i.e., essentially Newton's mechanics and Maxwell's electromagnetic theory, cannot be extended to the realm of atomic phenomena. Indeed, in this area, these concepts must be replaced by abstractions which are not describable in terms of common experience.

The purpose of this paper is, first of all, to indicate the manner in which these quantum concepts were introduced into physical theory and finally coordinated into a consistent interpretation, the Copenhagen interpretation. This is accomplished by an introductory examination of the development of quantum mechanics.

In the second section of this paper, the intention is to examine some of the key features of the Copenhagen interpretation, such as the Uncertainty Principle and the role of probability, in order to compare them with classical concepts and to indicate the extent to which the Copenhagen interpretation has changed our conception of reality.

Any physical theory makes more philosophical and physical assumptions than the facts alone give or imply, and the Copenhagen interpretation is particularly rich in epistemological, ontological, and metaphysical implications. This paper makes no pretention of being a philosophical analysis of quantum mechanics, but it is the author's hope that it can raise some questions in the reader's mind as to the nature of what is casually referred to as 'reality'.

INTRODUCTION: THE DEVELOPMENT OF QUANTUM MECHANICS

THE OLD QUANTUM THEORY

Quantum mechanics originated in connection with attempts to explain a phenomenon known as black-body radiation. Thermal radiation from a perfect radiator or 'ideal black body' does not occur at a single frequency but covers a range with maximum energy emmission at a frequency which depends upon temperature. However, classical theory was unable to predict the shape of the thermal radiation curve (energy vs. wavelength or frequency).[1]

In 1896 Wien attempted to obtain the correct formula for the radiation curve by noting the similarity between the curve and Maxwell's distribution law for speeds of molecules in a gas. By means of classical methods of computing the energy of a wave of a given frequency he tried to distribute these energies just as energies of molecules of a gas are distributed. By this process he arrived at a formula which described the curve well except at the low frequency range where it was increasingly in error.[2]

Wien's law: $\psi_\lambda = C_1 \lambda^{-5} \exp\left(-\frac{C_2}{\lambda T}\right)$

Lord Rayleigh, with further extension by Jeans, attempted to find the correct formula by using the idea of equal division of energy between degrees of freedom in a gas, applying it to radiation in a perfectly black enclosure. He applied classical methods to find just those wavelengths which could form standing waves in the enclosure. However, there were too many possibilities at short wavelengths and the energy represented

tended towards infinity. Still, the formula succeeded toward the long wavelengths where Wien's failed.[3]

Rayleigh-Jeans law: $\psi_\lambda = 8\pi KT\lambda^{-4}$

Then, in 1900 Max Planck developed empirically a formula which described the full curve accurately.

Planck's law: $\psi_\lambda = \dfrac{8\pi ch\lambda^{-5}}{\exp\left(\frac{ch}{\lambda KT}\right)-1}$

Classical theory failed to describe completely a phenomenon which should have had a simple explanation by means of the classical laws of heat and radiation. Therefore it became vital to find a theoretical interpretation of Planck's successful empirical formula. However all attempts to incorporate it into the classical structure failed. Planck then attempted to find a physical interpretation for his formula. His theoretical development turned the problem from the radiation to the radiating atom (atomic oscillator). By analyzing the energy exchanges between the radiation and the oscillator he was led to the conclusion that the oscillator could emit radiant energy only in finite quantities proportional to the frequencies. Specifically, the energy was limited to the following:

$E = nh\nu$ where E = energy, $n = 1,2,3,\ldots$, ν = frequency, h = a constant

This notion of discrete 'quanta' of energy was completely contrary to classical concepts i.e. oscillator should exhibit a continuous range of energies. However, at the time it was not generally recognized that the introduction of Planck's constant, h, would lead to a fundamental revision of classical concepts and it was hoped that classical theory caould be extended to accept it.[4]

The fundamental nature of Planck's 'quantum of action', h, was first indicated strongly by its successful applications to the photoelectric

effect and to the theory of specific heats.

In the photoelectric effect, which was first observed by Hertz in 1887,[5] electromagnetic radiation incident on a metal surface can cause the emission of electrons from the metal. Careful observations had led to the following experimental 'laws':

'Laws' of the Photoelectric effect:[6]

(1) The number of photoelectrons emitted per second varies directly as the intensity of the incident light.

(2) Emission from a given surface does not occur unless the frequency of the incident light is equal to, or greater than, a particular value known as the threshhold value.

(3) The maximum energy of the photoelectron after emission from a surface does not depend upon the intensity of the light.

(4) The maximum energy of the photoelectron after emission increases with increase in the frequency of the light.

(5) Emission of the photoelectron occurs at once regardless of the light intensity.

These 'laws' had been verified to a high degree of accuracy and thus could not be doubted; however, they could not be explained satisfactorily by classical theory. For example, according to the accepted wave theory of light, energy should be absorbed by the electron over a period of time, related to the intensity of the radiation, and emission should occur when the electron absorbed enough energy to escape from the metal. However emission was always observed to be instantaneous. No time lag was detectable between incidence of radiation and emission of electrons, no matter how weak the intensity.

In 1905 Einstein proposed an explanation involving Planck's

constant, h. Einstein assumed that light transmitted energy in discrete packets or quanta called 'photons' which conveyed the energy directly to the electrons, and that this energy of these photons depended upon the frequency of the light, i.e. $E_q = h\nu$, where h is Planck's constant. By this assumption he was able to explain the observed effects completely. In terms of the photoelectric 'laws':

(1) The intensity of the incident light measures the number of photons arriving per second and hence determines the number of electrons emitted.

(2) The energy absorbed by the electron must be greater than the work necessary to escape from the surface. Hence below a certain frequency, the electron would not absorb enough energy to be emitted.

(3) The maximum energy of the emitted electron would be that which remained after passing the potential barrier and would depend only on the frequency of the incident light.

(4) Increasing the frequency would increase the energy delivered to the electron.

(5) Energy would be delivered to the electron in discrete 'bundles' and would not have to be absorbed over a period of time.

Einstein then expressed his interpretation in a simple equation concerning the kinetic energy of the emitted electron.[7]

$$\frac{1}{2} m v^2 = h\nu - W_o$$

Where W_o is the work required to pass the potential barrier of the metal, i.e. the 'work function'

Despite its success Einstein's interpretation contained, from the classical point of view, some contradictory features. In assuming radiant

energy to consist of localized 'bundles' Einstein was, in effect, endowing radiation with particle-like properties. However the expression for the energy delivered by the photons, $E_q = h\nu$, contained a term for frequency which was a wave property. Also, a particle-like description of radiation was in direct contradiction to such phenomena as interference and diffraction, which could only be explained in terms of waves. Einstein offered no explanation for this contradiction, taking it as something which would be understood only in the light of future developments.[8] This wave-particle duality gave the first strong indication of the difficulties which would be involved in a consistent description of physical phenomena involving the quantum concept.

By 1907 Planck's quantum hypothesis had found suscessful application in another area, that of the specific heats of solid bodies. Traditional theory of specific heats led to values which agreed with observations at higher temperatures but disagreed with them at lower temperatures. Applying the quantum hypothesis Einstein, with further developments by Debye, was able to show that this phenomena could be understood in terms of elastic vibrations of the atoms in a solid body.[9] Planck's quantum of action was thus revealed in phenomena having nothing directly to do with the thermal radiation problem in which it initially appeared, indicating that its existence was not related solely to the explanation of a particular physical phenomenon.

The quantum hypothesis found further application in the area of atomic structure. In 1911 Rutherford's scattering experiments had led to a 'planetary' model of the atom in which the electrons were pictured as orbiting about a central nucleus, which contained most of the mass of the system, much as the planets orbit about the sun. No other model was

consistent with experimental results. However this model presented serious difficulties for classical theory. According to classical mechanics the orbiting electrons undergo an acceleration due to centripetal force and, according to Maxwell's electromagnetic theory, an electric charge, when accelerated must emit radiation i.e. it must continuously lose energy. Therefore the orbiting electron should emit radiation in a continuous manner, steadily losing energy until it spirals down into the nucleus. In other words, according to classical theory applied to the planetary model, the atom should collapse in a very short time, emitting radiation of continuously varying frequencies during the process.[10]

This classical prediction conflicted with two very obvious facts. First, atoms do not collapse but instead show great stability. Second, atoms do not emit radiation of continuously varying frequency but instead exhibit specific, discrete, line spectra which are characteristic for a particular element. Classical theory was therefore confronted with a serious dilemma. There was, on one hand, a model which had very strong experimental support and on the other a theory which also had strong confirmation.[11] Yet the two were not compatible.

Niels Bohr directed himself to this problem and in 1913 put forth an explanation of the structure of the hydrogen atom which involved Planck's constant. Bohr adopted the Rutherford model of the atom and to this model applied conditions of quantitization (see appendix III).[12]

According to the classical theory the orbiting electron should continuously lose energy as it circled the nucleus. In losing energy the radius of its orbit would decrease in a continuous fashion until it crashed into the nucleus, as was mentioned earlier. The energy of the electron was a function of the orbital radius. Bohr postulated that, in

orbits. Later he was able to include relativistic mechanics in the theory. As a result Sommerfield was able to explain the existence of the fine structure in line spectra which was not accounted for in the original Bohr formulation.[15] Apparently the quantum hypothesis indicated the generally correct approach to take in describing atomic phenomena.

However, despite its great success, the Bohr-Sommerfield theory still possessed important weaknesses. The intensites of the spectral lines were not predictable from the theory and the fine structure phenomena were not described completely.[16]

In addition, the very formulation of the theory was not logically consistent. In his development Bohr had assumed a description in terms of classical dynamics for the electron in its orbit. Then, after arbitrarily introducing the quantum hypothesis in order to determine energy levels, he required the electron to make abrupt transitions or jumps from one level to another in a manner impossible to describe by the same dynamics. In effect, the theory started by assuming classical dynamics and ended in its denial. Bohr was aware of the contradictions and considered the planetary model to be a fiction that was necessary at the time.[17]

Thus, before 1920, there were extremely strong indications that classical concepts were not applicable in the realm of atomic phenomena and that a consistent formulation incorporating the quantum hypothesis was necessary. What was needed was a system of quantum mechanics.

QUANTUM MECHANICS

The first consistent formulation of a system of quantum mechanics was accomplished by Werner Heisenberg in 1925. Aware of the difficulties of the Bohr theory he attempted a formulation in terms of only those

quantities which were observable. Since his aim was to explain atomic structure he began with the basic data of spectroscopy, the frequencies and intensities of spectral lines. Upon analysis, the frequencies appeared as transitions between definite energy levels while the intensities were related to the probabilities of transitions between the various levels. Heisenberg then expressed relationships between these quantities in terms of a matrix algebra which was not, in general, commutative. In particular, if P and Q are matrices representing canonically conjugate variables then $QP - PQ = \frac{h}{2\pi i}$.[18]

After determining the properties of the matrices used to represent the physical quantities, Heisenberg attempted to set up a system of dynamics. He accomplished this by assuming that the matrices obeyed equations which were identical in from to those of classical mechanics. When Heisenberg's formalism was applied, it not only duplicated the successes of the Bohr theory but also succeeded in areas where the Bohr theory had failed. Quantum mechanics had arrived.[19]

Heisenberg's novel approach was not to prove the only method of developing a quantum mechanical formalism, however. After twenty-five years without a consistent formalism, physicists were soon to find themselves in the embarassing position of possessing <u>two</u> systems of quantum mechanics.

Einstein's explanation of the photoelectric effect had indicated electromagnetic radiation possessed a strange dualism of wave and particle properties. A strong indication that this wave-particle duality was indeed a fundamental characteristic came in 1921 in Compton's experiments in X-ray scattering. According to classical theory X-rays scattered from a surface should exhibit the same frequency as the incident

wave, any energy lost in the process being at the expense of the wave amplitude. While Compton did observe such scattering he also observed scattered X-rays of a lower frequency, a result completely unexplainable by classical theory. However, according to the quantum hypothesis, $E_q = h\nu$, a decrease in frequency must occur when energy is lost since h remains constant. Using Einstein's idea of discrete energy 'bundles' Compton was able to explain the observed effect in terms of a collision process between photon and electron, i.e. the photon was conceived as possessing a momentum $p = \frac{h}{\lambda}$ and of losing energy in the collision just as a particle would.[20] Apparently electromagnetic radiation did possess certain particle-like properties!

The successes of the particle-like description of radiant energy led Louis de Broglie to formulate an idea of fundamental importance for the development of quantum theory. De Broglie suggested that the wave-particle duality in radiation was a result of the strange nature of Planck's constant and that such a duality might exist whenever Planck's Constant was a significant factor. In particular, he suggested that it might be possible to associate a wave train with the electron.[21] Basing his reasoning upon the formal analogy between Fermat's principle in optics and Hamilton's principle in dynamics[22] and applying relativity considerations he was able to relate a wave train to the dynamic quantities of the particle.[23] In fact the results were equivalent to those for the photon.

$E = h\nu$ $\lambda = \frac{h}{p}$ were $p = mv$, the classical momentum

A natural application of de Broglie's hypothesis was the hydrogen atom. The stationary states of the atom then were seen to correspond to a set of stationary waves associated with the electron. However, de Broglie's original analysis had been based to some extent on a

correspondence between the classical trajectories of a particle and the rays of a wave propagation, and could not be considered complete or rigorous.

A full solution would require the formulation of an equation for the propagation of the waves associated with an electron and the solution of this equation in terms of the boundary conditions necessary for stationary waves within the atom. In other words a system of wave mechanics was necessary.[24]

Only one year after the appearance of Heisenberg's matrix mechanics, Schrodinger published his papers outlining the methods of wave mechanics. Schrodinger's approach was somewhat similar to Heisenberg's in one respect; he also utilized an equation from classical mechanics. However all similarity ended there. Schrodinger's procedure was to begin with the classical equation for the energy of the system, the Hamiltonian. Then mathematical operators with complex coeffecients involving Planck's constant were substituted for the dynamical variables (see appendix IV), and were applied to a wave function, Ψ. These operators followed from analysis of quantitization requirements. When Schrodinger's wave equation was applied to the hydrogen atom it was found that the energy levels derived by Bohr could indeed be described in terms of stationary electron waves. In fact, the wave mechanical approach was also able to explain the failures of the Bohr-Sommerfield theory.[25]

The success of wave mechanics provided some consternation to physicists at the time, however, since Heisenberg's equally successful matrix mechanics seemed to be based on an entirely different foundation. Two dissimilar systems of quantum mechanics were really no more satisfactory than none at all. Fortunately, Schrodinger was able to show that the two

systems were mathematically equivalent and that in fact there was actually only one quantum mechanical formalism.[26]

However there still remained the problem of providing a consistent physical interpretation of the mathematical formalisms. Heisenberg had developed on his own matrix algebra which, though well known to mathematicians, was not generally familiar to physicists. As he put it: "Now the learned Gottingen mathematicians talk so much about Hermitian matrices, but I do not even know what a matrix is."[27] Most physicists of the time knew even less than Heisenberg, and consequently a physical interpretation of the matrix method did not seem promising. Then in 1927 Davisson and Germer published results of experiments which showed direct evidence of the wave properties of matter.[28] Their investigations of the reflection of electrons from a cleavage surface of a nickel crystal showed that the electrons behaved as waves with a wavelength equal to that suggested by de Broglie. Thus it seemed that a definite physical signifigance could be attributed to the concept of matter waves. However the wave function Ψ was a complex quantity and it seemed difficult to relate it to a physical wave. It was still necessary to answer such questions as: How can the wave function Ψ be interpreted physically? What physical meaning can be attributed to the wave-particle duality? From attempts to answer questions such as these emerged the Copenhagen interpretation of Quantum mechanics.

As early as 1924 Bohr, Kramers, and Slater had attempted to find a solution to the wave-particle duality. They interpreted electromagnetic waves not as 'real' waves but as probability waves whose intensity at every point determined the probability of absorbtion or induced emission of a light quanta. Their conclusions were not correct, however, but they

did introduce the distinctive idea of a probability interpretation. Later Born took up the idea of a probability wave and was able to give a clear interpretation of Ψ . He interpreted Ψ not as a 'real' wave but as a wave in many-dimensional configuration space. The square of the amplitude, $|\Psi|^2$, was then interpreted as indicating the probability of finding a particle in a particular area.[28]

After Born's work it fell to the Copenhagen group of physicists, primarily Bohr and Heisenberg, to provide a complete and, according to most physicists, a satisfactory clarification of the situation.[29] The conclusions which they reached were not arrived at easily. As Heisenberg expressed it:

> I remember discussions with Bohr which went through many hours till very late at night and ended almost in despair; and when at the end of the discussion I went alone for a walk in the neighboring park I repeated to myself again and again the question: Can nature possibly be as absurd as it seemed to us in these atomic experiments? 30

To Heisenberg's question the answer was apparently 'Yes, nature is absurd in the realm of atomic phenomena'.

From Heisenberg's studies emerged a relationship which was to shake the foundations of physics and to provide physicist, philosopher, and layman alike with a new perspective on the physical world. From his analysis of the nature of the relationship between the wave and particle descriptions of matter Heisenberg came to a conclusion that was, in terms of classical concepts, at least as radical as Planck's hypothesis. Heisenberg concluded that it was impossible, <u>in principle</u>, to measure simultaneously with unlimited precision the values of two conjugate quantities. That is, according to the relationships derived by Heisenberg, there was no way to measure simultaneously quantities such as the

position and the momentum of a particle with unlimited accuracy. In particular, according to Heisenberg, the product of the uncertainties in the measurements of two such quantities must always be equal to, or greater than $\hbar$; i.e. $\Delta x \Delta p_x \geq \frac{h}{2\pi}$.[31] Indeed the uncertainty principle seemed to refute the classical ideas of cause and effect relationships.

Niels Bohr contributed what might be called a key analysis in the Copenhagen interpretation. According to Bohr contradictions such as the wave-particle duality are due to the limitations of our language and concepts when applied to the atomic realm. In these terms the wave-particle duality was seen not as a contradiction but rather as a manifestation of two 'complementary' descriptions of reality. That is, in Bohr's view , since our concepts and language were not applicable to the area of atomic phenomena, our knowledge of the phenomena could only be completely expressed by two 'complementary' descriptions in classical terms. When coordinated together the ideas of Bohr and Heisenberg, including the probability concept, constitited the basics of the Copenhagen interpretation.

This interpretation of quantum mechanics did not find immediate acceptance. Heisenberg's Uncertainty Principle was vigorously criticized and subjected to careful examination, particularly by Einstein, due to its rejection of strict causality in physics. At the Solvay Conference of 1927 this interpretation of quantum mechanics was subjected to the closest scrutinty by Einstein and others. However, no inconsistencies were found. No one could disprove the uncertainty relations which were recognized as the keystone of the interpretation. The Copenhagen interpretation stood firm.[33]

The essentials of the development of quantum mechanics up to the Copenhagen interpretation can be summarized as follows: The study of

atomic phenomena had led to discoveries which could not be explained by classical physics. In order to describe atomic phenomena accurately it was found necessary to introduce ideas which were contrary to basic classical concepts of nature, e.q. quantum 'jumps' which could not be described by classical dynamics. These revolutionary quantum ideas then formed the basis for the development of a consistent mathematical formalism which successfully described atomic phenomena, but in extremely abstract terms. A consistent physical interpretation of the formalism was finally put forth by the Copenhagen group of physicists, primarily Bohr and Heisenberg.

Of course, quantum mechanics (As used in this paper, the term does not include the quantum field theories.) continued to develop during the years following the formulation of the Copenhagen interpretation, and many new discoveries were made. However, these new developments have not resulted in any essential modification of the Copenhagen interpretation.

DEFINITION: AN IDEAL EXPERIMENT

In order to clarify the ideas and arguments used in the development of their theories physicists often rely upon ideal or 'thought' experiments. This has been the case in the development of quantum mechanics. Therefore, before beginning an examination of the Copenhagen interpretation it is important to understand just what is meant by the term 'ideal experiment'.

In imagining an ideal experiment one is allowed to bypass restrictions of a practical nature as long as it is possible in principle to carry out the experiment. For example, one could discuss a hypothetical space ship capable of approaching the velocity of light, but not a space ship capable of exceeding that velocity, for that would violate the accepted theory of relativity. In practice it might be extremely difficult to carry out the experiment; at the present a space ship cannot be built that can approach the velocity of light. However, there is nothing which prohibits, in principle, the construction of such a ship, as far as we know now.

An ideal experiment, then, is a mental test of physical ideas which must be possible in principle to carry out.

AN EXAMINATION OF THE COPENHAGEN INTERPRETATION OF QUANTUM MECHANICS

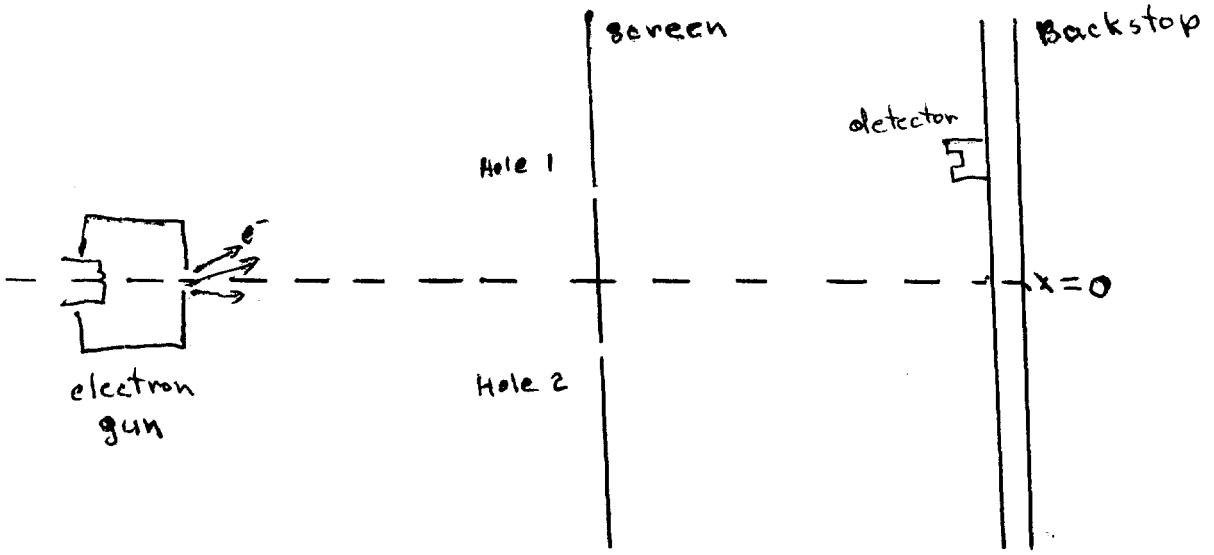
AN IDEAL EXPERIMENT

Quantum mechanics is difficult to understand. Quantum mechanics deals primarily with atomic phenomena. These two statements go hand in hand. Quantum mechanics is difficult to understand because it does deal with the very small world of the atom; and atoms, and the electrons and other elementary particles which make up atoms, do not, simply do not, behave as our ordinary experiences would lead us to expect. The following ideal experiment with electrons[34] illustrates this clearly. (This particular experiment has not actually been performed but it lends itself to a suitably simple description. However other experiments have been done which indicate what the results of this one would be).[35]

First of all let us remind ourselves of how we normally think about an electron; an electron is a very small piece of electrically charged matter, i.e. it is a very tiny particle to which can be attributed the usual properties of particles. Therefore, it can be predicted that an electron will behave in an understandable manner since the behavior of particles is quite well known. (After all, a particle is just like a very tiny baseball, isn't it)?

To conduct the experiment the equipment is set up as follows: First an electron gun is constructed. (A television picture tube is a common example of an electron gun. However ours need not be so complex.) The gun consists of a filament which will emit electrons when an electric current is passed through it surrounded by a metal box at a negative

potential with respect to the filament. The box has a small hole in it through which the emitted electrons can pass. After passing through the hole the electrons continue forward with a constant velocity. In front of the box is installed a screen with two small holes in it. On the other side of the screen a backstop is located equipped with a device to detect the arrival of the electrons.



[36]

Fig. I An Experiment With Electrons

The object of the experiment is to determine the probability that an electron will arrive at the backstop at some distance X from the center indicated by X=0 in Figure I. Since we know that an electron is a particle, the results can be predicted fairly accurately before the experiment is conducted. Some of the electrons emitted will go through hole number one resulting in probability distribution P_1. Others will go through hole number two resulting in the distribution P_2. The total probability of detecting an electron at some position X then must be simply the sum of P_1 and P_2, giving the distribution P_{12} in Figure II.

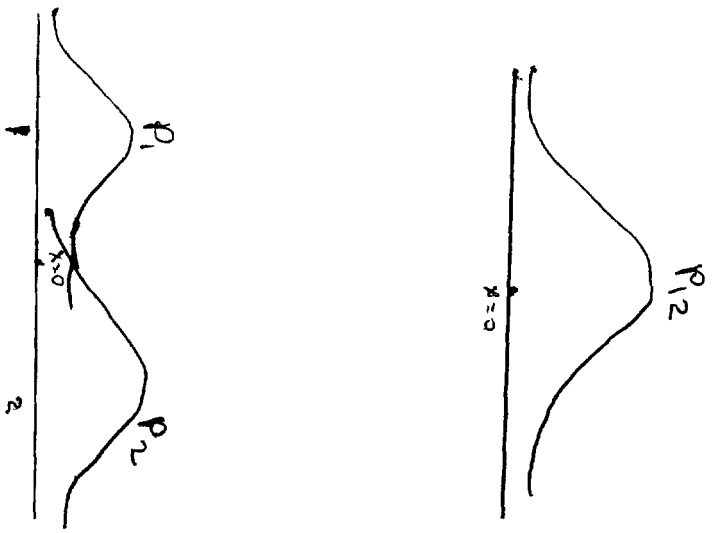


Fig. II Expected Results

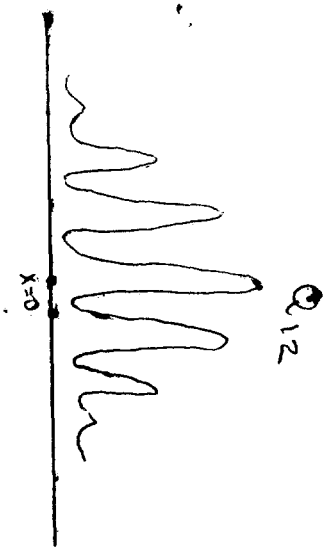


Fig. III Actual Results

However, when the experiment is performed we do not obtain the expected distribution indicated in Fig. II, but instead the one shown in Fig. III. But this curve does not represent a simple probability distribution. Instead it is like the curve obtained from analysis of the interference of light waves and is related to the intensities of the waves. The electrons have behaved, not like particles as expected, but like waves! Something must be wrong!

A little reflection, however, clarifies the situation. Remember, in 1923, de Broglie said something about waves being associated with electrons. These electron waves must be what has been observed.

But if the experiment is examined carefully something still does not seem quite right. Initially it was said that some electrons would go through hole number one and some through hole number two; but if they did, the distribution P_{12} would have resulted. Obviously, an electron has to go through one hole or the other so something must be wrong in the analysis of the experiment. Therefore the experiment must be performed again, but this time with provisions for determining which hole the electrons go through.

A light source is now added to the original apparatus, located between the screen and the backstop. Electric charges are known to scatter light so when an electron passes through hole number one a light flash will be observed in that vicinity and similarly for hole number two. The occurrence of these flashes can be recorded tnd thus it can be determined when an electron has passed through a particular hole.[37]

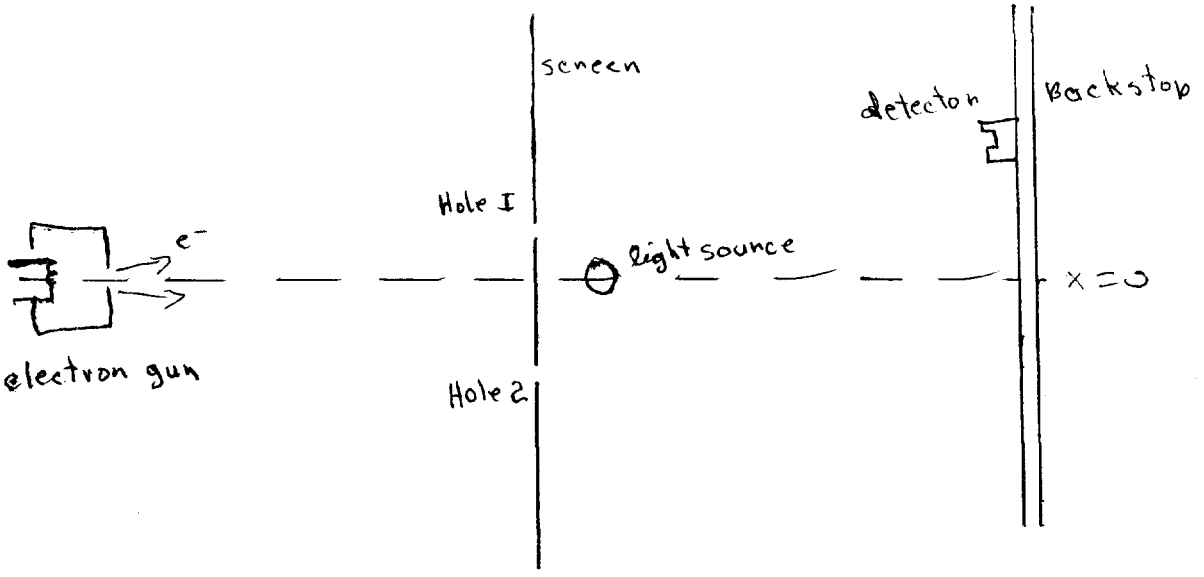


Fig. IV[38] Watching the Electrons

When the results of the experiment are analyzed, the distribution P_{12} indicated in Fig. IV is obtained. P_{12} is equal to the distribution P_{12} as was originally predicted. Therefore it has been confirmed that the electrons either go through hole number one or hole number two.

A little thought shows that there is still a problem. According to the work of Einstein and Compton, light consists of photons which impart momentum to the electrons upon collision. Therefore, perhaps the action of the light changed the motions of the electrons in such a way that the interference effect was no longer observable.

Altering the light source does not help resolve the problem at all. If the intensity of the light is lowered, the energy of the photons, $E_q=h\nu$, is not changed, only the number of photons is decreased. Therefore, some electrons are observed at hole number one and some at hole number two, but because of the smaller number of photons some electrons arrive at the detector without being observed at all. The observed electrons exhibit distribution P_{12} while the unobserved electrons show the interference pattern. If a light source of lower frequency is used a different problem occurs. As is well known in optics there is a limitation on how close together two points can be and still be distinguishable with a given wavelength. If the frequency is made small enough so that the electrons are negligibly affected by the photons, the wavelength is then too long to determine which hole the electrons have passed through.

The results of the experiment lead to some rather odd conclusions. If the electrons are not observed they produce an interference pattern, i.e., they behave like waves. If they are observed they behave like particles, i.e., it is then possible to say that they go through one hole or the other. Consequently it must be concluded that what an electron does depends upon whether it is observed or not. More properly, the

visual and tactile, play a vital role. The concepts of classical physics are basically abstractions from common experience precisely defined by mathematical relationships. Nevertheless, physical events are still describable in terms of mechanical models based upon ordinary experience.[42] Implicit in the structure of classical physics is the assumption that the basic concepts are universally applicable, i.e., the material world is assumed to behave in the same manner at all levels. However, as has been shown, in the very small world of atomic phenomena nature does not behave in a way describable by classical concepts.

Bohr realized that the failure of classical concepts was due to the existence of Planck's 'quantum of action'. Planck's quantum designates a property of nature, not of particular events. It sets up a limiting scale of measurement.[43] The theories of classical physics are simply idealizations which are valid in the limit of phenomena where the quantities involved are very large compared to the elementary quantum.[44] Consequently the application of classical concepts to atomic events results in contradictions, e.g., the wave-particle duality.

Bohr also recognized another vital point. The physicist himself is a part of the world of classical physics, and classical concepts provide the foundation for his thoughts and for the description of his experiments.[46] Therefore, classical concepts cannot simply be discarded. As Bohr said:

> Here it must above all be recognized that, however far quantum effects transcend the scope of classical physical analysis, the account of the experimental arrangement and the record of the observations must always be expressed in common language supplemented with the concepts of classical physics. 47

In other words, although atomic events cannot be described completely in classical terms, the experiments which are set up to observe these events,

and the results obtained from the experiments, must be expressed in classical terms in order for the physicist to be able to talk about the experiment at all.

In the ideal experiment which was discussed earlier results were obtained which indicated that electrons behave like waves; also other results were obtained which showed that electrons are, indeed, particles. However, when considered in the light of the limitations pointed out by Bohr the two descriptions are not necessarily contradictory.

> Such empirical evidence exhibits a novel type of relationship which has no analogue in classical physics and which may conveniently be termed complementary in order to stress that in the contrasting phenomena we have to do with equally well-defined knowledge about the objects. [48]

The contradictions arise when the results of an experiment are considered without reference to the experiment from which they were obtained. Since atomic events cannot be described in classical terms, the only information available is the description of an experimental arrangement and of the results which were obtained. If the experimental results are then described in terms of a classical concept such as wave or particle behavior, the description is valid only in reference to that particular experimental arrangement. The fact that quantum phenomena cannot be described classically implies that they cannot be separated from the experimental arrangement in which they appear.

In other words, atomic events exhibit what might be called an 'indivisibility'. It is not possible to separate the event into successive or parallel steps because setting up experiments to determine these steps alters the character of the event completely.[50] We cannot determine which hole the electron passed through and observe interference with the same experimental arrangement.

Thus the principle of Complementarity contains two key points for understanding quantum mechanics. First, the concepts of common language are subject to limitations when applied to atomic phenomena and different concepts must be used at different times to gain a complete picture. Second, as a result of these limitations it becomes impossible to make a sharp distinction between the observation and the event.

The impossibility of separating the behavior of objects from the interaction with the instruments used in experiments is in direct contradiction to one of the most basic tenets of physical science, that there are objective events which take place in space and time independently of whether they are observed or not.[51] More will be said about this in connection with the Uncertainty Principle.

THE UNCERTAINTY PRINCIPLE

The interaction between an atomic event and the measuring instruments used to observe it is expressed quantitatively by Heisenberg's Uncertainty Principle. Heisenberg found that there is a limit to the precision with which two conjugate quantities can be measured simultaneously. The most familiar expression of the Uncertainty Principle is in terms of the position and the momentum of a particle, $\Delta x \, \Delta p_x \geq h/2\pi$. According to the Uncertainty Principle, it is not possible to know both the position and the momentum of a particle with unlimited accuracy.[52]

The following ideal experiment[53] illustrates the basic nature of the Uncertainty Principle.

The object of the experiment is to determine the position and momentum (or velocity) of a free electron. The equipment consists of two very sensitive microscopes as indicated in Fig. V. Light scattered from the electron can thus be detected by the microscope.

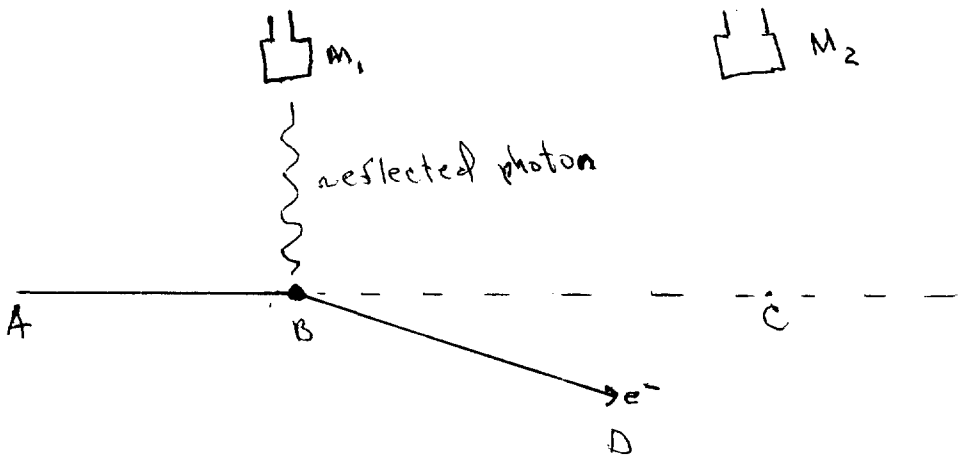


Fig. V[54] Determining the Position and Momentum of an Electron

The electron is moving along some path AB. A photon is reflected from the electron and received at M, locating the electron at B. To determine the velocity, however, a second measurement is needed, at M_2. By noting the times of the two observations the velocity (and therefore the momentum) can be obtained. However the photon which was reflected into M, imparted a momentum to the electron and deflected it from its original path. Hence the electron no longer has the momentum which it had at point B. Therefore any additional measurement can not result in the value of the momentum possessed at point B. The momentum is indeterminate.

This experiment is, of course, a very crude example but it does indicate the essential ideas of the Uncertainty Principle. A complete mathematical analysis yields the results given previously.[55] It should be pointed out here that the Uncertainty Principle has nothing to do with the uncertainties inherent in any physical measurement. The Uncertainty Principle is an expression of a fundamental property of quantum phenomena.[56]

There are also uncertainty relations involving other conjugate

quantities. The complete uncertainty relations are:[57]

(1) $\Delta X \, \Delta p_x \geq h/2\pi$ position and momentum

(2) $\Delta E \, \Delta t \geq h/2\pi$ energy and time

(3) $\Delta A \, \Delta \gamma \geq h/2\pi$ angular momentum and angular position

(4) $\Delta I \, \Delta w \geq h/2\pi$ moment of inertia and angular velocity

The existence of these uncertainty relations has important implications for the concepts of classical mechanics, as shall be shown.

According to the concepts of classical mechanics a particle has, at any moment in time, a definite position in space, and a definite velocity, in agreement with Newton's Laws. Thus a particle moving through space travels along a continuous path possessing at each instant of time a definite position and velocity. Similarly, a particle such as a bound electron describes a well defined orbit with a definite angular momentum and angular position.

In the light of the uncertainty relations, however, the situation appears much different. According to (1), the position and the velocity of a particle cannot be known simultaneously with arbitrary accuracy. Therefore, the concepts of precise position and velocity lose their meaning. Equation (1) implies the impossibility of sharp localization in space.[58] If (2) is examined it becomes obvious that the uncertainty relation denies the possibility of well-defined location in time, for a particle located at some precise instant of time would then have no definite energy.[59]

Equations (3) and (4) refer to rotational motion such as a bound electron would exhibit. In (3) if the angular momentum is completely determined the position of the electron on its orbit is completely undetermined; this is equivalent to the wave mechanical view of a 'standing

electron wave'. When the angular position takes on its maximum uncertainty of 2 , Bohr's quantitization requirements for the angular momentum are found. Since the electron cannot be given a precise position, the idea of a definite orbit seems contradictory. Further analysis shows that the formula for the radii of the orbits also contains a term subject to uncertainty considerations. Therefore (3) disallows the concept of a well-defined continuous orbit.[60]

According to (4), precise determination of the moment of inertia of a particle renders the angular velocity completely indeterminate. However, the moment of inertia depends upon quantities that are normally considered constants, i.e., mass, size, etc. Thus if a classical picture of a particle is maintained, the angular velocity cannot have any value at all, not even zero. Therefore it must be concluded that such properties as a precise radius, etc., cannot be attributed to 'particles' such as electrons.

Thus far it has been shown that some of the most basic tenets of classical physics have to be discarded or revised when dealing with quantum phenomena. To summarize some of the more important points:

(1) The results of an experiment cannot be separated from the experimental arrangement in which they were observed.

(2) Quantum phenomena can not be strictly located in space and time.

(3) Such concepts as well-defined continuous orbits, and rigid solid bodies are not applicable to quantum phenomena.

According to the Copenhagen interpretation, isolated events precisely located in space and time do not exist in the realm of quantum phenomena.

This interpretation would seem to imply that physics has given up the idea of precisely defining events and of predicting exactly the results of

experiments. This is, in fact, true.[62] We shall see what this means as we examine the role of probability and the status of causality.

THE ROLE OF PROBABILITY

In classical mechanics the state of a system is defined by the values of the instantaneous positions and velocities of the particles composing it. If the state of a system is known at some instant it is then possible to derive the state at any other time.[63] However, the uncertainty relations prohibit the precise determination of the state variables, at any one instant, as has been shown. Moreover, determination of the variables within the limits of the uncertainty relations cannot serve as the basis for future calculations, due to the unknown perturbations produced by the original measurements.[64] Therefore it is necessary to introduce the concept of probability into the description of atomic events. The quantum definition of state includes a term for probability.

Probability also plays a role in classical physics. As used there, the concept of probability refers to the extent of the observers knowledge about an event.[65] Its place is in the theory of errors, and in the statistical descriptions of complex mechanical events such as the motions of molecules in a gas.

However, in quantum mechanics the situation is quite different. Here the concept of probability refers, not to the extent of knowledge, but to subject matter of knowledge. It refers to what is knowable.[66] As was pointed out, the uncertainty relations do not refer to uncertainties in the physical process of measurement, but to a fundamental feature of quantum phenomena. This is expressed by a probability function (Born's interpretation of $|\Psi|^2$) which permits predictions about quantum events.

The theoretical interpretation of an experiment in quantum mechanics requires three steps:[67]

(1) the translation of the initial experimental situation into a probability function. (This also includes possible errors of measurement).

(2) the following up of this function in the course of time.

(3) the statement of a new measurement to be made of the system, the result of which can be calculated from the probability function.

Step (2) is not describable in classical terms. There is no description of what happens to a system between the initial observation and the next measurement. Attempting to find a description leads to contradictions, as was pointed out in the first ideal experiment.

The role of probability in quantum mechanics is clearly stated by Heisenberg:[68]

> When the probability function in quantum theory has been determined at the initial time from the observation, one can from the laws of quantum theory calculate the probability function at any later time and can thereby determine the probability for a measurement giving a specified value of the measured quantity. We can, for instance, predict the probability for finding the electron at a later time at a given point in the cloud chamber. It should be emphasized, however, that the probability function does not in itself represent a course of events in the course of time. It represents a tendency for events and our knowledge of events. The probability function can be connected with reality only if one essential condition is fulfilled: If a new measurement is made to

determine a certain property of the system. Only then does the probability function allow us to calculate the probable result of the new measurement. The result of the measurement again will be stated in terms of classical physics.

THE STATUS OF CAUSALITY

One of the most basic beliefs of science is that of cause and effect. The very idea of experiment is based upon the assumption of meaningful connections between events. Therefore to state that causality does not hold in quantum mechanics would seem to be destroying the very foundation of science.[69] And this is what the Copenhagen interpretation asserts.

The philosophical problems involved in the idea of causality and cause and effect relations are extremely complex and quite beyond the scope of this paper. However a few comments will be made in order to clarify the position of the Copenhagen interpretation.

When the statement is made that A 'causes' B, unless 'cause' is well-defined problems of interpretation arise. For example; Is the connection between A and B one of absolute necessity, or is A simply followed in time by B? In classical physics the idea is expressed concisely: If the state of a system is known precisely at any one time it can then be calculated at any other time. And in principle it is always possible to determine the state of a system at any instant. Thus classical physics is completely determinate.

Since the uncertainty relations prevent precise definition of state, in principle, strict causality cannot be considered to hold true in quantum mechanics. It is only in the sense of a probability connection, as expressed by statistical regularity in experimental results, that

causality can be said to hold in quantum physics.

FURTHER COMMENTS

The mathematical formalism of quantum mechanics has been shown to be consistent and to give results agreeing with experiment.[70] However, the Copenhagen interpretation of this formalism, though logically consistent, cannot be shown to be logically necessary.[71] Consequently criticisms of this interpretation generally are based on philosophical reasons.

The Copenhagen interpretation raises questions as to the extent to which it is possible to go from an observed phenomena to an objective conclusion independent of the observer; also it poses questions as to the physical reality of such concepts as position and velocity of an electron. It is these points which form the basis of most criticisms of the Copenhagen interpretation. The critics, among them de Broglie[72], Schrodinger[73], Planck[74], and Einstein[75], believe that it is desirable to return to a completely objective, deterministic description of nature. Einstein expressed this view in a famous statement: Der lieber Gott würfelt nicht.[76] — "The good Lord doesn't play dice."

Attempts to modify the mathematical of quantum mechanics in order to permit a derterministic description of phenomena have not been generally successful. However, Bohm has developed an approach which has achieved some success.[77] Bohm divides Schrodinger's equation into two real parts, one of which can be used to define a path for the electron. Disturbance of the path is regarded as due to a non-classical field created by the electrons presence and not to the measuring process.[78] The extent to which the non-classical field is to be regarded as 'real' is not obvious, however.

An interesting interpretation has been presented by Margenau[79] and with modifications by McKnight[80], in which it is denied that the electron has a position at all. Margenau attributes basic reality to the mathematical forms.

> ---this acknowledgement does not destroy our right to affirm the electron's presence as an objective component of reality. For it merely substitutes certain abstract qualities for those which we deemed obvious and immediate; it substitutes mathematical models for mechanical ones. Logically, there is no reasons why the character of an entity should be described by a visual image rather than a Hamiltonian.[81]

However, in general, these criticisms and interpretations are merely restatements of the Copenhagen interpretation in another language. No basically different, workable, interpretation has been presented and it is generally believed that a complete replacement of the present structure of quantum mechanics would be necessary in order to encompass such an interpretation.[82]

SUMMARY

The development of quantum mechanics showed that the concepts developed from common experience do not apply in the realm of atomic phenomena. In attempting to describe these phenomena quantum physics has developed a language of mathematical abstractions which is not completely describable in physical terms. The Copenhagen interpretation attempts to provide some understanding of these abstractions, and of the phenomena to which they refer, by examining the concepts of classical physics and attempting to define the limitations of application of those concepts. This is indicated by the uncertainty relations.

According to the Copenhagen interpretation, in the realm of atomic phenomena, 'physical reality' cannot be precisely defined in common terms, and the fundamentals of nature are expressed by probability functions.

This interpretation has been criticized due to its introduction of 'subjectivity' into physics. However no successful alternative has been proposed. Apparently only a complete revision of quantum mechanics can lead to the fall of the Copenhagen interpretation.

FOOTNOTES

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9. Ibid.
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51. Heisenberg, Range, p. 86.

52. Heisenberg, Physical Principles, p. 3.

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55. Heisenberg, Physical Principles, pp. 16-18.

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60. Ibid., pp. 291-292.

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63. Bohr, Essays, p. 1.

64. Heisenberg, Physical Principles, p. 20.

65. Heisenberg, Physics and Philosophy, p. 7.

66. Ibid., pp. 5-9.

67. Ibid., p. 47.

68. Ibid., p. 46.

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70. Bohr, Science, 111: pp. 51-54.

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A P P E N D I X

I. CHRONOLOGICAL LIST OF SOME IMPORTANT EVENTS IN THE DEVELOPMENT OF QUANTUM MECHANICS TO 1932

1895 --- X-rays (Roentgen)

1896 --- Radioactivity (Becquerel)

1897 --- Discovery of the electron (Thomson)

1900 --- Quantum hypothesis (Planck)

1905 --- Special Theory of Relativity (Einstein)
--- Quantum explanation of the photoelectric effect (Einstein)

1907 --- Quantum interpretation of specific heats (Einstein, Debye)

1911 --- Planetary model of the atom (Rutherford)

1913 --- Quantum hypothesis applied to the Hydrogen atom (Bohr)
--- Discovery of isotopes (Thomson)

1915 --- General Theory of Relativity (Einstein)

1916 --- Extension of the Bohr Theory (Sommerfeld, Wilson)
--- Correspondence Principle (Bohr)

1919 --- Artificial radioactivity (Rutherford)

1921 --- The Compton effect (Compton)

1923 --- Wave nature of particles (de Broglie)
--- Quantum theory of light dispersion (Kramers, Heisenberg)

1925 --- Matrix mechanics (Heisenberg)

1926 --- Wave mechanics (Schrodinger)
--- Equivalence of wave and matrix mechanics (Schrodinger, Echart)
--- Probability interpretation of (Born)

1927 --- Uncertainty relations (Heisenberg)
--- Experimental proof of wave character of particles (Davisson and Germer)
--- Copenhagen interpretation of quantum mechanics (Bohr and Heisenberg)

1930 --- Complete relativistic theory of the electron (Dirac)

1931 --- Discovery of the neutron (Bothe, Becker, Chadwick)

1932 --- Discovery of the proton (Anderson, Blackett and Occhialini)

II. SOME IMPORTANT PHYSICAL CONSTANTS

Velocity of light (vacuum)	c	2.997×10^{8} m/sec
Electron charge	e	1.60×10^{-19} coulomb
Electron mass	m	9.11×10^{-31} kg
Proton mass	M	1.673×10^{-27} kg
Planck's constant	h	6.626×10^{-34} joule sec
Boltzmann's constant	k	1.381×10^{-23} joule/molecule $^{\circ}$K

III. BOHR'S THEORY OF THE HYDROGEN ATOM

Assumptions: (A) The electron revolves about the nucleus in a circular orbit, subject to coulomb force.

(B) The center of mass of the system is essentially at the nucleus.

(C) The atom exists in certain stationary states in which it does not radiate.

(D) Radiation occurs only when the atom makes a transition from one state, with energy E_k, to a state with lower energy E_j.

(1) $h\nu = E_k - E_j$

(2) $F = ma$ Newton's Law

$F = e^2/4\pi\epsilon_0 r^2$ Coulomb's Law

$F = mv^2/r$ Centripetal force

(3) Kinetic Energy $K = \frac{1}{2}mv^2$ from (2), $v^2 = e^2/4\pi m\epsilon_0 r$

$K = e^2/8\pi\epsilon_0 r$

Potential energy $U = V(-e)$ where V is the potential of the proton at the radius of the electron. $V = e/4\pi\epsilon_0 r$

$U = -e^2/4\pi\epsilon_0 r$

(4) Total energy $E = K + U = -e^2/8\pi\epsilon_0 r$ If the radius r is quantitized then so is E.

(5) Rotational frequency $V_0 = v/2\pi r = \sqrt{\frac{e^2}{16\pi^3\epsilon_0 m r^3}}$

(6) Linear momentum $p = mv = \sqrt{\frac{me^2}{4\pi\epsilon_0 r}}$

(7) Angular momentum $L = mvr = \sqrt{\frac{me^2r}{4\pi\epsilon_0}}$

Bohr hypothesized that L can only have certain values:

(8) $L = nh/2\pi$ where n = 1,2,3,.....

combine (7) and (8)

(9) $r = n^2h^2\epsilon_0/\pi me^2$ This gives the allowed orbital radii.

substitute (9) into (4)

(10) $E = -me^4/8\epsilon_0^2h^2n^2$

combine (10) and (1)

(11) $\nu = \left(me^4/8\epsilon_0^2h^3\right)\left(\frac{1}{J^2} - \frac{1}{K^2}\right)$ J and K intergers referring to the lower and upper states respectivety.

This is the theoretical formula for the frequencies of the hydrogen spectrum.

IV. THE POSTULATES OF QUANTUM MECHANICS FOR A SYSTEM CONSISTING OF TWO PARTICLES

(1) The wave function, Ψ, belonging to a system composed of two particles, is a function of x_1, y_1, z_1, x_2, y_2, z_2, and t.

(2) The wave equation for Ψ is obtained from the classical expression for the total energy of the system by replacing the dynamical variables by operators:

dynamical variable		operator
x_1 (position)	$\longrightarrow$	x_1
p_{x_1} (momentum)	$\longrightarrow$	$\frac{\hbar}{i}\frac{\partial}{\partial x_1}$
W (total energy)	$\longrightarrow$	$-\frac{\hbar}{i}\frac{\partial}{\partial t}$

similarly for y_1, z_1, x_2, y_2, z_2

and by the insertion of the wave function Ψ as the operand.

(3) Ψ, and $\frac{\partial \Psi}{\partial x_1}$,..., $\frac{\partial \Psi}{\partial z_2}$, must be everywhere continuous, finite, and single valued, throughout "configuration space" (the full range of all six variables).

(4) Ψ is normalized, that is,

$$\int_{\text{all configuration space}} \Psi^* \Psi \, dx_1 \, dy_1 \, dz_1 \, dx_2 \, dy_2 \, dz_2 = 1$$

(5) The expectation value of the observable, α , is calculated from the wave function by the formula.

$$\bar{\alpha} = \int_{\text{all configuration space}} \Psi^* \alpha_{(\text{operator}} \Psi \, dx_1 \, dy_1 \, dz_1 \, dx_2 \, dy_2 \, dz_2$$